Optimal Charging for General Equivalent Electrical Battery Model, and Battery Life Management

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Abstract

The contributions of this paper are threefold. First, we present the linear quadratic solution to optimally charge a Li-ion battery in a general form. Although the battery model considered here is a circuit comprised of an open-circuit voltage (OCV), a series resistance, and an RC circuit, the methodology is applicable to any electrical circuit model of a battery. A combination of different cost functions is considered including: time-to-charge, energy loss, and temperature rise index. We discuss the effect of different weightings in the cost functions on the current and voltage profiles. Second, we present two models for normalized battery capacity as a function of the number of cycles and two charge control parameters, viz., maximum terminal voltage of the battery and maximum charge current. These models are compared to a bi-exponential capacity model. The effectiveness of the proposed models for forecasting the battery capacity is validated using the experimental data. Third, these models are used for battery life management by developing an optimal charging parameter selection method which provides the best setpoint values for the control variables to achieve a pre-specified desired “useful cycle life” while attaining the fastest possible time-to-charge. The proposed optimal charging parameter selection method is illustrated via numerical results.

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Index Terms

battery charging, optimal charging, capacity modeling, capacity fade, battery life management.

I. INTRODUCTION

The ever-increasing market for battery-operated smart devices and electric vehicles is a catalyst for the efficient operation and health monitoring of batteries. In this vein, researchers tackle this challenge, broadly, in two ways: optimal battery charging and life cycle management. The latter requires the tracking of battery capacity. In this paper, we propose a general framework for optimal battery charging and life management. The contributions of this paper are three-fold:

- Proposing a methodology for solving the optimal charging problem, by considering a generic equivalent electrical model for battery.
- Proposing two models for normalized battery capacity as a function of the number of cycles and two charge control parameters, viz., maximum terminal voltage of the battery and maximum charge current.
- Proposing a battery life management scheme, to select the best setpoint values for the control variables.

First, we look at the charging problem using an equivalent electrical circuit, more realistic than the model used in [1]. There are different approaches for charging batteries in the literature, including: traditional methods of charging such as the constant trickle current charge strategy, constant current strategy and the constant-current constant-voltage (CC-CV) [2], multi-step constant-current charging [3], Taguchi-based methods [4], [5], boost charging [6], pulse-charging [7], [8], [9], [10], ant-colony based optimization of multistage constant current strategy [11], optimal-control based approaches [12], neural network [13], Grey-predicted charging system [14]. [For a review of different charging methods, the interested reader may consult \[2\], \[15\], \[16\], \[17\].](#)

The goal is to find the optimal current profile that minimizes a specific cost function. The cost function is considered as a weighted sum of time-to-charge (TTC), energy loss (EL), the square of the difference of the state-of-charge (SOC) from the final desired SOC, and a temperature rise index. The model, shown in Fig. 1, consists of an SOC-dependent OCV, a resistance R_0 , and a parallel RC circuit with resistance R_1 and capacitance C_1 . Consistent with [18], we call this

model as "model III". Note that the method, however, is generic and applicable to any equivalent electrical circuit model, with an enlarged state space. As we will see, the optimal strategy is composed of two stages: the first stage is the solution of a linear-quadratic (LQ) problem and the second stage is a constant voltage. Therefore, the strategy is called a Linear Quadratic-Constant Voltage (LQ-CV) policy.

Fig. 1: Model III circuit for the battery

Second, we present two models for normalized battery capacity (or, equivalently, capacity fade) as a function of the number of cycles and two charge control parameters, viz., maximum terminal voltage of the battery and the maximum charge current. [This stage is a precursor to the third stage in which we develop a control policy for managing the battery life, where we need a capacity model that includes the two charge control parameters.](#) There are a number of capacity models in the literature, including bi-exponential [19], physics-based electrochemical [20], [21], [22], and methods that are based on support vector machines [23], relevance vector machines (RVM) [24], [sample entropy-based](#) [25], extended Kalman filter [26], weighted total least squares [27], and SOC lookup [28]. A robust real time capacity estimation approach based on recursive least squares, total least squares, and OCV lookup has been proposed in [29]. A good review of several capacity estimation techniques can be found in [30]. [For more information on capacity fade, the interested reader may refer to \[31\], \[32\].](#)

Third, we analyze the optimal charging problem in a two-level strategy (level-I and level-II) that provides fast charging, while considering the battery life management by achieving a pre-specified desired "useful cycle life". At a lower and more detailed level (level-II), the objective is to find the optimal current profile to minimize the time-to-charge and energy loss during charging. The

optimal charging algorithms require two parameters: maximum acceptable terminal voltage of the battery and the maximum current during charge. As observed by other researchers as well [33], these two parameters affect the life of the battery. Other factors that affect the life of the battery include depth of discharge, variations in charge and discharge currents, temperature, and duty cycle and pulse duration in the case of pulse charging, to name a few.

The block diagram of level-I optimization is shown in Fig. 2. The measured capacity data from aging experiments is fed into a capacity fade model that uses the present cycle and the capacity of the battery obtained from the battery fuel gauge (BFG) to select the optimal charging parameters $v_{\max}$ and $i_{\max}$ to be used as control parameters in the charging algorithm.

Fig. 2: Block diagram of level-I optimization

The paper is organized as follows. In section II, we present the formulation of the LQ-CV (Linear Quadratic-Constant Voltage) approach and we derive the corresponding optimal charging profile. In section III, we present the simulation results for the optimal charging strategy and we discuss the effect of different weightings in the cost function on the current and voltage profiles. In section IV, we discuss the aging experiments and propose two models for battery capacity fade. In section V, we present a methodology that combines fast charging and battery life management via optimal charging parameter selection. Numerical illustrations for the optimal charging parameter selection are presented in section VI. Finally, in section VII, we conclude the paper.

NOMENCLATURE

α	Parameter in the calculation of current through R_1 , (1)
α_1, α_2	Parameters in the LS-BE model, (49)
$\underline{\alpha}, \alpha_1, \alpha_2 \alpha_3$	Parameters in the CVD model, (55)
a, b_0, b_1	Coefficients in the temperature rise model, (13)
AIC	Akaike Information Criterion
β	Parameter in the LAR- $\alpha\beta\gamma$ model, (48)
β_1, β_2	Parameters in the LS-BE model, (49)
$\underline{\beta}, \beta_1, \beta_2 \beta_3$	Parameters in the CVD model, (55)
c_h	Parameter in coulomb counting, (4)
$C[n]$	Battery capacity at cycle n , (47)
C-Rate	Rate at which battery is discharged relative to maximum capacity
C_1 (F)	Capacitor in model III, as shown in Fig. 1, (2)
CF	Capacity Fade
$C_{\text{norm}}[n]$	Normalized capacity, (47)
C_{batt} (Ah)	Battery Capacity, (4)
$C_{\text{norm}, i_{\text{max}}, v_{\text{max}}}[n]$	CVD model for capacity, (55)
$C_{h, \text{batt}}$ (J/(kg·K))	Heat capacity of the battery, [1], (14)
$C_{\text{norm}, kl}[n]$	Simulated capacity, (63)
$C_{\text{norm}, \underline{u}}[n]$	CVD model for capacity, (56)
C_{PC}	Capacity at the present cycle, (67)
CVD	Control variable-dependent
ΔAIC	AIC difference
Δ (s)	Sampling time, (2)
η	Charging efficiency
EUCL_{kl}	Expected useful cycle life, (69)
EL	Energy loss
γ	Parameter in the LAR- $\alpha\beta\gamma$ model, (48)
$\underline{\gamma}, \gamma_1, \gamma_2 \gamma_3$	Parameters in the CVD model, (55)
$\underline{g}[k], \underline{u}$	Vectors used in the formulation of level-II optimization method, (42), (82)

