Optimization Models for Placement of an Energy-Aware Electric Vehicle Charging Infrastructure

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Abstract

This paper addresses the problem of optimally placing charging stations in urban areas. Two optimization criteria are used: maximizing the number of reachable households and minimizing overall e-transportation energy cost. The decision making models used for both cases are mixed integer programming with linear and nonlinear energy-aware constraints. A multi-objective optimization model that handles both criteria (number of reachable households and transportation energy) simultaneously is also presented. A number of simulation results are provided for two different cities in order to illustrate the proposed methods. Among other insights, these results show that the multi-objective optimization provides improved placement results.

Keywords: Charging infrastructure placement, electric vehicles, energy-aware optimization models, multi-objective optimization.

1. Introduction

In order to achieve emission reduction targets and reduce dependency on (foreign) oil and fossil fuels in general, electric vehicles have drawn more and more attentions from governments and the general public. Developing electric vehicles and creating an electrified transportation system is an effective way to promote urban sustainable development as pointed out by (Eberle and von Helmolt (2010), Bouscayrol et al. (2011), Khaligh and Krishnamurthy (2012) and Bilgin et al. (2015)). Therefore policies to facilitate the growth
and market penetration of electric vehicles (EVs) have been developed in almost every nation of the industrialized world. While over the last decade several car producers made EV models commercially available in the US, the market share trajectory of EVs in the US has been below predicted levels.

The acceptance of EVs by the public depends on a large variety of factors. An efficient, convenient and economic charging infrastructure system can enhance the willingness of consumers to purchase and promote industry development (Hatton et al. (2009), Guo and Zhao (2015)). The availability and convenient locations of charging stations in metropolitan environments is a key factor that globally affects not only the adoption process of EVs but also the sustainability of transportation. The energy-efficiency criterion is one of the principle considerations for enhancing sustainability (Kates et al. (2005)). Therefore, it is necessary to employ proper methods to determine the optimal energy-aware charging station locations. Aiming to construct an energy-efficient charging infrastructure system for sustainable urban transportation systems, multiple objective energy-aware decision-making models are introduced in this paper. Two different criteria are proposed in the optimization models, which consider several energy-aware constraints:

First, we address the problem of optimal charging station placement from the viewpoint of reaching the most customers or households by providing an energy cost constraint, i.e. something a private charging station owner would typically consider. On the other hand, this problem is also of interest for municipalities, power companies, and federal agencies such as the Environmental Protection Agency (EPA) and the Department of Transportation (DOT). Given an energy bound, the corresponding reachable contours in Google Maps for different possible charging station centers are determined by using the energy model in (Yi and Bauer (2014a)). Maximizing the number of households, i.e. EV users, in this range is discussed subsequently.

Second, another criterion, namely minimizing overall transportation energy consumption to perform charging actions, is addressed. This is an important aspect to construct future sustainable transportation systems, which is of utmost interest for agencies such as EPA and DOT. Suppose all EVs charge once a day at the charging station away from home. Each EV has a corresponding energy consumption when it travels to a public charging station from home. The objective is to find a subset of locations from potential positions to achieve minimum overall transportation energy cost considering all charging actions. This objective is crucial for establishing future sustainable cities. Some other constraints, e.g. service capability of charging
station, etc. also are included in the optimization model.

Third, we combine both criteria to get a more realistic decision making framework. The corresponding multiple objective optimization model will be proposed to obtain more balanced planning strategies under energy cost constraints. This multi-objective model will balance both introduced energy related requirements.

This paper is organized as follows. Section 2 provides a literature review, the main contributions of this paper, and a brief description of how this paper addresses open research problem in relative to other studies. In Section 3, the energy-aware charging station placement framework will be formulated. The optimization model for the maximum number of reachable households will be introduced in Section 4. In Section 5, the charging station placement problem considering minimum overall transportation energy cost will be proposed and discussed. In Section 6, the multiple objective optimization model will be constructed by considering both of the introduced requirements simultaneously. Conclusions are provided in Section 7.

2. Literature Review

The problem of charging infrastructure placement has been investigated by many researchers. The most powerful and popular techniques are decision-making methods such as linear/nonlinear programming, multilayer programming and mixed-integer programming. Many very different criteria and constraints have been applied in these models.

A maximal coverage model to optimize the demand covered within an acceptable level of service has been investigated in Frade et al. (2011). The work in Worley et al. (2012) formulates the problem of locating charging stations as a discrete integer programming optimization problem based on the classic vehicle routing problem. In Liu (2012), an assignment model for different charging infrastructure assignment strategies was proposed by estimating the charging demand of the early electric vehicle market in Beijing. In Xi et al. (2013) an optimization model was developed to maximize total fleet-wide charging levels for the location of a public EV charging infrastructure. In Lam et al. (2013, 2014), the electric vehicle charging station placement problem was formulated to minimize the total construction cost subject to the constraints for the charging station coverage and the driver convenience for EV charging. Environmental factors and service radius are considered in Liu et al. (2013) to determine the optimal charging station
locations. In Wang et al. (2013b), an optimal location model of charging stations is established based on electricity consumption along city roads. A mixed-integer programming model was developed in Chen et al. (2013) to determine optimal location assignments of charging stations in Seattle, which minimized the station access cost of EV users and took the parking demand, local job, population density and trip attributes as constraints. In Pasha-javid and Golkar (2013) the charging stations were allocated by minimizing energy loss and voltage deviation in the distribution system. In Wang et al. (2013a), the optimal location and size of charging stations were determined by maximizing the EV traffic flow under the constraint of battery capacity. The work in Xu et al. (2013) proposed a mathematical model with minimum total transportation distance to determine the optimal charging station locations. The work in Ghamami et al. (2014) formulated this problem as a fixed charge facility location model with charging capacity constraints, allowing unserved demands and considering driver preference for parking lots. A mixed-integer nonlinear optimization approach was proposed for determining the optimal place and size of fast charging stations in Sadeghi-Barzani et al. (2014), which took the station development cost, EV energy loss, electric loss and the location of electric substations as well as urban roads as constraints. In Yao et al. (2014), an equilibrium-based traffic assignment model was proposed to maximize the annual traffic flow captured by fast charging stations. The work in Cavadas et al. (2014) tried to plan the location of charging stations for EVs in a city by maximizing the number of vehicles served under a fixed budget for building charging stations. In Khalkhali et al. (2015), the optimal location of plug-in hybrid electric vehicle charging stations were determined by maximizing the benefit of the distribution system manager.

