PLASTICITY MODELING OF CONCRETE CONFINED WITH NITINB SHAPE MEMORY ALLOY SPIRALS

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ABSTRACT

Concrete confinement is proved to be an effective method to improve concrete strength and ductility. In recent years, a novel confinement technique using NiTiNb shape memory alloy (SMA) spirals was proposed and proved to be promising for seismic retrofit of concrete structures. Despite the experimental work that has been done in this area, there is few computational tools available that are capable of describing the three-dimensional constitutive behavior of NiTiNb-SMA confined concrete. This paper proposes a plasticity-based NiTiNb-SMA confined concrete constitutive model, which is capable of predicting and simulating three-dimensional stress-strain behavior of NiTiNb-SMA confined concrete under both monotonic and cyclic loading. New hardening/softening function, dilation rate function and damage parameter are derived and validated based on experimental results. The proposed model is able to closely simulate the axial and lateral stress-strain behaviors of NiTiNb-SMA confined concrete when compared to experimental results.

Key Words: Shape Memory Alloys, Confinement, Concrete, Cyclic, Plasticity, Modeling

31 List of Notations

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A_f	Austenite finish temperature
A_s	Austenite start temperature
A_{SMA}	SMA wire' cross section area
a	Coefficient in the proposed NiTiNb-SMA confined concrete damage parameter model
b	Coefficient in the proposed NiTiNb-SMA confined concrete damage parameter model
c	Coefficient in secant dilation ratio model of NiTiNb-SMA confined concrete
D	Concrete cross section diameter
d	Coefficient in secant dilation ratio model of NiTiNb-SMA confined concrete
d_{SMA}	Diameter of SMA wire
E_0	Elastic modulus
E_r	Reloading modulus
E_0^k	Initial slope of NiTiNb-SMA confined concrete hardening/softening function
E_p^k	Secant slope of NiTiNb-SMA confined concrete hardening/softening function
E_0^α	Initial slope of NiTiNb-SMA confined concrete dilation rate function
E_p^α	Secant slope of NiTiNb-SMA confined concrete dilation rate function
f_{co}'	Unconfined concrete strength
f_l	Lateral confining pressure
$f_{l,active}$	Active confining pressure
$f_{l,total}$	Total lateral confining pressure (active + passive)
$f_{l,total}'$	Total lateral confining pressure (active + passive) when SMA confined concrete reaches its peak stress
f_{co}'	Unconfined concrete strength
f_{cc}'	Peak stress of confined concrete
f_{SMA}	Average stress developed along SMA wires
f_r	SMA recovery stress
G	Potential function describing the flow rule
I_1	First stress invariant

ΔI_1^p	Incremental volumetric plastic strain
J_2	Second deviatoric stress invariant
ΔJ_2^p	Incremental second deviatoric plastic strain
k	Hardening/softening function in Drucker-Prager yield criterion
k_0	Initial value in NiTiNb-SMA confined concrete hardening/softening function
k_e	Confinement effectiveness factor
k_{peak}	Peak value in NiTiNb-SMA confined concrete hardening/softening function
k_{res}	Residual plateau in NiTiNb-SMA confined concrete hardening/softening function
M_f	Martensite finish temperature
M_s	Martensite start temperature
m	Parameter in the proposed NiTiNb-SMA confined concrete hardening/softening function, which controls the ascending slope
n	Parameter in the proposed NiTiNb-SMA confined concrete hardening/softening function, which controls the descending slope
p	Parameter in the proposed NiTiNb-SMA confined concrete dilation rate function, which controls the ascending slope
q	Parameter in the proposed NiTiNb-SMA confined concrete dilation rate function, which controls the descending slope
s	Pitch spacing of SMA spiral
s'	Clear pitch spacing between SMA spirals
U	Normalized dilation rate function for NiTiNb-SMA confined concrete
V	Normalized equivalent plastic strain in the proposed dilation rate function for NiTiNb-SMA confined concrete
x	Normalized axial strain in the proposed NiTiNb-SMA confined concrete secant dilation ratio function
X	Normalized equivalent plastic strain in the proposed hardening/softening function for NiTiNb-SMA confined concrete
$x_{\mu_{max}}$	Normalized axial strain corresponding to maximum secant dilation ratio
Y	Proposed normalized hardening/softening function for NiTiNb-SMA confined concrete
α	Dilation rate
α_0	Initial value in NiTiNb-SMA confined concrete dilation rate function
α_{peak}	Peak value in NiTiNb-SMA confined concrete dilation rate function

α_{res}	Residual plateau in NiTiNb-SMA confined concrete dilation rate function
$\Delta \hat{\varepsilon}^p$	Incremental equivalent plastic strain
$\Delta \varepsilon_c^p$	Incremental axial plastic strain
$\Delta \varepsilon_l^p$	Incremental lateral plastic strain
ε_c	Axial strain
ε_c^p	Axial plastic strain
ε_{co}	Concrete axial strain corresponding to unconfined concrete peak stress
ε_{SMA}	Strain along NiTiNb-SMA spiral
ε_l	Lateral strain
ε_l^p	Lateral plastic strain
$\hat{\varepsilon}^p$	Equivalent plastic strain
$\hat{\varepsilon}_{\alpha, peak}^p$	Equivalent plastic strain at the peak of dilation rate function
$\hat{\varepsilon}_{k, peak}^p$	Equivalent plastic strain at the peak of hardening/softening function
θ	Frictional angle in Drucker-Prager yield criterion
$\Delta \lambda$	Magnitude of plastic deformation
μ	Secant dilation ratio
μ_0	Initial secant dilation ratio
$\mu_{asymptotic}$	Asymptotic value of secant dilation ratio as axial strain tends to infinity
μ_{max}	Maximum secant dilation ratio
ξ	Damage parameter
ρ_{cc}	Longitudinal reinforcement ratio
σ_c	Axial stress
σ_i	Principal stress in direction i
σ_l	Lateral stress
ν	Poisson's ratio
ϕ	Internal friction angle
ψ	Potential function parameter based on the deformation characteristics
ω	Coefficient representing confinement effectiveness

