Fast Selection of Geologic Models Honoring CO₂ Plume Monitoring Data Using Hausdorff Distance and Scaled Connectivity Analysis

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ABSTRACT

The spatial and temporal characteristics of observed CO₂ plumes obtained from 4D seismic surveys can be honored by finding the most probable models in which the migration of CO₂ plumes are spatially and temporally similar to the observation. A computationally efficient scheme is necessary to assess the dissimilarity between CO₂ plumes simulated over a large suite of geologic models, and subsequently to select the subset of models exhibiting characteristics similar to the observation.

The Euclidean distance is the most common way to measure dissimilarity between vector representations of CO₂ plumes. However, the Euclidean distance yields an average measure of dissimilarity between plume shapes and does not take the spatial relation between vector elements into account. For this reason, we measure the shape dissimilarity between CO₂ plumes using the Hausdorff distance. In our examples, CO₂ plumes selected using the Hausdorff distance are more similar to observed CO₂ plumes than those selected using the Euclidean distance.

However, it is impractical to conduct complex flow simulations on a large suite of geological models in order to find the most probable models honoring the spatial characteristics of an observed CO₂ plume. We save about 80% of the computational cost of selecting the most probable models by screening the large suite of geological models using scaled connectivity analysis. The paper demonstrates the combined use of scale

connectivity analysis and assessment of dissimilarity using Hausdorff distance on realistic field examples.

1. INTRODUCTION

4D seismic images that reflect CO₂ plume migration both spatially and temporally are important for calibrating simulation models for CO₂ storage. Once simulated CO₂ plumes are matched to observed CO₂ plumes by adjusting the model parameters of simulation models, reliable prediction of future CO₂ plume migrations can be achieved. This would be invaluable for risk assessment of a CO₂ storage project and its management.

Chadwick and Noy (Chadwick and Noy, 2010) calibrated aquifer models to honor the spatial and temporal characteristics of CO₂ plumes obtained using 4D seismic surveys for the Sleipner project. They manually matched simulated CO₂-water contacts (CWC) to an observed CWC by adjusting the aquifer model parameters such as CO₂ density, permeability, and the depth topography of the top surface. Singh et al. (2010) compared observed CWCs to CWCs simulated using different commercial simulators for the Sleipner benchmark model released by Statoil. Chadwick and Noy (2010) and Singh et al. (2010) compared the observed and simulated CWCs manually for a small number of models. However, an automatic measure to compute the mismatch between the observed and simulated plumes is necessary to screen a large number of models.

Chadwick and Noy (2015) suggested several geometrical measures to compute dissimilarity between observed and simulated plumes such as plume footprint area, maximum lateral migration distance, plume volume, and etc. Lüth et al. (2015) compared the geometrical measures to the similarity index suggested by Dice (1945). The similarity index is calculated using the footprint areas of two plumes and their intersecting area. Zhu et al. (2015) used plume footprint area to match simulated plumes to an observed

plume inverted from 4D seismic data in the Sleipner benchmark model. However, because these methods do not consider the spatial distribution of the mismatch between CO₂ plumes, models may be selected that exhibit the same level of dissimilarity even though they have different spatial characteristics. An example of plume footprint area will be discussed later.

Techniques for measuring image dissimilarity in computer vision literature can be considered. However, most of those techniques are not applicable to measuring the dissimilarity between the shapes of CO₂ plumes during sequestration. In computer vision literature, rotation, scale, and location invariances are essential properties when investigating image dissimilarity because images differ according to the perspective even though they include the same object (Nixon and Aguado, 2012). The objective for using such computer vision techniques is to detect and compare objects regardless of rotation, scale, and location of objects. However, in sequestration applications, differences in terms of rotation, scale, and location of CO₂ plumes should be discriminated because they result in different spatial characteristics of CO₂ plumes. In this regard, techniques measuring image dissimilarity regardless of rotation, scale, and location are inappropriate for measuring the dissimilarity in shapes of CO₂ plumes.