Several data-based methods for analyses of driving activities and travel behaviors were employed in order to improve the placement strategies. In the work of Sweda and Klabjan (2011, 2015), a decision support system was presented for identifying patterns in residential EV ownership and driving activities to enable strategic deployment of a new charging infrastructure. A similar decision support system was also developed in (Wagner et al. (2013, 2014) and Cai et al. (2014)). It was achieved by analyzing large-scale trajectory data to obtain travel patterns. The work in Andrews et al. (2013) proposed an optimization model based on a user charging model to find locations for charging stations by analyzing the needs of vehicle owners. In Dong et al. (2014), an activity-based assessment method was proposed to evaluate electric vehicle feasibility for the heterogeneous traveling population in the
context of real world driving, and then determined the optimal location for public charging station sites of electric vehicles. The work in Cavadas et al. (2015) considered successive activities of travelers and then proposed an improved mathematical model for locating EV charging stations. In Lee et al. (2014), a optimal location model for fast charging stations was proposed by taking battery state of charge and user charging and traveling behavior into account. The work in You and Hsieh (2014) determined the optimal location of charging stations by analyzing the round-trip itineraries to maximize the number of people who can complete their trips.

Several other methods have also been investigated to improve the charging station placement strategies, such as grid partition method (Ge et al. (2011)), clustering methods (Ip et al. (2010) and Montazpour et al. (2012)), Voronoi diagrams for equilibrium arrangements (Koyanagi and Yokoyama (2010), Koyanagi et al. (2001) and Feng et al. (2012)), simulation-based methods (Ma et al. (2014) and Hess et al. (2012)), dynamic spatial and temporal models (Wirges et al. (2012)), equilibrium modeling frameworks (He et al. (2013)), continuous facility location models (Sathaye and Kelley (2013)) and flow-refueling location models (Hodgson (1990), Kuby and Lim (2005), Wang and Lin (2013), Chung and Kwon (2015)). All these methods have provided important contributions from different viewpoints on the optimal charging station placement problem.

From the above literature review, it becomes obvious that the placement of charging stations has been approached from a number of different angles. A large variety of optimization models for decision-making with different criteria were used. Unfortunately, the previous research paid almost no attention to transportation energy cost of electric vehicles. Very few research results thoroughly addressed the energy consumption requirements in the context of charging station placement. This is an important aspect for achieving a sustainable transportation system. The main difference and contributions of our work relative to other studies are as follows:

- An energy-aware optimization framework is introduced that provides a solution to the problem of charging infrastructure placement. Two different energy-based criteria are proposed: (1) maximum number of reachable household with travel energy cost bound; (2) minimum overall transportation energy cost for charging actions. Several energy-aware constraints are also incorporated, e.g. reachable range constraint represented by travel energy cost, a service capability, etc... Further-
more, an energy-aware multi-objective decision-making model is pro-
posed to find better equilibrium placement strategies. In the literature,
some work also discussed the maximum number of vehicles, e.g. in
(Cavadas et al. (2014); You and Hsieh (2014)), but the travel energy
cost was omitted in these models.

• A detailed energy consumption estimation model for electric vehicles is
employed to construct optimal decision-making models. The work in
Sebastiani et al. (2014) and Baouche et al. (2014) provided the optimal
placement model by considering the energy cost of electric vehicles, al-
though the energy consumption estimation models used in this work are
not explicit and cannot consider the real geographic and environmental
information to obtain an accurate estimation. The energy consumption
estimation model in our previous work (Yi and Bauer (2014a)) con-
siders realistic information and is applied to provide a more accurate
energy cost for finding energy-aware optimal locations.

• In order to solve the proposed mixed nonlinear integer program-
ning problem with nonlinear constraints, a highly efficient algorithm called
mesh adaptive direct search (Audet and Dennis Jr (2006)) has been
utilized. It is a powerful tool to aid in solving the large-scale problem.

• A data analysis technique, i.e. a clustering algorithm, is used as a
pre-selection procedure to determine the potential charging station set.
This proposed preprocessing method will aid in finding a suitable sub-
set of potential locations for charging stations and then reducing the
computational complexity in the final decision-making process.

• Real world datasets are investigated, including datasets for a large city
(Chicago, IL) and a medium to small size city (South Bend, IN). By
using real world data, the capability and performance of the developed
framework is demonstrated.

3. Problem Formulation

3.1. Notation

The notation used throughout this paper is listed in Table 1. In addition,
variables and parameters are explained in the text where they are first used.
Table 1: Notations

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( U = {u_1, \ldots, u_j, \ldots, u_N } )</td>
<td>EV set and ( u_j = (u_{jh}, u_{jw}, u_{jt}) )</td>
</tr>
<tr>
<td>( U_t )</td>
<td>EV type set and ( U_t = {u_1^t, \ldots, u_l^t, \ldots, u_L^t } )</td>
</tr>
<tr>
<td>( \alpha_e )</td>
<td>Average market penetration rate for EVs</td>
</tr>
<tr>
<td>( \alpha_l )</td>
<td>Market share for EV type ( u_l^t )</td>
</tr>
<tr>
<td>( z_j )</td>
<td>The variable to describe whether ( y_j ) is covered</td>
</tr>
<tr>
<td>( I_l )</td>
<td>Coverage indicator matrix</td>
</tr>
<tr>
<td>( X_0 )</td>
<td>Needed number of charging station</td>
</tr>
<tr>
<td>( p_{ij} )</td>
<td>Probability of EV ( j ) to choose charging station ( i )</td>
</tr>
<tr>
<td>( d_{ij} )</td>
<td>Distance between work place of EV ( j ) and charging station ( i )</td>
</tr>
<tr>
<td>( S_i )</td>
<td>Service ability of charging station ( i )</td>
</tr>
</tbody>
</table>