1 Introduction

Lateral concrete confinement is commonly used to delay the failure of concrete and improve its ductility; a feature that is critically important for structures subjected to extreme loads such as earthquakes. Passive confinement has been widely used for both new and existing structures to improve its strength and ductility, such as internal transverse steel reinforcement and external steel jackets and fiber-reinforced polymer (FRP) jackets (Richart *et al.* 1928; Mander *et al.* 1988; Priestley *et al.* 1994; Mirmiran and Shahawy 1997; Jiang and Teng 2007). Passive confining pressure develops as a result of concrete dilation during loading. However, previous research demonstrates that active confinement is much more superior to passive confinement (Cheong and Perry 1991; Krstulovic-Opara and Thiedeman 2000). This is owing to the active confining pressure applied on the concrete before loading and has been proved to be more effective in delaying the dilation of concrete structures. Recently, a new approach for applying active confinement to concrete using spirals made of a class of smart materials known as shape memory alloys (SMAs) has been proposed and studied (Andrawes and Shin 2008; Shin and Andrawes 2010 & 2011; Destrebecq *et al.* 2010; Park *et al.* 2011; Choi *et al.* 2011; Dommer and Andrawes 2012; Chen *et al.* 2014a & 2014b; Chen and Andrawes 2014a&2014b; Tran *et al.* 2015; Chen and Andrawes 2017). The technique is based on applying the lateral confining pressure through the thermal-prestressing of prestrained SMA spiral. The lateral prestressing pressure will delay the dilation of concrete under axial loading; hence the increase in concrete ductility. Despite the experimental work done on the new SMA confinement technique over the last few years, there is very few work done on the analytical modeling of the constitutive behavior of concrete confined with SMA.

Most existing concrete models can be mainly divided into two types. The first type is based on using empirical equations to fit stress-strain relationships obtained from experimental testing, while the second is based on the theory of plasticity to describe the

three-dimensional (3-D) constitutive relationship of concrete. Various empirical equations have been developed and widely applied due to their simplicity (e.g. Richart *et al.* 1928; Popovics 1973; Scott *et al.* 1982; Mander *et al.* 1988; Tsai 1988; Collins and Porasz 1989). Most of these empirical equations assumed constant confining pressure under axial compression as is the case in active confinement. This assumption is valid for cases using steel hoops or spirals as transverse reinforcement, since once the steel yields, the strength of steel can be considered constant. However, this assumption is not valid for cases where the confining pressure changes continuously with loading (i.e. passive confinement) such as in the case of fiber reinforced polymer (FRP) confined concrete (Mirmiran *et al.* 2000; Oh 2002; Yu *et al.* 2012) and SMA spirals confined concrete (Chen and Andrawes 2017).

It is important to note that the behavior of SMA confined concrete is quite different from the behaviors of both actively or passively confined concrete since SMA confined concrete is essentially a combination of active confinement applied through heating of the SMA spiral prior to loading, and passive confinement that develops and keeps increasing as concrete dilates during loading. Furthermore, the induced passive confinement is governed by the type of SMA used and its stress-strain behavior in the prestressed state (Chen and Andrawes 2017). Most of the SMA concrete confinement studies to date utilized spirals made of the ternary NiTiNb alloy in its martensite phase (Dommer and Andrawes 2012). This is primarily attributed to its wide thermal hysteresis, which enables the NiTiNb alloy to maintain its heat-triggered recovery stress (prestress) within wide range of ambient temperature. Due to the unique stress-strain behavior of NiTiNb alloy, a combination of active and passive confinement, none of the existing concrete models can represent the behavior of NiTiNb confined concrete. With the aim of modeling the unique 3-D behavior of NiTiNb-SMA confined concrete, this paper presents a new plasticity-based model, including yield criterion, hardening/softening function, dilation rate function and damage parameter.

The model is derived within the Drucker-Prager plasticity framework (Chen 1982) using the experimental results by Chen and Andrawes (2017).

2 Behavior of NiTiNb-SMA Confined Concrete

Chen and Andrawes (2017) conducted a series of cyclic compression tests on concrete cylinders confined by NiTiNb-SMA spirals with active confining pressures ranging from 0.91 (0.132 ksi) to 3.92 MPa (0.569 ksi) and with unconfined concrete strength ranging from 30.6 MPa (4.4 ksi) to 55.3 MPa (8.0 ksi). Ni-39Ti-9Nb (wt pct) wires used in this study were prestrained to about 6% strain by the manufacturer Memry Corporation (Bethel, Connecticut). In this experiment, the prestrained NiTiNb-SMA wires were first wrapped around concrete cylinders with a diameter of 152.4 mm (6 in) and height of 305 mm (12 in) as spirals. Then, the shape memory effect of the SMA spirals was triggered through torch heating. As the SMA wires were heated above austenite finish temperature A_f of NiTiNb-SMA wires, the shape recovery was activated. Attributed to the restraint from concrete cylinders, the NiTiNb-SMA spirals were not able to recover their original shape and therefore, recovery stress developed along the SMA wires as prestressing force (active confinement) applying on concrete structures prior to loading. The transformation temperatures of the NiTiNb wires used in this study were tested by Dommer and Andrawes (2012) through differential scanning calorimetry (DSC) testing, showing that the martensite start M_s and finish M_f temperatures were both below $-105^{\circ}C$ and the austenite start A_s and finish A_f temperature were $68^{\circ}C$ and $76^{\circ}C$, respectively. Therefore, NiTiNb-SMA wires can be stored, transported and installed under ambient temperature without losing their prestrain and able to maintain their recovery stress within ambient temperature after shape memory effect is activated. Chen and Andrawes (2017) found in their experiment that (1) the

improvement of strength and ductility increases as the NiTiNb-SMA confinement increases;
(2) residual stress of NiTiNb-SMA confined concrete after reaching the peak stress is
independent of concrete strength and relies only on active confinement provided by SMA; (3)
NiTiNb-SMA confined concrete stiffness reduces logarithmically as axial strain increases.

