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Abstract:

Ultrahigh molecular weight polyethylene (UHMWPE) has remained the primary polymer used in hip, knee and shoulder replacements for over 50 years. Recent case studies have demonstrated that catastrophic fatigue fracture of the polymer can severely limit device lifetime and are often associated with stress concentration (notches) integrated into the design. This study evaluates the influence of notch geometry on the fatigue of three formulations of UHMWPE that are in use today. A linear-elastic fracture mechanics approach is adopted to evaluate crack propagation as a function of notch root radius, heat treatment and Vitamin E additions. Specifically, a modified stress-intensity factor that accounts for notch geometry was utilized to model the crack driving force. The degree of notch plasticity for each material/notch combination was further evaluated using finite element methods. Experimental evaluation of crack speed as a function of stress intensity was conducted under cyclic tensile loading, taking crack length and notch plasticity into consideration. Results demonstrated that crack propagation in UHMWPE emanating from a notch was primarily affected by microstructural influences (cross-linking) rather than differences in notch geometry.

Keywords: UHMWPE, notch fatigue, fatigue crack propagation, cross-linking, Vitamin E

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Introduction

Ultrahigh molecular weight polyethylene (UHMWPE) has remained the polymer of choice for total joint replacements (TJRs) in hips and knees for over half a century [1,2]. The polymer's long chains (with a molecular weight of 4-6 million g/mol) are arranged in a semi-crystalline structure, with roughly 50% as ordered crystalline lamellae and the balance occupying an amorphous state [3–5]. This morphology contributes to UHMWPE's impressive strength and ductility particularly at an *in vivo* operating temperature of 37°C, well above its glass transition temperature, T_g , of approximately -80°C [6,7]; specifically, at physiological temperatures, virgin UHMWPE typically displays yield and tensile strengths of, respectively, $\sigma_y \sim 24$ MPa and $\sigma_{UTS} \sim 50$ MPa, with a failure strain of $\epsilon_f \sim 4$. The polymer's high strength and ductility, coupled with its biocompatibility, has fueled its continued use as the primary soft bearing component in total joint replacements (TJRs) since the late 1960s.

Even with this long history, UHMWPE has often been blamed for limiting the lifespan of TJRs. This has motivated modern microstructural modifications to improve its durability, such as irradiation cross-linking to enhance its wear resistance and post-irradiation annealing to mitigate oxidative embrittlement [3,4,6,8]. Despite these changes, reports of fatigue and fracture of UHMWPE components in hips and knees have been well-documented in the recent literature (**Figure 1**) [9–19]. Specifically, an increased susceptibility to fatigue-crack propagation in UHMWPE has been linked to the changes in microstructure that accompany both irradiation cross-linking and above-melt ($> T_{melt} = 137^\circ\text{C}$) annealing; indeed, this has contributed to higher incidences of catastrophic device failure over the past two decades [6,7,20–22].

Typical examples where the design of components can play a critical role in the fatigue failure of TJRs are shown in **Figure 1**. These devices are often designed with notches which play a particular role in enabling fixation (hip acetabular liner locking mechanisms) or kinematic stability (knee tibial posts) in hip and knee prostheses; however, cracks can often be initiated at these locations, leading to catastrophic failure of the device [11,23]. Despite this, few studies have examined the specific role of notch geometry in the fatigue behavior of this polymer. In fact, Sobeiraj and coworkers have presented the only known analysis of notch effects on the