### 3.2. Problem Statement

As shown in Fig. 1, a possible location for a charging station may initially be determined by the power infrastructure limit. A potential location set for charging stations should also consider the concerns of EV users, e.g. driving pattern, most popular parking locations, etc. A “driving pattern” denotes often used travel routes, i.e. popular sequences of distance segments. The obtained potential charging station locations can then improve the selection of locations. As an example, EV owners can charge their vehicles at the parking lots of their work places. In city planing, DOT and EPA also have an interest in the transportation energy consumption caused by charging actions. Global energy savings will be significant if the charging stations are placed optimally. This is an important aspect for developing sustainable transportation systems and green cities. An energy-aware set of optimal charging station locations needs to be determined by both, maximizing the number of reachable EV home locations under energy cost constraints and minimizing the overall transportation energy cost for all charging actions. In order to achieve the goal of this energy-aware optimality, the optimization framework will involve EVs and potential charging stations and depend on the real characteristic information of both entities.
3.2.1. EVs

Define the EV set as \( U = \{u_1, ..., u_j, ..., u_N\} \). The cardinality of \( U \) is \( N \), i.e. there are \( N \) EVs. Each \( u_j \) in \( U \) is a tuple with three elements and \( u_j = (u^j_h, u^j_w, u^j_t) \), where \( u^j_h, u^j_w, u^j_t \) are the EV home address, the work place address and the type of EV for vehicle \( j \), respectively. In the following analyses, simulations and case studies, the initial state of battery for electric vehicles is assumed to be 100\% state of charge (SOC) at home, which is known to usually be the case (Smart and Schey (2012)).

3.2.2. Charging Stations

Define the potential charging station set as \( C = \{c_1, ..., c_i, ..., c_M\} \). The cardinality of \( C \) is \( M \), i.e. there are \( M \) potential charging station positions. Each \( c_i \) in \( C \) is a tuple with three elements, i.e. \( c_i = (c^i_p, c^i_{cap}, x_i) \), where \( c^i_p \) is the location and \( c^i_{cap} \) is the capacity for the potential charging station \( i \). The variable \( x_i \) is an indicator with binary value to show whether the charging station \( c_i \) will be placed \((x_i = 1)\) or not be placed \((x_i = 0)\). Assume that public transportation is available in all potential charging station locations and expected to be used to reach final destinations. Also assume that this additional transport cost is neglected or approximately equal.

Suppose \( C_0 \) is the set of all possible charging stations in the investigated area, and the potential charging station location subset is \( C, C \subset C_0 \). Assume a selection function \( g : C = g(C_0) \). This function serves as the pre-selection for the charging station locations according to some additional criteria, e.g. parking time and driving patterns of EVs, available infrastructure and power limits. Therefore our model can combine many other constraints into the
optimization framework. In the following study, the average parking time will be used to determine the potential charging station set $\mathcal{C}$.

3.2.3. EV Energy Consumption

In order to obtain an accurate value of the energy consumption, we need to know the route followed by the vehicle, the traffic condition along the route, the driving pattern and environmental information as shown in Fig. 2. Transportation energy consumption $E_{ij}$ for EV $u_j$ at charging station $c_i$ can be calculated by the following function

$$E_{ij} = f(c^i_p, u^j_h, u^j_t, \text{environmental information})$$  \hspace{1cm} (1)

The energy consumption when EVs travel to a charging station depends on the location of the charging station $c^i_p$, the home location of EV $u^j_h$, the EV type $u^j_t$ and the corresponding environmental information along the route. The environmental information can include geographic information, the traffic conditions, weather conditions such as wind speed, temperature and road surface conditions. The energy consumption can be computed by integrating out a differential equation which provides instantaneous power of an EV. This has been discussed in detail in our previous work in (Yi and Bauer (2014a), Bauer et al. (2012)), where a detailed energy consumption model was proposed for a single EV:

$$P_{bat}(t) - P_{para}(t)\eta_{bat}\eta_{conv}\eta_{contr}\eta_{mp} = P_{air}(t) + P_{roll}(t) + P_{hill}(t) + P_{ac}(t)$$  \hspace{1cm} (2)

Where $P_{bat}(t)$ is the power provided by the EV battery, $P_{para}$ are parasitic power losses at the battery (lights, heater, stereo, etc.), $P_{air}(t)$ is the air drag power component, $P_{roll}(t)$ is the rolling resistance power component, $P_{hill}(t)$ is the hill climbing power component, and $P_{ac}(t)$ is the acceleration power.
component at the wheel. The efficiencies of battery, power converter, e-motor controller, e-motor and mechanical powertrain are denoted by $\eta_{bat}$, $\eta_{conv}$, $\eta_{contr}$, $\eta_{m}$, $\eta_{mp}$ respectively, i.e. we are using a lumped model for efficiencies. By obtaining the route information from Google Maps, typically the shortest path and using the software EVRE that was developed in Yi and Bauer (2014a), we can estimate the energy cost between starting location and destination.

4. The Maximum Number of Reachable Households Problem

In this framework, we focus on maximizing the number of reachable households with a given battery energy bound. This is a very crucial consideration for charging station owners. Given the energy constraint, the reachable range can be derived from Google Maps (Yi and Bauer (2014a)). In order to get the amount of covered EV users in the reachable range, the population distribution for the area considered must be known. A reasonable way to divide the city into subareas is by ZIP code zones and subsequently by using the population in each ZIP code area.

