Incorporating Food Microstructure and Material Characteristics for Developing Multiscale Saturated and Unsaturated Transport Models

Pawan S. Takhar*
Food Science and Human Nutrition,
1302 W Pennsylvania Ave
University of Illinois at Urbana-Champaign,
Urbana, IL, 61801, U.S.A.
Tel.: +1-(217) 300-0486
E-mail: ptakhar@illinois.edu

Abstract

This communication discusses hybrid mixture theory and how it could be utilized for food science applications by presenting examples on drying, frying and expansion of biopolymers. Hybrid mixture theory, which has a continuum mechanics basis can be used to merge the quality changes in food biopolymers with processes such as transport of heat, water, vapors, oil, etc. in food systems. Once a developed model has been validated, obtaining the solution via numerical simulations yields a large amount of data on transport of fluids, heat, species, etc. in the food matrix and food's rheological and microstructural characteristics. Hybrid mixture theory and some other continuum mechanics based theories have advanced to a stage that they can be used to describe the fate of multiphases and multiconstituents in complex foods undergoing phase and state transitions during processing. The developed modeling equations also help to serve as a guide for designing parameter estimation experiments, and solution of the model helps to fill gaps in experimental knowledge on underlying physical mechanisms. The solution of developed model provides information on optimum processing conditions needed to improve the quality of food products and efficiency of processes.

Keywords: Hybrid mixture theory, Darcy's law, non-Fickian transport, unsaturated transport, drying, frying

Nomenclature

Latin Symbols

D^α	Coefficient of diffusivity of the phase α [m^2/s]
$\beta \hat{e}^\alpha$	Net rate of mass transfer from the phase β to the phase α [$kg/(m^3s)$]
$\dot{E}_{KL}^s$ (m)	First order material time derivative of E_{KL}^s [s^{-1}]
E_{KL}^s	m^{th} order material time derivative of E_{KL}^s [s^{-m}]
E	Coefficient of elasticity of the biopolymeric matrix [Pa]
K^α	Permeability of the α phase [m^2]
M	Moisture content on mass dry basis [g/g solids]
$\mathcal{M}^\alpha$	$= M^\alpha / R^\alpha$, Memory function in Darcy's law eq. (2) [$m^5/(kgs)$]
M_g	Glass transition moisture content [g/g solids]
p^α	Physical pressure in the phase α [Pa]
p_c	Capillary pressure [Pa]
REV	Representative elementary volume [m^3]
T	Temperature [$^\circ K$]
T_g	Glass transition temperature [$^\circ K$]

* Author has previously published as Pawan P. Singh

M_g	Glass transition moisture content [g/g solids]
v_l^α	Velocity of the α phase [m/s]
<i>Greek Symbols</i>	
ε^α	Volume fraction of the α phase (dimensionless)
ϕ	Porosity of the biopolymeric matrix (dimensionless)
ρ^α	Density of the phase α [kg/m ³]
<i>Subscripts</i>	
k, l	Indices for Eulerian coordinates
K, L	Indices for Lagrangian coordinates
<i>Superscripts</i>	
α	General representation of a phase
β	General representation of a phase
m	Order of the material time derivative of E_{KL}^s that varies from 0 to p
s	Solid phase
w	Water phase
o	Oil phase
g	Gas phase
<i>Special Symbols</i>	
D^α/Dt	Material time derivative with respect to velocity of the α phase particle [s^{-1}]
$v_l^{\alpha,s}$	Velocity of the α phase relative to the solid phase ($= v_l^\alpha - v_l^s$) [m/s]
Dot (.)	$= D^s/Dt$, material time derivative with respect to velocity of the solid phase particle

1 Introduction

Continuum mechanics involves treating the matter as continuous, and studying its motion and deformation as a function of internal or external forces. Continuum mechanics also utilizes the laws of thermodynamics by including energy balance and entropy inequality [19]. The field has helped to enhance our understanding of the behavior of materials, their interaction with the environment, design new materials, and improve their quality by predictably modifying the processing conditions. Continuum mechanics involves developing mathematical models describing material behavior during processing by utilizing the laws of conservation of mass, momentum and energy balance that are universal as they apply to all materials; including the nature of material via constitutive theory; and applying restrictions using the second law of thermodynamics. Using the framework of continuum mechanics, a large number of engineering models can be obtained via mathematical derivations. Examples include thermoviscous, thermoelastic and thermoviscoelastic relations, integro-differential viscoelasticity relations, generalized Fourier's law of heat transfer, Navier-Stokes equation, Fick's law, Darcy's Law, magnetoelasticity relations, etc. (see e.g. [11]).