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4 fatigue resistance of several modern UHMWPE material formulations using a stress-total life ($S-N$) approach [24]. However, significant scatter in their results severely limits the statistical
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7 interpretation of the data, suggesting that such an $S-N$ approach may not be the best
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9 methodology for the notch fatigue evaluation of UHMPWE [24,25].
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13 Linear-elastic fracture mechanics (LEFM) provides an alternative means to evaluate the
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15 fatigue behavior of materials. Fatigue-crack propagation studies, including several that have
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17 successfully been performed on modern formulations of UHMWPE [6,7,20,26,27], utilize LEFM
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19 to evaluate the growth of ‘sharp’ cracks in standard specimen geometries under cyclic loading.
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21 Modification of these standard specimens, to include varying stress concentrations from which
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23 cracks advance, must be accompanied by an adjustment to LEFM theory to consider the
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25 changing local stress environment surrounding the notch. Several approaches have been
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27 developed to calculate the stress intensity for such situations that consider not only the far-field
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29 applied stress and crack length, but also the local plastic field associated with the notch
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31 geometry [28–33]. However, a lack of consistency across notch and specimen types in these
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33 analyses presents difficulties in choosing an appropriate method for the study for UHMWPE.
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35 Furthermore, such linear elastic approaches can be called into question when considering the
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37 degree of notch plasticity that can accompany a propagating crack [28].
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40 Here, we utilize an approach, initially used by Dowling [28], to compare the rates of fatigue-
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42 crack propagation at fixed intensity ranges ahead of several clinically-relevant notch-root radii.
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44 Experimental evaluations of each notch geometry are presented for three modern material
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46 formulations of UHMWPE to isolate the individual effects on crack behavior of design *vs*
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48 material composition and structure. Uniaxial stress-strain data are also examined to calculate
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50 the degree of notch-tip plasticity for each material-notch combination. Based on this
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52 experimental study, we demonstrate how the influence of microstructural on crack growth
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54 behavior far outweighs the design factors (notch geometries) for UHMWPE materials employed
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56 as components in TJRs.
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Methods

Materials: Three clinically-relevant modern material formulations of compression-molded UHMWPE (GUR 1020 resin, Orthoplastics, U.K.) were investigated:

- virgin UHMPWE (hereafter termed **UHMWPE**)
- highly cross-linked (75 kGy) and remelted UHMWPE (termed **RXLPE**)
- Vitamin E blended (0.1 wt.%)/highly cross-linked (75 kGy) UHMPWE (termed **VXLPE**).

Uniaxial tensile properties: True stress-strain data were obtained for all three UHMPWE materials. Five dog-bone specimens from each material cohort were displaced at 30 mm/min and 150 mm/min in accordance with ASTM and ISO recommendations (ISO 527-1 [34], ISO 527-2 [35], ASTM D638 [36]) on an Shimadzu AGS-X electromechanical load frame (Kyoto, Japan), with displacement data measured using a custom dual video extensometer system to obtain true stress/strain values. The raw data were processed in a custom Matlab code (R2012a, Nattick Massachusetts) to extract material properties. Statistical comparison of values was performed using a Student's t-test (95% confidence interval).

Fatigue Testing: Fatigue-crack propagation rates were measured on compact-tension, C(T), specimens (**Figure 2**), with dimensions in accordance with previous studies on UHMWPE [6,20,37] based on ASTM Standard ASTM E647 [38,39], *i.e.*, with a width $W \sim 25.4$ mm, a length $L \sim 31.8$ mm, and an out-of-plane thickness $B \sim 8.2$ mm. Initial notch lengths were kept consistent at $c_0 = 8.89$ mm, although notch geometries were varied to achieve the following five notch-tip radii (ρ): 0.13 mm ("sharp" specimen), 0.75 mm, 1 mm, 2 mm and 3 mm. The latter four radii were determined from measurements of notch-like features seen on hip acetabular liners and knee tibial inserts from a variety of manufacturers.

Before testing, a pre-crack 0.3 - 0.5 mm in length was created at the tip of each notch using a razor blade [4]. Samples were cycled in an Instron 8871 servo-hydraulic system (Norwood, MA) using a load-controlled sinusoidal wave function at a frequency of 5 Hz. Testing was performed at room temperature with an air cooling system to minimize hysteretic heating [4,5]. The cyclic load was increased periodically to produce an increasing ΔK , while always maintaining a

constant load ratio (minimum load / maximum load) of $R = 0.1$. Testing was paused at regular intervals to measure crack lengths using a variable magnification optical system (Infinivar CFM-2/S, Boulder, Colorado, 5 $\mu\text{m}/\text{pixel}$), a digital CCD video camera (Sony XCD-SX910, Tokyo, Japan) and custom LabVIEW (v. 13.0.0, Austin, Texas) image capture code. Crack speed (da/dN) was calculated using a secant method [38]. For each material and notch combination, 3-5 specimens were tested. The Paris equation was used to map the fatigue-crack propagation rates as a function of stress-intensity range, ΔK [6].