4.1. Optimization Model

Suppose the possible charging station set is $\mathcal{C} = \{c_1, \ldots, c_M\}$ and there are $N$ ZIP code zones: $\mathcal{Y} = \{y_1, \ldots, y_N\}$. ZIP codes are used to simplify the location information for EV home locations. The EVs in the same ZIP code zone have the same home address information. The model for a ZIP code zone is given by $y_j = (y_{cp}^j, y_{pop}^j)$, where $y_{cp}^j$ is the center position of $y_j$ and $y_{pop}^j$ is the population of $y_j$. Define the EV type set as $\mathcal{U} = \{u^1, \ldots, u^l, \ldots, u^L\}$, which means $L$ types of EVs are considered in total. Suppose the average market penetration rate for EVs is $\alpha_e$ and the market share for EV type $u^l$ in overall EV sales is $\alpha^l_e$, then $\sum_{l=1}^L \alpha^l_e = 1$. For each possible charging station $c_i$, the function $Range(c^l_p, u^l_t, E_{bound})$ is used to obtain its reachable range $R^l_i$ for EV type $u^l_t$ where $E_{bound}$ is the energy constraint. Furthermore $R_i = \bigcap_{l=1}^L R^l_i$. This range can be obtained by the software tool we designed based on Google Maps Javascript API in Yi and Bauer (2014a). Assume $X_0$ charging stations will be placed in the city area. The corresponding optimization model can be constructed as:
Max $\sum_{l=1}^{L} \sum_{j} z_j \alpha_e \alpha_e^l y_{lep}$

s. t. $\sum_{i=1}^{M} x_i = X_0, \quad x_i \in \{0, 1\}$

$\sum_{y_{lep} \in R_i} x_i \geq z_j, \quad z_j \in \{0, 1\}$

$i = 1, \ldots, M; \quad j = 1, \ldots, N.\quad (3)$

where the binary value $x_i$ is an indicator for filling a possible charging station $c_i$ (i.e. $x_i = 0$: not placed; $x_i = 1$: placed). $z_j$ describes whether $y_j$ is covered: If $z_j = 1$, $y_j$ is covered. The objective is to find the indicator variables $x_i$ for determining the set of charging station that covers the maximum number of reachable households.

The optimization problem (3) is a discrete problem and essentially solves a coverage problem on the real map. The union of coverage ranges $R_t$ is always difficult to obtain when $X_0$ is large. In order to make this problem solvable and decrease the computational complexity, the following procedures are proposed to relax this problem:

1. A coverage indicator vector for each charging station can be pre-calculated. Suppose the coverage indicator vector for ZIP code zone $y_j$ under EV type $u_l$ is $\vec{I}_j^l$ and $\vec{I}_j^l \in \mathbb{R}^M$, the element values in $\vec{I}_j^l$ should be 0 or 1. For example, for the $kth$ element in this vector, if its value is one, it is covered by charging station $c_k$. If its value is zero, it is not covered by $c_k$.

2. There are $N$ ZIP code zones, then $N \vec{I}_j^l$s will be constructed. Then the coverage indicator matrix is given by $I_t = [\vec{I}_1^l, \ldots, \vec{I}_j^l, \ldots, \vec{I}_N^l]^T$ and $I_t \in \mathbb{R}^{N \times M}$.

3. Let $\vec{x} = [x_1, x_2, \ldots, x_i, \ldots, x_M]^T$, then $I_t \vec{x} \in \mathbb{R}^N$ represents the covered number of charging stations for each ZIP code zone. So if ZIP code zone $k$ is covered at least by one charging station, the corresponding value of $(I_t \vec{x})_k$ will be equal or larger than one. By considering a step function $U(.)$, we have:

$$\text{Max} \quad \sum_{l=1}^{L} \alpha_e \alpha_e^l \vec{p}^T U(I_t \vec{x} - 0.5)$$

s. t. $|x| = X_0, \quad x_i \in \{0, 1\}$

$$\vec{p} \in \mathbb{R}^N, \quad I_t \in \mathbb{R}^{N \times M}, \quad \vec{x} \in \mathbb{R}^M\quad (4)$$
Where $\vec{p}$ is the population vector for $N$ ZIP code zones. If we want to
maximize the number of reachable households, each ZIP code zone can only
be calculated at one time. The step function $U(I; x - 0.5)$ is used to guarantee
this.

4.2. Case Study: South Bend

South Bend, IN, a midsize city in the US, is selected as an example. As
stated in the above optimization models, in order to model the population
distribution, the area can be divided according to ZIP codes. There are 13
ZIP codes in the South Bend area, which means $N = 13$. Table 2 includes
the ZIP codes and their corresponding population information. For possible
charging station positions, five possible places are selected as examples, e.g.
$M = 5$. They are listed in Table 3. We need to choose one or more from these
cfive possible positions to maximize the reachable population. According to
the data from DOT, we have the approximate average penetration rate in
the US of $\alpha_e = 0.001$. The most popular EVs in the US are the Nissan Leaf
and Tesla Model S. All others are neglected in our simulation. So we have
$L = 2$ and $\alpha_1^e = 0.66$ for Nissan Leaf, $\alpha_2^e = 0.34$ for Tesla Model S according
to the data in Wikipedia (2015).

4.2.1. Single Charging Station Placement Case

<table>
<thead>
<tr>
<th>ZIP Code</th>
<th>Population</th>
</tr>
</thead>
<tbody>
<tr>
<td>46544</td>
<td>30695</td>
</tr>
<tr>
<td>46545</td>
<td>28445</td>
</tr>
<tr>
<td>46556</td>
<td>7424</td>
</tr>
<tr>
<td>46601</td>
<td>8460</td>
</tr>
<tr>
<td>46613</td>
<td>11526</td>
</tr>
<tr>
<td>46614</td>
<td>27041</td>
</tr>
<tr>
<td>46615</td>
<td>16905</td>
</tr>
<tr>
<td>46616</td>
<td>6431</td>
</tr>
<tr>
<td>46617</td>
<td>11644</td>
</tr>
<tr>
<td>46619</td>
<td>22489</td>
</tr>
<tr>
<td>46628</td>
<td>25319</td>
</tr>
<tr>
<td>46635</td>
<td>4172</td>
</tr>
<tr>
<td>46637</td>
<td>13829</td>
</tr>
</tbody>
</table>

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Table 3: Charging Station Placement in South Bend

<table>
<thead>
<tr>
<th>( C )</th>
<th>Possible Position</th>
<th>Nissan Leaf</th>
<th>Tesla Model S</th>
<th>Total Coverage</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c_1 )</td>
<td>Century Center</td>
<td>73</td>
<td>64</td>
<td>137</td>
</tr>
<tr>
<td>( c_2 )</td>
<td>University of Notre Dame</td>
<td>73</td>
<td>33</td>
<td>106</td>
</tr>
<tr>
<td>( c_3 )</td>
<td>McKinley Town Center</td>
<td>65</td>
<td>43</td>
<td>108</td>
</tr>
<tr>
<td>( c_4 )</td>
<td>University Park Mall</td>
<td>56</td>
<td>21</td>
<td>77</td>
</tr>
<tr>
<td>( c_5 )</td>
<td>South Bend Airport</td>
<td>44</td>
<td>25</td>
<td>69</td>
</tr>
</tbody>
</table>