The most common and intuitive shape dissimilarity measure considering the rotation, scale, and location is the Euclidean distance between vectors representing CO₂ plumes. The Euclidean distance, which is also called L₂ norm, is computed as the square root of the sum of the squared difference between each element of the vectors representing CO₂ plumes. For example, Figure 1 shows an observed and two simulated CO₂ saturation images. Each image consists of 600 by 600 pixels, and each pixel has a value that ranges from 0 to 1. The observed CO₂ plume and the 2nd simulated CO₂ plume corresponding to Figure 1(a) and Figure 1(c) migrate radially while the 1st simulated CO₂

plume corresponding to Figure 1(b) migrates elliptically. The Euclidean distances between the observed and the 1st CO₂ plumes, and between the observed and the 2nd CO₂ plumes are 64.31 and 66.93, respectively. In terms of the Euclidean distance, it is not true that the visual observation that the observed CO₂ plume looks more similar to the 2nd CO₂ plume than 1st simulated CO₂ plume. The spatial characteristics of CO₂ saturation images can be compared quantitatively using the Euclidean distance. However, the Euclidean distance is not appropriate to compare the overall shape of CO₂ plumes if the spatial distribution of CO₂ saturations is significantly different within the CO₂ plumes. This is because the Euclidean distance does not consider the spatial relation between vector elements representing CO₂ saturations at specific locations. In this study, we propose using a binary image and an alternative to the Euclidean distance to measure shape dissimilarity between CO₂ plume images.

Figure 1 Observed and two simulated CO₂ saturation images

The quantitative image of the observed CO₂ saturations shown in Figure 1(a) has uncertainty because of different interpretations of seismic data such as velocity models. If CO₂ saturations inverted from seismic data have high uncertainty, then it can be considered as a binary image rather than a quantitative CO₂ saturation image. CO₂

plumes can be also described as binary images when observed CO₂ saturations are not in good agreement with simulated CO₂ saturations. Zhu et al. (2015) compared the area of the CO₂ plumes instead of the quantitative CO₂ saturations for the Sleipner benchmark model because the simulated CO₂ saturations are not fairly matched to the observed CO₂ saturations.

Obidegwu et al. (2015) measured the dissimilarity between binary images of gas distributions obtained from seismic data using the Hamming distance (Hamming, 1950). The Hamming distance between binary images is equivalent to the square of the Euclidean distance. However, both the Euclidean distance and the Hamming distance do not indicate the difference between the vector elements at individual locations rather they focus on the aggregate of the plume mismatch. In the Euclidean distance, the mismatch between locations that are spatially close or far are treated equally. Even though simulated CO₂ plumes are spatially or visually different from an observed CO₂ plume, they can have the same Euclidean distance.

For example, Figure 2 shows the observed (left) and two simulated CWCs of the CO₂ plumes shown in Figure 1. The inside of the CWCs are regarded as the binary images of the CO₂ plumes. In Figure 2, the observed CWC is a circle with a radius, 15 units. In Figure 2(b), the first simulated CWC is an ellipse with a horizontal radius, 20 units, and a vertical radius, 10 units. In Figure 2(c), the second simulated CWC is a circular plume with a radius, 16.60 units. In Figure 2(b) and Figure 2(c), the green area represents the mismatch between the observed and simulated CWCs, and it is equivalent to the Euclidean distance to the observed CO₂ plume. The two simulated CO₂ plumes shown in Figure 2(b) and Figure 2(c) have the same green area, and this implies that they have the same Euclidean distance to the observed CO₂ plume shown in Figure 2(a). The two simulated CO₂ plumes have the same shape dissimilarity in terms of the Euclidean

distance, but the 2nd CO₂ plume looks more similar to the observed CO₂ plume than the 1st CO₂ plume. This is because the Euclidean distance considers only the absolute value of the green area. The Euclidean distance does not take the spatial distribution of the green area into account. Thus another measure is necessary to consider the spatial difference between the CO₂ plumes.

Figure 2 Observed and two simulated CO₂-water contacts. The areas marked in green show the mismatches between the simulated and observed contacts.