$H[k]$	Hamiltonian function, (76)
$i[k]$ (A)	Charging current, (1)
$i_{\max}$	Maximum allowed current during charging, (31)
$i_{\max}^*$	Optimal charging current
I_3	3x3 Identity matrix, (42)
J_E (J)	Energy loss cost function, (6)
J_s	sum of SOC error squared cost function, (6)
J_t (s)	Time-to-charge cost function, (6)
J_{tEsT} (J)	Cost function as a combination of cost functions of TTC, energy los, sum of SOC error squared, temperature rise, (6)
$J_{\text{tEsT}}^{\text{new}}$ (J)	Calculated cost function in the current step
$J_{\text{tEsT}}^{\text{old}}$ (J)	Calculated cost function in the previous step
J_T (K.s)	Temperature rise cost function, (6)
k	Time index, (1)
k_1	Time when the terminal voltage reaches the maximum allowed value, (36)
k_f	Final value of time index, (5)
$\underline{\lambda}$	Co-state vector, (75)
$l_{\text{LS-BE}}$	Relative likelihood of LS-BE model
$L[k]$	Lagrangian function, (75)
LAR- $\alpha\beta\gamma$	Least absolute residuals-alpha beta gamma, (48)
LPNC	Least permissible normalized capacity, (65)
LQ-CV	Linear Quadratic-Constant Voltage
LS	Least squares
LS-BE	Least squares bi-exponential
m_{batt} (kg)	Battery mass, (14)
ν	Lagrange multiplier, (42)
n	Cycle index, (47)
NCL	Nominal cycle life, (71)
$\omega[k]$	Variable used in the formulation of method, (43)
$\Phi, \Gamma, \Psi[k], P[k]$	Matrices used in the formulation of level-II optimization method, (24), (24), (42), (42)

PC	Present cycle, (69)
ρ	Weight on the TTC for the OtE algorithm
ρ_s	Weight for sum of SOC error squared cost function, (6)
ρ_t	Weight for the TTC cost function, (6)
ρ_T	Weight for the temperature rise cost function, (6)
R_{Eff} (K/W)	Effective thermal resistance, (20)
R_0, R_1 (Ω)	Resistors in model III as shown in Fig.1, (8), (2)
$R_{0\text{eq}}$ (Ω)	Equivalent resistance due to R_0 and heating effects, (18)
R_{0T} (Ω)	Heating equivalent resistance due to R_0 , (19)
$R_{1\text{eq}}$ (Ω)	Equivalent resistance due to R_1 and heating effects, (18)
R_{1T} (Ω)	Heating equivalent resistance due to R_1 , (21)
RUCL_{kl}	Remaining useful cycle life, (68)
RLS	Recursive least squares
s	State of charge, (3)
s_1	State of charge when the terminal voltage reaches the maximum allowed value, (43)
s_{kf}	Final SOC, (5)
s_{th}	State of charge threshold
SFCS	Set of feasible control settings, (72)
SOC	State of charge
T (K)	Battery core temperature, (11)
T_{amb} (K)	Ambient temperature, (11)
$\tilde{T}[k]$ (K)	Temperature rise, (10)
TRI	Temperature rise index
TLS	Total least squares
TTC	Time-to-charge
$\underline{u}$	Control variables v_{max} and i_{max} , (56)
$\underline{u}^*$	Optimal control setting, (74)
UCL_{kl}	Useful cycle life, (65)
$v[k]$ (V)	Terminal voltage of battery, (29)
v_c (V)	Voltage corresponding to a SOC of 1, (32)

$v_{\max}$	Maximum allowed value of terminal voltage during charging, (30)
$v_{\max}^*$	Optimal charging voltage
V_0	Open-circuit voltage, (29)
VPC_{kl}	Virtual present cycle, (67)
$x[k]$ (A)	Current through R_1 , (1)
$\underline{z}[k]$	State vector for the temperature model, (23)

II. LINEAR QUADRATIC- CONSTANT VOLTAGE (LQ-CV) STRATEGY FORMULATION

In this section, we investigate the battery charging problem using the equivalent electrical circuit model (Fig. 1), which is termed model III in [18]. This model, compared to the model discussed in [1], includes an extra RC circuit (R_1 and C_1). Let the current (in continuous time) through R_0 and R_1 be $i(t)$ and $x(t)$, respectively, and let the current through and the voltage across the capacitor C_1 be $i_c(t)$ and $v_c(t)$, respectively. Then, we have: $i_c(t) = C_1 \dot{v}_c(t)$, $i_c(t) = i(t) - x(t)$, $\dot{v}_c(t) = R_1 \dot{x}(t)$, $\dot{x}(t) = \lim_{\Delta \rightarrow 0} \frac{x(t+\Delta) - x(t)}{\Delta}$. Using these relations and inserting $\frac{1}{R_1} \lim_{\Delta \rightarrow 0} \frac{\Delta}{1 - \exp(-\Delta/(R_1 C_1))}$ for C_1 , one can derive the following relation in the discrete form:

$$x[k+1] = \alpha x[k] + (1 - \alpha)i[k] \quad (1)$$

where $x[k]$ and $i[k]$ are $x(k\Delta)$ and $i(k\Delta)$, respectively; Δ is the sampling interval (typically 0.1 to 1 second, and assumed constant for simplicity of notation), and α is as follows:

$$\alpha = \exp\left(-\frac{\Delta}{R_1 C_1}\right) \quad (2)$$

The dynamic evolution of the battery state-of-charge (SOC), s , is given by

$$s[k+1] = s[k] + c_h i[k] \quad (3)$$

where $s[k]$ is the SOC at time k and c_h (in 1/Amperes) is the Coulomb counting coefficient [29], given by

$$c_h = \frac{\Delta}{3600 C_{\text{batt}}} \quad (4)$$

and C_{batt} (in Ah) is the battery capacity, assumed to be known. It is assumed that the charging process takes $k_f \Delta$ seconds (k_f unknown and to be optimized), to attain a specified final desired SOC of s_{k_f} . That is,

$$s[k_f] = s_{k_f} \quad (5)$$

We consider a cost function composed of time-to-charge (TTC), energy loss (EL) in R_0 and R_1 , and a temperature rise index (TRI). To have more flexibility in controlling the time to charge and the trajectory of SOC during charging, we also include a term including the sum of squares of the difference of SOC at various times of the charging interval from the final desired SOC.

The cost function is, therefore, [as follows](#):

$$J_{\text{tEsT}} = \rho_t J_t + J_E + \rho_s J_s + \rho_T J_T \quad (6)$$

where

$$J_t = k_f \Delta \quad (7)$$

$$J_E = \Delta \sum_{k=0}^{k_f-1} (R_0 i^2[k] + R_1 x^2[k]) \quad (8)$$

$$J_s = \sum_{k=0}^{k_f-1} (s[k] - s_{k_f})^2 \quad (9)$$

Remark: One can replace s_{k_f} with $s_d(k)$, the desired SOC at time step k , for the charging process to follow a desired sequence of SOC $\{s_d(k)\}_{k=0}^{k_f}$.

$$J_T = \Delta \sum_{k=0}^{k_f} \tilde{T}[k] \quad (10)$$

where $\tilde{T}[k]$ is the temperature rise from the ambient temperature, T_{amb} ; that is

$$\tilde{T}[k] = T[k] - T_{\text{amb}} \quad (11)$$

Since it is the ratio of the weights that matters, we have assigned a weight of 1 to the energy loss cost function, i.e., J_E .