With these ZIP codes and their population information, EVRE in Yi and Bauer (2014a) is used to calculate the covered households with EVs for each possible position. The energy bound \( E_{\text{bound}} \) for the reachable range is assumed to be 2 kWh. The energy bound is set to such a low value so that the reachable range can be obtained accurately in an acceptable computation time. It also provides a good option for a mid size city. Fig. 3 shows the coverage of two possible charging station positions with EV type Tesla Model S, one is the Century Center and the other is the University Park Mall. The markers with “A” are the centers of the covered ZIP code zones, and the marker with “C” is the possible charging station position. The proposed five locations have different reachable ranges and will cover different ZIP code zones. After we know the covered ZIP code zones, we can calculate the corresponding number of covered EVs. This number is listed in Table 3. We can see that the Century Center in South Bend has the largest number of covered EVs, which means that under this gain function and the 2 kWh battery energy bound constraint, it is the optimal location.

4.2.2. The Multiple Charging Stations Case

As discussed in the single charging station case, there are 5 potential charging station positions. Two of them will be selected in order to maximize the number of reachable households, e.g. \( X_0 = 2 \). According to the relaxed optimization model in Equation (4), the population vector \( \mathbf{p} \) has 13 elements and the values can be obtained from Table 2. The energy bound for coverage is 2kWh. By calculating the energy cost for each ZIP code zone center to the five potential positions, we can construct the corresponding coverage indicator matrix \( \mathbf{I} \). Only the Nissan Leaf and Tesla Model S are considered.
and indicator matrices are \(I_1\) and \(I_2\), respectively. \(I_2\) is provided as follows:

\[
I_2^T = \begin{bmatrix}
0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 1 \\
0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 1 \\
1 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0
\end{bmatrix}
\]

The element \(I_{ij}\) in \(I\) indicates whether \(y_j\) is covered by \(c_i\). \(I_{ij} = 1\) means it is covered and \(I_{ij} = 0\) means it is not. For example, \(I_{32} = 1\) ZIP code zone No. 3 is covered by \(c_2\).

By solving (4), we can obtain the optimal solution \(x^* = (0, 1, 0, 1, 0)^T\), i.e. the optimal pair of charging station locations are the University of Notre Dame and the University Park Mall.

5. The Minimum Overall Transportation Energy Problem

The objective of minimizing the overall transportation energy required when EVs owners commute between home and charging station can reduce the global transportation energy cost. This is a crucial criterion for deciding the location of charging stations in future sustainable and green cities.

5.1. Probability Model for Charging Station Selection

In the multiple charging stations placement case, \(X_0(X_0 \geq 2)\) charging stations will be placed. Each EV owner can only choose one of them for
charging service each time. The decision is affected by many factors, for example, the distance $d_{ij}$ between the charging station $c_i$ and work place $u_j$, the waiting time for charging service, the transportation energy consumption $E_{ij}$, etc.. This is shown in Fig. 4.

Suppose each EV owner has a certain probability to visit anyone in the potential charging station set. Assume the probability of EV $j$ to choose charging station $i$ is $p_{ij}$, where $p_{ij} \geq 0$. In total, $M$ charging stations can be selected by each EV $j$. So the probability set for EV $j$ is \{${p_{ij}}, \ldots, p_{ij}, \ldots, p_{Mj}$\}. If $\sum_{i=1}^{M} p_{ij} = 1$, each EV definitely will select one of $M$ charging stations. Of course, $\sum_{i=1}^{M} p_{ij}$ could be less than 1 if not everyone always charges his/her EV. For example, $\sum_{i=1}^{M} p_{ij} = 0.5$ means that EV $j$ only have the probability of 50% to charge in the commute. Our proposed decision making models can handle this case, too.

The transportation energy consumption $E_j$ of EV $j$ at the charging station is

$$E_j = E_{ij} \text{ w. r. t. } p_{ij}$$ \hspace{1cm} (5)

The expected value of the transportation energy consumption for EV $j$ is

$$\mathbf{E}(E_j) = \sum_{i=1}^{M} E_{ij}p_{ij}$$ \hspace{1cm} (6)

The total transportation energy consumption for all EVs in an urban area is therefore:

$$E_{total} = \sum_{j=1}^{N} \mathbf{E}(E_j) = \sum_{j=1}^{N} \sum_{i=1}^{M} E_{ij}p_{ij}$$ \hspace{1cm} (7)

For establishing the probability model, the key point is the construction of a method to calculate the probability values \{${p_{ij}}$\}. Two methods are introduced:
(a) Suppose the distance from parking place to work place is $d_{ij}$ and the probability $p_{ij}$ has an inverse relationship with $d_{ij}$. This is because EV owners may not (or have low probability to) go to charge at a location which is far away from the work place. An exponential dependency is highly useful for modeling such a relationship and for generating the probability values respectively. The exponential model can be expressed as follows:

$$p_{ij} = \frac{x_i e^{-d_{ij}}}{\sum_{i=1}^{M} x_i e^{-d_{ij}}} \quad (8)$$

Where $x_i$ is the indicator variable to show whether the charging station is placed. Each $p_{ij} \geq 0$ and $\sum_{i=1}^{M} p_{ij} = 1$.

