Model development and analysis of the evolution of coal permeability under different boundary conditions

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Abstract: Coal permeability is an important parameter for coalbed methane (CBM) production and CO₂-enhanced coalbed methane (ECBM) recovery. Coal permeability is mainly controlled by the structure of the coal. In this study, based on the pore-elasticity principle and the coal matrix-fracture interactions, we developed an analytical coal permeability model and its five forms under the corresponding specific boundary conditions, and four of them are validated by matching the corresponding field data or experimental data. The study presents a discussion of the permeability evolution for the different forms. The results show that the coal permeability under constant volume and constant effective stress conditions are controlled by the matrix sorption deformation and that the coal permeability under uniaxial strain and constant external stress conditions is governed by both the effective stress and the matrix sorption deformation. The constant volume model may be a special form of the constant effective stress model. For the same pressure drawdown, the permeability that is predicted under constant external stress conditions decreases more than that the permeability that is predicted under uniaxial strain conditions, and the permeability change induced by the matrix sorption deformation depended on the values of the internal swelling partition. The boundary conditions are important to the model, and different boundary conditions will lead to different evolutions of coal permeability. The permeability model can only be used to predict the evolution of permeability under the corresponding boundary conditions.

Keywords: Coal permeability; Boundary conditions; Internal swelling partition; Matrix sorption deformation; Effective stress

1 Introduction

Coalbed methane (CBM), known as gas, is generated during the formation of coal [1], and the CBM is stored in the coal seam [2, 3]. Threatening seriously on the safety of coal mine, methane is a clean and efficient energy [4]. China has a rich array of CBM resources that are buried at depths of less than 2000 m. China's geological CBM resources are $36.8 \times 10^{12} \text{ m}^3$, which is ranked third in the world [5, 6].

Coal permeability is an important parameter that is associated with CBM production and ECBM recovery. Coal permeability is mainly controlled by the structure of the coal. Coal consists of fractures and matrixes. The matrix contains many pores and significant amount of adsorbed methane, while the methane that is present in the fractures is free gas. The matrix permeability of coal is typically far less than the fracture permeability [7]. Thus, the permeability of CBM is dependent on its fracture system, which is a key property for understanding CBM production. Because of the gas desorption-shrink of coal matrix, the pore-elastic processes that are associated with CBM reservoirs are different from the processes that are associated with traditional oil and gas reservoirs [8]. Similar to traditional oil and gas reservoirs, the pressure of a CBM reservoir will gradually decrease during production, and the effective stress will increase as the methane is extracted from the coal. The coal permeability will decrease as a result of this process. However, much of the adsorbed methane diffuses from the matrix to the

fracture, which causes the coal matrix to shrink and the coal permeability to increase. Therefore, the effective stress and the sorption-induced matrix deformation have the opposite effect on the coal permeability.

When coal permeability is studied, the simplify coal structure can be helpful to highlight the impact of the fractures. The structure of coal can be simplified into regular models, such as a spherical model, a capillary tube model, a matchstick model, and a cube model (Fig. 1). In addition, the boundary conditions of the reservoir can be simplified. For example, they can be simplified to uniaxial strain with constant vertical external stress conditions or constant volume conditions. Uniaxial strain conditions are more widely used than constant volume conditions, and they can provide more convenience for the study of coal permeability evolution in a CBM reservoir. With continued CBM production, the overlying load remains approximately constant, and the horizontal displacements remain unchanged due to the horizontal stresses. Under these conditions, the boundary conditions can be assumed to be uniaxial strain with a constant vertical external stress. During the CBM production, the vertical displacement of the CBM reservoir is almost unchanged [9]. Under these conditions, the boundary conditions can be assumed to be constant volume conditions. Using a theoretical analysis and field surveys, Massarotto et al. [10] thought that the constant volume conditions are more appropriate for methane reservoirs. Due to limitations in the experimental conditions, in the laboratory, we always test the coal permeability under tri-axial stress conditions.

Fig. 1 The structure of coal and several simplified models

People established lots of coal permeability models over the past few decades, and their number is still growing. Based on their boundary conditions, the models can be divided into uniaxial strain conditions, constant volume conditions and tri-axial stress conditions (Fig. 2). Using the matchstick model and uniaxial strain conditions, Gray [11] is first one, who considered both the effective stress and the matrix swelling, when developing the permeability model. Two most common models (P&M and S&D) [12, 13] were developed using the same assumptions. Using a matchstick model, Ma et al. [9] developed their permeability equation for coal under constant volume assumption. Using a cube model and tri-axial stress assumption, several researchers [14-16] have proposed their coal permeability models. Using a porous medium and tri-axial stress assumption, other researchers [17-18] have proposed coal permeability models. Liu et al. [19] used a simple “free expansion + push back” approach to explain the relations of the different boundary conditions, and a series of permeability models have been developed to analyze the evolution of coal permeability under different boundary conditions. Constant volume conditions or uniaxial strain conditions are only assumed boundary conditions of CBM reservoirs. However, tri-axial stress boundary conditions are used in laboratory experiments of coal cores. By substituting specific boundary conditions, the tri-axial stress model can be converted into a model with constant volume, uniaxial strain or other boundary conditions.