Remark: The cost function can include other functions of the SOC and the method that we discuss is still applicable. One such candidate is the following term, which corresponds to sum of squares of cumulative SOC difference from the final desired SOC.

$$J_S = \sum_{k=0}^{k_f-1} \left(\sum_{l=0}^{k-1} (s[l] - s_{k_f}) \right)^2 \quad (12)$$

The temperature dynamics are as follows (For more details about thermal models refer to [1], [34], [35], [36], [37], [38], [39], [40]):

$$\tilde{T}[k+1] = (1-a)\tilde{T}[k] + b_0 i^2[k] + b_1 x^2[k] \quad (13)$$

where a is the cooling coefficient (see [1], eq. (42))

$$b_0 = \frac{R_0 \Delta}{m_{\text{batt}} C_{h,\text{batt}}} \quad (14)$$

$$b_1 = \frac{R_1 \Delta}{m_{\text{batt}} C_{h,\text{batt}}} \quad (15)$$

and $C_{h,\text{batt}}$ is the heat capacity of the battery (see [1], eq. (42)). The solution of (13) is as follows:

$$\hat{T}[k] = \sum_{l=0}^{k-1} (1-a)^{k-1-l} (b_0 i^2[l] + b_1 x^2[l]) \quad (16)$$

Similarly to [1], the cost function due to temperature, i.e., J_T , can be written as follows:

$$J_T = \Delta \sum_{k=0}^{k_f-1} \left(\left(\sum_{l=0}^{k_f-k-1} (1-a)^l \right) (b_0 i^2[l] + b_1 x^2[l]) \right) \quad (17)$$

Calculating the summation of $(1-a)^l$ and noting that $b_0/a = R_0 R_{\text{Eff}}$ and $b_1/a = R_1 R_{\text{Eff}}$, we can write

$$J_T = \Delta \sum_{k=0}^{k_f-1} R_{0\text{eq}}[k] i^2[k] + R_{1\text{eq}}[k] x^2[k] \quad (18)$$

$$R_{0\text{eq}}[k] = R_0 + R_{0T}[k] \quad (19)$$

$$R_{0T}[k] = \rho_T R_0 R_{\text{Eff}} (1 - (1-a)^{k_f-k}) \quad (20)$$

$$R_{1\text{eq}}[k] = R_1 + R_{1T}[k] \quad (21)$$

$$R_{1T}[k] = \rho_T R_1 R_{\text{Eff}} (1 - (1-a)^{k_f-k}) \quad (22)$$

where $R_{0T}[k]$ and $R_{1T}[k]$ are the heating equivalent resistances respectively due to R_0 and R_1 . To simplify the equations, we define the following state vector:

$$\underline{z}[k] = \begin{bmatrix} s[k] - s_{kf} \\ x[k] \end{bmatrix} \quad (23)$$

The dynamics of $\underline{z}[k]$ with its initial and final states could be written as follows:

$$\underline{z}[k+1] = \Phi \underline{z}[k] + \Gamma i[k] \quad (24)$$

$$\Phi = \begin{bmatrix} 1 & 0 \\ 0 & \alpha \end{bmatrix} \quad (25)$$

$$\Gamma = [c_h, 1 - \alpha]^T \quad (26)$$

$$\underline{z}[0] = [s_0 - s_{kf}, 0]^T \quad (27)$$

$$\underline{z}[k_f] = [0, \text{free}]^T \quad (28)$$

The battery terminal voltage is then as follows:

$$v[k] = V_0(s[k]) + R_0 i[k] + \begin{bmatrix} 0 & R_1 \end{bmatrix} \underline{z}[k] \quad (29)$$

with following constraints:

$$v[k] \leq v_{\max} \quad (30)$$

$$i[k] \leq i_{\max} \quad (31)$$

Selecting the optimal values of $i_{\max}$ and $v_{\max}$ is discussed in section V. In this section, without loss of generality, we use v_c for $v_{\max}$, where v_c is the voltage corresponding to SOC of 1; that is

$$v_c = V_0(1) \quad (32)$$

Using the above notations, we can write the cost function as follows:

$$J_{\text{tEsT}} = \rho_t k_f \Delta + \sum_{k=0}^{k_f-1} (\Delta R_{0\text{eq}}[k] i^2[k] + \underline{z}^T[k] Q[k] \underline{z}[k]) \quad (33)$$

where

$$Q[k] = \begin{bmatrix} \rho_s & 0 \\ 0 & \Delta R_{\text{leq}}[k] \end{bmatrix} \quad (34)$$

$$(35)$$

The charging process can be divided into two stages: in the first stage that lasts for k_1 samples, the battery terminal voltage is less than v_c . That is

$$v[k] < v_c \quad k = 0, 1, \dots, k_1 - 1 \quad (36)$$

and in the second stage which starts at time k_1 onwards, the battery terminal voltage is equal to v_c . That is

$$v[k] = v_c \quad k = k_1, k_1 + 1, \dots, k_f \quad (37)$$

As the voltage hits the boundary value of v_c at time k_1 , and remains constant from then on, the current in stage 2 is already determined by the constrained dynamics of the system, as follows:

$$i[k] = \frac{v_c - V_0 - R_1 x[k]}{R_0} \quad k = k_1, k_1 + 1, \dots, k_f \quad (38)$$

$$x[k] = \alpha x[k-1] + (1 - \alpha) i[k-1] \quad k = k_1, k_1 + 1, \dots, k_f \quad (39)$$

with the following initial values

$$i[k_1 - 1] = i_{k_1-1} \quad (40)$$

$$x[k_1 - 1] = x_{k_1-1} \quad (41)$$

where i_{k_1-1} and x_{k_1-1} are the initial values of the currents for stage 2, which are equal to the final values of currents for stage 1.

The current profile in stage 1 is obtained by solving a linear quadratic (LQ) problem. The following algorithm finds the optimal current profile in stage 1.

Algorithm: For a given k_1 value, the optimal current profile in stage 1 (the LQ stage) is obtained as follows:

$$i[k] = -\frac{\Gamma^T}{2\Delta R_{\text{leq}}[k]} \left(P[k+1] \left((I_3 + \Psi[k]P[k+1])^{-1} (\Phi \underline{z}[k] - \Psi[k]\underline{g}[k+1]\nu) \right) + \underline{g}[k+1]\nu \right) \quad (42)$$

where ν and $\Psi[k]$ are given by:

$$\nu = \frac{s_1 - s_{k_f} - \underline{g}^T[0]\underline{z}[0]}{\omega[0]} \quad (43)$$

$$\Psi[k] = \frac{\Gamma \Gamma^T}{2\Delta R_{0eq}[k]} \quad (44)$$

$\underline{z}[0]$ is the initial state, and $\underline{g}[0]$ and $\omega[0]$ are calculated by solving the following backward set of recursions:

$$\begin{aligned} P[k] &= 2Q[k] + \Phi^T P[k+1] (I_3 + \Psi[k] P[k+1])^{-1} \Phi \\ \underline{g}[k] &= \Phi^T (I_3 + P[k+1] \Psi[k])^{-1} \underline{g}[k+1] \\ \omega[k] &= \omega[k+1] - \underline{g}^T[k+1] (I_3 + \Psi[k] P[k+1])^{-1} \Psi[k] \underline{g}[k+1] \end{aligned} \quad (45)$$

with the following terminal values:

$$\begin{aligned} P[k_1] &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \\ \underline{g}[k_1] &= \begin{bmatrix} 1 & 0 \end{bmatrix}^T \\ \omega[k_1] &= 0 \end{aligned} \quad (46)$$

The derivation of this algorithm is presented in the Appendix.