To account for the presence of the notches, Dowling's approach for blunt-notched C(T) specimens was determined to be the most appropriate methodology for approximating a modified stress-intensity factor, K [28]. Within the vicinity of the notch, the driving force was modeled as that for a short edge crack (K_s):

$$K_s = 1.12 k_t \sigma \sqrt{\pi l} , \quad (1)$$

where k_t is the stress-concentration factor of the notch, σ is the far-field (applied) stress, and l is the crack length emanating from the notch tip. Cracks that propagated outside the "notch-affected zone" conform to standard linear-elastic stress-intensity solutions for long cracks (K_L) in C(T) specimens under an applied load, P [40], *via*:

$$K_L = \frac{P}{B\sqrt{W}} f(\alpha) , \quad (2)$$

where $\alpha = \frac{a}{W}$, the ratio of crack length a to width W , $a = c_0 + l$, and

$$f(\alpha) = \frac{(2+\alpha)}{(1-\alpha)^{1.5}} (0.886 - 4.64 \alpha - 13.32 \alpha^2 - 14.72 \alpha^3 - 5.6 \alpha^4) . \quad (3)$$

The notch-affected zone size for a given notch geometry extends to a point where the stress intensities determined from Eq. 1 for the notch (K_s) and Eq. 2 for the long crack are equal; the crack length here is known as the "transition length" (**Figure 3**). The notch-affected zone sizes for each of the four blunt-notch C(T) specimens shown in **Figure 2** are listed in **Table 1**. Further details are given in the Appendix.

The notch geometry not only influences the linear elastic stress fields as addressed by Dowling [28], but also the degree of notch plasticity that engulfs a propagating crack. The size of the notch plastic zone as a function of maximum applied load was determined computationally using 3-D finite element models of each C(T) specimen. Uniaxial true stress-strain data recorded at 150 mm/min (more aligned with fatigue testing conditions) were used to construct constitutive behavior for all three materials. Models of the four blunt C(T) specimens in **Figure 2** contained on average 14,000 to 30,000 20-node quadratic elements, with a biased mesh near the crack tip (**Figure 4**). A load was applied in the vertical direction at the pinhole and the local equivalent (von Mises) stresses compared to the yield strength of each material. The notch plastic-zone size was defined as the horizontal distance ahead of the notch root where the equivalent stresses first drop below the yield strength. Notch plastic-zone sizes were recorded as a function of applied loads for each material-notch combination for loads varying from 200 to 500 N (the maximum loads used during experimental evaluation).

Where cracks lengths were longer than the notch plastic-zone size at a given applied maximum load, their growth was assumed to be subjected to small-scale yielding conditions and their extension was categorized as “elastic” crack growth. At a given maximum load, cracks that were smaller in length than the notch plastic-zone size tended to be engulfed by notch plasticity; their extension was termed as “plastic”. **Figure 5** summarizes these categorizations of crack growth based on the degree of influence of the notch field.

Fractography: Fracture surfaces of C(T) specimens from each material cohort were examined using scanning electron microscopy, SEM, (Hitachi TM-1000, Tokyo, Japan). Prior to imaging, each surface was sputter-coated with gold-vanadium to enhance the visualization of the surface.

Results

Tensile Properties: Uniaxial tensile properties for the three UHMWPE materials were computed from true stress/strain curves (**Figure 6**) and are listed in **Table 2**. The VXLPE material displayed the highest modulus and yield strength, followed by UHMWPE and RXLPE. Both