Fig. 2 Different boundary conditions for coal permeability models

Assuming a constant volume, all the coal matrix sorption deformation should be used to the change of coal permeability [20]. Under uniaxial strain with a constant vertical external stress conditions, almost all of the

horizontal coal swelling deformation contributes to a change in permeability [21]. Most laboratory, many experiments are performed by keeping the external stress unchanged. Under these conditions, the external boundaries can be expanded both the horizontally and vertically. Robertson and Christiansen [7] found that P&M and S&D models, which were developed under uniaxial strain conditions, could not achieve good matches with the permeability data obtained by keeping the external stress unchanged. It is because that the interactions of fractures and matrixes are not considered [19], and further improvements have been demonstrated by Liu and his co-workers [20, 22-25]. Several studies [14, 16, 17, 21] have developed coal permeability models based on the assumption that only part of the deformation that is caused by swelling of the gas sorption-induced matrix contributes to the change in permeability, and the evolution of coal permeability under constant stress conditions was successfully explained by these proposed permeability models. Including the concept of only part of the swelling deformation contributing to the fracture aperture, Chen et al. [26] developed a phenomenological permeability model by combining the assumption of natural strain to define the contribution of effective stress to the fracture aperture, and his model can match the permeability data under constant stress conditions very well.

The boundary conditions are important to the permeability model, and the boundary conditions of permeability model need to be carefully considered. This study develops a new permeability model that considers the effective stress and matrix sorption deformation under tri-axial stress conditions, is developed. Five forms of the model are then derived using specific boundary conditions. Four of them are validated by matching the field data or experimental data that were obtained under the corresponding boundary conditions. Finally, the permeability evolution and the relationships of the model forms under different boundary conditions are discussed.

2 Development of the permeability model

2.1 Gas sorption-induced swelling deformation

Coal is a porous medium that is composed of both matrixes and fractures. Liu and Rutqvist [21] believe that the coal matrixes are connected with others by a coal-matrix bridge. Pone et al. [27] found that the coal matrix swelling would make the aperture of the fracture decrease. However, there are many inorganic minerals in the fractures, and they may prevent the coal matrix swelling to close the coal fractures [28, 29]. As a result, not all the matrix sorption deformation is used to change the fracture volume. We define the partition of the matrix sorption deformation that changes the fracture volume as the internal swelling partition (f), which ranges from 0 to 1. If the coal does not contain any fracture just like a matrix block, f is equal to 0. If coal matrixes completely separate between each other, f is equal to 1. The value for f is decided by the coal structure, and it may be controlled by the coal structure [17]. The parameter f could be obtained by contrasting the matrix sorption deformation and the coal block swelling deformation under the same conditions, and it also can be obtained by using permeability model fitting data.

Under unconstrained conditions, the relation between the matrix sorption strain and gas pressure is satisfied

with a Langmuir-type [30], as follows:

$$\varepsilon_m^S = \frac{\varepsilon_{\max}^S p}{p + p_\varepsilon} \quad (1)$$

$$\Delta \varepsilon_m^S = \varepsilon_m^S - \varepsilon_{m0}^S = \frac{\varepsilon_{\max}^S p}{p + p_\varepsilon} - \frac{\varepsilon_{\max}^S p_0}{p_0 + p_\varepsilon} = \frac{\varepsilon_{\max}^S p_\varepsilon (p - p_0)}{(p + p_\varepsilon)(p_0 + p_\varepsilon)} \quad (2)$$

The matrix volume is defined as V_m , and the fracture volume is defined as V_f . The bulk volume of the coal (V_b) can be expressed as $V_b = V_m + V_f$. The porosity of the coal (ϕ) is defined as $\phi = V_f / V_b$. The gas sorption-induced matrix sorption strains are included into the bulk and fracture strains [14, 16, 17]. The strain increments of bulk volume and fracture volume for the matrix sorption deformation can be described in terms of $d\varepsilon_b^S$ and $d\varepsilon_f^S$, respectively, as (positive in compression) [14]

$$d\varepsilon_b^S = -\frac{dV_b^S}{V_b} = -\frac{(1-f)V_m d\varepsilon_m^S}{V_b} = -(1-f)(1-\phi)d\varepsilon_m^S \quad (3)$$

$$d\varepsilon_f^S = -\frac{dV_f^S}{V_f} = \frac{fV_m d\varepsilon_m^S}{V_f} = \frac{f(1-\phi)d\varepsilon_m^S}{\phi} \quad (4)$$

2.2 Permeability model development

According to the effective stress principle, the mechanistic strains can be included in the bulk strain and the fracture strain. The strain increment of bulk volume ($d\varepsilon_b^E$) and the strain increment of fracture volume ($d\varepsilon_f^E$) for the effective stress of $\bar{\sigma}$ and p can be expressed as [31]

$$d\varepsilon_b^E = -\frac{dV_b^E}{V_b} = \frac{1}{K}(d\bar{\sigma} - \alpha dp) = \frac{1}{K}d\bar{\sigma} - \left(\frac{1}{K} - \frac{1}{K_m}\right)dp \quad (5)$$

$$d\varepsilon_f^E = -\frac{dV_f^E}{V_f} = \frac{1}{K_f}(d\bar{\sigma} - \beta dp) = \frac{1}{K_f}d\bar{\sigma} - \left(\frac{1}{K_f} - \frac{1}{K_m}\right)dp \quad (6)$$

