Theoretical and experimental study on the penetration rate for roller cone bits based on the rock dynamic strength and drilling parameters

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Abstract: Roller cone bits have been used extensively in petroleum and natural gas mining, and the accurate prediction of their rate of penetration (ROP) is of crucial importance for improving drilling quality and reducing drilling cost. In this study, a new prediction model of the ROP considering the combined effect of the main drilling parameters and rock dynamic compressive strength is developed for roller cone bits. The model is derived based on the mechanism of rock fragmentation under a single tooth impact indentation. The newly introduced ROP model is different from others in that it replaces the rock static strength with rock dynamic compressive strength and can reflect the real process of rock dynamic crushing by a roller cone bit. The results of theoretical analysis show that the ROP linearly correlates with the bit rotary speed; the relationship between the ROP and weight on bit (WOB) is $3/2$ power, and the relationship between the ROP and rock dynamic compressive strength is $-3/2$ power. Furthermore, lab drilling tests on sandstone and limestone are carried out with a full-scale bit drilling testing machine, and

the influence laws of WOB, rotary speed and rock dynamic strength on the ROP are analyzed. In addition, the theoretical ROP values calculated based on the rock dynamic compressive strength and static compressive strength are compared with the experimental ROP values. The results from the tests indicate that the experimental ROP values are basically consistent with the theoretically derived values, and the comparative results show that the theoretical ROP values calculated according to the rock dynamic compressive strength are closer to the actual values with, an average relative error below 15%, signifying that the newly established ROP model in this paper is valid and can be used to predict the drilling rate with reasonable accuracy and can be used to provide useful guidance for optimizing drilling parameters.

Key words: roller cone bit; rate of penetration; rock dynamic compressive strength; weight on bit; bit rotary speed

1. Introduction

Roller cone bits are widely used in the oil and gas drilling industry, and their performance directly affects the drilling quality and cost (Winters et al., 1987; Warren, 1987; Kahraman et al., 2000). The drilling rate or rate of penetration (ROP) is one of the most important bit performance parameters, and a series of studies on ROP models for roller cone bits have been performed by a number of scholars and valuable research results have been obtained. For example, Maurer (1962) studied crater formation under a single tooth and identified the relationships among the load, depth and rock breaking volume and then concluded the drilling rate equation under conditions of perfect cleaning. Walker et al. (1986) obtained a prediction equation for roller cone bits as a function of bit weight, well depth, in-situ compressive strength, and rock porosity by adopting the statistical regression method. Ma et al. (1995) developed a computer simulation program on the bit-rock interaction, but it is impossible to use in real-time drilling simulation. Hareland et al. (2010) introduced a ROP model considering the bit cutting structure and drilling parameters based on the interaction mechanism between the drill bit and rock. Rashidi et al. (2015) investigated the effect of the drilling parameters, cutting structure of bits and bit wear on the drilling rate and developed a ROP model for roller cone bits. However, because of the complexities and questionable assumptions of the existing models, errors exist, to some extent, between the model predictions and field observed data or experimental data.

The theoretical and experimental studies on ROP modeling cited above primarily concentrated on the drilling parameters, bit structure and rock static mechanical properties; few studies have considered the rock dynamic properties. However, the main working principle of roller cone bits is to impact and break up rock by using teeth; this is a dynamic fragmentation process (Eronini et al., 1982; Xu et al., 1984; Ma et al., 1985; Liu, 1988; Luiz, 2011), and the rock strength under impact loading will be improved significantly compared with the static strength (Olsson, 1991; Cho et al., 2003; Li et al., 2005; Demirdag et al., 2010). Thus, it is not suitable to use ROP models based on rock static properties to predict the drilling rate.

Therefore, this paper is focused on developing a simple and practical ROP prediction model that considers the rock dynamic compressive strength and presents a theoretical analysis and results of lab drilling experiments. The test results show that the actual drilling rate is in good agreement with the theoretical values, demonstrating the validity of the proposed model.

2. Theoretical analysis of rock fragmentation by a roller cone bit

When bits are drilling, the primary working parameters that are involved include the weight on bit (WOB), bit rotary speed and rate of penetration (ROP), and the main rock mechanical parameters that are involved are the compressive strength and tensile strength. Considering the rock at breaks in compression under an axial impact load generated by bit teeth, the influence of the rock tensile strength can be ignored. Thus, this paper concentrates on establishing the quantitative relationships among the ROP, WOB, bit rotary speed and rock dynamic compressive strength.