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4 RXPLE and VXPLE exhibited significantly lower ultimate tensile properties than UHMWPE;
5 similar to reports in the literature [3,5,6,41], this reflects the reduced ductility of these
6 formulations due to cross-linking.
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11 *Fatigue-crack propagation behavior:* The variation in fatigue-crack propagation rates, da/dN , with
12 the stress-intensity range, ΔK , for the three formulations of UHMWPE is plotted in **Figure 7**.
13 Distinctions in crack growth behavior between all three formulations are apparent, with virgin
14 UHMWPE demonstrating the most resistance to fatigue-crack propagation, RXLPE the least
15 resistance to crack growth, and VXLPE in between the two other materials. Crack growth data
16 obtained ahead of varying notch-root radii demonstrate clear overlap with sharp notch data
17 within each material cohort.
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25 To isolate the influence of the notch-affected zone, short and long crack growth relative to
26 the notch-affected zone are remapped in **Figure 8** for the RXLPE and virgin UHMWPE
27 materials. The majority of short crack growth occurred within the notch plastic zone for all
28 materials, leading to significantly more scatter than “elastic” data, especially for UHMWPE. As
29 seen in **Figure 7**, this scatter does generate more overlap in fatigue-crack propagation rates at
30 lower ΔK values, but does not eliminate distinctions between the different material
31 formulations.
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40 *Fractography:* SEM analysis of representative fracture surfaces from each specimen cohort are
41 shown in **Figure 9**. All images were recorded for cracks roughly 1 to 1.5 mm ahead of the notch
42 tip, and are thus representative of “elastic” crack growth ahead of the notch-affected zone. The
43 fracture surfaces of the virgin UHMWPE show more evidence of ductility than the RXLPE and
44 VXLPE fracture surfaces in the form of distinct features, with criss-cross markings aligned at 45°
45 angles and demonstrated rippling associated with larger surface strains before rupture
46 [6,7,27,42–44]. Such features have been associated with local Mode III damage and the gradual
47 accumulation of damage during cyclic loading in a region ahead of the crack tip known as a
48 “process zone” [45,46]. Fracture surfaces from both the RXLPE and VXLPE materials show a
49 marked reduction in these ductile features, consistent with their higher strength from cross-
50 linking. It should be noted that the striations apparent on all SEM images are not fatigue
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4 striations concomitant with the demarcation of the cyclic crack advance each cycle; the size of
5 these striations do not increase with increasing ΔK and they have been recorded for both cyclic
6 [42] and quasi-static [43] crack extension in UHMWPE polymeric materials. Such striations have
7 been seen in many semi-crystalline polymers, and have been attributed to accumulated damage
8 or discontinuous crack growth during cyclic loading or time-dependent creep [47].
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15 **Figure 10** compares fracture surfaces for short cracks growing inside the notch plastic zone
16 with long “elastic” cracks for the RXLPE and UHMWPE materials. No obvious distinctions
17 between “plastic” and “elastic” growth are apparent; distinctions between material
18 formulations are similar to those seen in **Figure 9**. Similar results were obtained for all notch
19 geometries.
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24 25 Discussion

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28 Most UHMWPE components used in TJRs contain stress concentrations in their design to
29 improve stability or enable fixation of devices. However, these features can also serve as sites
30 for crack initiation and subsequent propagation, resulting in catastrophic *in vivo* fracture and
31 the need for immediate revision surgery [10]. Changes to UHMWPE microstructure have
32 exacerbated the incidence of TJR fracture in recent years, leading many to suggest that changes
33 in notch geometry may offset tradeoffs in material performance [11,23,48,49]. This work
34 represents the first LEFM-based effort to characterize notch geometry effects on cyclic crack
35 growth in modern UHMWPE formulations. Despite the fact that such notches can certainly
36 promote the initiation of fatigue cracks, our results indicate that the crack-propagation behavior
37 in UHMWPE materials is primarily influenced by the nature of the microstructure, regardless of
38 notch geometry or degree of notch plasticity.
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51 Dowling’s approach was adopted to ascertain the local crack-driving forces near a notch tip,
52 which assumed the existence of a notch-affected zone that scaled with the notch geometry [28].
53 Fatigue-crack propagation data gathered for long cracks outside this theoretical region of
54 influence of the notch were found to be congruent across all notch geometries, including
55 standard sharp notches, consistent with Dowling’s analysis. Short cracks growing within the
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notch-affected zone were often engulfed by notch plasticity, yet still exhibited reasonable overlap with sharp notch data when mapped with Dowling's modified LEFM parameters. This implies that while the local driving force (K) near a notch is dependent on notch geometry, the inherent mechanisms of cyclic crack growth in UHMWPE do not change significantly for cracks growing under the influence of the elasticity and plasticity associated with the notch field.

Short crack growth within the notch-affected zone exhibited significant scatter at low ΔK values. This is likely due to limitations in using linear elastic parameters to characterize stress intensity within the notch plastic zone, as well as limitations with using an optical crack growth measurement technique. Nonetheless, short cracks did not exhibit markedly higher crack velocities compared to long cracks at a given ΔK in these UHMWPE materials, as has been documented in metals for short cracks growing ahead of notches [50–52] or even for sharp small cracks within the material bulk [53]. ~~This implies that within areas of notch plasticity, short crack growth nominally obeys the crack length similitude conditions imposed by LEFM, and thus may not be subject to microstructural influences or crack closure effects that have been shown to accelerate or decelerate short crack growth in other materials [51–53].~~ It is possible that the plastic zone surrounding short cracks causes deceleration of crack growth, counteracting microstructural influences or lack of crack closure effects that have been shown to accelerate short crack growth in other materials [48–50], but not necessarily polymers. This remains to be seen for short crack growth within elastic notch stress fields of UHMWPE, which may be determined using larger notch-root radii, materials with a higher yield strength or an alternate specimen geometry. However, such experimental changes do not necessarily reflect the materials, design or stress fields of TJRs in use today.