Where $\alpha = 1 - K/K_m$ and $\beta = 1 - K_f/K_m$. From Eq. (3) and (4), we obtain

$$d\varepsilon_b^E - d\varepsilon_f^E = \left(\frac{1}{K} - \frac{1}{K_f}\right)(d\bar{\sigma} - dp) \quad (7)$$

If we consider $K \gg K_f$, the simplified expression is derived as

$$d\varepsilon_b^E - d\varepsilon_f^E = -\frac{1}{K_f}(d\bar{\sigma} - dp) \quad (8)$$

Considering that $1 \gg \phi$ ($\phi < 10\%$), the relationships between $d\varepsilon_b^S$ and $d\varepsilon_f^S$ can be derived as

$$d\varepsilon_b^s - d\varepsilon_f^s = -\frac{f}{\phi} d\varepsilon_m^s \quad (9)$$

The strain increments of bulk volume and fracture volume for the matrix sorption deformation and effective stress can be described as [17]:

$$d\varepsilon_b = d\varepsilon_b^E + d\varepsilon_b^s \quad (10)$$

$$d\varepsilon_f = d\varepsilon_f^E + d\varepsilon_f^s \quad (11)$$

The relationships between fracture porosity and strain can be expressed as[17, 32]:

$$\frac{\phi}{\phi_0} = \exp \left[\left(\int_{\varepsilon_{b0}}^{\varepsilon_b} d\varepsilon_b - \int_{\varepsilon_{f0}}^{\varepsilon_f} d\varepsilon_f \right) \right] \quad (12)$$

By substituting Eq. (10) and Eq. (11) into Eq. (12), Eq. (12) can be rewritten as follows:

$$\frac{\phi}{\phi_0} = \exp \left[\left(\int_{\varepsilon_{b0}}^{\varepsilon_b} d\varepsilon_b^E - \int_{\varepsilon_{f0}}^{\varepsilon_f} d\varepsilon_f^E \right) + \left(\int_{\varepsilon_{b0}}^{\varepsilon_b} d\varepsilon_b^s - \int_{\varepsilon_{f0}}^{\varepsilon_f} d\varepsilon_f^s \right) \right] \quad (13)$$

Then, by solving Eq. (8), (9) and (13), we obtain

$$\frac{\phi}{\phi_0} = \exp \left[-\frac{1}{K_f} (\Delta\bar{\sigma} - \Delta p) - \frac{f}{\phi} \Delta\varepsilon_m^s \right] \quad (14)$$

If $K_m \gg K$ and $\alpha \approx 1$, based on the Betti–Maxwell reciprocal theorem [28], we obtain

$$K = \phi K_f \quad (15)$$

By substituting Eq. (15) into Eq. (14), and Eq. (14) can be rewritten as

$$\frac{\phi}{\phi_0} = \exp \left[-\frac{1}{K_f} (\Delta\bar{\sigma} - \Delta p + fK\Delta\varepsilon_m^s) \right] \quad (16)$$

The relationship between permeability and fracture porosity can be described by the cubic law as:

$$\frac{k}{k_0} = \left(\frac{\phi}{\phi_0} \right)^3 \quad (17)$$

Then, substituting $K = \frac{E}{3(1-2\nu)}$, $C_f = \frac{1}{K_f}$, Eq. (16) and Eq. (2) into Eq. (17), and Eq. (17) can be rewritten

as

$$\frac{k}{k_0} = \exp \left\{ -3C_f \left[(\bar{\sigma} - \bar{\sigma}_0) - (p - p_0) + f \frac{E}{3(1-2\nu)} \frac{\varepsilon_{\max}^s p_\varepsilon (p - p_0)}{(p + p_\varepsilon)(p_0 + p_\varepsilon)} \right] \right\} \quad (18)$$

According to Eq. (18), if all the gas sorption-induced matrix deformation is considered by analogy to the thermos-elastic deformation, the internal swelling partition is equal to 0. Then, the Eq. (18) can be rewritten as

$$\frac{k}{k_0} = \exp \left\{ -3C_f \left[(\bar{\sigma} - \bar{\sigma}_0) - (p - p_0) \right] \right\} \quad (19)$$

Eq. (19) is equal to the permeability model developed by Cui and Bustin [32]. As we know, the permeability data are obtained under specific boundary conditions. Therefore, only the forms of permeability model under specific boundary conditions can be validated by these data. Under different specific boundary conditions, the different corresponding forms of Eq. (18) are derived.

(a) Constant volume conditions

For the constant volume conditions, the vertical and horizontal strain increments are equal to zero ($d\varepsilon_{bx} = d\varepsilon_{by} = d\varepsilon_{bz} = 0$). Then, the bulk volumetric strain increment is equal to zero ($d\varepsilon_b = d\varepsilon_{bx} + d\varepsilon_{by} + d\varepsilon_{bz} = 0$).