2.1 Relationship between the volume generated by a single tooth and rock dynamic compressive strength

The stress state of rock under a single tooth indentation can be simplified as a problem of semi-infinite space subjected to concentrated force, namely, Boussinesq's problem (Fig. 1), and the elastic solution of this problem under a cylindrical coordinate system is as follows (Boussinesq, 1885; Xu et al., 1984):

Fig. 1 Stress state model of rock under a single tooth indentation

$$\begin{cases} \sigma_r = \frac{F}{2\pi R^2} \left[(1-2\mu) \frac{R}{R+z} - \frac{3r^2 z}{R^3} \right] \\ \sigma_\theta = \frac{(1-2\mu)F}{2\pi R^2} \left[\frac{z}{R} - \frac{R}{R+z} \right] \\ \sigma_z = -\frac{3Fz^3}{2\pi R^5} \\ \tau_{rz} = -\frac{3Frz^2}{2\pi R^5} \end{cases} \quad (1)$$

where F is the concentrated force, R is the distance of a point in space to the point of force, and r and z are the coordinates under the cylindrical coordinate system. μ is the Poisson's ratio of rock; σ_r , σ_θ , σ_z , τ_{rz} are the normal stress and shear stress, respectively, in the corresponding direction.

Based on the geometrical relationship in Fig. 1 and equation (1), the axial stress can also be expressed as:

$$\sigma_z = -\frac{3Fz^3}{2\pi R^5} = -\frac{3F}{2\pi} z^3 (r^2 + z^2)^{-5/2} \quad (2)$$

When the axial stress exceeds the rock ultimate dynamic compressive strength ($\sigma_z \geq \sigma_d$), the rock will be fractured. Substituting $\sigma_d = \sigma_z$ into equation (2), the solution of the fracture radius parallel to the boundary condition is:

$$r^2 = \left(\frac{3F}{2\pi\sigma_d} z^3 \right)^{2/5} - z^2 \quad (3)$$

Introducing $r=0$ into equation (3), breaking depth z_0 is obtained:

$$z_0 = \left(\frac{3F}{2\pi\sigma_d} \right)^{1/2} \quad (4)$$

Then, the volume of rock fragmentation generated by a single tooth can be obtained by integration:

$$V_0 = \int_0^{z_0} \pi r^2 dz = \int_0^{z_0} \pi \left[\left(\frac{3F}{2\pi\sigma_d} z^3 \right)^{2/5} - z^2 \right] dz = \frac{4\pi}{33} \left(\frac{3F}{2\pi\sigma_d} \right)^{3/2} \quad (5)$$

2.2 Modeling of ROP for roller cone bit

The indentation model under a single tooth developed above is the basis for ROP modeling. Assuming the weight on bit is W and the number of teeth embedded in three cones generatrix in contact with rock at the same time is k_1 , k_2 and k_3 (Fig. 2), then the force F generated by a single tooth can be written as:

$$F = \frac{W}{k_1 + k_2 + k_3} \quad (6)$$

Fig. 2 Schematic diagram of a bit tooth in contact with rock

Assuming the volume of rock fragmentation generated by each tooth is equal to V_0 , then the total volume ΔV generated by k_i teeth embedded in one cone is:

$$\Delta V = k_i V_0 \quad (7)$$

The rock failure volume generated by one cone rotating one circle around its axis is:

$$V_i = \sum_{j=1}^{m_i} (\Delta V)_i = \sum_{j=1}^{m_i} (k_i V_0)_j \quad (8)$$

Thus, the total failure volume generated by three cones making one revolution can be given as follows:

$$V = \sum_{i=1}^3 V_i = V_1 + V_2 + V_3 \quad (9)$$

Combining equations (8) and (9), the total failure volume generated by three cones rotating in one circle can also be expressed as:

$$V = \left(\sum_{j=1}^{m_1} (k_1)_j + \sum_{j=1}^{m_2} (k_2)_j + \sum_{j=1}^{m_3} (k_3)_j \right) V_0 \quad (10)$$

The coefficient of equation (10) is defined as:

$$K = \sum_{j=1}^{m_1} (k_1)_j + \sum_{j=1}^{m_2} (k_2)_j + \sum_{j=1}^{m_3} (k_3)_j \quad (11)$$

where m_1 , m_2 , and m_3 stand for the number of generatrices on the three cones.