The approaches to measure similarity between discrete features such as facies have been mainly discussed in stochastic simulation using multi-point statistics. Arpat and Caers (2007) measured similarity between patterns for conditional simulations using the Manhattan distance, which is also called L_1 norm. However, the Manhattan distance does not distinguish the two simulated plumes shown in Figure 2 because it does not consider the spatial distribution of the misfit like the Euclidean distance. Mirowski et al. (2009) measured similarity between training images using stationary scored for statistical distribution, orientation, and scale. Zhang et al. (2006) scored dissimilarity between patterns in training images using filters quantifying direction, average, gradient, and curvature. However, the approaches that Mirowski et al. (2009) and Zhang et al. (2006) presented do not consider all of rotation, scale, and location at the same time.

Another possibility for comparing CO₂ plume migrations is to measure shape dissimilarity using the Hausdorff distance. The Hausdorff distance is used to match the shape of objects in binary images (Huttenlocher et al., 1993). The Hausdorff distance is not only sensitive to rotation, scale, and location (Zhang and Lu, 2004), but it also considers the spatial mismatch between images.

The Hausdorff distance has been applied to measure shape dissimilarity of geologic features and water fronts (Abadpour et al., 2013; Lee et al., 2013; Suzuki and Caers, 2008). However, the Hausdorff distance has not been used to measure the shape dissimilarity between CO₂ plumes in CO₂ sequestration. In this study, the Hausdorff distance is compared to the Euclidean distance in finding the most probable models honoring the spatial characteristics of an observed CO₂ plume. The Hausdorff distance presented more accurate shape dissimilarity between CO₂ plumes than the Euclidean distance.

The final goal of this study is to select the most probable models in a large suite of geologic models honoring the spatial characteristics of an observed CO₂ plume. However, it is computationally expensive to run flow simulations in the entire suite in order to find the most probable models. Jeong and Srinivasan (2016) proposed scaled connectivity analysis (SCA) that quickly approximates CO₂ plume migrations. The entire suite of prior geologic model can be quickly screened by comparing the observed CO₂ plume to the approximate ones obtained using SCA. Only a set of geologic models whose approximate CO₂ plumes are similar to the observed one are processed through a flow simulator. The most probable models are selected by comparing the observed CO₂ plume to those simulated on the set of geologic models. We saved about 80% of the computational cost of finding the most probable models honoring the spatial characteristics of an observed CO₂ plume using the Hausdorff distance and SCA.

2. HAUSDORFF DISTANCE

The Euclidean distance between two binary images is given by:

$$Euclidean(\mathcal{A}, \mathcal{B}) = \|\mathcal{A} - \mathcal{B}\| \dots\dots\dots \text{Equation (1)}$$

where $\mathcal{A}$ and $\mathcal{B}$ represent the vectors describing two spatial objects such as a CO₂ plume and $\|\cdot\|$ is a norm operator. Each element of the vector corresponds to a binary value at a location e.g. either filled with CO₂ or not computed by applying a threshold. The norm operator is applied over all elements of the vectors regardless of their spatial location and so in that sense, the Euclidean distance measures the dissimilarity between images regardless of the spatial relation between the elements of the vectors representing the images.

Contrary to the Euclidean distance, the Hausdorff distance considers the spatial relation between two point sets. The Hausdorff distance is given by (Huttenlocher et al., 1993):

$$H(A, B) = \max(h(A, B), h(B, A)) \dots\dots\dots \text{Equation (2)}$$

where

$$h(A, B) = \max_{a \in A} \min_{b \in B} \|a - b\| \dots\dots\dots \text{Equation (3)}$$

and A and B represent the point sets of non-zero elements of the vectors $\mathcal{A}$ and $\mathcal{B}$, respectively. a and b represent the points of A and B , and each point has a N-dimensional location. In Figure 3, A and B consists of five and four points, respectively. Figure 3 shows the four distances from a_3 to B . The distance from a_3 to b_1 is the smallest and the other minimum distances from a to b are also calculated. The maximum distance among the minimum distances from a to b is $h(A, B)$ in Equation (3). Likewise, $h(B, A)$ is computed and then the maximum of $h(A, B)$ and $h(B, A)$ is the Hausdorff distance between A and B .