This paper aims at proposing a plasticity model that is able to simulate the stress-strain behavior of NiTiNb-SMA analytically through capturing the varying lateral confining pressures of NiTiNb-SMA confined concrete, which is essentially a combination of active (applied prior to loading) and passive confinement (develops during loading). Before deriving the plasticity model, it is critical to understand the confining pressure development of NiTiNb-SMA confined concrete and its relation to the axial strain of concrete. Therefore, a lateral-axial strain model was proposed based on the test results by Chen and Andrawes (2017). This lateral-axial strain model is one of the essential components for the plasticity model because: (1) it helps to determine the actual lateral confining pressure corresponding to every loading step (axial strain); (2) it determines the dilation feature of NiTiNb-SMA confined concrete.

2.1 Computation of Lateral Confinement

After prestressing the SMA spiral through the thermal activation of shape memory effect, as loading is applied, additional passive confining pressure is induced as concrete dilates. Hence, the total confining pressure sustained by concrete is a combination of active and passive confining pressure, which depends on the recovery stress of SMA upon heating, dilation of concrete, diameter of SMA wires, size of concrete section, and the pitch spacing of the SMA spirals. The total lateral confining pressure applied on circular concrete section can be computed using the following equation proposed by Mander *et al.* (1988):

$$f_{l,total} = \frac{2f_{SMA}A_{SMA}k_e}{sD} \quad (1)$$

where, f_{SMA} is the average stress developed along SMA wires; A_{SMA} is SMA wire' cross section area; D is concrete cross section diameter; s is pitch spacing of SMA spiral; k_e is the confinement effectiveness factor, which is computed as $k_e = (1 - s'/2D)/(1 - \rho_{cc})$, where $s' = s - d_{SMA}$ is the clear pitch spacing between SMA spirals; and ρ_{cc} is the longitudinal reinforcement ratio. f_{SMA} can be calculated using Eq. (2), which was derived using the NiTiNb-SMA tensile coupon test carried out by Chen and Andrawes (2017). In the tensile coupon test, two ends of the NiTiNb-SMA wire were first gripped by the hydraulic load frame, and then the NiTiNb-SMA wire was heated using electrical current to above 200 °C in order to fully activate the shape memory effect. Since the SMA wire was fixed at the two ends, recovery stress f_r developed along the wire. Then, the SMA specimen was tested under uniaxial tension with a strain rate of 0.005 mm/sec. The experiment shows that recovery stress f_r and the proportional limit of the test specimen were 607 MPa (88 ksi) and 744 MPa (108 ksi), respectively. The proportional strain was found to be 0.003 mm/mm. Therefore, the Young's modulus of the NiTiNb-SMA can be calculated using Hooke's Law as 45660 MPa (6622 ksi). The hardening modulus was approximated as 2980 MPa (432 ksi) (see Figure 1). Note that in Eq. (2), ϵ_{SMA} is the strain along NiTiNb-SMA spiral. Fig. 1 shows good agreement between the proposed and the experimental stress-strain behaviors of prestressed NiTiNb-SMA.

$$f_{SMA} = \begin{cases} 45660\epsilon_{SMA} + f_r & (\epsilon_{SMA} \leq 0.003) \\ 2980(\epsilon_{SMA} - 0.003) + 137 + f_r & (\epsilon_{SMA} > 0.003) \end{cases} \text{ (MPa)} \quad (2)$$

Fig. 1. Comparison of prestressed NiTiNb-SMA stress-strain model with test results.

2.2 Lateral and Axial Strain Relation

It is critical to understand the concrete lateral and axial strain relation in order to understand the NiTiNb-SMA confined concrete behavior as well as to develop a plasticity model that can represent their relation. None of the existing concrete axial-lateral strain models (e.g. Elwi and Murray 1979; Mirmiran and Shahawy 1997; Harries and Kharel 2002; Jiang and Teng 2007; etc.) is capable of describing the NiTiNb-SMA confined concrete lateral and axial strain relation, since: (1) these models were not developed to describe the active confinement and passive confinement combined effect, and (2) most of these models were developed for concrete with confinement ratio (ratio between lateral confining pressure f_l and unconfined concrete strength f'_{co}) much higher than that of NiTiNb-SMA confined concrete. In this paper, secant dilation ratio, $\mu = \varepsilon_l / \varepsilon_c$, in which ε_l is the lateral strain and ε_c is the axial strain of concrete, is used to describe the relation between axial and lateral strain. As proven by the experimental results discussed by Chen and Andrawes (2017), important features of NiTiNb-SMA confined concrete lateral-axial strain behavior include (1) the initial dilation ratio is equal to Poisson's ratio of unconfined concrete; (2) as concrete dilates, the secant dilation ratio reaches the peak dilation ratio μ_{max} , and then decreases and stabilizes at an asymptotic value $\mu_{asymptotic}$. Eq. (3) was suggested by Mirmiran and Shahawy (1997) to

describe secant dilation ratio of confined concrete. Since Eq. (3) is able to capture the previously discussed NiTiNb-SMA confined concrete dilation features, it is adopted in this study to describe the secant dilation ratio of NiTiNb-SMA confined concrete.

$$\mu(x) = \frac{\mu_0 + \mu_0 cx + \mu_{asymptotic} dx^2}{1 + cx + dx^2} \quad (3)$$