The reduced fatigue-crack growth resistance of both cross-linked materials (RXLPE and VXLPE), as compared with virgin UHMWPE, is consistent with previous evaluations of standard sharp-notched specimens [6,7,20,26,27,44]. Covalent bonding within the amorphous regions of the polymer due to cross-linking results in reduced ductility during deformation (Figure 3, Table 1) [3,5]. This manifests as limited crack-tip plasticity, as seen in the fractography images shown in Figure 9; specifically, fibrillation and rippling on the fracture

surfaces associated with stretching, rupture and recoil are dramatically diminished for cross-linked formulations, as compared with virgin UHMWPE [6,7,27,42–44].

The smaller distinction between the fatigue behavior of the RXLPE and VXLPE materials is likely associated with differences in the cross-linking density and post-irradiation annealing treatments. Exposure to irradiation has been shown to release free radicals within the UHMWPE bulk, increasing the polymer's susceptibility to oxidation and subsequent embrittlement [54,55]. Post-irradiation above-melt annealing ("remelting") eliminates these free radicals, but at the cost of reduced crystalline quality and quantity for RXLPE, resulting in reduced fatigue-crack propagation performance [6,8,54,56]. Vitamin E blended materials seek to retain crystallinity by absorbing free radicals with an antioxidant during the irradiation process without subsequent annealing [57,58]. However, measurements of cross-linking density of Vitamin E blended materials have been shown to have an overall reduced cross-linking density compared with non-Vitamin E materials for a given irradiation dose [58]. However, this leads to an overall reduced cross-linking density for a given irradiation dose [59]. VXLPE's apparent improved fatigue-crack propagation resistance thus may also be the result of decreased cross-linking density, despite receiving the same irradiation dose as RXLPE.

Notches are frequently cited as initiation sites for fracture in case reports of failed hip and knee components. Fully intact implants retrieved in as little as one month after implantation have demonstrated the existence of short cracks (0.1 to 0.3 mm) initiated from notches, which fall within the notch-affected zone as predicted by Dowling [28]. Our work demonstrates that such cracks grow at the rate dictated by the underlying microstructure of the UHMWPE, regardless of the notch geometry, although clearly sharper notches would promote easier crack initiation. Thus, changes to implant design may not mitigate the tradeoffs in crack propagation imposed by wear-resistant cross-linked UHMWPE formulations. Ultimately, further study is warranted to evaluate the impact that notch geometry has on crack initiation under cyclic or static loading. Our work establishes the Dowling approach as a viable methodology for further LEFM-based notch fatigue studies, either for crack initiation or investigation of novel UHMWPE materials.

Summary and Conclusions

Based on an experimental study of the behavior of fatigue cracks emanating from notches in various formulations of ultrahigh molecular weight polyethylene (UHMWPE), the following conclusions can be made:

1. Crack growth in medical grade UHMWPE is primarily dictated by microstructural features, with highly cross-linked formulations (RXLPE) promoting the fastest crack-growth rates under cyclic loading and virgin UHMWPE provided the slowest.
2. No distinctions in the crack-growth rates in UHMWPE as a function of the stress-intensity range were detected between various notch geometries for all materials tested; notch crack-growth data was congruent with that emanating from sharp notches.
3. Cracks growing within the plastic zone of a notch exhibited consistent behavior with those growing in predominantly elastic regions, implying the applicability of linear elastic methods in assessing notch fatigue in UHMWPE polymeric materials. However, the potential retarding effect of notch-tip plasticity on short crack growth warrants further investigation, especially in the case of polymers such as UHMWPE.