Field equations in continuum mechanics (conservation of mass, momentum and energy) apply to all types of materials when the hypothesis of continuum holds. However, there are more variables than the number of equations. This is expected because we know that different materials behave differently under deformation. The system of equations is closed by including additional relations by formulating the constitutive theory, which involves incorporating the nature of materials (e.g. elastic, viscoelastic, viscous, plastic, conductive, diffusive, etc.). Most relations available were obtained in pure fields of engineering for materials with simpler nature than foods. Foods are complex mixtures of biopolymers (e.g. carbohydrates, proteins, lipids), solutes (e.g. sodium) and fluids (water, vapors, oil, etc.); exhibit porous microstructure with scale hierarchy (e.g. pores in cell walls at microscale, cell cytoplasm at mesoscale and tissue at macroscale); undergo phase and state transitions; and transform their constitutive behavior, structure and composition during processing. Thus, when continuum mechanics based models are derived for food materials, more general and often novel relations are obtained [1, 31–33, 39].

To include the hierarchical porous structure of materials and mixture of species (e.g. solutes and biopolymers), various theories have evolved within the framework of continuum mechanics (see [8] for an overview). Examples of these theories are –volume averaging the smoothed transport equations with material coefficients from micro to macroscales [40] (see [9, 14, 15] for the food science

application); homogenizing the microscale differential equations describing transport processes using asymptotic methods [20]; and hybrid mixture theory that involves upscaling the microscale field equations to meso and macro scales, formulating the constitutive equations at macroscale to close the system and exploiting the entropy inequality to impose restrictions on the equations to make them physically viable [17].

The focus of this communication will be on hybrid mixture theory (HMT) in, which the material properties in a system appear at the macroscale during the mathematical derivation process. This makes HMT specifically suitable for foods as it is difficult to measure their material properties at the micro and meso scales due to structural and compositional heterogeneity; tedious sample preparation of soft materials; and a wide contrast in properties and structure in different regions of a food (e.g. crystalline and amorphous regions in starch [22]).

2 Hybrid Mixture Theory (HMT)

Fig. 1 shows three-scales in a porous material involving interactions between different phases at three scales. HMT allows including physical mechanisms at different scales in the developed models. It involves volume averaging the laws of mass, momentum, energy and entropy at the microscale to obtain equations at the meso and macro scales. First, microscale equations are averaged over the solid-vicinal fluid domain to obtain the mesoscale equations. At mesoscale, the equations are further averaged over the solid-fluid mixture and bulk phase to obtain equations at the macroscale. At macroscale, the constitutive equations are formulated by exploiting the entropy inequality using Coleman and Noll [7]’s method.

HMT was developed by Hassanizadeh and Gray [17, 18] to study the transport processes in soils. Achanta and Cushman [1], Bennethum and Cushman [5] and Bennethum et al. [6] applied HMT to swelling and shrinking systems with dissolved species (solutes) and included interactions between phases and species. They treated the solid phase as elastic and the liquid phase as viscous. In earlier studies, HMT was used for either swelling clays [5, 6], or adapted for polymeric systems from clay systems [1] for which the assumption of elastic solid phase holds. The system depicted viscoelastic behavior with short-memory at the macroscale, which resulted from the interaction of the elastic solids with the viscous fluids at microscale. Earlier HMT based studies ignored the microscale relaxation processes within the polymeric matrix, which occur due to conformational changes in the flexible thread-like polymers at the molecular scale [12]. These relaxation processes can cause a food material to exhibit viscoelastic behavior even in the absence of water or a plasticizing agent. These relaxation processes provide a force term, which affects liquid movement through the porous matrix [16, 25, 26, 28]. This force term plays a fundamental role when the time-scale of relaxation is of the same order as the time scale of the liquid movement, which occurs when the polymers are undergoing glass-transition.