In Eq. (3), $x = \varepsilon_c / \varepsilon_{co}$ is the normalized axial strain; ε_c is the axial strain; ε_{co} is the unconfined concrete axial strain corresponding to its peak stress; μ_0 is the initial secant dilation ratio, which is equal to the Poisson's ratio; μ_{max} and $x_{\mu_{max}}$ are the maximum secant dilation ratio and its corresponding normalized axial strain; $\mu_{asymptotic}$ is the asymptotic value of secant dilation ratio as axial strain tends to infinity; c and d are coefficients, which can be obtained from the relation that dilation ratio reaches its peak μ_{max} at a normalized axial strain of $x_{\mu_{max}}$, and can be computed as $c = -2 / x_{\mu_{max}}$, and $d = c^2 (\mu_{max} - \mu_0) / 4 (\mu_{max} - \mu_{asymptotic})$. Using experimental data by Chen and Andrawes (2017), secant dilation ratio for each tested NiTiNb-SMA confined concrete specimen was calculated and then regression analysis was conducted for each parameter ($x_{\mu_{max}}$, μ_{max} , and $\mu_{asymptotic}$) in the dilation ratio function [Eq. (3)]. Fig. 2 presents the regression analysis results of the asymptotic secant dilation ratio $\mu_{asymptotic}$ [Fig. 2(a)], peak secant dilation ratio μ_{max} [Fig. 2(a)] and its corresponding normalized axial strain $x_{\mu_{max}}$ [Fig. 2(b)] as functions of active confinement ratio $f_{l,active} / f_{co}'$, which is the ratio between active confining pressure $f_{l,active}$ and strength of the unconfined concrete f_{co}' . The mathematical functions resulting from this regression analysis are presented in Eqs. (3b) - (3d) with their corresponding R-square values. From Fig. 2, it is clear that all the undetermined parameters in Eq. (3) solely depend on the

active confinement ratio; hence the secant dilation ratio can be determined by knowing the active confinement ratio and the axial strain of concrete. Fig. 3 compares the experimental results of secant dilation ratio (by Chen and Andrewes 2017) with the proposed model described in Eq. (3). The abscissa axis represents normalized axial strain ($x = \varepsilon_c / \varepsilon_{co}$), while the ordinate axis represents secant dilation ratio ($\mu = \varepsilon_l / \varepsilon_c$). The number shown in the legend for each specimen denotes its active confinement ratio ($f_{l,active} / f'_{co}$). Fig. 3 demonstrates that the proposed model is capable of simulating the lateral and axial strain relation of NiTiNb-SMA confined concrete with an acceptable accuracy for various active confinement ratios. Note that with the assumption that lateral strain of concrete is equal to the strain along SMA spiral, one can also calculate the axial strain-lateral stress relation, which is an important component to define the three-dimensional stress state of NiTiNb-SMA confined concrete.

$$x_{\mu_{\max}} = 31.935 \frac{f_{l,active}}{f'_{co}} + 2.852 \quad (R^2 = 0.994) \quad (3b)$$

$$\mu_{\max} = 0.0462 \left(\frac{f_{l,active}}{f'_{co}} \right)^{-1.121} \quad (R^2 = 0.926) \quad (3c)$$

$$\mu_{asymptotic} = 1.432 \exp \left(-14.78 \frac{f_{l,active}}{f'_{co}} \right) \quad (R^2 = 0.966) \quad (3d)$$

(a)

(b)

Fig 2. Effect of active confinement ratio on the asymptotic secant dilation ratio (a), peak secant dilation ratio (a) and its corresponding normalized axial strain (b).

Fig. 3. Comparison between the experimental (solid) and analytically predicted (dashed) secant dilation ratio.

3 Proposed Plasticity Model

Plasticity models can be described as constitutive models developed based on internal-variable theory of plasticity (Lubliner 2008), which are characterized by yield criterion, hardening/softening function and flow rule. Yield criterion uses stress invariants to describe yield surfaces. As material reaches the first yield surface, plastic deformation starts to accumulate. After first yield, subsequent yield surfaces are utilized to describe hardening/softening of material. The direction and magnitude of plastic deformation are determined by flow rule. These components are based on the multi-axial stress-strain states and for isotropic materials they are described by the principal stresses or the stress invariants to represent the coordinate system independence. Some previous researches have been done on using plasticity model to simulate the three-dimensional behavior of confined concrete (Lubliner *et al.* 1989; Oh 2002; Malvar *et al.* 1994; Yu *et al.* 2010). In this paper, the proposed plasticity model was derived within the framework of Drucker-Prager plasticity model and in order to consider both the plasticity and stiffness degradation effect on concrete

nonlinearity, a damage parameter was introduced. Note that in this study, unless specified, compression is negative and tension is positive.

3.1 Yield Criterion

The Drucker-Prager yield function presented below was utilized in this study:

$$\sqrt{J_2} + \theta I_1 - k = 0 \quad (4)$$

where J_2 and I_1 are the second deviatoric stress invariant and the first stress invariant, respectively; k is the hardening/softening function; θ is the frictional angle and its calculation method is described in Chen (1982), Oh (2002) and Yu *et al.* (2010). This frictional angle is a material property that can be derived from experimental results. For example, frictional angle was computed by Richart *et al.* (1928) as 0.2934 for actively confined concrete and by Teng *et al.* (2007) as 0.2634 for fiber reinforced polymer (FRP) confined concrete. Based on the relationship between Mohr-Coulomb yield surface and Drucker-Prager yield surface, the frictional angle θ in the Drucker-Prager model can be expressed as a function of internal friction angle ϕ in the Mohr-Coulomb model as shown below:

$$\theta = \frac{2 \sin \phi}{\sqrt{3}(3 + \sin \phi)} \quad (5)$$

where $\sin \phi = \frac{\omega - 1}{\omega + 1}$; and $\omega = \frac{f'_{cc} / f'_{co} - 1}{f'_{l,total} / f'_{co}}$, which represents the confinement effectiveness;

f'_{cc} is the peak stress of SMA confined concrete; $f'_{l,total}$ denotes the total lateral confining pressure (active + passive) when SMA confined concrete reaches its peak stress. Fig. 4 presents the total confinement ratio $f'_{l,total} / f'_{co}$ at peak versus normalized peak stress f'_{cc} / f'_{co} at the peak of the stress-strain curve, in which dots represent experimental data and the solid line represents the model derived using regression analysis. Based on regression

analysis, with an ω value equal to 5.63, one can find that θ is equal to 0.3503 for NiTiNb-SMA confined concrete.