According to equations (10) and (11), the total failure volume V becomes:

$$V = K V_0 \quad (12)$$

when K is a constant related to the bit type.

It is assumed that the bottom hole angle is equal to 0 and the cone angle is $2\varphi_0$, and then, the relationship between the cone rotary speed n_c and bit rotary speed n_b can be represented as:

$$\frac{n_c}{n_b} = \sin \varphi_0 \quad (13)$$

According to the principle of the cumulative volume of rock fragmentation generated by all teeth equivalent to the total volume calculated by the means of bit penetration, the following equation is derived:

$$\frac{\pi R_0^2 v}{n_b} = V \frac{1}{\sin \varphi_0} \quad (14)$$

By substituting equations (12) and (13) into equation (14) and performing the appropriate transformation, the rate of penetration v can be shown as follows:

$$v = \frac{K n_b}{\pi R_0^2 \sin \varphi_0} V_0 \quad (15)$$

where R_0 is the radius of the bit.

Inserting equations (5) and (6) into equation (15), the ROP model can be presented as:

$$v = \frac{K n_b}{(k_1 + k_2 + k_3)^{3/2} \pi R_0^2 \sin \varphi_0} \frac{4\pi}{33} \left(\frac{3W}{2\pi \sigma_d} \right)^{3/2} = \frac{0.126K}{(k_1 + k_2 + k_3)^{3/2} \pi R_0^2 \sin \varphi_0} n_b \left(\frac{W}{\sigma_d} \right)^{3/2} \quad (16)$$

The coefficient of equation (16) can be defined as:

$$A = \frac{0.126K}{(k_1 + k_2 + k_3)^{3/2} \pi R_0^2 \sin \varphi_0} \quad (17)$$

Finally, the developed ROP model is simplified as follows:

$$v = A n_b \left(\frac{W}{\sigma_d} \right)^{3/2} \quad (18)$$

where coefficient A depends on the bit type, and when the bit is selected, A can be determined. Equation (18) is the new ROP model for a roller cone bit considering the drill bit structure parameters, main drilling parameters and rock dynamic compressive strength that can be conveniently applied to the drilling rate prediction. For different bits, the values of coefficient A are different, and the bit performance can be compared easily based on coefficient A ; the larger the value of A , the higher the performance of the bits.

As seen from the theoretical model, the relationship between the drilling rate and bit rotary speed is linear; the relationship between the drilling rate and weight on bit is 3/2 power, and the relationship between the drilling rate and rock dynamic compressive strength is -3/2 power.

3. Experimental section

3.1 Materials and preparation

The rock materials used in lab drilling tests are sandstone and limestone with good integrity and homogeneity. Before the drilling tests, static and dynamic uniaxial compression tests are conducted by using the RTR-1500 Rock Triaxial Testing System and the $\phi 50$ mm Split Hopkinson Pressure Bar (SHPB) Test System to obtain static and the dynamic compressive strength of the two types of rock. Considering that the maximum impact velocity of the bit teeth in the drilling process is approximately 5 m/s (Liu, 1988), the impact velocity in the SHPB test is selected to be 5 m/s. The average values of rock static and dynamic strength are summarized in Table 1.

Table 1 Dynamic and static mechanical parameters of different rocks

Rock sample	Density (kg/m ³)	Static compressive strength (MPa)	Dynamic compressive strength (MPa)
Sandstone	2630	60.57	74.49
Limestone	2450	44.81	58.70