Figure 3 An example of calculating the Hausdorff distance

In Figure 2, the Hausdorff distances between the observed CO₂ plume and the 1st model, and between the observed CO₂ plume and the 2nd one are 5 and 1.6, respectively. This is consistent with the observation that the 2nd CO₂ plume looks more similar to the observation than the 1st CO₂ plume.

3. RANKING MODELS USING THE HAUSDORFF DISTANCE

The final goal of this study is to select the most probable models honoring an observed CO₂ plume over a large suite of geologic models. The observed CO₂ plume has to be compared to the CO₂ plumes that are estimated in the large suite. The geologic models that exhibit similar CO₂ plume migrations to the observed data are chosen as the most probable models.

A flow simulator such as CMG-GEM (CMG, 2012) can be used as an estimator of CO₂ plume migrations on a large suite of geologic models. Figure 4 shows the procedure of ranking models using the Hausdorff distance.

Figure 4 Procedure of ranking models using the Hausdorff distance

In the 2nd step, if an observed CO₂ plume is given as a 2D image, the simulated CO₂ saturation images are converted to 2D images by vertically averaging and only the characteristics of the horizontal migration are compared. In the 3rd step, the CO₂ plumes are described as binary images where 0 and 1 represent pixels or voxels filled and unfilled with CO₂, respectively.

In the 4th step, the dissimilarity between CO₂ plumes is measured using the Euclidean distance and the Hausdorff distance, and then their results will be compared in the application section. In the last step, a set of models is selected based on the smallest dissimilarity. For example, if the smallest dissimilarity is 100 and M is 110 % in the last step, then the models whose dissimilarity is less than 110 are selected as the most probable ones.

4. RANKING USING SCA AND THE HAUSDORFF DISTANCE

SCA proposed by Jeong and Srinivasan (2016) is a statistics-based proxy to quickly approximate CO₂ plume migrations in 3-dimensional heterogeneous reservoirs

where viscous forces are dominant over capillary forces. In SCA, connectivities between neighbor grid cells are calculated with the structure, fluid properties, permeability, and porosity. SCA determines the sequence in which grid cells are filled with CO₂ based on the connectivities. SCA provides good approximations of simulated CO₂ plumes for different operating conditions such as fluid properties, rock properties, and aquifer structures. SCA is also successful in approximating CO₂ plumes for two real field cases, the Johansen Formation model (Eigestad et al., 2009) and the Sleipner L9 model released by Statoil in 2011.

In the previous section, the CO₂ migration over the entire model was simulated using a flow simulator in order to find the most probable models honoring an observed CO₂ plume. However, it is impractical to conduct flow simulations over entire models. The most probable models can be quickly selected through a ranking process using proxy models (Ballin et al., 1992; Deutsch and Srinivasan, 1996; Scheidt and Caers, 2009). The entire models are quickly ranked based on proxy responses, and then a set of highly ranked models are preselected. The most probable models are selected based on the simulated results of the preselected models.

A set of models that are expected to have similar CO₂ plumes to the observed one are preselected by comparing the observed CO₂ plume to the SCA approximations using the Hausdorff distance. The preselected models are processed through a flow simulator, and then the preselected models are ranked based on the Hausdorff distance between the observed and simulated CO₂ plumes. Figure 5 shows the procedure of ranking model using SCA and the Hausdorff distance. In the 1st step of the procedure shown in Figure 5, the SCA approximations do not need to be thresholded because they are already available as binary images. This procedure is demonstrated in the examples of the application section.