Based on the above, the numerical calculation of the optimal current is computed via the following steps:

- ❶ Set J_{tEsT}^{old} to a big value (for example, in Matlab, set it as "Inf").
- ❷ Initialize k_1 with 1.
- ❸ Solve the set of equations (45) with the terminal values in (46).
- ❹ Calculate the current in stage 2 by assuming the terminal voltage as v_c until the SOC reaches the desired value of s_{k_f} .
- ❺ Calculate the cost function based on the current profiles of stage 1 and stage 2 and assign it to J_{tEsT}^{new} .
- ❻ If $J_{tEsT}^{new} > J_{tEsT}^{old}$, the optimal k_1 is the previous value and the optimal current profile was obtained in the previous step; otherwise, go to next step.
- ❼ Solve the set of recursions (45) for one step with the terminal values obtained in the previous step.
- ❽ Go to step "❹".

It is important to note that although the optimal current profile is derived for model III (see Fig. 1), the method is generic and it is applicable to any equivalent electrical circuit model, including all electrical circuit models discussed in [18], with an enlarged state vector $z[k]$.

It should be noted that the practical meaning of the parameters of optimization problem (e.g., ρ_t , ρ_s , and ρ_T in (6)) is to use them in an iterative design procedure to reach the desired performance. For example, if the maximum allowed energy loss is $E_{\max}$ and the maximum acceptable time-to-charge is $TTC_{\max}$, then in the design procedure, ρ_t should be selected proportional to $\frac{E_{\max}}{TTC_{\max}}$; that is $\rho_t \propto \frac{E_{\max}}{TTC_{\max}}$ and then iterate on the proportionality factor.

III. SIMULATION RESULTS FOR THE OPTIMAL CHARGING STRATEGY

In this section, we present simulations based on the theoretical foundations of the previous section.

A. Effect of Weights

In this subsection, we use different cost functions and find the corresponding optimal profiles. First, we set $\rho_s = \rho_T = 0$, and change the values of ρ_t from 0.1 to 0.5. Figures 3(a), 3(b), and 3(c), respectively, show the profiles of current, state of charge and terminal voltage. Fig. 3(a) shows that low values of ρ_t result in low values of current in stage 1 (LQ stage). In other words, a low ρ_t puts less emphasis on charging time and more emphasis on the energy losses; hence, it results in low level of current which provides low energy loss and higher charging efficiency. On the other hand, by increasing ρ_t , more emphasis is placed on the charging time.

Fig. 3: Effect of different weights on the current, voltage, and SOC profiles

Next, we consider the effects of ρ_T and ρ_s , and we compare the results to the case when these weights are zero. Figure 3(d) shows the optimal current profile for three different weight sets. The blue curve represents the optimal profile when $\rho_t = 0.5, \rho_s = 0, \rho_T = 0$. Here there is no penalty on the temperature rise and the squared difference of the SOC and the final desired SOC (s_{k_f}). When we change ρ_T from zero to 0.5, the emphasis on the temperature rise results in a decrease the magnitude of current, as depicted by the blue and red curves in Figure 3(d). The green curve shows the optimal current profile when we set ρ_s to 0.1. Here, the penalty on the squares difference of the SOC and the final desired SOC (s_{k_f}) results in an increase in the current level.

IV. PROPOSED CAPACITY MODELS

In this section, we propose two models, namely LAR- $\alpha\beta\gamma$ and CVD, to estimate the normalized capacity (and, equivalently, the capacity fade) as a function of the cycle number. The motivation for developing a capacity fade model is that, for the level-I control policy (to be presented in

section V), we need a capacity model that considers both the current and the terminal voltage as control parameters of the model. In other words, this section is a precursor to section V. These models are developed by exploring a number of models with different fitting methods. The development of the models is based on the data obtained from the aging experiments, which are detailed in subsection A. Let n denote the cycle number. The capacity at cycle n , denoted by $C[n]$, is calculated by trapezoidal integration of the injected current during the charging process, which is also referred to as the Coulomb counting. In each experiment, the calculated capacities $C[n]$ are normalized by dividing them by the capacity of the battery at the first cycle, i.e., $C[1]$, and the resulting normalized capacities are denoted by $C_{\text{norm}}[n]$; that is

$$C_{\text{norm}}[n] = \frac{C[n]}{C[1]} \quad n = 1, 2, \dots \quad (47)$$

These normalized capacities are used for fitting the capacity models.

In the following, after a brief presentation of the conducted aging experiments, we propose LAR- $\alpha\beta\gamma$ model, study LS-BE model, compare LAR- $\alpha\beta\gamma$ and LS-BE models, propose CVD model, and compare it with LS-BE. The CVD model, which is a function of control variables v_{max} and i_{max} , is then utilized in section V, in developing the level-I control policy.

A. Aging Experiments

In order to study the effects of aging on capacity, ten different experiments were performed. The batteries used in these experiments were Samsung GS4 with a capacity of 2600 mAh and a nominal terminal voltage of 4.35 volts. The batteries were exposed to several (from 25 to 200) cycles of charging and discharging with a 10 minute rest after any charge or discharge process. The experiments were performed at room temperature (25 °C) using Keithley 2651A [41]; descriptions of the experiments are provided in Table I. Seven different charge profiles (Ch1-Ch7) were used and are described in Table II. Note that 1.0C for GS4 battery corresponds to 2.6 amperes. All the charge profiles are terminated once the charge current falls below $\frac{1}{20}$ C or, equivalently, 130 mA. Three different discharge profiles, namely Disch1-Disch3, were used. Table III describes the discharge profile Disch1, which has been used in all experiments except 2 and 10. The profile Disch2 is similar to Disch1, except that 0.25C is used for stage 1, 0.5C for stage 2, and 0.2C for stage 3. The profile Disch3 consists of only one stage: discharge at 0.2C until the terminal voltage reaches 2.8V.

TABLE I: List of aging experiments

Experiment ID	Charge Profile	Discharge Profile	Number of Cycles
1	Ch1	Disch1	200
2	Ch2	Disch1	125
3	Ch3	Disch1	75
4	Ch3	Disch3	75
5	Ch4	Disch1	50
6	Ch4	Disch2	50
7	Ch5	Disch1	50
8	Ch2	Disch1	50
9	Ch6	Disch1	25
10	Ch7	Disch1	25

TABLE II: Description of charging profiles

Charge Profile	Description
Ch1	Multi-level current at 1.25C for 1200 seconds, 0.833C for 1100 seconds then 0.5C until the battery terminal voltage hits 4.35, afterwards the CV stage is activated with 4.35 volts
Ch2	CC-CV with current at CC stage equal to 1.0C and voltage at CV stage equal to 4.35 volts
Ch3	CC-CV with current at CC stage equal to 0.7C and voltage at CV stage equal to 4.35 volts
Ch4	Multi-level CV at 4.45, 4.40, 4.35, 4.30, 4.25 volts each for 0.5 seconds with tapering down the voltage to 4.35 volts after SOC of 70%
Ch5	CC-CV with current at CC stage equal to 1.3C and voltage at CV stage equal to 4.35 volts
Ch6	CC-CV with current at CC stage equal to 1.3C and voltage at CV stage equal to 4.25 volts
Ch7	CC-CV with current at CC stage equal to 1.0C and voltage at CV stage equal to 4.25 volts

TABLE III: Description of discharge profile of Disch1

Stage	Description
1	discharge at 0.5C for 1 hour
2	discharge at 1C for 15 minutes
3	discharge at 0.5C for few seconds (6 seconds)
4	discharge at 0.2C until battery terminal voltage hits 2.8 volts

Figure 4(a) shows a sample charge-discharge profile for one of the experiments and Figure 4(b) shows all of its 125 voltage profiles. It can be seen that, as the battery ages, the time-to-charge

and time-to-discharge decrease and hence the total time of a charge-rest-discharge-rest cycle decreases. This decrease in time-to-charge and time-to-discharge is due to capacity fade.