Singh et al. [32] modeled the solid polymers as viscoelastic and obtained an integro-differential fluid transport equation with long-term memory effects [33]. This equation predicted that fluid transport in food biopolymers would follow Fick’s law only when a food is in the rubbery or the glassy state. In the vicinity of glass-transition, the transport becomes non-Fickian [33]. Kim et al. [21] made similar conclusions by performing dodecane sorption experiments in a polystyrene matrix. Xing et al. [41] verified these predictions by performing drying experiments with pasta in an NMR tube to observe moisture distribution profile as it changed from a rubbery to a glassy state. Moisture distribution profile across the cross-section of pasta had round profiles when the pasta was in the rubbery state, which indicated Fickian transport. As a large part of pasta underwent glass-transition due to drying in the NMR tube, the moisture distribution profile became sharper, which indicated non-Fickian transport.

Most of the previous work on using continuum-thermodynamics based frameworks was for obtaining transport and thermomechanical relations for saturated biopolymeric systems. The work was relatively scarce for unsaturated transport, which involves the movement of the gas phase (mixture of air and vapors) in the system. In both high and low temperature applications such as frying, expansion during extrusion, and freezing, the physicochemical structure of food biopolymers changes rapidly, and fluid transport is of the unsaturated type because of the involvement of gas phase (mixture of water and vapors). Unsaturated transport is significantly more complex than saturated transport (e.g.

drying of a wet food) because the gas phase is compressible, can leak from the porous matrix, and results in free surfaces in the pores for which tensile and capillary forces need to be incorporated.

In a macroscale representative elementary volume of a food, hydrophilic (e.g. polysaccharides), hydrophobic (e.g. lipids, cholesterol and triglycerides) and amphiphilic (e.g. proteins) molecules may be present [39]. Thus, various microscale regions in an elementary volume may interact with both water and lipids. For example, proteins and minerals in fried meat products are hydrophilic, and the lipids are lipophilic. The hydrophilic and hydrophobic effects can be included using hybrid mixture theory. Recently, Takhar [39] used hybrid mixture theory's framework to develop unsaturated transport equations for food biopolymers, which include viscoelastic relaxation in biopolymers, capillary and swelling pressure relations, hydrophilic and hydrophobic interactions, non-Darcian flow and thermomechanical stress relations.

Next, the mass balance and generalized Darcy's law relations of [39] are presented and their application to one saturated fluid transport (drying) and two unsaturated transport applications (frying, expansion during extrusion) is discussed.

3 Unsaturated Fluid Transport

The two-scale mass balance equation derived using hybrid mixture theory [5] includes material time derivative of upscaled phase density ($\epsilon^\alpha \rho^\alpha$) by considering the phase (α) itself as a reference frame. This equation can be transformed to make the material time derivative with respect to the solid polymers as a reference frame. Calculating velocities of various phases with respect to solid polymers, makes it easier to solve the equation for swelling or shrinking foods systems during processing. The transformed mass balance equation for various phases can be written using indicial notation as [39]

$$\frac{D^s(\epsilon^\alpha \rho^\alpha)}{Dt} + (\epsilon^\alpha \rho^\alpha v_l^{\alpha,s})_{,l} - \epsilon^\alpha \rho^\alpha \frac{\dot{\epsilon}^s}{\epsilon^s} = \sum_{\substack{\beta=w, o, g \\ \beta \neq \alpha}} \beta \hat{e}^\alpha, \quad \text{where } \alpha = w, o, g \quad (1)$$

Here, $\alpha = w, o, g$ represents the equation for water, oil and gas phase, respectively. If one of the phases is missing in a given food system, the corresponding phase equation would not be included (see nomenclature for definition of various symbols). The solid phase volume fraction, ϵ^s is related to porosity, $\epsilon^s = 1 - \phi$. For phase change problems such as frying and extrudate expansion, ${}^w \hat{e}^g$ represents mass lost by the liquid water, which is equal to the mass gained by the vapors. The rate of evaporation can be calculated by simplifying Gibbs free energy based relation resulting from hybrid mixture theory based derivation for unsaturated transport [39]. Since for problems like drying and frying, oil and solid phases are not considered to exchange mass with other phases, $\beta \hat{e}^\alpha = 0$ for these phases (Fig. 1).