Substituting $d\varepsilon_b = 0$ into Eq. (10), we can obtain $d\varepsilon_b^E = -d\varepsilon_b^S$. According to Eq. (3) and Eq. (5), we obtain

$$(1-f)(1-\phi)d\varepsilon_m^S = \frac{1}{K}d\bar{\sigma} - \left(\frac{1}{K} - \frac{1}{K_m} \right)dp \quad (20)$$

Considering that $K_m \gg K$ and $1 \gg \phi$, we can find $d\bar{\sigma} - dp = (1-f)Kd\varepsilon_m^S$. Substituting this relation into Eq. (18), it can be rewritten as

$$\frac{k}{k_0} = \exp \left\{ -3C_f \left[\frac{E}{3(1-2\nu)} \frac{\varepsilon_{\max}^S p_\varepsilon (p - p_0)}{(p + p_\varepsilon)(p_0 + p_\varepsilon)} \right] \right\} \quad (21)$$

Under constant volume, both the two parts of the gas sorption-induced matrix deformation ($d\varepsilon_b^S$ and $d\varepsilon_f^S$) contribute to the change of coal permeability in different manners. Therefore, under constant volume conditions, there is no need to treat the two parts differently when the permeability is calculated.

(b) Uniaxial strain with constant vertical external stress conditions

The horizontal strain increments are equal to zero ($d\varepsilon_{bx} = d\varepsilon_{by} = 0$), and the vertical external stress remains constant ($d\sigma_z = 0$). Considering that $\alpha \approx 1$, the horizontal effective stress increments can be calculated by

$d\sigma_z^E = d\sigma_z - dp = -dp$. The bulk volumetric strain increment can be calculated by $d\varepsilon_b = d\varepsilon_{bx} + d\varepsilon_{by} + d\varepsilon_{bz}$. According

to Eq.(3), Eq.(5) and Eq.(10), and by assuming an isotropic sorption strain ($d\varepsilon_{mx}^S = d\varepsilon_{my}^S = d\varepsilon_{mz}^S = \frac{d\varepsilon_m^S}{3}$), $d\varepsilon_{bx}$, $d\varepsilon_{by}$

and $d\varepsilon_{bz}$ can be calculated by

$$\begin{cases} d\varepsilon_{bx} = d\varepsilon_{bx}^E + d\varepsilon_{bx}^S = \frac{d\sigma_x^E - \nu d\sigma_y^E - \nu d\sigma_z^E}{E} - (1-f) \frac{(1-\phi)}{3} d\varepsilon_m^S \\ d\varepsilon_{by} = d\varepsilon_{by}^E + d\varepsilon_{by}^S = \frac{d\sigma_y^E - \nu d\sigma_x^E - \nu d\sigma_z^E}{E} - (1-f) \frac{(1-\phi)}{3} d\varepsilon_m^S \\ d\varepsilon_{bz} = d\varepsilon_{bz}^E + d\varepsilon_{bz}^S = \frac{d\sigma_z^E - \nu d\sigma_x^E - \nu d\sigma_y^E}{E} - (1-f) \frac{(1-\phi)}{3} d\varepsilon_m^S \end{cases} \quad (22)$$

Substituting $d\varepsilon_{bx} = d\varepsilon_{by} = 0$ and $d\sigma_z^E = -dp$ into Eq. (22), by assuming problem symmetry ($\sigma_x^E = \sigma_y^E$) and

$1 \gg \phi$, $d\sigma_x^E = d\sigma_y^E$ can be calculated by

$$d\sigma_x^E = d\sigma_y^E = -\frac{\nu}{1-\nu} dp + \frac{E}{3(1-\nu)} (1-f) d\varepsilon_m^S \quad (23)$$

Then, substituting $d\sigma_z^E = -dp$ and Eq. (23) into Eq. (18), and Eq. (18) can be rewritten as

$$\frac{k}{k_0} = \exp \left\{ -3C_f \left\{ -\frac{(1+\nu)}{3(1-\nu)} (p - p_0) + \left[(1-f) \frac{2E}{9(1-\nu)} + f \frac{E}{3(1-2\nu)} \right] \frac{\varepsilon_{\max}^S p_\varepsilon (p - p_0)}{(p + p_\varepsilon)(p_0 + p_\varepsilon)} \right\} \right\} \quad (24)$$

If the internal swelling partition is equal to 0, the Eq. (18) can be rewritten as

$$\frac{k}{k_0} = \exp \left\{ -3C_f \left[-\frac{(1+\nu)}{3(1-\nu)} (p - p_0) + \frac{2E}{9(1-\nu)} \frac{\varepsilon_{\max}^S p_\varepsilon (p - p_0)}{(p + p_\varepsilon)(p_0 + p_\varepsilon)} \right] \right\} \quad (25)$$

Eq. (25) is equal to the permeability model developed by Cui and Bustin [32] under uniaxial strain with constant vertical external stress conditions.