(b) Similarly, when an EV goes to a charging station, it has a smaller probability to charge at this station if this increases the energy cost. The model in terms of energy can be expressed:

$$p_{ij} = \frac{x_i e^{-E_{ij}}}{\sum_{i=1}^{M} x_i e^{-E_{ij}}} \quad (9)$$

5.2. Optimization Models

Assume $X_0$ charging stations are placed. Equation (10) formulates the constructed optimization problem that finds the optimal charging station positions by minimizing the total expected transportation energy cost.

$$\min \sum_{i=1}^{M} \sum_{j=1}^{N} p_{ij} E_{ij}$$

s. t. $E_{ij} = f(c_i^j, u_h^i, u_j^i); \quad p_{ij} = \frac{x_i e^{-d_{ij}}}{\sum_{i=1}^{M} x_i e^{-d_{ij}}} \forall i, j$

$$\sum_{i=1}^{M} x_i = X_0 \text{ where } x_i = 0 \text{ or } 1 \quad (10)$$

$$\sum_{j=1}^{N} p_{ij} \leq S_i \forall i$$

$$c_i \in C, \ i = 1, \ldots, M; \ u_j \in U, \ j = 1, \ldots, N$$

The optimization variable is $x_i$. The objective function is the expectation of the overall transportation energy consumption, which depends on $p_{ij}$. The
probability $p_{ij}$ is a nonlinear function with binary variables. A constraint $\sum_{j=1}^{N} p_{ij} \leq S_i$ is used to make sure the expected number of EVs at a specific charging station $i$ is smaller than its service capacity $S_i$. Such service capacity limits exist for many reasons, e.g., if there are infrastructure and power limitations in specific locations. The optimal solution of $x_i$ provided by this model can ensure the selected places have enough capacity to serve all EVs. This constraint is also nonlinear. Hence Equation (10) is a nonlinear integer programming problem.

The total distance between charging stations and work place is also a crucial factor. The charging station should be placed at the most convenient positions for EV owners to reach the work place. Equation (11) is the model that takes this factor into account. The coefficient $\lambda$ is used to adjust the weight of the total distance cost in the optimal decision making process.

$$
\begin{align*}
\min & \quad \sum_{i=1}^{M} \sum_{j=1}^{N} p_{ij}E_{ij} + \lambda \sum_{i=1}^{M} \sum_{j=1}^{N} p_{ij}d_{ij} \\
\text{s. t.} & \quad E_{ij} = f(c_i^u, u_h^j, u_t^j); \quad p_{ij} = \frac{x_i e^{-d_{ij}}}{\sum_{i=1}^{M} x_i e^{-d_{ij}}} \forall i, j \\
& \quad \sum_{i=1}^{M} x_i = X_0 \text{ where } x_i = 0 \text{ or } 1 \\
& \quad \sum_{j=1}^{N} p_{ij} \leq S_i \forall i \\
& \quad c_i \in C, \ i = 1, \ldots, M; u_j \in U, \ j = 1, \ldots, N 
\end{align*}
$$

Model (10) and (11) are nonlinear integer programming problems with nonlinear constraints. To solve such a problem, NOMAD in (Audet and Dennis Jr (2006); Le Digabel (2011)) is a good choice. It uses a Mesh Adaptive Direct Search algorithm to solve non-differentiable and global nonlinear programs. Algorithm 1 includes the detailed procedures to solve the problem of multiple charging station placement.

5.3. Simulations

The Simulated City (SimCity) in Fig. 5 will be used to demonstrate the proposed optimization models. It provides $50 \times 50$ avenues and streets. In total 2500 locations are considered as potential charging station positions.
Algorithm 1: Solver for Multiple Charging Station Problem via Minimum Overall Energy Consumption

1. Obtain the profile set $\mathcal{U}$ for all EVs.
2. Determine the potential charging station positions and their corresponding profiles using set $\mathcal{C}$.
3. Calculate the energy consumption set $\{E_{ij}\}$ and the distance set $\{d_{ij}\}$.
4. Construct the charging station selection probability model $\{p_{ij}\}$.
5. Establish the optimization model based on the obtained data and formulate it in matrix form with vector variable $x = (x_1, \ldots, x_M)^T$.
6. Apply the obtained optimization model using the NOMAD solver.
7. Transfer the optimal solution of $x^*$ into real charging station locations.

Eight suburban areas are connected to the city center by highways. Each suburban area has its own specific number of EVs, EV types, altitudes, etc, as shown in Fig. 5. By applying the SimCity data to the proposed optimization model, the model parameters can be obtained, e.g. $M=2500$, $N=940$. This is the complete set of possible charging station locations in this simulation, while in a real situation, only a subset of these locations would be used due to other practical considerations. The information such as home and work address of EVs and the type of EV will be used for the energy cost calculation.

1. The Single Charging Station Case

In this case, only one charging station will be placed according to the optimization model in Equation (10). Since all 2500 intersection points can be considered as a potential charging station location, the overall transportation energy consumption for all EVs to each possible position needs to be calculated. Then the position with minimum overall transportation energy consumption for all EVs can be solved according to the optimization model.

With the current suburban area information in Fig. 5, position (1,15)(at the lower left of the SimCity) is the optimal location for this single charging station placement problem to achieve minimum overall energy cost. Changing the number of EVs in some suburban areas, e.g. from 50 to 150 in Suburban Area 3, from 60 to 160 in Suburban Area 4, from 50 to 150 in Suburban Area 5, from 100 to 200 in Suburban Area 6, the same calculation and optimization procedures are performed. In this case the optimal location
(4,35) is obtained. The two different results show that the optimal charging infrastructure position definitely depends on the distribution of EVs in suburban areas.

(2) Multiple Charging Station Case

Multiple charging stations will be placed based on the model in (11). In order to make this problem scalable, a pre-selection has been applied. Only 100 positions (shown in Fig. 6) in SimCity have been used as the potential location set. Each potential charging station position has been randomly assigned with service capability $S_i$ (the maximum servable number of EV during one day). The other parameters for the optimization model are the same as those in the single charging station case. Then 10 optimal charging station positions will be selected from the 100 potential locations.

In model (11), the value of $\lambda$ serves to change the weight between energy cost and distance to the work place. Different values of $\lambda$ (e.g. 0, 0.01, 0.1, 1, 10) have been used to perform this simulation. Different values of $\lambda$ generate totally different placement strategies. The results in Fig. 6 show that the selected optimal charging station locations become more scattered when the value of $\lambda$ becomes larger. This is because the total distance cost related to work place location contributes more to the total optimization cost.
5.4. Case Study: Chicago

The Chicago metropolitan area is used to demonstrate the optimal charging station placement which aims to minimize overall transportation energy cost.