3.2 Experimental apparatus and method

The full-scale bit drilling testing machine is shown in Figure 3 and is composed of five parts: the console, power-driven system, axial loading system, data-processing system and other auxiliary devices. The maximum axial applied load and rotary speed of the testing machine are 300 KN and 250 RPM, respectively. In drilling tests, the type of roller cone bit is IADC 517, with a diameter of 215.9 mm. When conducting the drilling test, the rock sample is placed in a core gripper and the WOB is applied to bits by the axial loading system; then, the bit rotates and drills rock through a power-driven system. Additionally, the machine has a computerized data acquisition system that can continuously record the WOB, bit rotary speed, penetration depth and time data during the drilling test. The value of the weight on the bit is measured by a pressure sensor mounted on the dowel bar; the rotary speed can be read directly from the tachometer, and the ROP is obtained by the ratio of the penetration depth and drilling time. To demonstrate the validity of the theoretical model and predict the drilling rate, the drilling tests are divided into two groups and the specific scheme is as follows: 1) making the bit rotary speed constant and changing the weight on the bit to verify the relationship between ROP and WOB; 2) making the weight on

bit constant and changing the bit rotary speed to verify the relationship between ROP and bit rotary speed.

Fig. 3 Diagram of the full-scale bit drilling testing system

3.3 Results and discussion

3.3.1 Influence law of a WOB on the ROP

To fully investigate the influence rule of WOB on the ROP, lab drilling tests for sandstone and limestone under conditions of constant rotary speed of 90 RPM and different WOB were carried out, and the relevant drilling parameters are presented in Table 2.

Table 2 Drilling parameters under a constant rotary speed

Rock sample	Bit rotary speed (RPM)	WOB (KN)	ROP (m/h)
Sandstone	90	124.5	10.99
	90	133.3	12.69
	90	145.1	14.61
	90	155.6	17.75
	90	167.3	19.23
Limestone	90	90.6	9.51
	90	99.4	10.37
	90	109.3	12.79

90	114.4	13.96
90	124.6	16.14

According to the testing results in Table 2, the diagrams of the ROP versus WOB for the two types of rock are shown in Figure 4. As seen in Figure 4, there is a good power function relationship between the ROP and WOB for sandstone and limestone, and the power indexes are both 3/2. The statistical correlation coefficients of the fitting lines in these figures are 0.939 and 0.968, revealing that the test results are well matched with the theoretical model.

Fig. 4 Relationship between the ROP and WOB

3.3.2 Influence law of the rotary speed on the ROP

To study the influence rule of the bit rotary speed on the ROP, lab drilling tests for sandstone and limestone under the conditions of a constant WOB of 130 KN and five different bit rotary speeds were performed, and the relevant drilling parameters are indicated in Table 3.

Table 3 Drilling parameters under a constant WOB

Rock sample	Bit rotary speed (RPM)	WOB (KN)	ROP (m/h)
Sandstone	60	130	8.01
	70	130	8.76
	80	130	10.60
	90	130	12.11
	100	130	14.24
Limestone	60	130	11.52

70	130	12.38
80	130	14.81
90	130	16.98
100	130	19.17

On the basis of the data in Table 3, the relationship between the ROP and bit rotary speed is depicted in Figure 5. As shown in Figure 5, it is clearly observed that the ROP-bit rotary speed curves of sandstone and limestone are similar and there is a very remarkable linear relationship between them. With increasing bit rotary speed, the ROP increases significantly. Accordingly, the relation of the ROP with the bit rotary speed is quite consistent with the theoretical model.

Fig. 5 Relationship between the ROP and bit rotary speed

3.3.3 Influence law of the rock dynamic compressive strength on the ROP

To verify the relationship between the rock dynamic compressive strength and ROP, lab drilling tests for sandstone and limestone were conducted, and the relevant measuring parameters of the drilling process are shown in Table 2. Considering the combined effect of WOB and the rock dynamic compressive strength, the fitting process is conducted by utilizing equation (18), and the result is shown in Figure 6. As observed from the fitted curves in Figure 6, coefficient A in formula (18) is obtained. For sandstone, $A = 32.5$ and the fitting correlation coefficient is 0.996; for limestone, $A = 29.6$ and the fitting correlation coefficient is 0.998. Thus, it can be concluded that coefficient A remains constant when the bit type is determined. For roller cone bit IADC 517 used in the experiment, the theoretically calculated value of parameter A is 36.73 on the basis of formula (17) (where $k_1=k_2=k_3=3$; $R_0=0.10795$ m; $m_1=m_2=m_3=16$; $\phi_0=30^\circ$). Then, the

theoretical mathematical relationship between the working parameters (ROP, bit rotary speed and WOB) of the testing bit and rock dynamic compressive strength can be expressed as:

$$v = 36.73n_b \left(\frac{W}{\sigma_d} \right)^{3/2} \quad (19)$$

Notably, the experimental value of parameter A is close to the theoretically calculated value. Because of the tooth wear reducing bit efficiency in the drilling process, it is reasonable that the theoretical value of A is slightly larger than the experimental value.