Figure 5 Procedure of ranking models using SCA and the Hausdorff distance

5. APPLICATION

5.1. 2D Observed CO₂ Plume

The ranking process is applied to select the most probable models honoring CO₂ plumes imaged in an aquifer shown in Figure 6. This example will serve to emphasize the efficacy of the Hausdorff distance to capture the differences between models. The grid system consists of $151 \times 151 \times 10$ grid cells with $\Delta x = \Delta y = 100$ and $\Delta z = 5\text{m}$. A CO₂

injector is located in the middle of the aquifer. The CO₂ injection rate is 20,000 m³/day at bottom hole conditions over a period of 3 years. The initial average pressure and temperature of the aquifer is 18,145 kPa and 75 °C, respectively. The boundary is connected to an infinite aquifer allowing outflow of CO₂. The relative permeability curves are shown in Figure 7 and capillary pressure is not considered.

Figure 6 Structure of the aquifer and the location of the CO₂ injector in the 2D cases. The axes represent distance in km. The z-axis is exaggerated by factor of 10.

Figure 7 Relative permeability curves of CO₂ and water

Table 1 shows the geostatistical parameters for three datasets of 400 geologic models. Porosity of the 400 geologic models is generated using the sequential Gaussian simulation of SGeMS (Remy et al., 2011). Permeability is correlated with porosity, and it is generated using the sequential Gaussian co-simulation of SGeMS (Remy et al., 2011). The dataset names in Table 1 refer to short and long correlation lengths, and north-south

(NS) and east-west (EW) correlation directions. The ratio of vertical to horizontal permeability is 0.1.

Table 1 Geostatistical parameters used for generating the three sets of geologic models

Number of models	200 models	100 models	100 models
Dataset name	short NS	long NS	long EW
Mean and standard deviation of ϕ	(0.2, 0.05)	(0.2, 0.04)	(0.2, 0.04)
Minimum and maximum of ϕ	(0.05, 0.35)	(0.1, 0.3)	(0.1, 0.3)
Mean and standard deviation of $\log_{10} k$	(2.5, 0.35)	(2, 0.53)	(2, 0.53)
Minimum and maximum of $\log_{10} k$	(1.6, 3.4)	(0.8, 3.2)	(0.8, 3.2)
Major correlated direction of ϕ and k	north-south	north-south	east-west
Major, minor, and vertical correlation lengths of ϕ and k in terms of cells	(20, 15, 5)	(40, 10, 3)	(40, 10, 3)
Correlation of ϕ and k	0.8	0.7	0.7

Table 2 shows porosity, logarithm in base 10 of horizontal permeability, and the top and cross-sectional views of the corresponding CO₂ plumes simulated using CMG-GEM for one model in each of the three datasets. As evident by the simulated CO₂ saturations in Table 2, the injected CO₂ migrates along permeable features. In the cross-sectional view of CO₂ saturation of Table 2, CO₂ migrates first to the top of the structure and then spreads horizontally. The wedge shape of the CO₂ plumes is common in CO₂ plume migrations because of strong buoyancy. This implies that the simulated vertical characteristics of the CO₂ plumes are not likely to be too different for the 400 models.

Table 2 Porosity, logarithm in base 10 of horizontal permeability, CO₂ saturations computed using CMG-GEM for one model in each of the three datasets. The axes represent distance in km. The z-axes in the 3D views and the cross-sectional views are exaggerated by factors of 10 and 40, respectively.

2D Case #1

Figure 8 shows an observed CO₂ plume for the 2D case #1. The observed CO₂ plume shown in Figure 8 is given as a 2D image. As discussed in the ranking process of the previous section, simulated CO₂ plumes are converted into 2D binary images, and then they are compared to the observed CO₂ plume shown in Figure 8. The extent of the observed CO₂ plume in the east-west direction is longer than that in the north-south direction. The shape of the observed CO₂ plume is described as an ellipse in the center with a long and narrow protrusion toward the east. Although the protrusion seems to be relatively small compared to the ellipse, the length of the protrusion is not negligible (= about 2 km). The long protrusion is caused by long and narrow features with high permeability, so an unexpected CO₂ migration path can be formed along the highly permeable features in the future. Thus geologic models that reproduce the long and narrow protrusion in CO₂ plume migrations should be selected and considered.

Figure 8 Observed CO₂ plume for the 2D case #1. The axes represent distance in km.