Fig. 4: Current and voltage profiles

B. LAR- $\alpha\beta\gamma$ Model

Among the [explored models](#), the following model with the least absolute residuals (LAR) as [its fitting criterion](#) was found to [better](#) capture the trend in the normalized capacity. We refer to this model as "LAR- $\alpha\beta\gamma$ ".

$$C_{\text{norm}}[n] = \alpha\beta^n n^\gamma \quad n = 1, 2, \dots \quad (48)$$

The LAR method is robust in that it minimizes the sum of the absolute values of the residuals, rather than the squared differences as in ordinary least squares. Therefore, extreme values have substantially smaller influence on the fit, thereby achieving the desired robustness of the fit. The Matlab function of "fit" with the "StartPoint" of [1 1 1] was used for training the model.

C. LS-BE Model

Among the various models we had tried, the following bi-exponential model, which is frequently used in the literature [42], [43], [19], was chosen for comparison. We used the least squares approach as the fitting method; hence we refer to it as "LS-BE".

$$C_{\text{norm}}[n] = \alpha_1 \beta_1^n + \alpha_2 \beta_2^n \quad n = 1, 2, \dots \quad (49)$$

The estimation of the parameters is performed as follows. First, we can easily verify that the following relation holds.

$$C_{\text{norm}}[n+1] = x_1 C_{\text{norm}}[n] - x_2 C_{\text{norm}}[n-1] \quad (50)$$

where,

$$x_1 = (\beta_1 + \beta_2) \quad x_2 = \beta_1 \beta_2 \quad (51)$$

Since we have the profile of normalized capacity, the unknown variables of x_1 and x_2 in (50) can easily be estimated using LS method. Since x_1 and x_2 are, respectively, the sum and the product of β_1 and β_2 , we can find the estimates of β_1 and β_2 by solving the following quadratic equation, once we estimate x_1 and x_2 .

$$\beta^2 - x_1 \beta + x_2 = 0 \quad (52)$$

Having the estimates of β_1 and β_2 , equation (49) reduces to a linear equation with unknown variables α_1 and α_2 . Hence, by applying the LS method, we can estimate both α_1 and α_2 .

Note that since the capacity model is in the normalized form, the capacity fade (CF), which is defined in (53), can also be written as in (54).

$$\text{CF}[n] = \frac{C[1] - C[n]}{C[1]} \quad n = 1, 2, \dots \quad (53)$$

$$\text{CF}[n] = 1 - C_{\text{norm}}[n] \quad n = 1, 2, \dots \quad (54)$$

D. Comparison of LAR- $\alpha\beta\gamma$ Model and LS-BE Model

Figure 5(a) shows the data for normalized capacities in one of the experiments, along with the estimates of LAR- $\alpha\beta\gamma$ and LS-BE models. In this experiment, as Fig. 5(a) shows, the LAR- $\alpha\beta\gamma$ is better at capturing the trend in the normalized capacities than LS-BE. While the outliers affect

the LS-BE model significantly, they have minor effect on the $\text{LAR-}\alpha\beta\gamma$ and hence $\text{LAR-}\alpha\beta\gamma$ filters out the outliers. Figure 5(b) shows the result of $\text{LAR-}\alpha\beta\gamma$ and LS-BE with different sizes of fitting data (40, 100, and 160). It is seen that **predictions of $\text{LAR-}\alpha\beta\gamma$ are consistent with the experimental data even when just 40 samples are used in the fitting process.**

Fig. 5: (a) Comparison of $\text{LAR-}\alpha\beta\gamma$ model and LS-BE model. (b) Comparison of $\text{LAR-}\alpha\beta\gamma$, LS-BE models.

Next, in one of the experiments, we used the data from cycles 1-175 for **training the two models, and then we predicted** the normalized capacity for cycles 176-200. We found while the average of the relative error of LS-BE prediction is 0.99%, that of $\text{LAR-}\alpha\beta\gamma$ is only 0.35%; **in addition**, the maximum **values** of relative error for these models are 1.50% and .87%, respectively. This suggests that $\text{LAR-}\alpha\beta\gamma$ provides better performance in capturing the trend in capacity fade than LS-BE does. This observation was found to be consistently valid across all the experiments.

Next we compare $\text{LAR-}\alpha\beta\gamma$ and the LS-BE models using the Akaike Information Criterion (AIC). The AIC is used to rank multiple models in the Kullback-Leibler information sense [44]

and values the goodness-of-fit and parsimony, two often-counteracting factors. However, once a model, with fewer parameters (more parsimonious), provides a better goodness-of-fit than a model with more parameters, the AIC selects the model with less parameters. The number of parameters for the LAR- $\alpha\beta\gamma$ is 3 and for the LS-BE is 4, meaning that the LAR- $\alpha\beta\gamma$ model is more parsimonious than the LS-BE model. As for the goodness-of-fit, Fig. 6 shows the ratio of MSE (Mean Squared Error) of LAR- $\alpha\beta\gamma$ to that of LS-BE, where in all experiments, this ratio is higher than 1 with it being as high as 3.6 for the first experiment. In ranking models via AIC, what is important is the AIC differences rather than the AIC values themselves [44]. Table IV shows the AIC differences ($\Delta AIC = AIC_{LS-BE} - AIC_{LAR-\alpha\beta\gamma}$) for the 10 aging experiments. For eight of experiments the AIC difference is greater than 10 and for the remaining two experiments, it is greater than 4. As rule of thumb, the level of empirical support for a model with higher AIC is considerably less when the AIC difference is between 4 and 7 and it is essentially none when the AIC difference is greater than 10 [45]. The relative likelihood of LS-BE model (with respect to LAR- $\alpha\beta\gamma$) can be calculated using the AIC differences as $l_{LS-BE} = \exp(-\frac{\Delta AIC}{2})$ [44]. These values are shown in Table IV and it is seen that the highest value for relative likelihood of LS-BE model is 0.1, which demonstrates that the LAR- $\alpha\beta\gamma$ model is better than LS-BE model in the Kullback-Leibler information sense.

Fig. 6: MSE ratio of LS-BE to LAR

TABLE IV: AIC differences of the LAR- $\alpha\beta\gamma$ and LS-BE models and relative likelihood of LS-BE model for all aging experiments

Experiment ID	1	2	3	4	5	6	7	8	9	10
$\Delta\text{AIC} = \text{AIC}_{\text{LS-BE}} - \text{AIC}_{\text{LAR-}\alpha\beta\gamma}$	257	128	32	16	7.5	16	4.5	11	25	11
$l_{\text{LS-BE}}$ (Relative likelihood)	0.00	0.00	0.00	0.00	0.02	0.00	0.10	0.00	0.00	0.00

Table V shows the parameters of LAR- $\alpha\beta\gamma$ model for the ten experiments.