Appendix

The adaptation of LEFM theory to blunt-notched specimens has been investigated by numerous authors [28–32]. These studies have led to several mathematical models that seek to integrate notch geometry into stress intensity calculations in a single universal model (Table A1). However, the development of these approaches differs greatly in notch and specimen geometry, leading to discrepancies in their predicted stress intensity values for different specimen types. As a result, finite element analysis (FEA) was used to determine the most appropriate stress intensity model for the CT specimen dimensions shown in Figure 2.

Both stress concentration factors (k_t) and stress-intensity factors (K) were determined using 2D plane-strain finite element models (ABAQUS, version 6.12). Each CT specimen was modeled as a half-plane, invoking symmetry conditions (Figure A1). The pin was loaded with a distributed vertical force, while the bottom plane was held fixed. Linear-elastic material deformation was assumed, and models for the four notched specimens contained 1300 to 4300

8-node quadratic elements, with a biased mesh near the crack tip. (Each model was optimized to ensure results were independent of the number of elements.) To calculate k_t , an arbitrary vertical load was applied as a distributed pressure along the top-half of the loading pin-hole and the corresponding peak notch tip stress in the vertical (S22) direction was noted. The global stress S was calculated by modeling the top-half of the specimen as a prismatic beam of thickness B , length c , and width W undergoing combined tension and bending due to the application of force P at the pin-hole [60,61]:

$$S = \frac{2P(2W+c)}{B(W-c)^2} . \quad (A1)$$

The same 2D half-plane models and far-field stress calculation were used to determine stress intensity factors for varying crack lengths (Figure A1, Inset). Cracks of lengths 0.1 to 6 mm were inserted into each model by relaxing boundary conditions on near-notch tip nodes. Corresponding K values were extracted for each crack length while keeping the far-field load constant. Element failure was not modeled; rather, the stress intensity prior to crack advance was recorded.

Stress intensities determined through FEA were compared to those predicted by the models in Table A1 for the same crack length and far-field stress (Figure A2). No single analytical solution provided a completely accurate prediction of stress intensity for all cracks lengths ahead of each of the four blunt-notched C(T) specimens evaluated. The primary deviation for all expressions occurred near the transition of cracks from near-notch to bulk conditions. Most analyses tended to provide a strong correlation for very short cracks (under 0.5 mm for crack-like notches; under 1 mm for keyhole notches), with expressions by Kujawski [31], Lukas & Klesnil [32] and Schijve [29] presenting the best predictions of the FEA solution of K in this region. Only Dowling [28] proposed a solution that reasonably predicts *both* short and long crack stress intensities.

Dowling's approach argues that within the notch-affected zone, cracks behave similarly to long cracks emanating from a free surface, driven by a global stress that is equivalent to the peak local stress ($k_t S$) with a free surface correction factor of 1.12. However, two limiting

assumptions are made with this interpretation: first, that the edge is a straight line and second, that the stress distribution ahead of the notch is not a linearly decreasing function [29]. These assumptions likely lead to the 17-40% deviations that can occur at the transition between “short” and “long” crack behavior. Nonetheless, compared to available LEFM theories shown in Table A1, the Dowling approach presents the most accurate predictor of stress intensity for cracks growing within and beyond any notch influence.

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Figure 1: Fractured knee (A) and hip (B) UHMWPE components. Both devices exhibit fracture at a stress concentration (notch) that serves to improve kinematic stability (vertical post on a tibial insert, A) or fixation (locking liner mechanism of an acetabular liner, B) [9,10].

Figure 2: Compact-tension, C(T), specimens with various notch geometries used for fatigue analysis. Specimens were tested with a 'sharp' crack, a 'crack-like' blunt notch (0.75 mm or 1 mm notch-root radius), and a blunt keyhole notch (2 or 3 mm notch-root radius).

Table 1: Computed size of the notch-affected zone (“transition length”) as a function of notch-root radii, as determined using Dowling (1979).

Notch Root Radius [mm]	0.75	1	2	3
Notch-Affected Zone Size [mm]	0.15	0.20	0.39	0.59

Figure 3: The stress intensity driving crack growth ahead of a notch (as determined using finite element analysis (FEA)) can be approximated using short edge crack (K_s) and long sharp crack (K_L) models, shown here for a 2 mm notch root radius (see Appendix A for further discussion).

The notch-affected zone size is equivalent to the transition crack length where K_s and K_L intersect (l^*).