Fig. 4. Total confinement ratio $f'_{l,total} / f'_{co}$ at peak versus normalized peak stress f'_{cc} / f'_{co} .

3.2 Hardening/Softening Function

Hardening/softening function k represents the first and subsequent yield surfaces and it is related to the plastic work. It has been proven by several researchers that hardening/softening function of confined concrete is dependent on concrete strength, lateral confinement and plastic deformation (Oh 2002; Yu *et al.* 2010; Chen and Andrawes 2014c). According to the Drucker-Prager model utilized in this paper, a hardening/softening function can be calculated using equation $k = \sqrt{J_2} + \theta I_1$ and the triaxial stress status from the experiment for each test specimen by Chen and Andrawes (2017). Fig. 5(a) displays a typical shape of hardening/softening function calculated from the experimental results. It increases linearly starting from an initial value k_0 , and after reaching a peak value k_{peak} , it decreases to a plateau k_{res} .

Fig. 5. Comparison of experimental and modeling (a) hardening/softening function; (b) dilation rate function.

Tsai (1988) recommended a generalized form of equation [Eq. (6a)] to describe the stress-strain relation of concrete, which can control both the initial slope and the descending branch of the stress-strain curve. Since the features of hardening/softening function are similar to the axial stress-strain relation of concrete, Eq. (6a) was adopted in the proposed model to describe the hardening/softening function at the pre-plateau range, with slight modification to account for the initial value k_0 in the hardening/softening function. After reaching the residual value k_{res} , the hardening/softening value is assumed to be constant.

$$Y = \frac{mX}{1 + \left(m - \frac{n}{n-1}\right)X + \frac{X^n}{n-1}} \quad (6a)$$

In Eq. (6a), $Y = k / k_{peak}$; $X = \hat{\epsilon}^p / \hat{\epsilon}_{k,peak}^p$; $\hat{\epsilon}^p$ is the equivalent plastic strain, which represents the 3-D plasticity accumulation in concrete, and the incremental equivalent plastic strain is defined as $\Delta \hat{\epsilon}^p = \sqrt{\Delta \epsilon_c^p + \Delta \epsilon_l^p + \Delta \epsilon_t^p}$, in which $\Delta \epsilon_c^p$ is the incremental axial plastic strain and $\Delta \epsilon_l^p$ is the incremental lateral plastic strain; k_{peak} is the maximum value in the hardening/softening function; $\hat{\epsilon}_{k,peak}^p$ is the equivalent plastic strain at the peak of hardening/softening function; parameter $m = \partial Y / \partial X = E_0^k / E_p^k$, controls the ascending

slope; E_0^k is the initial slope of the hardening/softening function, while $E_p^k = k_{peak} / \hat{\varepsilon}_{k,peak}^p$ is the secant slope of NiTiNb-SMA confined concrete hardening/softening function; n is a parameter that controls the descending slope of the hardening/softening function. Parameters E_0^k , k_0 , k_{peak} , $\hat{\varepsilon}_{k,peak}^p$, k_{res} , and n were first calculated from hardening/softening function of each specimens described above (see Fig. 5a), and then equations proposed for each parameter were derived using regression analysis [see Eqs. (6b) - (6g)]. It should be noted that although hardening/softening function k is written as a function of equivalent plastic strain $\hat{\varepsilon}^p$ only, the effect of concrete strength and SMA confinement on the hardening/softening function is represented in the parameters mentioned above. Fig. 5(a) compares the hardening/softening function experimental results published by Chen and Andrawes (2017) with the proposed model described in Eq. (6) to demonstrate that the proposed hardening/softening function is able to capture the experimental results with good agreement.

$$E_0^k = -270433 f_{l,active} / f_{co}' + 45972 \text{ (MPa)} \quad (R^2 = 0.834) \quad (6b)$$

$$k_0 = (-0.6958 f_{l,active} / f_{co}' + 0.1071) \cdot (-f_{co}') \text{ (MPa)} \quad (R^2 = 0.824) \quad (6c)$$

$$k_{peak} = 0.266(-f_{co}') \text{ (MPa)} - 1.421 \quad (R^2 = 0.876) \quad (6d)$$

$$\hat{\varepsilon}_{k,peak}^p = (7.461 f_{l,active} / f_{co}' + 1.036) \cdot (-\varepsilon_{co}) \quad (R^2 = 0.795) \quad (6e)$$

$$k_{res} = 0.725 f_{l,active} / f_{co}' + 1.077 \text{ (MPa)} \quad (R^2 = 0.795) \quad (6f)$$

$$n = 0.0144(-f_{co}') \text{ (MPa)} + 1.582 \quad (R^2 = 0.795) \quad (6g)$$

3.3 Flow Rule

Flow rule features the plastic deformation direction and magnitude. A non-associated flow rule was utilized in the proposed model, which was dependent on concrete strength,

plastic deformation and lateral confining pressure (Oh 2002; Yu *et al.* 2010; Chen and Andrawes 2014c). The incremental equivalent plastic strain can be described by Eq. (7a) and a Drucker-Prager type potential function G is adopted [Eq. (7b)].

$$\Delta \hat{\varepsilon}^p = \Delta \lambda \sqrt{\frac{\partial G}{\partial \sigma_i} \frac{\partial G}{\partial \sigma_i}} \quad (i=1,2,3) \quad (7a)$$

$$G = \sqrt{J_2} + \psi I_1 \quad (7b)$$

In Eq. (7), ψ is potential function parameter to be determined based on the deformation characteristics; $\Delta \lambda$ denotes the magnitude of plastic deformation in the direction of $\partial G / \partial \sigma_i$, in which σ_i is the principal stress of concrete in the direction i . To describe the potential flow for confined concrete, dilation rate α was used, which represents the relation between incremental volumetric strain and the incremental deviatoric strain. α equal to zero indicates that there is no volumetric change. α less than zero, indicates volumetric contraction occurs; and an α greater than zero represents volumetric dilation. Dilation rate α is defined by Eq. (8) (Oh 2002; Yu *et al.* 2010), was calculated for each specimen tested by Chen and Andrawes (2017) based on axial stress-strain relation from the test results, axial strain-lateral strain relation [Eq. (3)], and Prandtl-Reuss equations [Eq. (9)].