(1) Pre-selection of the Set of Potential Charging Station Position

In order to obtain a potential position set for optimal charging station placement, data mining techniques are applied to analyze the data set—Chicago Regional Household Travel Inventory (CRHTI) on CMAP (2006). This data set is a comprehensive study of the demographic and travel behavior characteristics of residents in the greater Chicago area. The criterion “Positions with larger average parking time have higher probability as potential charging station locations” is used to implement the pre-selection. The parking time analysis is perform on the whole data set to obtain the parking time distribution for all the places visited. Fig. 7 provides histogram statistics of parking times. The cases with abnormally low or high parking time of more than 12 hours or less than 30 minutes are discarded. Hence, initial cleaning actions are necessary to obtain more accurate analyses results. In order to obtain potential charging station positions, the following steps are performed:

a) Use the K-means clustering algorithm to classify the sample set into N classes.

b) Calculate the average parking time for each class.
c) Classes with larger parking time have higher priority for placing charging stations and will be selected more likely.

In order to determine the set of potential charging station locations in this case study, the K-means clustering algorithm with 50 classes is used to classify the whole data set. The corresponding centroid of each class is used to represent a whole class. The histogram of average parking times and the positions on the map for these 50 clusters are provided in Fig. 8. The result shows that most of these clusters have an average parking time between 3 and 4 hours.

(2) Charging Station Placement in Chicago

The Chicago metropolitan area is divided by the ZIP code areas. For each ZIP code area, the corresponding population is obtained from the dataset on UnitedStatesZipCodes.org (2015). Consider each ZIP code area as an
entirety with the location and population information. When we calculate the transportation energy for traveling to potential charging station positions, EVs in the same ZIP code area have the same location information. In the greater Chicago area, the same 10 counties as investigated in dataset CRHTI (CMAP (2006)) are considered and the detailed information is displayed in Table 4. In summary, there are totally 344 ZIP code areas with a total population of 7500299 in the entire city area.

<table>
<thead>
<tr>
<th>County</th>
<th>ZIP Code Number</th>
<th>Population</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cook County</td>
<td>163</td>
<td>4093345</td>
</tr>
<tr>
<td>DuPage County</td>
<td>36</td>
<td>869041</td>
</tr>
<tr>
<td>Grundy County</td>
<td>7</td>
<td>43937</td>
</tr>
<tr>
<td>Kane County</td>
<td>18</td>
<td>365911</td>
</tr>
<tr>
<td>Kendall County</td>
<td>6</td>
<td>86351</td>
</tr>
<tr>
<td>Lake County</td>
<td>51</td>
<td>978188</td>
</tr>
<tr>
<td>McHenry County</td>
<td>19</td>
<td>294188</td>
</tr>
<tr>
<td>Will County</td>
<td>29</td>
<td>552783</td>
</tr>
<tr>
<td>La Porte County</td>
<td>9</td>
<td>85518</td>
</tr>
<tr>
<td>Porter County</td>
<td>6</td>
<td>131037</td>
</tr>
</tbody>
</table>

Assume the entire greater Chicago area will be considered for the overall energy cost calculation. By considering the obtained potential charging station position set, we can use $M = 50$ and then $N = 7500299$. Each energy consumption value $E_{ij}$ is obtained by using the developed software tool EVRE (Yi and Bauer (2014a)). After all the energy components are calculated, the probability models will be constructed. Then the optimization model can be established to find the optimal position subset among the 50 potential charging station positions.

1) The Single Charging Station Case

Only one particular position will be selected from the potential set. Fig. 9(a) gives the optimal position (red marker) among the 50 potential positions. The result appears reasonable due to the optimal location in the center of downtown which has the highest population density. The other blue markers are the potential positions in Cook county, and the corresponding energy consumption for travel to these positions are provided in the sensitivity analysis in Fig. 9(b). The result shows that an increase in energy consumption has an approximately quadratic relationship with driving distance to the opti-
Figure 9: (a) Single optimal charging station placement in Chicago; (b) Sensitivity analysis

mal location. If the charging stations are placed far away from the optimal positions, energy cost for EVs will increase dramatically. This shows that the energy cost component has a strong dependency on location.

2) Multiple Charging Stations Case

In this case, multiple positions will be selected for charging station placement. Particularly 10 out of 50 potential positions are selected in this case study. Fig. 10 provides the placement results. Fig. 10(a) shows the 10 optimal positions for charging station placement. More of the selected positions are located around the city center, which appears intuitively reasonable. The positions are scattered enough outside the city center to minimize the energy cost for EVs in suburban areas. This is due to the lower population density in the suburban areas.

The other results in Fig. 10(b)–Fig. 10(c) are used to show the sensitivity of our optimization model. The sensitivity is checked by increasing the desired location number from 10 to 12 and 15. The number propagation process shows that, most of the selected positions are the same, only a small part of positions are added or changed. All these results indicate a very good robustness of our optimization model to find the optimal subset even when the required number of charging stations are increasing over time.
Figure 10: Multiple charging station placement in Chicago: (a) 10 charging stations; (b) 12 charging stations; (c) 15 charging stations
6. Multi-objective Optimization Model for Charging Station Placement

6.1. Optimization Model

Two energy-aware objective functions have been discussed thoroughly in previous sections, including maximum number of reachable households and minimum overall transportation energy consumption for charging actions. Both are crucial for charging station owners and EV users and definitely play a role in real world placement strategies and planning. Indeed these two objective functions are in conflict with each other, because a larger reachable range causes more energy cost for charging actions of EVs that are far away from charging station.