Figure 4: Full 3D finite element model of the full-plane compact-tension C(T) specimen with a 2 mm notch-root radius used for the calculation of the notch plastic zone. Equivalent (von Mises) stresses (units: MPa) are shown.

"Elastic" Short Crack Growth

"Plastic" Short Crack Growth

"Elastic" Long Crack Growth

"Plastic" Long Crack Growth

Figure 5: Illustration of the four categories of crack growth, as would vary with the notch root radius, the material yield strength, the crack length and the applied load. K_s is defined in Eq. (1) as the short crack approximation, while K_L is defined in Eq. (2) as the long crack approximation.

Figure 6: True stress-strain data for all three UHMWPE formulations at 150 mm/min displacement rate. Cross-linked formulations show greater strain hardening and less ductility (strain at failure) than non-cross-linked samples. Similar trends were seen at the 30 mm/min rate (see **Table 2**).

Table 2: Tensile true stress-strain properties for three formulations of UHMWPE ($n = 5$). Values are reported as means and standard deviations, with ranges denoted in parentheses. Statistical significance is noted as numerical superscripts.

Material Property	Displacement Rate [mm/min]	Virgin UHMWPE (UHMWPE)	Cross-linked (75 kGy) & Remelted UHMWPE (RXLPE)	Vitamin E (0.1 wt.%) Blended & Cross-linked (75 kGy) UHMWPE (VXLPE)
Modulus [MPa]	30	378 ± 18 ^{2,3} (346 - 389)	334 ± 8 ^{1,3} (325 - 345)	460 ± 21 ^{1,2} (431 - 487)
	150	388 ± 24 ³ (365 - 424)	365 ± 8 ³ (361 - 378)	457 ± 17 ^{1,2} (431 - 476)
True Yield Strength [MPa]	30	17.8 ± 2.1 ^{2,3} (14.5 - 19.6)	15.3 ± 1.0 ^{1,3} (14.5 - 16.8)	20.6 ± 0.7 ^{1,2} (19.9 - 21.5)
	150	19.5 ± 1.3 ^{2,3} (18.1 - 20.9)	17.1 ± 1.9 ^{1,3} (14.8 - 19.6)	22.5 ± 0.9 ^{1,2} (20.9 - 23.1)
True Yield Strain	30	0.046 ± 0.003 (0.041 - 0.049)	0.045 ± 0.004 (0.042 - 0.051)	0.044 ± 0.002 (0.04 - 0.046)
	150	0.051 ± 0.004 (0.044 - 0.055)	0.049 ± 0.010 (0.038 - 0.064)	0.049 ± 0.003 (0.043 - 0.052)
True Ultimate Strength [MPa]	30	183 ± 20 ^{2,3} (156 - 203)	130 ± 37 ¹ (83 - 172)	150 ± 32 ¹ (105 - 191)
	150	187 ± 26 ^{2,3} (161 - 226)	127 ± 31 ¹ (91 - 170)	146 ± 19 ¹ (118 - 168)
True Ultimate Strain	30	4.0 ± 0.3 ^{2,3} (3.5 - 4.3)	3.0 ± 0.4 ¹ (2.5 - 3.4)	3.0 ± 0.3 ¹ (2.7 - 3.4)
	150	4.1 ± 0.2 ^{2,3} (3.8 - 4.4)	2.9 ± 0.3 ¹ (2.6 - 3.4)	3.0 ± 0.2 ¹ (2.7 - 3.3)

¹ $p < 0.05$ vs. UHMWPE; ² $p < 0.05$ vs. RXLPE; ³ $p < 0.05$ vs. VXLPE

Figure 7: Fatigue-crack propagation rate (da/dN) data as a function of the stress-intensity range (ΔK) for RXLPE (left, blue), VXLPE (middle, red) and UHMWPE (right, black). Filled symbols represent crack growth outside the influence of notch plasticity ("elastic" data), while open symbols demarcate cracks growing within the notch plastic zone ("plastic" data). Notch-root radii are differentiated by symbol geometry as seen in the illustrative legend left of the graph.

Figure 8: Fatigue-crack propagation rate (da/dN) data as a function of the stress-intensity range (ΔK) data delineated by short and long crack growth, *i.e.*, cracks growing within or outside the notch-affected zone, respectively.

Figure 9: SEM images of fracture surfaces for cracks growing outside the notch-affected zone and notch-plastic zone. UHMWPE samples demonstrate greater ductile features (rippling, criss-cross features) than both RXLPE and VXLPE.