The velocity of a phase, $v_l^{\alpha,s}$, can be calculated using generalized Darcy's law of [39]

$$v_l^{\alpha,s} = - \left[\frac{K^\alpha}{\mu^\alpha} \epsilon^\alpha p_{,l} + \epsilon^\alpha D^\alpha \epsilon_{,l} + \epsilon^\alpha \rho^\alpha \int_0^t \mathcal{M}^\alpha(t - \tau) \frac{\partial E_{MM,l}^s}{\partial \tau} d\tau \right], \quad \text{where, } \alpha = w, o, g \quad (2)$$

where, K^α is the coefficient of permeability, D^α is the coefficient of diffusivity of phase α , and $\mathcal{M}^\alpha$ is the non-Fickian memory term. It can be deduced that K^α and D^α are related to each other by the relation [2], $K^\alpha = D^\alpha \mu^\alpha / E$, where, (E) represents coefficient of elasticity of the biopolymeric mixture. Therefore, experimentally determined diffusivity can be converted to permeability (or *vice versa*), if E of the biopolymeric mixture is known.

The first term on the right-hand side of (2) is the classical pressure gradient term of Darcy's law; the second term denotes diffusive flux caused by the concentration gradient of a given fluid in the food material. The integral term is novel in Darcy's law, and describes flow caused or resisted by polymers relaxing in a time-dependent manner near glass-transition. Following [33], the memory function can be represented as $\mathcal{M}^\alpha(t) = B_c G(t)$, where B_c is the material coefficient relating the viscoelastic relaxation to fluid flow, and $G(t)$ is the stress relaxation function. B_c can be estimated

inversely during fluid transport simulations [34] (by varying the B_c value to determine its effect on a calculated variable compared against experimental data) or by analyzing the asymptotic behavior of the ratio of diffusivity to the stress relaxation function of materials in glassy and rubbery states [37].

The integral term of (2) describes the effect of viscoelastic relaxation in the poroviscoelastic matrix on flow of water and oil phases. Due to its non-local form, it is able to incorporate long-memory effects of strain in the biopolymers on the flow of fluids. From continuum mechanics it is known that E_{MM}^s of (2) is equal to the first invariant of strain (I_E^s) [11], which is further related to porosity (or solid phase volume fraction) of the material [39], $E_{MM}^s = I_E^s = (1 - \epsilon^s)/\epsilon^s = \phi/(1 - \phi)$. For a food matrix saturated by water, ϕ can be replaced by ϵ^w . Singh et al. [33] showed that for foods undergoing glass-transition, the integral term takes different forms in different states. In glassy and rubbery states (sufficiently far from glass-transition regime), the integral term merges with the diffusivity term, and the flow follows Fick's law. Near glass-transition, when the relaxation time of biopolymers is of the order of the diffusion time, the integral term retains its form, which implies non-Fickian (or non-Darcian) mode of fluid transport. Singh et al. [33] made these conclusions for saturated transport problems such as drying and sorption. Similar deductions can also be made for unsaturated transport such as frying of foods or expansion of biopolymers exiting an extruder [39].

4 Examples of Food Processing Applications of Hybrid Mixture Theory

This section discusses one saturated transport (drying) and two unsaturated transport (frying and expansion during extrusion) examples. The discussion presented here focuses on eqs. (1) and (2) and Fig. 1 to describe the idea behind continuum mechanics based modeling equations. Modeling of these applications requires numerous additional equations (e.g. heat transfer, water activity, capillary pressure, etc.), material properties (e.g. coefficient of diffusivity, stress relaxation function), and initial and boundary conditions, which are presented in the cited references.

4.1 Drying of Foods

To calculate moisture distribution in foods during drying, Fick's law is often used. However, Fick's law is able to describe fluid transport in polymers only when they are in glassy or rubbery states sufficiently far from the glass-transition regime [21, 33]. In glassy and rubbery states, moisture distribution across the cross-section of a food matrix shows round curvature, which can be modeled using Fick's law. In the vicinity of glass transition, the transport becomes non-Fickian resulting in sharp moisture profiles [33]. Eqs. (1) and (2) can be solved to calculate moisture profiles in food in not only glassy and rubbery states, but also when it is undergoing glass-transition. The normalized stress developed in the food as a consequence of the process can also be calculated by keeping track of the integral-term in (2), which further provides information on process conditions that may initiate stress-cracks in a material [37, 38].

During drying of foods such as gels and pasta, the matrix collapses to compensate for loss of water [2, 41]. In soybean seeds, air phase has not been observed for large part of drying [24]. For these cases, drying can be treated as a saturated transport problem for which, (2) can be simplified by dropping the pressure term. Singh et al. [34, 35] solved the saturated form of (1) and (2) for drying and water sorption of soybeans. Takhar [37] solved these equations for drying of corn kernels. By keeping track of the magnitude of the integral in (2) while calculating the numerical solution, viscoelastic stress in food biopolymers was determined, which is related to the quality changes at polymeric scale Takhar et al. [38].