$$\alpha = \frac{\Delta I_1^p}{\Delta \sqrt{J_2^p}} = \sqrt{3} \frac{\Delta \varepsilon_c^p + 2\Delta \varepsilon_l^p}{|\Delta \varepsilon_c^p - \Delta \varepsilon_l^p|} = \sqrt{3} \frac{1 + 2\Delta \varepsilon_l^p / \Delta \varepsilon_c^p}{\Delta \varepsilon_l^p / \Delta \varepsilon_c^p - 1} = 6\psi \quad (8)$$

$$\varepsilon_c^p = \varepsilon_c - \frac{1}{E_0} (\sigma_c - 2\nu \sigma_l) \quad (9a)$$

$$\varepsilon_l^p = \varepsilon_l - \frac{1}{E_0} [(1-\nu) \sigma_l - \nu \sigma_c] \quad (9b)$$

In the above equations, ΔI_1^p is the incremental volumetric plastic strain; ΔJ_2^p is the incremental second deviatoric plastic strain; $\Delta \varepsilon_c^p$ is incremental axial plastic strain; $\Delta \varepsilon_l^p$ is

329 incremental lateral plastic strain; ε_c^p is axial plastic strain; ε_l^p is lateral plastic strain; E_c
 330 is total axial strain; E_l is total lateral strain; σ_c is axial stress; σ_l is lateral stress; E_0 is
 331 elastic modulus and ν is Poisson's ratio. Fig. 5(b) shows a typical shape of the dilation rate
 332 function α calculated from the experimental results published by Chen and Andrawes
 333 (2017). Similar to the hardening/softening function, the dilation rate function increases
 334 linearly starting from an initial value α_0 ; after reaching a peak value α_{peak} , it decreases to a
 335 plateau α_{res} . To represent this behavior, Eq. (10a) [similar to Eq. (6a)] was found to be
 336 suitable and was chosen to describe the dilation rate function at the pre-plateau stage. After
 337 reaching the residual value α_{res} , the dilation rate value was assumed to be constant.

$$338 \quad U = \frac{pV}{1 + \left(p - \frac{q}{q-1} \right) V + \frac{V^q}{q-1}} \quad (10a)$$

339 In Eq. (10a), $U = (\alpha - \alpha_0) / (\alpha_{peak} - \alpha_0)$; $V = \hat{\varepsilon}^p / \hat{\varepsilon}_{\alpha, peak}^p$; $\hat{\varepsilon}_{\alpha, peak}^p$ is the equivalent plastic
 340 strain at the peak of dilation rate function; parameter $p = \partial U / \partial V = E_0^\alpha / E_p^\alpha$, controls the
 341 ascending slope of the dilation rate function; E_0^α is the initial slope of the dilation rate
 342 function, while $E_p^\alpha = (\alpha_{peak} - \alpha_0) / \hat{\varepsilon}_{\alpha, peak}^p$ is the secant slope of NiTiNb-SMA confined
 343 concrete dilation rate function; q is a parameter that controls the descending slope of the
 344 dilation rate function. The initial dilation rate α_0 can be determined using Eq. (10a) by
 345 substituting Poisson's ratio (dilation ratio in the elastic range) $\nu = -\Delta \varepsilon_l^p / \Delta \varepsilon_c^p$. Assuming
 346 $\nu = 0.18$, one can obtain $\alpha_0 = -0.9394$. Parameters E_0^α , α_{peak} , $\hat{\varepsilon}_{\alpha, peak}^p$, α_{res} , and q were
 347 first calculated from the dilation rate of each specimen described above (see Fig. 5b) and then
 348 equations proposed for each parameter were derived using regression analysis [see Eq. (10b)]

- (10e)]. Fig. 5(b) compares the dilation rate function experimental results (Chen and Andrawes 2017) with the proposed model described in Eq. (10) to demonstrate that the proposed dilation rate function is able to capture the experimental results.

$$E_0^\alpha = \begin{cases} -11722 f_{l,active} / f_{co}' + 1417.3 \text{ (MPa)} & (f_{l,active} / f_{co}' \leq 0.1) \\ 245.1 \text{ (MPa)} & (f_{l,active} / f_{co}' > 0.1) \end{cases} \quad (R^2 = 0.770) \quad (10b)$$

$$\alpha_{peak} = \frac{0.1357}{(f_{l,active} / f_{co}' + 0.0261)} \quad (R^2 = 0.980) \quad (10c)$$

$$\hat{\varepsilon}_{\alpha,peak}^p = \frac{(-\varepsilon_{co})(3.708 f_{l,active} / f_{co}' + 0.0212)}{(f_{l,active} / f_{co}' - 0.0118)} \quad (R^2 = 0.948) \quad (10d)$$

$$\alpha_{res} = -16.468 f_{l,active} / f_{co}' + 1.176 \quad (R^2 = 0.993) \quad (10e)$$

$$q = \begin{cases} -111.6 f_{l,active} / f_{co}' + 8.464 & (f_{l,active} / f_{co}' < 0.04) \\ 4 & (f_{l,active} / f_{co}' \geq 0.04) \end{cases} \quad (R^2 = 0.961) \quad (10f)$$