Fig. 6 Relationship between the ROP and $(W/\sigma_d)^{3/2}$

3.3.4 ROP prediction

To elucidate that it is more appropriate to use the rock dynamic strength rather than the static strength in the ROP model, lab drilling tests for sandstone and limestone under the conditions of a constant WOB of 100 KN and five different rotary speeds were performed, and the test data are listed in Table 4. The measured rock static and dynamic strength are substituted into formula (19) to calculate the theoretical ROP values. The theoretical ROP values based on the rock dynamic strength, static strength and actual ROP values are also given in Table 4. According to the data in Table 4, the comparative diagrams of the three types of ROP values are shown in Figure 7.

Table 4 Calculated values and actual values of the ROP

Rock sample	Rotary speed	WOB (KN)	Actual ROP	ROP based on dynamic strength	Relative error	ROP based on static strength	Relative error

	(RPM)		(m/h)	(m/h)	(%)	(m/h)	(%)
Sandstone	50	100	4.92	5.42	10.14	7.39	50.24
	60	100	5.78	6.50	12.50	8.87	53.47
	70	100	6.53	7.58	16.17	10.35	58.48
	80	100	7.42	8.67	16.85	11.83	59.39
	90	100	8.47	9.75	15.15	13.30	57.09
Limestone	50	100	6.98	7.75	11.00	10.57	51.42
	60	100	8.17	9.30	13.80	12.68	55.24
	70	100	9.14	10.85	18.67	14.79	61.89
	80	100	10.62	12.39	16.73	16.91	59.23
	90	100	11.73	13.94	18.89	19.02	62.18

Fig. 7 Comparative graphs of the theoretical value and actual value of the ROP

As observed from Table 4 and Figure 7, it is obvious that the ROP values calculated based on the rock dynamic strength are closer to the actual ROP values than the ROP values calculated based on the rock static strength and that the average relative error of the ROP based on the rock dynamic strength is controlled within 15%, while the relative error of the ROP based on the rock static strength is above 50%, indicating that it is not applicable to predict the ROP for a roller cone bit using the rock static strength. Therefore, the ROP model considering the drilling parameters and rock dynamic compressive strength proposed in this paper is valid and can be used to predict the ROP with reasonable accuracy.

4. Conclusions

Roller cone bits are widely used in mining and oil and gas drilling. An accurate prediction of the ROP based on the rock properties and drill operational parameters is of vital importance for efficient drilling planning. In this study, a new and simple ROP model is established that is different from others in that the model considering the rock dynamic strength can truly reflect the practical working action of a roller cone bit to impact and break up rock on the bottom hole during the drilling process. Furthermore, lab drilling tests on sandstone and limestone were performed by using a full-scale bit drilling testing machine, and the influence rules of the WOB, bit rotary speed and rock dynamic strength on the ROP are analyzed. Summaries of the main conclusions that can be drawn from this study are as follows:

(1) The quantitative relationship between the volume of rock fragmentation generated by a single tooth and rock dynamic compressive strength is derived based on the elastic solution of Boussinesq's problem, and then, the new ROP model is developed, indicating that the relationship between the ROP and bit rotary speed is linear and the relationship between the ROP and ratio of WOB and rock dynamic compressive strength is $3/2$ power.

(2) The mathematical model is validated against the collected lab data, and the calculated data and lab data match closely, confirming that the developed relationship between the working parameters of the bit is reasonable.

(3) The error of the static model based on the rock static strength is approximately 45 to 50%, while the average error of the dynamic model based on the rock dynamic strength is approximately 15%, indicating that the dynamic model proposed in this paper shows a great advantage over the static model in ROP prediction.

(4) The newly developed ROP model integrating the main drilling parameters as well as rock dynamic strength is appropriate and can be utilized to conveniently and easily predict the drilling rate and is expected to provide useful guidance for optimizing drilling parameters required for enhancing overall drilling performance.

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