Figure 10: SEM images of fracture surfaces for cracks growing outside the notch-affected zone and notch-plastic zone. UHMWPE samples demonstrate greater ductile features (rippling, criss-cross features) than both RXPLE and VXLPE.

Figure A1: Plane-strain 2-D model of half-plane C(T) specimen with a 2 mm notch-root radius and 1 mm crack (inset) used for the calculation of the linear-elastic stress-intensity factors. Equivalent (von Mises) stresses (units: MPa) are shown.

Figure A2: Comparison of analytical expressions with finite element analysis solutions for the stress intensity of cracks emanating from the four notch-root radii (for C(T) specimen dimensions shown in **Figure 2**). Expressions derived by Smith & Miller [30], Dowling [28] and Schijve [29] present approximate estimates for short crack stress intensity; these solutions transition to the limiting long crack solution once crack extension is outside the notch field, as seen in **Table A1**. Kujawski [31] and Lukas & Klesnil [32] each represent this transition in a single expression. The transitions from “short” to “long” crack behavior (*i.e.*, the size of the notch-affected zone, as delineated by the abscissa) are noted by the circular symbols.

Table A1: Summary of expressions for the stress-intensity solutions (K for all crack lengths or K_S/K_L ratio for short or long cracks, respectively) developed to describe crack growth emanating from a notch.

Reference	Stress Intensity Factor (K_s)
Dowling [28]	$K_S = \sigma_{local} \sqrt{\pi l} = 1.12 k_t S \sqrt{\pi l} \quad \text{for } l \leq l^*$ $K_L = FS \sqrt{\pi a} = FS \sqrt{\pi(c_o + l)} \quad \text{for } l > l^*$ $K_S(l^*) = K_L(l^*) \quad \text{OR} \quad l^* = \frac{c_o}{\left(\frac{1.12 k_t}{F}\right)^2 - 1}$
Schijve [29]	$K_S = \sigma_{local} \sqrt{\pi l} = C k_t S \sqrt{\pi l} \quad \text{for } l \leq l^*$ $K_L = FS \sqrt{\pi a} = FS \sqrt{\pi(c_o + l)} \quad \text{for } l > l^*$ $K_S(l^*) = K_L(l^*) \quad \text{OR} \quad l^* = \frac{c_o}{\left(\frac{C k_t}{F}\right)^2 - 1}$ $C = 1.1215 - 3.21 \left(\frac{l}{\rho}\right) + 5.16 \left(\frac{l}{\rho}\right)^{1.5} - 3.73 \left(\frac{l}{\rho}\right)^2 + 1.14 \left(\frac{l}{\rho}\right)^{2.5}$
Smith & Miller [30]	$K_S = [1 + 7.69 \sqrt{c_o/\rho}]^{1/2} S \sqrt{\pi l} \quad \text{for } l < 0.13 \sqrt{c_o \rho}$ $K_L = S \sqrt{\pi a} = S \sqrt{\pi(c_o + l)} \quad \text{for } l \geq 0.13 \sqrt{c_o \rho}$
Kujawski [31]	$K = \sigma_{local} \sqrt{\pi l} = Q f \frac{k_t S}{2} \left[\left(1 + 2 \frac{l}{\rho}\right)^{-\frac{1}{2}} + \left(1 + 2 \frac{l}{\rho}\right)^{-3/2} \right] \sqrt{\pi l}$ $f = 1 \quad \text{for } x/\rho < 0.2$ $f = 1 + \frac{\tan(\pi/2 k_t)}{2.8} \left(\frac{x}{\rho} - 0.2\right) \quad \text{for } x/\rho \geq 0.2$ $Q = 1.22 \text{ for a through-thickness crack}$
Lukas & Klesnil [32]	$K = \sigma_{local} \sqrt{\pi l} = \frac{F k_t}{\sqrt{1 + 4.5(l/\rho)}} S \sqrt{\pi l}$
K_L formula for compact tension specimen geometry shown in eq. A1 (Tada et al. 1985).	
ρ = notch root radius l = crack length emanating from notch S = far-field stress c_o = notch depth k_t = stress-concentration factor F = specimen geometry factor [40] l^* = transition length between long crack (K_L) and short crack (K_S) approximations Q = shape factor, as found in [40]	