(c) Constant external stress conditions

The increments of total external stresses are equal to zero ($\Delta\sigma_x = \Delta\sigma_y = \Delta\sigma_z = 0$). The increment of the mean stress is equal to zero, so $\Delta\bar{\sigma} = \bar{\sigma} - \bar{\sigma}_0$. Eq. (18) can be rewritten as

$$\frac{k}{k_0} = \exp \left\{ -3C_f \left[-(p - p_0) + f \frac{E}{3(1-2\nu)} \frac{\varepsilon_{\max}^S p_\varepsilon (p - p_0)}{(p + p_\varepsilon)(p_0 + p_\varepsilon)} \right] \right\} \quad (26)$$

(d) Constant fracture pressure conditions

The increment of the fracture pressure is equivalent to zero, $\Delta p = p - p_0 = 0$. Eq. (18) can be rewritten as

$$\frac{k}{k_0} = \exp \left\{ -3C_f (\bar{\sigma} - \bar{\sigma}_0) \right\} \quad (27)$$

(e) Constant effective stress conditions

The increment of the effective stress is equal to zero in the x, y or z directions. $\Delta\sigma_x^E = \sigma_x - p = 0$;

$\Delta\sigma_y^E = \sigma_y - p = 0$; $\Delta\sigma_z^E = \sigma_z - p = 0$. Then, the difference between the mean stress and the fracture pressure is

equal to zero, $(\bar{\sigma} - \bar{\sigma}_0) - (p - p_0) = 0$. Eq. (18) can be rewritten as

$$\frac{k}{k_0} = \exp \left\{ -3C_f \left[f \frac{E}{3(1-2\nu)} \frac{\varepsilon_{\max}^s p_e (p - p_0)}{(p + p_e)(p_0 + p_e)} \right] \right\} \quad (28)$$

3 Model validation

Field and experimental permeability data were used to validate the different forms of the model. The San Juan Basin lead the world in CBM production, and much of the available permeability data were obtained during CBM production. The boundary conditions of the actual reservoir could also be simplified to uniaxial strain conditions or constant volume conditions. The constant volume assumption requires more limiting conditions to be used, and the constant volume assumption should be further investigated [3]. The uniaxial strain assumption has been more widely used in the past, so the permeability data that were provided by McGovern [33] were only used to validate the model under uniaxial strain conditions. Thus, three sets of experimental data, which were obtained under constant external stress, constant fracture pressure and constant effective stress conditions, were used to validate the model under the corresponding boundary conditions.

3.1 Uniaxial strain conditions

A series of field permeability data in the San Juan basin were reported by McGovern [33]. These data have been used by many modelers to validate their permeability models [1, 9, 34-36]. Because of the limiting field conditions, it was impossible for all of the input parameters of the model to be directly obtained from the field. The specific values of the input parameters may directly influence the evaluation of the validity of permeability models. However, during the model validation process, the values that were used by the modelers were often varied within a specified range. To make the parameters more representative, we summarized the range of the parameters for coal in the San Juan Basin. The values and possible range for each parameter are shown in Table 1.

Table 1 Possible ranges of input parameters

In this paper, the mechanical properties ($E=2900$ MPa, $\nu=0.35$), adsorption constants ($\varepsilon_{\max}=0.01266$, $p_e=4.31$ MPa) and fracture compressibility ($C_f=0.1392$ MPa⁻¹) were taken directly from the Ref. [35]. The internal swelling partition (f) is assumed as a constant with a range from 0 to 1. A match was obtained by adjusting the internal swelling partition, and the results of this match are shown in Fig. 3. To further investigate the influences of matrix sorption deformation on fracture, the permeability curves, which were calculated for $f=0$ and $f=1$, are shown in Fig. 3, too.

Fig. 3 Field and modeled permeability data under uniaxial strain conditions

As shown in Fig. 3, the model with $f=0.20$ matches the permeability data well, which suggests that it is reasonable to assume f is a constant. Under uniaxial strain conditions, approximately 20% of the matrix sorption deformation contributes to a change in the fracture aperture. No matrix sorption deformation that contributes to the change in the coal permeability when $f=0$, and all the total matrix sorption deformation contributes to the change in

the coal permeability when $f=1$. Therefore, the change of permeability when $f=1$ is much larger than that when $f=0$ in the late period of CBM production. The coal permeability decreases with the effective stress decreasing, and the matrix sorption deformation has the opposite effect. The coal permeability ratio increases with a decrease in the gas pressure. During a drawdown in the gas pressure, the matrix sorption deformation has a greater impact on the permeability than on the effective stress.

3.2 Constant external stress conditions

Experimental permeability data for the Sulcis coal from Monte Sinni coal mine were reported by Pini et al. [45]. These data have been used by many modelers to validate their permeability models [14, 16, 26]. This study uses the data with CO_2 under constant external stress conditions (10MPa) to verify the proposed model. The mechanical properties ($E=1119$ MPa, $\nu=0.26$) were taken directly from Ref. [45]. Based on the sorption swelling data, the adsorption constants for CO_2 were calculated by the method which we have used in Ref. [14]. The fracture compressibility was also calculated with the permeability data under constant fracture pressure (0.5MPa and 1.8MPa) conditions provided by Pini et al. [45]. The input parameters for the model are listed in Table 2. Only the internal swelling partition (f) was assumed to a constant with a range from zero to one, and the results of the best-fit curve that were calculated with $f=0$ and $f=1$ are shown in Fig. 4.