Solution of saturated form of (1) and (2) for drying of soybeans showed that moisture distribution across the cross-section exhibits round profiles in the temperature range of (70-93 °C), which is routinely used in the industry [34, 35]. The round profiles indicated Fickian transport. Soybeans exhibit glass transition at 25 °C at 10% moisture content, and T_g increases with reduction in moisture content. Since, in a single soybean seed moisture is not uniformly distributed, even if the average moisture content is above the glass transition moisture content (M_g), the regions between center and surface are expected to undergo glass-transition. Moisture profiles exhibited sharp shapes near glass-transition, which indicated non-Fickian transport. Calculation of the integral term of (2), which is related to viscoelastic stress, provided information about temperature-humidity combinations of drying air resulting in more uniform stress distribution, and lower stress-magnitudes in soybean biopolymers. Using such profiles, optimum drying conditions were identified to obtain soybeans with desirable

198 rubbery/glassy domains, which would result in desirable physical characteristics during further pro-
199 cessing via conditioning, flaking and oil extraction.

200 Takhar et al. [38] discussed the moisture transport and stress development mechanisms in corn
201 kernels by solving saturated form of (1) and (2). The 3D kernel structure of corn kernels was imaged
202 using micro X-ray computed tomography, segmented and imported into a commercial finite elements
203 package, where the multiscale transport model was also implemented. Numerical simulations pro-
204 vided information on moisture distribution in germ, soft-endosperm, hard-endosperm and pericarp of
205 corn kernels, which was in agreement with magnetic resonance imaging based study of Song et al.
206 [36]. The highest moisture content was observed to be in a region spanning parts of soft-endosperm
207 and germ. A high moisture flux was observed in various regions such as under the pericarp, in soft-
208 endosperm near the top and in the region connecting soft-endosperm and germ. The germ retained
209 higher moisture and had lower moisture flux in comparison to other corn components due to its lower
210 diffusivity. Calculation of integral term of (2) for intermittent drying simulations helped to determine
211 fan on/off strategies, which farmers can easily implement in their driers for obtaining corn with fewer
212 stress cracks.

213 4.2 Frying of Foods

214 In unsaturated transport applications in foods, both pressure and concentration driven flow can
215 occur simultaneously. For example during frying of foods, water diffuses from high moisture to low
216 moisture regions and also moves due to pressure gradient in the system [15]. The first and second
217 terms on the right-hand side of (2) combine Darcy's and Fick's law in a single relation, which can
218 be used to study water, oil and vapor transport during frying of foods [39]. Takhar [39] solved a
219 model composed of (1) and (2) and numerous other equations (e.g. capillary pressure, water activity,
220 saturated vapor pressure, psychrometric relations, multiscale heat transport equation, diffusivity rela-
221 tions, surface heat and mass transfer relations, etc.) to calculate moisture, oil, gas (air+vapor mixture)
222 and heat transport in French fries. Bansal et al. [3] solved the model for frying of rice crackers, and
223 Bansal et al. [4] solved it for frying of chicken nuggets. For obtaining the solution of gas transport,
224 diffusive and integral terms of (2) were neglected as the pressure gradient in gas is expected to be the
225 dominating forcing term. Since, frying involves simultaneous solution of a large number of relations
226 as noted above (see references for the complete list), a short-memory assumption of the integral term
227 of (2) was made to obtain the solution in real time. The short memory assumption allows changing
228 the integral term to a polymer mixture-viscosity based differential term [39].

229 Positive gas pressure developed inside the food due to evaporation (Fig. 2a). The gas pressure
230 increases in the beginning and then starts decreasing with its magnitude varying with the type of food
231 [3, 4, 39]. The gradient of gas pressure near the surface was negative with peak being at a point of
232 highest evaporation at a given time (Fig. 2a). The gas pressure gradient was noted to be crucial for
233 resisting oil uptake from the large pool of oil outside the food. Fig. 2b shows capillary pressure (p_c)
234 in a potato cylinder obtained during simulations [39]. Similar data was obtained for rice crackers and
235 chicken nuggets by Bansal et al. [3] and Bansal et al. [4], respectively.