The dissimilarity between the observed and simulated CO₂ plumes is measured using the Euclidean distance and the Hausdorff distance for 2D CO₂ plumes. The CO₂ saturations simulated using CMG-GEM are thresholded with a CO₂ saturation of 0.1. Table 3 shows the three most similar CO₂ plumes to the observed CO₂ plume and their dissimilarities obtained using the Euclidean distance and the Hausdorff distance for 2D

CO₂ plumes. The red curves in the CO₂ plumes of Table 3 are the perimeter of the observed CO₂ plume shown in Figure 8.

In Table 3, the magnitude of the dissimilarities measured using the Hausdorff distance is much larger than that of the dissimilarities measured using the Euclidean distance. This is because the Hausdorff distance is calculated based on the spatial distance between elements while the Euclidean distance is on the square root of the sum of the difference between binary values.

Table 3 Three most similar simulated CO₂ plume obtained using the Euclidean distance and the Hausdorff distance for the 2D case #1. The number below each figure is the numeric values of dissimilarity between the observed and simulated CO₂ plumes. The red perimeter represents the observed CO₂ plume shown in Figure 8. The axes represent distance in km.

Rank	1	2	3
Euclidean distance	19.67	19.70	19.98
Hausdorff distance	800.00	860.23	900.00

The most probable models are selected based on the smallest dissimilarity measure. In this case, the models whose dissimilarity is less than 110 % of the smallest

dissimilarity in each approach are selected as the most probable models. For example, the smallest dissimilarity obtained using the Hausdorff distance in Table 3 is 800.00, and then the models whose dissimilarity is less than 880.00 are selected. Ten most probable models are selected using the Euclidean distance and two using the Hausdorff distance, respectively. The 4th through 10th most similar models obtained using the Euclidean distance are not presented here, but their spatial characteristics are almost the same to those in 1th to 3rd most similar models.

The most similar CO₂ plumes selected using the Euclidean distance do not have the long and narrow protrusion toward the east. This is because the Euclidean distance measures the dissimilarity regardless of the spatial distribution of the mismatch between CO₂ plumes.

On the contrary, the 1st and 2nd most similar CO₂ plumes selected using the Hausdorff distance have an ellipse in the center and a long and narrow protrusion toward the east. The 1st and 2nd most similar CO₂ plumes selected using the Hausdorff distance are more similar to the observed CO₂ plume in terms of shape than those selected using the Euclidean distance. This is because the Hausdorff distance considers the spatial distribution of the mismatch between CO₂ plumes.

The most probable models honoring the observed CO₂ plume shown in Figure 8 are selected through the ranking process using SCA. 400 proxy CO₂ plumes are computed for the prior 400 models using SCA, and then they are converted into 2D binary images. The 2D binary images are compared to the observed CO₂ plume using the Euclidean distance and the Hausdorff distance. The 80 prior models corresponding to the 80 most similar CO₂ plumes (20% of the 400 prior models) are sampled, and then flow simulations are conducted in the 80 sampled models using CGM-GEM. The 80 simulated CO₂ plumes are converted into 2D binary images, and then shape dissimilarity between

the observed CO₂ plume and the 2D binary images are measured using the Euclidean distance and the Hausdorff distance. Table 4 and Table 5 show the three most similar proxy CO₂ plumes computed using SCA and corresponding CO₂ plumes compute using CMG-GEM. Table 6 shows the three most similar CO₂ plumes selected through the ranking process using SCA. The results shown in Table 6 are exactly the same to those shown in Table 3 for the Euclidean distance and the Hausdorff distance. The 1st and 2nd most similar CO₂ plumes selected through the ranking process using SCA and the Hausdorff distance have the long and narrow protrusion toward the east while the three most similar CO₂ plumes selected through the ranking process using SCA and the Euclidean distance do not have.

Table 4 Three most similar SCA approximations obtained using the Euclidean distance for the 2D case #1, and corresponding CO₂ plumes computed using CMG-GEM. The number below each figure is the numeric value of dissimilarity between the observed and simulated CO₂ plumes. The red perimeter represents the observed CO₂ plume shown in Figure 8. The axes represent distance in km.