Table 2 Input parameters for the model under constant external stress conditions

Fig. 4 Experimental and modeled permeability data under constant external stress conditions

When $f=0$, only the effective stress affects the coal permeability, and the coal permeability ratio will increase with an increase in the gas pressure. This result usually occurs because all of the external stresses are constant, and the effective stress increases during this process. When $f=1$, the effective stress and matrix sorption deformation both affect the permeability. The permeability will decrease slowly during the early stages. Then, it will increase at a higher pressure stage (Fig. 4). During this process, the effective stress decreases linearly, and the matrix sorption deformation shows a Langmuir-type trend. During an increase in the gas pressure, the impact of the effective stress on permeability is larger than that of the matrix sorption deformation. As the pressure increases, the matrix sorption deformation will gradually approach a specific value, and the change in the permeability ratio will decrease. The model matches the experimental data when the internal swelling partition is equal to 0.15, which also suggests that not all of the matrix sorption deformation contributes to the change in fracture aperture.

3.3 Constant fracture pressure and constant effective stress conditions

Pan et al. [46] measured the gas (CH_4 , CO_2 and He) permeability under different gas pressures and external pressures. These data have been used by many modelers to validate their permeability models [16, 17]. Two series of data for CH_4 at a pore pressure of 3.5 MPa and an effective stress of 4MPa were chosen to evaluate the model expressed as Eq. (27) and Eq. (28). The mechanical parameters and the adsorption constants were taken directly from Ref. [46], and the average fracture compressibility for CH_4 was selected (Table 3). We should note that the

swelling deformation of coal block with fractures were measured. Therefore, based on Eq. (3), the maximum matrix sorption-induced strain (ε_{max}^s) can be calculated by $\varepsilon_{max}^s = \varepsilon_{bmax}^s / (1-f)$. Because the swelling deformation of coal block is not equal to 0, the internal swelling partition is not equal to 1. The matching result when $f=0$ is shown in Fig. 6.

Table 3 Input parameters for the model under the two conditions

Fig. 5 Experimental and modeled permeability data under constant fracture pressure conditions

Fig. 6 Experimental and modeled permeability data under constant effective stress conditions

The model for the two boundary conditions can match the corresponding permeability data very well (Fig. 5 and Fig. 6). Wang et al. [16] used the data in Fig. 5 to evaluate their proposed permeability model and found that the modeled permeability results were lower than the measured ones. However, this result does not occur when our proposed permeability model is used, which suggests that our proposed model can make a better description of the processes. Only 0.33% of the matrix sorption deformation contributed to the change in fracture aperture when keeping the effective stress unchanged (Fig. 6). Neither the effective stress nor the matrix sorption deformation affect the permeability when $f=0$, and therefore, the permeability ratio will be one. Comparing Eq. (28) and Eq. (21), Eq. (28) is equivalent to Eq. (21) when $f=1$. Therefore, the constant volume model may be a special form of the constant effective stress model.

4 Discussions

Many permeability models have been developed over the past 30 years [47-49]. These permeability models were verified by using field or experimental data. During the verification process, appropriate parameters should be used by the models to match the data. However, the reasonable values for each input parameter often vary within a certain range. Without accurate values for the input parameters, the models can only be validated to a limited extent, which makes it difficult to compare the advantages of different permeability models. As we have observed, the ability of new permeability models to match the data is often satisfactory.

The primary purpose of developing models is not to compare the advantages with other permeability models. The more important purpose is better for understanding the coal permeability evolution. Thus, the different forms of the model were compared. All of the input parameters are listed in the last line of Table 1, and the initial gas pressure ($p_0=11.03$ MPa) was selected. The internal swelling partition ($f=0.20$) has been obtained under uniaxial strain conditions. We think that its value is decided by the coal structure, so the same values are chosen under other boundary conditions. Fig. 7 shows the results of the coal permeability evolution.

Fig. 7 Evolution of coal permeability under different boundary conditions

Because of the differing boundary conditions, the change of the permeability ratio can vary even with the same input parameters. This behavior occurs because of differences in the interactions between the coal fractures and the matrix under different boundary conditions. Comparing with the other four conditions, the permeability under constant fracture pressure conditions only affects by the external stress, so the permeability ratio is constant

of 1. The permeability ratios for the constant volume and constant effective stress conditions both increase with the decrease of gas pressure, and the change of the permeability ratio is greater constant volume conditions than that for constant effective stress conditions. The permeability ratios of both the two boundary conditions are only affected by the matrix sorption deformation. Only 20% of the matrix sorption deformation contributes to the change in the fracture aperture when keeping the effective stress unchanged, and the other part of the matrix sorption deformation contributes to change coal bulk. However, both the two parts of the gas sorption-induced matrix deformation contribute to the change of coal permeability in different manners under constant volume. Comparing Eq. (28) with Eq. (21), Eq. (28) is equal to Eq. (21), When $f=1$. As shown in Fig. 7, the permeability ratios for uniaxial strain and constant external stress both decrease and then increase as the gas pressure decreases. This behavior is a combined result of the effective stress and the matrix sorption deformation. The permeability switches instantaneously because the matrix sorption deformation is the leading factor for the permeability at a high gas pressure. Then, the effective stress becomes the leading factor due to gas pressure decrease.

To further evaluate the impacts of the two factors on the permeability under the two boundary conditions, the rebound pressures (p_{rb}) are calculated (Table 4). The effects of f on permeability are analyzed and shown in Fig. 8.