3.4 Damage Parameter

As explained earlier, modeling both concrete damage and plasticity is essential to capture the nonlinear stress-strain behavior of NiTiNb-SMA confined concrete. With proper yield criterion, hardening/softening function and flow rule, the model can simulate the monotonic stress-strain behavior or the envelope of the cyclic stress-strain curve with good agreement. In order to capture the unloading and reloading stiffness of NiTiNb-SMA confined concrete, it is important to define a damage parameter, which is derived based on the stiffness degradation during cyclic loading. In the proposed model, based on experimental observations (Chen and Andrawes 2017), the following assumptions were made: (1) unloading and reloading stiffness is identical; (2) stress and strain are linearly related between unloading and reloading points. Damage parameter can be calculated as $\xi = 1 - E_r / E_0$, where

369 E_r is the reloading modulus of NiTiNb-SMA confined concrete at a certain axial strain and
 370 E_0 is the elastic modulus of the concrete. Chen and Andrawes (2017) demonstrated that the
 371 stiffness of NiTiNb-SMA confined concrete decreases logarithmically as axial strain
 372 increases and the degradation rate only depends on the active confinement level, regardless of
 373 concrete strength. Therefore, the logarithmic function presented in Eq. (11a) was utilized to
 374 model the damage parameter ξ and the two undetermined parameters can be obtained based
 375 on experimental results presented by Chen and Andrawes (2017). Substituting $\xi = 0$ into
 376 Eq. (11a), one can find the axial strain at the onset of stiffness degradation
 377 $\varepsilon_c = \exp[(1-b)/a]$. Parameters a and b were obtained based on curve fitting from each
 378 confinement level (see Fig. 6). Therefore, parameters a and b can be calculated using Eq.
 379 (11b) and Eq. (11c), respectively based on regression analysis. Fig. 6 compares experimental
 380 results of damage parameter ξ and the proposed model, and verifies that the proposed model
 381 can closely predict the stiffness degradation under cyclic loading. In Fig. 6, S2, S3, S4 and S5
 382 denote specimens with active confining pressures of 1.23 MPa (0.178 ksi), 1.92 MPa (0.278
 383 ksi), 2.85 MPa (0.413 ksi) and 3.92 MPa (0.569 ksi), respectively (see Chen and Andrawes
 384 2017).

$$385 \quad \xi = 1 - \frac{E_r}{E_0} = -a \ln(\varepsilon_c) + (1-b) \quad (11a)$$

$$386 \quad a = 0.0085 f_{l,active} (MPa) - 0.1717 \quad (R^2 = 1.0) \quad (11b)$$

$$387 \quad b = -a \ln(0.1764 f_{l,active} (MPa) + 0.0082) \quad (R^2 = 1.0) \quad (11c)$$

(a)

(b)

(c)

(d)

Fig. 6. Comparison of the damage parameter model prediction and the experimental results under various confinement pressures of: (a) 1.23 MPa (0.178 ksi); (b) 1.92 MPa (0.278 ksi); (c) 2.85 MPa (0.413 ksi); (d) 3.92 MPa (0.569 ksi).

3.5 Comparison of Proposed Model and Experimental Results

To evaluate the response of the proposed constitutive model and examine whether it is capable of simulating the SMA confined concrete experimental results, the numerical results were compared with the experimental results presented in Chen and Andrawes (2017). The proposed model was implemented using a modified Newton-Raphson algorithm and a numerical procedure described by Ortiz and Popov (1985) using the trapezoidal rule. Fig. 7 compares the experimental and numerical results produced using the proposed model for both the axial stress-axial strain and axial stress-lateral strain behaviors for all the NiTiNb-SMA confined specimens presented by Chen and Andrawes (2017). One can notice that the proposed plasticity model was able to predict the stress-strain relation of NiTiNb-SMA

confined concrete with acceptable accuracy. Table 1 shows the ratios between numerically
 predicted results and experimental results for peak axial stress, and its corresponding axial
 strain, ultimate stress and ultimate strain, as well as the axial stress at 2.5% axial strain
 (residual stress). The average prediction to experiment ratios of peak stress, ultimate stress
 and axial stress at 2.5% are 0.998, 1.028, and 0.985, respectively, with standard deviation not
 greater than 6.5%. The ultimate strain prediction also has very good agreement with the
 experimental results. The 12.6% average error of the ultimate strain came from two
 specimens, SMA-B1S5C and SMA-B3S3C, which have been identified as outliers by Chen
 and Andrawes (2017). The average error reduces to 0.7%, when excluding these two outliers.
 Although the average prediction to experiment ratio of peak strain is high, the error in the
 prediction is relatively minor when the absolute value is considered. Therefore, the proposed
 model has proven to be in good agreement with the experimental results and is deemed able
 to capture the NiTiNb-SMA confined stress-strain behavior for both axial and lateral
 direction under both cyclic and monotonic loading.

Fig. 7. Comparison of axial stress-axial strain and axial stress-lateral strain experimental and numerical results using the proposed model for specimens: (a) SMA-B1S2C; (b) SMA-B1S3C; (c) SMA-B1S5C; (d) SMA-B2S2C; (e) SMA-B2S3C; (f) SMA-B2S5C; (g) SMA-B3S2C; (h) SMA-B3S3C; (i) SMA-B3S5C; (j) SMA-B4S4C; (k) SMA-B5S4C; (l) SMA-B2S2M; (m) SMA-B2S3M.