The multiple-objective optimization is a promising technique to obtain better decision making when dealing with different objective functions. It provides a compromise between the considered objectives. Assume the new objective function vector is \( f(x) \) with two required scalar objective functions: 

\[
\begin{align*}
  f_1(x) &= -p^T u(I \bar{x} - 0.5) \\
  f_2(x) &= \sum_{i=1}^{M} \sum_{j=1}^{N} p_{ij} E_{ij}
\end{align*}
\]

where \( f_1(x) = \sum_{i=1}^{M} \sum_{j=1}^{N} p_{ij} E_{ij} \) is for maximizing number of reachable households, \( f_2(x) = \sum_{i=1}^{M} \sum_{j=1}^{N} p_{ij} E_{ij} \) is for minimizing overall transportation energy consumption for charging actions. Due to the maximizing requirement for \( f_1(x) \), we use the negative objective function for reachable households in the overall function vector. This operation aims to use the two conflicting objective functions in the same vector function. Then the multiple objective optimal charging station placement decision making framework is given by:

\[
\begin{align*}
  \min & \quad f(x) = (-p^T u(I \bar{x} - 0.5), \sum_{i=1}^{M} \sum_{j=1}^{N} p_{ij} E_{ij})^T \\
  \text{s. t.} & \quad |x|_1 = X_0 \quad \text{where } x_i = 0 \text{ or } 1 \\
  & \quad \sum_{j=1}^{N} p_{ij} \leq S_i \quad \forall i \\
  & \quad c_i \in \mathcal{C}, \ i = 1, \ldots, M; u_j \in \mathcal{U}, \ j = 1, \ldots, N
\end{align*}
\]

where the corresponding constraints are the combination of required constraints in two single objective optimization models. Hence, this optimization model aims to find the optimal placement strategy \( x^* \) for charging stations to satisfy both objective requirements.
6.2. Scalar Method for Optimal Solution

The Pareto optimal solution will be used as the optimal strategy \( x^* \) for Equation (12). Pareto optimality is a resource allocation method in which it is impossible to make any one individual objective component better without making at least one other objective component worse.

Several methods can be used to solve multiple objective optimization problems for Pareto optimal solutions (Collette and Siarry (2003)). The scalar method is one of the highly efficient methods. The weighted sum of objective functions method in the scalar method framework is simple to implement for multi-objective optimization. This type of method will be used in this paper in order to solve the multiple objective charging station placement problem.

The goal is to transform Equation (12) to a single objective optimization problem. In case the transformed problem is within the single objective optimization framework, we can use the same solver proposed in Algorithm 1 to find the optimal solution. Suppose there are two weights \( \omega_1 \) and \( \omega_2 \), \( \omega_1 \geq 0 \), \( \omega_2 \geq 0 \) and \( \omega_1 + \omega_2 = 1 \). Then the corresponding scalar objective function can be expressed as

\[
f_w(x) = -\omega_1 f_1(x) + \omega_2 f_2(x) \tag{13}
\]

Given a pair of weights \((\omega_1, \omega_2)\), we can solve a single objective optimization problem to obtain the optimal solution. This single objective function has objective function \( f_w(x) \) and the same constraints as in Equation (12). Different pairs of \((\omega_1, \omega_2)\) provide different optimal solutions for single objective optimization problems. From these obtained solutions, the Pareto optimal solution for the original multiple objective problem will be found. Algorithm 2 is proposed to summarize the procedure for solving the charging station placement problem.

6.3. Case Study: Chicago

As in the minimum transportation energy consumption case, the greater Chicago area is used to demonstrate our multiple objective optimization model. The same 50 locations in Fig. 8 are used as potential charging station positions. Fig. 11 provides the results for the single objective optimization model with each single criterion applied separately, including the maximum number of reachable household and minimum overall transportation energy cost. Both of the results are the 10 optimal locations selected out of the
Algorithm 2: Pareto Optimal Solution for Multiple Objective Charging Station Placement

1. Transform the multiple objective optimization problem into the single objective optimization problem by the weighted sum of objective function method.
2. $N$ different values of $\omega_1$ in $[0, 1]$ will be used and $\omega_1 + \omega_2 = 1$.
3. For each pair $(\omega_1^i, \omega_2^i), i = 1, ..., N$, the scaled single objective optimization problem is solved to obtain the optimal solution $f_{wp}^i$.
4. For each $f_{wp}^i$, the optimal values of $f_{1p}^i$ and $f_{2p}^i$ for two objective values can be found.
5. With all $N$ pairs of $(f_{1p}^i, f_{2p}^i)$, the Pareto optimal solution point can be obtained.

Figure 11: Charging station placement in Chicago for two different criteria

potential 50 locations. We can see the result for maximum reachable households is much more scattered, while the selected positions will become more dense by using the minimum overall transportation energy cost. Both are reasonable results due to the corresponding characteristic of the used criterion.

In order to obtain the Pareto optimal solution for the proposed multiple objective decision-making model, different $\omega_1$s are used to find different optimal solutions for all the obtained single objective optimization problems in the weighted Algorithm 2. The corresponding values of $-f_1(x)$ and $f_2(x)$ are shown in Fig. 12. The results provide the possible optimal solutions for all the checked weights. Then the Pareto optimal solution point is the one with the red circle in Fig. 12.

With respect to this Pareto optimal solution point, we can obtain the corresponding optimal value of $x^*$, and then mark it on Google Maps, as
Figure 12: Two objective optimization with different values of $\omega_1$ and $\omega_2$

Figure 13: Charging station placement in Chicago for multiple objectives

shown in Fig. 13. Comparing this result to Fig. 11, the selected charging station locations in Fig. 13 have an equilibrium (not too scattered and at same time not too dense) that is between the results from application of a single criterion. This result provides a more balanced decision making strategy for charging station placement by considering two important requirements simultaneously.

7. Conclusions

In order to accelerate transportation electrification for future sustainable transportation systems, an optimal energy-aware charging infrastructure placement framework has been introduced. Two main objective requirements are proposed: maximizing the number of reachable households under an energy constraint and minimizing the overall transportation energy consumption for charging actions. Single objective optimization models for
the two requirements have been discussed. The corresponding algorithms for obtaining optimal decision variables have been proposed. Detailed simulations and real city case studies have been provided to demonstrate these models. The results show that the introduced models are promising to provide good strategies for charging station placement. Additionally, a multiple objective optimization model is established to handle both proposed criteria simultaneously and the corresponding case study for Chicago demonstrates significant improvements by considering two requirements simultaneously. The here presented method allows to incorporate additional constraints that originate from criteria such as power infrastructure, popularity of sites, etc. by apriori reducing the size of the set of all possible locations. Therefore additional criteria are easily integrated into the presented approach. Finally, we would like to point out that quick charging stations are not considered in this treatise.

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