TABLE V: Parameters of LAR- $\alpha\beta\gamma$ model for all experiments

Experiment ID	α	β	γ
1	0.9992	0.9995	-0.0037
2	1.0015	0.9996	-0.0027
3	1.0007	0.9996	-0.0018
4	1.0007	0.9996	-0.0006
5	1.0013	0.9995	-0.0042
6	1.0008	0.9996	-0.0043
7	1.0005	0.9996	-0.0028
8	1.0016	0.9996	-0.0032
9	1.0004	0.9996	-0.0021
10	1.0004	0.9996	-0.0009

E. CVD Model

In order to be able to apply the level-I control policy (as described in section V) to select the optimal values for the maximum allowable current and the maximum allowable terminal voltage, we need a capacity fade model that incorporates these two parameters. For this reason, we developed a control variable-dependent model (CVD model), in which the $\alpha\beta\gamma$ parameters are dependent on $v_{\max}$, and $i_{\max}$. The model chosen for this purpose is as follows:

$$C_{\text{norm}, v_{\max}, i_{\max}}[n] = (\alpha_1 v_{\max}^{\alpha_2} i_{\max}^{\alpha_3}) (\beta_1 v_{\max}^{\beta_2} i_{\max}^{\beta_3})^n n^{(\gamma_1 + \gamma_2 v_{\max} + \gamma_3 i_{\max})} \quad n = 1, 2, \dots \quad (55)$$

The fitting of model (55) is performed in three stages:

- ❶ The data for each experiment are used to find the corresponding LAR- $\alpha\beta\gamma$ model.
- ❷ For each experiment, the corresponding LAR- $\alpha\beta\gamma$ is used to generate the normalized capacity data for 500 cycles.

③ The generated normalized capacity data in stage 2 for all experiments as well as the $v_{\max}$ and $i_{\max}$ of the experiments are used to fit the CVD model using the LS approach.

The CVD model of (55) can be written as follows:

$$C_{\text{norm}, \underline{u}}[n] = f(\underline{\alpha}, \underline{\beta}, \underline{\gamma}, \underline{u}, n) \quad (56)$$

$$\underline{\alpha} = \begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 \end{bmatrix}^T \quad (57)$$

$$\underline{\beta} = \begin{bmatrix} \beta_1 & \beta_2 & \beta_3 \end{bmatrix}^T \quad (58)$$

$$\underline{\gamma} = \begin{bmatrix} \gamma_1 & \gamma_2 & \gamma_3 \end{bmatrix}^T \quad (59)$$

$$\underline{u} = \begin{bmatrix} v_{\max} & i_{\max} \end{bmatrix}^T \quad (60)$$

where $\underline{\alpha}$, $\underline{\beta}$, and $\underline{\gamma}$ are fixed parameters, [once the model is trained](#). Therefore, the trend in capacity (and consequently capacity fade) over cycles depends on the values of control variables $\underline{u}$. Note that $v_{\max}$ and $i_{\max}$ are expressed in normalized form [with their nominal values being 4.35 V and 2.6 A, respectively](#).

Table VI shows the parameters of the CVD model.

TABLE VI: Parameters of CVD model

α_1	α_2	α_3
1.0007	0.0539	-0.0051
β_1	β_2	β_3
0.9996	-0.0012	-1.0407×10^{-4}
γ_1	γ_2	γ_3
0.0749	-0.0767	-4.9293×10^{-4}

F. Comparison of CVD and LS-BE Models

Figure 7(a) shows a comparison of the LAR- $\alpha\beta\gamma$, CVD and LS-BE models in capturing the trend in capacities for two of the experiments. The CVD model and the LAR- $\alpha\beta\gamma$ model are indistinguishable. Figure 7(b) shows the relative error of the CVD and the LS-BE models with respect to the LAR- $\alpha\beta\gamma$ model. As is evident from these experiments, compared to LS-BE, the CVD model has a lower relative error.

Fig. 7: (a) Comparison of LAR- $\alpha\beta\gamma$ model and LS-BE model with different fitting data lengths. (b) Relative error of LS-BE, and CVD models with respect to LAR- $\alpha\beta\gamma$.

V. FAST CHARGING WITH BATTERY LIFE MANAGEMENT VIA OPTIMAL CHARGING PARAMETER SELECTION

In this section, we present a fast charging strategy, coupled with battery life management policy, by presenting a formulation for the optimal selection of control variables, i.e., $v_{\max}$ and $i_{\max}$. The goal is to find the optimal terminal voltage and maximum current so that we preserve the "Least Permissible Normalized Capacity" LPNC (typically 80%) at the end of the "Nominal Cycle Life" NCL (typically 500) cycles of battery usage, while minimizing the time to charge. In this paper, for illustrative purposes, we use $\text{LPNC} = 80\%$ and $\text{NCL} = 500$ as the constraints for the optimal charging parameter selection.

Suppose we have K different candidate values for $v_{\max}$ and L different candidate values for $i_{\max}$ as follows:

$$v_{\max} \in \{v_1, v_2, \dots, v_K\} \quad (61)$$

$$i_{\max} \in \{i_1, i_2, \dots, i_L\} \quad (62)$$

This results in $K \times L$ different curves as follows:

$$C_{\text{norm},kl}[n] = f(\underline{\alpha}, \underline{\beta}, \underline{\gamma}, \underline{u}_{kl}, n) \quad (63)$$

$$\underline{u}_{kl} = \begin{bmatrix} v_k & i_l \end{bmatrix}^T \quad k \in \{1, 2, \dots, K\} \quad l \in \{1, 2, \dots, L\} \quad (64)$$

Corresponding to curve $C_{\text{norm},kl}[n]$, we define the "Useful Cycle Life" of UCL_{kl} as the cycle number at which the normalized capacity hits the "Least Permissible Normalized Capacity" LPNC (say, 80%). Thus,

$$C_{\text{norm},kl}[n] = f(\underline{\alpha}, \underline{\beta}, \underline{\gamma}, \underline{u}_{kl}, n) \geq \text{LPNC} \quad \text{for } n \leq \text{UCL}_{kl} \quad (65)$$

$$C_{\text{norm},kl}[n] = f(\underline{\alpha}, \underline{\beta}, \underline{\gamma}, \underline{u}_{kl}, n) < \text{LPNC} \quad \text{for } n > \text{UCL}_{kl} \quad (66)$$

Assume that we have estimates of the number of cycles the battery has been exposed to and the normalized capacity of the battery at any time (for example through BFG [29]). Let PC denote the "Present Cycle" of the battery and let C_{PC} be the estimated normalized capacity at cycle PC. If this battery is going to be charged using the control setting $\underline{u}_{kl}$, its normalized capacity should correspond to the curve $C_{\text{norm},kl}[n]$. Corresponding to the control setting $\underline{u}_{kl}$ and

the normalized capacity of the battery at cycle C_{PC} , we define the "Virtual Present Cycle" of the battery VPC_{kl} as follows:

$$C_{PC} = f(\underline{\alpha}, \underline{\beta}, \underline{\gamma}, \underline{u}_{kl}, VPC_{kl}) \quad (67)$$