Table 4 Coefficients and rebound pressures of the model under uniaxial strain and constant external stress conditions

For uniaxial strain or constant external stress, permeability is controlled by the two competing parts. The coefficient of the effective stress term is $\frac{1+\nu}{3(1-\nu)}$ for uniaxial strain conditions, and one for constant external stress conditions. In Table 1, Poisson's ratio ranges from 0.2 to 0.46, and the coefficient of one is greater than $\frac{1+\nu}{3(1-\nu)}$ (0.5 to 0.94). For the same pressure drawdown, the predicted permeability decreased more under constant external stress conditions more than under uniaxial strain conditions.

The coefficients of the matrix sorption deformation term are $(1-f)\frac{2E}{9(1-\nu)} + f\frac{E}{3(1-2\nu)}$ and $f\frac{E}{3(1-2\nu)}$. Because $0 \leq f \leq 1$, the range of the coefficient for matrix sorption deformation under uniaxial strain conditions is between 913.21 to 3222.19, that under constant external stress conditions is between 0 to 3222.19. Therefore, the range of the coefficient for matrix sorption deformation under constant external stress conditions is larger than that under uniaxial strain conditions.

The rebound pressure is the switch that occurs when the permeability first decreases and then increases as the gas pressure is drawn down, and it can describe whether the effective stress or the matrix sorption deformation plays a major role in the resulting permeability. When $0 \leq f \leq 1$, the rebound pressure for uniaxial strain ranges from 4.53 to 11.63 MPa. When $f=0$, only the effective stress affects the permeability under constant external stress conditions, so the rebound pressure could be regarded as infinitesimal. Therefore, the rebound pressure for constant external stress conditions is less than or equal to 8.95 MPa.

Fig. 8 Evolution of coal permeability under uniaxial strain and constant external stress conditions

As shown in Fig. 8, an intersection in the permeability is predicted under the two boundary conditions with decreasing gas pressure. The permeability ratios of the two boundary conditions are both affected by the two factors (effective stress and matrix deformation), and only the coefficients are different (Table 4). Because the forms are so similar, some researchers have used models that are derived under uniaxial strain assumptions to match data that were tested by keeping the constant external stress unchanged [7, 14, 20, 50]. Although the permeability ratios of the two boundary conditions are affected by the same factors, and the matrix sorption deformation factors do not completely contribute to the fracture aperture. However, it is unreasonable that data that were obtained under constant external stress conditions are matched by a uniaxial strain model. The boundary conditions of the permeability model need to be carefully considered to avoid similar mistakes in the future. Each permeability model has its own application conditions and can only be used to predict the permeability evolution under the corresponding boundary conditions.

5 Conclusions

Coal permeability is an important parameter for CBM production and ECBM recovery. The permeability is dependent on the coal's fracture system. The interactions of fractures and the matrix make coal permeability more mysterious. In this study, the gas sorption-induced matrix sorption strains are included in the bulk and fracture strains. Based on the principle of pore-elasticity, we developed an analytical coal permeability model and its five forms under the corresponding specific boundary conditions. Four of them were validated by matching the model results to the field data or experimental data, which were obtained under the corresponding boundary conditions. All of the model results agreed well with the data.

The evolutions of permeability for the different forms of the model are discussed using parameters from the San Juan Basin. The change of the permeability ratio differs under different boundary conditions due to the differences in the interactions between the coal fractures and the matrix. Under constant volume or constant effective stress conditions, the coal permeability is only controlled by the matrix sorption deformation, and the constant volume conditions may be a special form of the constant effective stress conditions. Under uniaxial strain and constant external stress conditions, the coal permeability is governed by both the effective stress and the matrix sorption deformation. For the same pressure drawdown, the permeability that is predicted under constant external stress conditions decreases more than that the permeability that is predicted under uniaxial strain conditions. The coefficient for matrix deformation under constant external stress conditions is larger than that under uniaxial strain conditions, so the changes of permeability that are induced by the matrix sorption deformation depended on the values of f . With $0 \leq f \leq 1$, the rebound pressure for uniaxial strain is from 4.53 to 11.63 MPa, and the rebound pressure for constant external stress conditions is less than or equal to 8.95 MPa.

The goal of this paper was not only to obtain a better permeability model by matching field or experimental data, instead, it was to reveal the mechanisms that are associated with the evolution of coal permeability. The

boundary conditions of the permeability model must be carefully considered carefully, and the model can only be used to predict the evolution of permeability under the corresponding boundary conditions.