Table 1. Ratios between plasticity model predictions and experimental results

	Strength (MPa)	Pitch (mm)	Peak Stress	Peak Strain	Ultimate Stress	Ultimate Strain	Axial Stress @ 2.5%
SMA-B1S2C	30.6	15.9	0.823	1.278	1.118	1.048	0.929
SMA-B1S3C		10.2	1.024	1.157	0.872	0.852	0.957
SMA-B1S5C		5.1	0.949	1.014	1.038	2.840	1.015
SMA-B2S2C	39.6	19.1	1.042	1.255	1.021	1.190	0.985
SMA-B2S3C		12.7	1.057	1.205	1.112	0.910	0.921
SMA-B2S5C		6.4	1.086	1.036	0.980	0.895	1.051

SMA-B3S2C		19.1	1.035	1.308	1.042	1.009	0.999
SMA-B3S3C	50.0	12.7	1.001	1.182	0.971	0.655	0.958
SMA-B3S5C		6.4	0.956	1.073	1.080	1.322	1.136
SMA-B5S4C	36.1	7.6	1.022	0.984	1.085	1.016	0.984
SMA-B6S4C	55.3	7.6	0.941	1.174	0.981	0.937	1.044
SMA-B2S2M		19.1	1.010	1.239	1.052	1.039	0.890
SMA-B2S3M	39.6	12.7	1.025	1.216	1.009	0.931	0.935
Average			0.998	1.620	1.028	1.126	0.985
Standard Deviation			0.065	1.560	0.065	0.518	0.063

The proposed plasticity model was also validated by using experimental data published in the literature. Shin and Andrawes (2010) tested several 152 mm $\times$ 305 mm (6 in $\times$ 12 in) NiTiNb-SMA confined concrete cylinders under uniaxial compression. One of their tested specimens was used herein for model validation. The concrete strength reported for that specimen was 39.3 MPa (5.7 ksi) and the SMA spiral pitch used was 13 mm (0.5 in). NiTiNb-SMA wires used in the test had a diameter of 2 mm (0.0787 in). Shin and Andrawes reported that the NiTiNb-SMA wires used in the test were partially heated, i.e. only portion of the wire length was heated. For these partially heated wires they measured a recovery stress of 447 MPa (64.8 ksi). This value was modified to be 558 MPa (89.1 ksi) in order to account for the full wire length of the NiTiNb-SMA wires as per the modification method described in Chen *et al.* (2014b). Accordingly, and based on this modified value, the active confining pressure used in the model was 1.92 MPa (0.278 ksi).

Fig. 8 compares the axial stress-strain relation from Shin and Andrawes (2010)' test and the proposed plasticity model prediction. For the ascending branch, the model prediction has the same elastic modulus as the experimental result. The peak axial stress from the test and from the model prediction was 47.2 MPa (6.8 ksi) and 51.7 MPa (7.5 ksi), respectively resulting in 9.5% difference. This difference can partially be attributed to the method of heating adopted by Shin and Andrawes, which involved placing the cylinder inside an oven prior to loading at a temperature of 160°C for 15 min (Shin and Andrawes 2010), which

might result in reduced concrete strength due to the high temperature. For the descending branch, Fig. 8 shows that, the experimental and analytical slopes of descending branch agree well, especially after axial strain level of 0.9%. As concluded earlier, the residual stress and the ultimate stress are independent of concrete strength and are only related to the level of active confinement. Hence, despite the difference in the experimental and analytically predicted strength, Fig. 8 shows very good agreement on both residual stress and ultimate stress.

Comparing the ultimate strain it is clear that the model overestimated the ultimate strain of the specimen. This can also be attributed to the heating method which could have possibly affected the concrete ductility. Moreover, in Fig. 8, the experimental axial stress-strain curve by Shin and Andrawes (2010) is also compared with existing concrete stress-strain models by Mander *et al.* (1988) and by Attard and Setunge (1996). Mander *et al.* (1988) model is one of the most commonly used concrete stress-strain models for traditional steel transversely reinforced concrete. Attard and Setunge (1996)'s model was developed based on triaxial test results for actively confined high-strength concrete (60-130 MPa [8.7-18.9 ksi]). Note that in Fig. 8 for both Mander *et al.* (1988) and Attard and Setunge (1996) models, ultimate strain is plotted up to 0.04 mm/mm since ultimate strain is not clearly defined for SMA confined concrete in these two models. Fig. 8 demonstrates that due to the constant confining pressure assumption in both Mander *et al.* (1988) and Attard and Setunge (1996) models, they are unable to capture the combination effect of active and passive confinement of NiTiNb-SMA confined concrete. They both underestimated the residual stress of NiTiNb-SMA confined concrete, as a result of underestimating the total confining pressure of NiTiNb-SMA confined concrete. That also proves that existing concrete models are unable to capture the unique behavior of NiTiNb-SMA confined concrete and the

proposed plasticity model is essential in understanding and predicting NiTiNb-SMA confined concrete behavior.

Fig. 8. Comparison between the proposed model and experimental and analytical results presented by other researchers

6. Conclusions

This paper presented a new plasticity-based constitutive model that can accurately predict the axial stress-axial strain and axial stress-lateral strain behaviors of NiTiNb-SMA confined concrete. Experimental results presented by Chen and Andrawes (2017) were utilized to derive the proposed model. First, a dilation ratio model was proposed to model the axial-lateral strain relation. Then, the three components of a plasticity model—yield criterion, hardening/softening function, and flow rule were derived based on the experimental results. A Drucker-Prager type yield function was used to describe the effect of deviatoric stress on hydrostatic pressure. Both the hardening/softening function and the dilation rate function were developed as functions of equivalent plastic strain, and both were able to capture the influence of concrete strength and SMA confinement on the shapes of the hardening/softening function and the dilation rate function. In addition, a damage parameter was derived based on the stiffness degradation of concrete during cyclic loading, which accounts for the stiffness degradation effect on concrete nonlinearity. Through comparing the experimental results with the model predictions, it was found that the proposed model was able to capture the stress-strain behavior of NiTiNb-SMA confined concrete accurately

including peak stress (0.2% error on average), axial stress at 2.5% strain (1.5% error on average), ultimate stress (2.8% error on average), ultimate strain (0.7% error on average), as well as the stiffness changes during cyclic loading. The proposed constitutive model for SMA confined concrete was also validated using experimental data published in the literature. Hence, this plasticity model is deemed suitable to predict and simulate the three-dimensional stress-strain behavior of NiTiNb-SMA confined concrete. Finally, it should be noted that this model was developed based on SMA confined concrete experimental results using one type of SMA, namely NiTiNb alloy. Further studies are needed to explore the feasibility of using this model for concrete confined by other SMA types.

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