Then, for a charge profile with control setting u_{kl} , the "Remaining Useful Cycle Life" $RUCL_{kl}$ is defined as the difference between useful cycle life and the virtual present cycle; that is

$$RUCL_{kl} = UCL_{kl} - VPC_{kl} \quad (68)$$

Since the battery has already experienced PC cycles of usage, the "Expected Useful Cycle Life" of the battery $EUCL_{kl}$ corresponding to the control setting u_{kl} is:

$$EUCL_{kl} = PC + RUCL_{kl} \quad (69)$$

Inserting (68) into (69), we have

$$EUCL_{kl} = (PC - VPC_{kl}) + UCL_{kl} \quad (70)$$

The control setting u_{kl} is feasible if its corresponding expected useful cycle life is greater than or equal to "Nominal Cycle Life" NCL (say, 500 cycles); that is

$$EUCL_{kl} \geq NCL \quad (71)$$

Thus, the "Set of Feasible Control Settings" (SFCS) is

$$SFCS = \{\underline{u}_{kl} | k \in \{1, 2, \dots, K\}, l \in \{1, 2, \dots, L\}, EUCL_{kl} \geq NCL\} \quad (72)$$

Among all feasible control settings, we select the one that minimizes the time to charge. This can be done as follows by using TTC_{kl} . For this purpose, the model-I [1] of the battery is simulated for different values of feasible control parameters, and among them the policy that results in the lowest TTC is selected. In the simulations, the CC-CV charging is used with the current in CC stage is selected as i_l and the terminal voltage in CV stage as v_k . This is equivalent to running the optimal charging algorithm with the objective of weighted sum of time-to-charge (TTC) and energy loss (EL) (referred to as OtE algorithm in [1]) with weight on TTC, i.e. ρ_t , being equal to $R_0 i_l^2$, where R_0 is the series resistance of the equivalent electrical circuit model. Note that for running model-I we need to know an estimate of the resistance of the battery, i.e. R_0 , which can be obtained through BFG [18]. In the numerical results of this paper, however, we assume that R_0 is known.

Remark 1: Note that we can set a threshold SOC, s_{th} , for which we want to have the fastest TTC among all feasible strategies. The s_{th} threshold may be different from the desired final SOC after charging, i.e., s_{kf} . For example, if we want to find the best policy, which maintains the life cycle constraints and results in the lowest time to charge for reaching an SOC of 50%, then we set $s_{th} = 0.50$.

Remark 2: Note that any capacity model can be used in place of the CVD model, as long as the model incorporates the control variables (the maximum terminal voltage and the maximum charge current).

The optimal strategy is then

$$(k^*, l^*) = \arg \left(\min_{k,l} (\text{UCL}_{kl}) \right) \quad (73)$$

The optimal control setting is

$$\underline{u}^* = \begin{bmatrix} v_{\max}^* & i_{\max}^* \end{bmatrix}^T = \begin{bmatrix} v_{k^*} & i_{l^*} \end{bmatrix}^T \quad (74)$$

Figure 8 shows the detailed block diagram of the level-I optimization problem. The setpoints of the diagram are LPNC and NCL, and s_{th} , the inputs to the optimization algorithm are PC, C_{PC} , and R_0 . These inputs may be estimated by BFG. In the numerical results of this paper, we assume these inputs as known. The outputs of the diagram are $i_{\max}^*$ and $v_{\max}^*$, which are used in the level-II algorithms.

Fig. 8: Detailed block diagram of level-I optimization

VI. NUMERICAL ILLUSTRATION OF OPTIMAL CHARGING PARAMETER SELECTION

In this section, we illustrate the proposed optimal charging parameter selection method by numerical examples. Resolution of candidate values for control variables $v_{\max}$ and $i_{\max}$ are selected as 0.01 and 0.1, respectively. Note that these control variables are in normalized form with the nominal value of 4.35 V and 2.6 A for voltage and current, respectively. In other words, the resolution of control variables are 0.0435 volts and 0.26 amperes. Six different scenarios, each specified by four parameters (PC , C_{PC} , R_0 , and s_{th}), are illustrated. In all scenarios, we assume $LPNC = 0.80$ and $NCL = 500$. For a fair comparison of the effect of changing battery resistance R_0 and threshold SOC s_{th} , we have used the same PC (that is $PC = 50$) for all scenarios. Table VII shows the parameters and the optimal policy for each scenario. Due to space limitation, only for “Scenario 6” the table of expected useful cycle life (EUCL) is shown (Table VIII), in which the EUCL of SFCS are shown in boldface, and the optimal control settings and the corresponding EUCL values are underlined. Note that the values for i_l and v_k are in the form “normalized (actual)” and the actual values for currents and voltages are in amperes and volts, respectively.

TABLE VII: Scenarios: parameters and optimal policy

Scenario	Parameters				Optimal Policy (normalized)		Optimal Policy (actual)	
	PC	C_{PC}	$R_0(\text{m}\Omega)$	s_{th}	$v_{\max}^*$	$i_{\max}^*$	$v_{\max}^*(\text{V})$	$i_{\max}^*(\text{A})$
1	50	0.96	150	0.50	0.97	1.3	4.22	3.38
2	50	0.99	150	0.50	1.00	1.3	4.35	3.38
3	50	0.96	150	0.80	0.98	1.1	4.26	2.86
4	50	0.96	200	0.50	0.98	1.1	4.26	2.86
5	50	0.96	250	0.50	0.99	0.9	4.31	2.34
6	50	0.96	300	0.50	1.00	0.8	4.35	2.08

TABLE VIII: EUCL for Scenario 6

i_l v_k	0.7 (1.82)	<u>0.8 (2.08)</u>	0.9 (2.34)	1.0 (2.6)	1.1 (2.86)	1.2 (3.12)	1.3 (3.38)
0.97 (4.22)	590	568	550	536	523	512	502
0.98 (4.26)	567	547	531	517	505	494	485
0.99 (4.31)	546	527	512	498	488	478	469
<u>1.00 (4.35)</u>	526	<u>508</u>	495	482	472	462	454
1.01 (4.39)	507	491	477	466	456	448	440
1.02 (4.44)	489	474	461	450	442	433	426

The following insights can be drawn from Tables VII and VIII:

- 1) For scenarios with mild usage of battery and low values of s_{th} (as in “Scenario 1” and “Scenario 2”), the optimal strategy is the one among the feasible candidates (members of SFCS) that has the highest current.
- 2) If the battery is treated mildly, the optimal strategy yields higher values of v_{max}^* and i_{max}^* . For example, the battery in “Scenario 2” is treated more mildly than the battery in “Scenario 1”, and consequently, the v_{max}^* (1.00 normalized or 4.35 volts) in “Scenario 2” is higher than that (0.97 normalized or 4.22 volts) of “Scenario 1”.
- 3) For scenarios with higher values of s_{th} (as in “Scenario 3”), the optimal strategy yields lower values of i_{max}^* and higher values of v_{max}^* .
- 4) The increase in battery resistance (compare Scenarios 4, 5, and 6) results in lower values of i_{max}^* and higher values of v_{max}^* .

VII. CONCLUSION

In this paper, we presented a linear quadratic optimization approach and its solution to optimally charging a Li-ion battery in a general form. The optimal profile was derived based on a cost function, which is a weighted sum of time-to-charge (TTC), energy loss (EL), sum of the squares of the differences of the state-of-charge (SOC) from the final desired SOC, and temperature rise index (TRI). The presented solution strategy is generic and it is applicable to any equivalent electrical circuit model of a battery. The effect of changing the weights in the cost function was discussed.