Nomenclature

C_f	Fracture compressibility (MPa^{-1})
E	Young's modulus (MPa)
f	Internal swelling partition of Matrix sorption deformation (dimensionless)
k	Coal permeability (Darcy)
K	Bulk modulus (MPa)
p	Fracture pressure (MPa)
p_e	Langmuir-type matrix sorption strain constant (MPa)
p_{rb}	Rebound pressure (MPa)
ν	Poisson's ratio (dimensionless)
V	Volume (m^3)
α	Biot's coefficient (dimensionless)
β	Effective coefficient of fracture (dimensionless)
ε	Strain(dimensionless)
ϕ	Fracture porosity (dimensionless)
$\overline{\sigma}$	Mean stress (MPa)
σ	Stress (MPa)

Subscripts and superscripts

0	The initial state
b	Coal block
f	Coal fracture
m	Coal matrix
S	Sorption-induced strain
E	Effective stress
max	Maximum
x, y, z	Coordinate axes

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Figure captions

Fig. 1 The structure of coal and several simplified models

Fig. 2 Different boundary conditions for coal permeability models

Fig. 3 Field and modeled permeability data under uniaxial strain conditions

Fig. 4 Experimental and modeled permeability data under constant external stress conditions

Fig. 5 Experimental and modeled permeability data under constant fracture pressure conditions

Fig. 6 Experimental and modeled permeability data under constant effective stress conditions

Fig. 7 Evolution of coal permeability under different boundary conditions

Fig. 8 Evolution of coal permeability under uniaxial strain and constant external stress conditions

Table captions

Table 1 Possible ranges of input parameters

Table 2 Input parameters for the model under constant external stress conditions

Table 3 Input parameters for the model under the two conditions

Table 4 Coefficients and rebound pressures of the model under uniaxial strain and constant external stress conditions

Figure1

(a) Real structure of coal

Sphere model

Capillary tube model

Matchstick model

Cube model

(b) Simplified models of coal

Figure2

Constant volume conditions

Uniaxial strain conditions

Tri-axial stress conditions

Figure3

Figure4

Figure5

Figure6

Figure7

Figure8

Table 1 Possible ranges of input parameters

Reference	Young's modulus (E), MPa	Poisson's ratio (ν)	Fracture compressibility (C_f), MPa ⁻¹	Maximum matrix sorption-induced strain (ϵ_{max})	Langmuir-type matrix sorption strain constant (p_c), MPa
Seidle et al. [37, 38]	3450	0.33	0.062-0.5133	0.0136	3.45
Harpalani and Chen [39]	-	-	-	0.027	3.45
Levine [30]	2070-4140	0.22-0.42	-	0.0075-0.0136	4.3-6.3
Zahner [40]	-	-	-	0.0051-0.0439	2.1-5.3
Mavor and Vaughn [41]	3593	0.21	-	0.0127	4.31
Palmer and Mansoori [12]	855-3070	0.39	-	0.0128	4.31
Palmer et al. [42]	-	0.2-0.46	-	0.0051-0.0439	2.1-5.31
Palmer [34]	2069	0.35	-	0.033	3.45
Shi et al. [13]	2900	0.3, 0.35	0.266-0.290	0.01266	2.55, 4.31
Shi and Durucan [35]	2900	0.35	0.09135-0.1392	0.01266	4.31
Shi and Durucan [36]	2900	0.35	0.116-0.58	0.01266	4.31
Shi and Durucan [43]	2350-2550	0.25-0.35	0.194-0.485	0.01075	4.16
Liu and Rutqvist [21]	2900	0.3	0.43	0.0129	2.55
Liu et al. [44]	3520	0.32	0.092	0.01075	4.16
Liu and Harpalani [1]	2900	0.35	0.075-0.15	-	-
Range	855-4140	0.2-0.46	0.062-0.5133	0.0051-0.0439	2.1-6.3
Selected value	2900	0.35	0.1392	0.01266	4.31

Table 2 Input parameters for the model under constant confining stress conditions

Parameters	Value
Young's modulus (E), MPa	1119
Poisson's ratio (ν)	0.26
Fracture compressibility (C_f), Mpa ⁻¹	0.19126
Maximum matrix sorption-induced strain (ϵ_{max})	0.05187
Langmuir-type matrix sorption strain constant (p_e), MPa	2.913
Internal swelling partition (f)	0~1

Table3

Table 3 Input parameters for the model under the two conditions

Parameters	Value
Young's modulus (E), MPa	791.2
Poisson's ratio (ν)	0.42
Fracture compressibility (C_f), Mpa ⁻¹	0.045325
Maximum coal block sorption-induced strain (ϵ_{bmax})	0.01
Langmuir-type matrix sorption strain constant (p_e), MPa	8.9
Internal swelling partition (f)	0~1

Table 4 Coefficients and rebound pressures of the model under uniaxial strain and constant confining stress conditions

Boundary conditions	Coefficient of effective stress	Coefficient of matrix swelling deformation	Rebound pressure equation	p_{rb} ($E=2900\text{Mpa}$, $\nu=0.35$)
Uniaxial strain condition	$\frac{1+\nu}{3(1-\nu)}$	$(1-f)\frac{2E}{9(1-\nu)}+f\frac{E}{3(1-2\nu)}$	$\sqrt{(1-f)\frac{2E}{3(1+\nu)}+f\frac{E(1-\nu)}{3(1-2\nu)(1+\nu)}\varepsilon_{\max}^s p_{\varepsilon}-p_{\varepsilon}}$	$4.53\leq p_{rb}\leq 11.63(0\leq f\leq 1)$
Constant confining stress condition	1	$f\frac{E}{3(1-2\nu)}$	$\sqrt{f\frac{E}{3(1-2\nu)}\varepsilon_{\max}^s p_{\varepsilon}-p_{\varepsilon}}$	$p_{rb}\leq 8.95(0\leq f\leq 1)$