236 A high magnitude of capillary pressure (in comparison to gas pressure) makes water pressure neg-
237 ative ($p^w = p^g - p_c$) during large part of frying. Negative capillary pressure further results in negative
238 gage pore pressure (average gage pressure experienced by the walls of a porous matrix) (Fig. 2d).
239 Simulations showed that large part of frying takes place under negative gage pore pressure, which was
240 also validated experimentally by Sandhu et al. [30]. In frying literature, the role of capillary pressure
241 in oil phase during post-process cooling stage has been emphasized. In simulations conducted in au-
242 thor's group [3, 4, 39] it was noted that capillary pressure in water phase is the key to obtain negative
243 pore pressure observed experimentally by [30], and plays an important role to affect oil uptake. Since,
244 it is difficult to measure oil penetration in real time during frying, many previous studies noted that
245 large part of oil penetrates during the cooling stage [27]. With hybrid mixture theory based simula-
246 tions it was shown that in high moisture and thicker foods like potatoes (French fries) and chicken
247 nuggets, negative pore pressure causes oil penetration during frying itself, but the oil stays only near
248 the surface (Fig. 2d). In low moisture and thin foods like rice crackers, oil can penetrate the food
249 throughout its cross-section during frying before the product is taken out of the fryer [3]. During
250 post-frying cooling, the surface oil is expected to penetrate deeper into a food product due to suction
251 caused by negative pore pressure.

4.3 Expansion of Polymers During Extrusion

Food biopolymers such as starch wetted with a plasticizer such as water undergo shearing, heating and mixing in the barrel of an extruder. It forms a viscous paste, which experiences a sudden drop in pressure upon passing through the die nozzle. Inside the extruder, plasticizer exists in a super heated state. Upon exiting the die, plasticizer evaporates in a short time to form vapors. The vapors expand to form starch cells. The phase change from liquid to vapor state causes local evaporative cooling at different points in the melt. During expansion and subsequent contraction due to vapor collapse and elastic recovery, starch's rheological characteristics pass through various stages such as viscoelastic, viscoplastic, elastoplastic, elastic, etc. [13, 23, 29]. The microstructural properties of starch also undergo a significant change [13, 23]. Different regions in the starch forms cells of different sizes. Cellular structure is affected by extrusion parameters such as temperature in different zones, barrel size, nozzle size and shape, amount of blowing agent, degree of grinding, pressure in different zones, etc. Due to complexity of the extrusion process and expansion upon exiting the die, controlling the process requires operation by experienced operators along with knowledge of scientific and engineering principles. Dit-u dompo and Takhar [10] solved hybrid mixture theory based unsaturated transport equations of Takhar [39] to model expansion of starch during extrusion. Simplified form of (1) and (2) are examples of equations solved (by taking $\alpha = w, g$) in this study. The model included capillary pressure, heat transfer, viscoelasticity relations, etc. similar to the frying studies discussed above.

An interesting aspect of this study was that porosity of the starch matrix could be calculated along the cross-section of starch using fundamental hybrid mixture theory based equations coupled to poro-viscoelasticity relations discussed in [39]. The model was also used to calculate temperature, moisture content, glassy crust formation, expansion ratio, pore-pressure and viscoelastic-stress distribution across the cross-section of extrudate as function of distance from the die nozzle [10]. The microstructural and rheological characteristics of model calculated by numerical simulations would require several years to measure experimentally. The model can be used to predictably modify the extrusion parameters for various food, feed and biomedical applications.

5 Conclusions

Continuum mechanics and thermodynamics based theories such as Hybrid Mixture Theory can be used to merge quality aspects of biopolymers with transport and thermomechanical relations in a fundamental manner. A large amount of theoretical and experimental work in porous media physics has advanced these theories to the level that complex physical phenomena such as drying, frying, expansion, etc. in complex materials such as foods can be described using mathematical equations. Solution of these equations via numerical simulations yields information on the effect of processing conditions on food's microstructural, rheological and physical characteristics, which the food industry can utilize to improve the efficiency of processes and quality of food products. The solution of these models also helps to fill gaps in experimental knowledge on underlying physical mechanisms.

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- Of special interest for learning about the continuum mechanics from the related material science fields.
- • Of special interest for learning about the continuum mechanics based models for food and biomaterials subjected to various processes.

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414 • • Author presents the development of unsaturated transport equations for food biopolymers.
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Figure 1: Micro, meso and macro scales in a porous food matrix.

Figure 2: (a) Gas pressure, (b) Capillary pressure, (c) Pore pressure, and (d) Oil distribution in a potato cylinder during frying. From [39]. Reprinted with permission.