As a precursor to developing a battery life management policy, we presented two models for estimating the normalized battery capacity, namely, the LAR- $\alpha\beta\gamma$ and the CVD. The former is a function of the number of cycles and the latter is a function of the number of cycles and two charge control parameters, viz., maximum terminal voltage of the battery ($v_{\max}$) and maximum charge current ($i_{\max}$). The accuracies of these models were explored by experimental data gathered from aging experiments performed on Samsung GS4 battery, and their dominance over the bi-exponential capacity model was demonstrated using the experimental data.

Next, we presented a fast charging strategy, coupled with a battery life management policy, by developing an optimal charging parameter selection. The proposed approach finds the best $v_{\max}$ and $i_{\max}$ to achieve a desired cycle life (for example, 500 cycles), while maintaining the normalized capacity above a desired threshold (for example, 80%) and attaining the fastest possible time-to-charge. The method was illustrated via numerical results. The optimal setpoint depends on the desired state of charge (denoted by s_{th}) for which we aim to attain the fastest charging time. It also depends on the resistance of the battery, i.e., R_0 . Simulation results show that low fast-charging threshold on state of charge (s_{th}) and low battery resistance result in an optimal setpoint with the highest $i_{\max}$ among the set of feasible control settings, while increasing s_{th} and the battery resistance result in moving the optimal setpoint to lower values of $i_{\max}$ and higher values of $v_{\max}$. We suggest robust optimal charging strategies that are insensitive to model order and model parameter variations as an avenue for future research.

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APPENDIX

In this appendix, we derive the algorithm stated in section II. We assumed that at time k_1 , the SOC reaches the value v_c . Therefore, we can relax the constraint $s[k_1] = s_1$ with Lagrangian multiplier ν . Also, the dynamics of the system are relaxed by co-state vector $\underline{\lambda}$. Hence, the LQ problem is to minimize the following Lagrangian:

$$L[k] = J_{\text{tEsT}} + \nu(s[k_1] - s_1) + \sum_{k=0}^{k_1-1} (\underline{\lambda}^T[k+1] (\Phi \underline{z}[k] + \Gamma i[k] - \underline{z}[k+1])) \quad (75)$$

By inserting (33) into (75), and using Hamiltonian function $H[k]$ and noting that $\begin{bmatrix} \nu & 0 \end{bmatrix} \underline{z}[k_1] = \nu(s[k_1] - s_{k_f})$, we can write $L[k]$ as in (76)

$$L[k] = \nu(s_{k_f} - s_1) + \left(\begin{bmatrix} \nu & 0 \end{bmatrix} - \underline{\lambda}^T[k_1] \right) \underline{z}[k_1] + H[0] + \sum_{k=1}^{k_1-1} (H[k] - \underline{\lambda}^T[k] \underline{z}[k]) \quad (76)$$

Applying the necessary conditions for optimality results in:

$$i[k] = -\frac{\Gamma^T}{2\Delta R_{0\text{eq}}[k]} \underline{\lambda}[k+1] \quad (77)$$

$$\underline{\lambda}[k] = \frac{\partial H[k]}{\partial \underline{z}[k]} = 2Q[k] \underline{z}[k] + \Phi^T \underline{\lambda}[k+1] \quad (78)$$

From (24), (77) and (78), we can write the dynamics of $\underline{z}[k]$ and $\underline{\lambda}[k]$ in the following forward/backward form:

$$\begin{bmatrix} \underline{z}[k+1] \\ \underline{\lambda}[k] \end{bmatrix} = \begin{bmatrix} \Phi & -\Psi[k] \\ 2Q[k] & \Phi^T \end{bmatrix} \begin{bmatrix} \underline{z}[k] \\ \underline{\lambda}[k+1] \end{bmatrix} \quad (79)$$

where

$$\Psi[k] = \frac{\Gamma \Gamma^T}{2\Delta R_{0\text{eq}}[k]} \quad (80)$$

In order to obtain the optimal current profile, we first consider the following evolutions:

$$\underline{\lambda}[k] = P[k] \underline{z}[k] + \underline{g}[k] \nu \quad (81)$$

$$s_1 = \underline{u}^T[k] \underline{z}[k] + \omega[k] \nu + d[k] \quad (82)$$

In order that the above equations hold at time k_1 , we need to have: $P[k_1] = \mathbf{0}_{2 \times 2}$, $\underline{g}[k_1] = \underline{u}[k_1] = \begin{bmatrix} 1 & 0 \end{bmatrix}^T$, $\omega[k_1] = 0$, and $d[k_1] = s_{k_f}$. Using (79) and (81), we can write:

$$\underline{\lambda}[k] = 2Q[k] \underline{z}[k] + \Phi^T (P[k+1] \underline{z}[k+1] + \underline{g}[k+1] \nu) \quad (83)$$

Using (79) and (81), we can derive $\underline{z}[k+1]$ in terms of $\underline{z}[k]$ and ν , which once inserted into (83) yields $\underline{\lambda}[k]$ as a function of $\underline{z}[k]$ and ν . Comparison of the so-obtained function for $\underline{\lambda}[k]$ to (81) and using Woodbury matrix identity [46] results in the following relations for the evolution of $P[k]$ and $\underline{g}[k]$:

$$P[k] = 2Q[k] + \Phi^T P[k+1] (I_3 + \Psi[k]P[k+1])^{-1} \Phi \quad (84)$$

$$\underline{g}[k] = \Phi^T (I_3 + P[k+1]\Psi[k])^{-1} \underline{g}[k+1] \quad (85)$$

Next, from (82) for times k and $k+1$, we can write two equivalent relations. Inserting in $\underline{z}[k+1]$, the function we obtained earlier in terms of $\underline{z}[k]$ and ν , and then equating the corresponding coefficients on both sides of the equations, we derive:

$$\underline{u}[k] = \Phi^T (I_3 + P[k+1]\Psi[k])^{-1} \underline{u}[k+1] \quad (86)$$

$$\omega[k] = \omega[k+1] - \underline{g}^T[k+1] (I_3 + \Psi[k]P[k+1])^{-1} \Psi[k] \underline{g}[k+1] \quad (87)$$

$$\underline{u}[k] = \underline{g}[k] \quad (88)$$

$$d[k] = s_{k_f} \quad \forall k \quad (89)$$

From (82), (88), and (89) we can write

$$\nu = \frac{s_1 - s_{k_f} - \underline{g}^T[k] \underline{z}[k]}{\omega[k]} \quad (90)$$

Knowing k_1 , we can solve the dynamical equations for $P[k]$, $\underline{g}[k]$, and $\omega[k]$ in a backward fashion to calculate $\underline{g}[0]$ and $\omega[0]$. Inserting these values into (90) yields:

$$\nu = \frac{s_1 - s_{k_f} - \underline{g}^T[0] \underline{z}[0]}{\omega[0]} \quad (91)$$

Finally, we can write the optimal current as follows:

$$i[k] = -\frac{\Gamma^T}{2\Delta R_{0eq}[k]} \left(P[k+1] \left((I_3 + \Psi[k]P[k+1])^{-1} (\Phi \underline{z}[k] - \Psi[k] \underline{g}[k+1] \nu) \right) + \underline{g}[k+1] \nu \right) \quad (92)$$

Note that this is a state feedback (closed-loop) policy.

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