Rank	1	2	3
Euclidean distance	Proxy CO ₂ plumes computed using SCA
	19.85	20.83	21.93
	Corresponding CO ₂ plumes computed using CMG-GEM

Table 5 Three most similar SCA approximations obtained using the Hausdorff distance for the 2D case #1, and corresponding CO₂ plumes computed using CMG-GEM. The number below each figure is the numeric value of dissimilarity between the observed and simulated CO₂ plumes. The red perimeter represents the observed CO₂ plume shown in Figure 8. The axes represent distance in km.

Rank	1	2	3
Hausdorff distance	Proxy CO ₂ plumes computed using SCA

	800.00	806.23	894.43
	Corresponding CO ₂ plumes computed using CMG-GEM

Table 6 Three most similar simulated CO₂ plumes obtained through the ranking process for the 2D case #1. The number below each figure is the numeric value of dissimilarity between the observed and simulated CO₂ plumes. The red perimeter represents the observed CO₂ plume shown in Figure 8. The axes represent distance in km.

Rank	1	2	3
Euclidean distance	19.67	19.70	19.98
Hausdorff distance	800.00	860.23	900.00

It took about 700 seconds and 2.2 seconds to run CMG-GEM and SCA, respectively for each model. The cost of measuring shape dissimilarity between CO₂ plumes using the Euclidean distance and the Hausdorff distance was negligible compared to the cost of running CMG-GEM and SCA. The total computational cost was $400 * 700 / 3600 = 77.78$ hours for the ranking process without using SCA and $(400 * 2.2 + 40 * 700) / 3600 = 15.80$ hours for the ranking process using SCA. About 80% of the computational cost was saved in selecting the most probable models honoring the observed CO₂ plume. The ranking process using SCA and the Hausdorff distance selected exactly the same models despite about 20% of the computational cost of the ranking process without using SCA.

2D Case #2

There is likely to be uncertainty associated with the imaging of CO₂ plumes. In the results presented in the previous section, this uncertainty is manifested in the form of uncertainty in the threshold used to separate the CO₂ contacted reservoir volume from the background. This is because the CO₂ plumes are imaged as binary images by thresholding the images of CO₂ saturation maps obtained through seismic data processing and flow simulation.

In this case, the extent of an observed CO₂ plume is assumed to be larger than that of the observed CO₂ plume shown in Figure 8 due to the uncertainty associated with the imaging of CO₂ plumes from seismic data. Figure 9 shows the observed CO₂ plume when a different threshold is employed. The extent of the observed CO₂ plume in the east-west direction is longer than that in the north-south direction. The shape of the observed CO₂ plume is described as an ellipse in the center. However, the size of the observed CO₂ plume shown in Figure 9 is larger than that shown in Figure 8. Thus the simulated CO₂ plumes shown in Table 2 require the measure for dissimilarity of CO₂ plumes to be able to discriminate and select the set of reservoir models that exhibit visual and spatial features consistent with the observed plume characteristics regardless of small difference between the extents of CO₂ plumes. The 400 prior models do not have CO₂ plumes that extend as far as the observed CO₂ plume shown in Figure 9.

Figure 9 Observed CO₂ plume for the 2D case #2. The axes represent distance in km.

Table 7 shows the three most similar CO₂ plumes to the observed CO₂ plume and their dissimilarities obtained using the Euclidean distance and the Hausdorff distance for 2D CO₂ plumes. The red curves in the CO₂ plumes of Table 7 are the perimeter of the observed CO₂ plume shown in Figure 9.

Table 7 Three most similar simulated CO₂ plumes and their dissimilarities obtained using the Euclidean distance and the Hausdorff distance for the 2D case #2. The number below each figure is the numeric value of dissimilarity between the observed and simulated CO₂ plumes. The red perimeter represents the observed CO₂ plume shown in Figure 9. The axes represent distance in km.

Rank	1	2	3
Euclidean distance	32.22	33.27	33.44
Hausdorff distance	1063.01	1503.33	1552.42

110% of the smallest dissimilarity is used as a criterion to select the most probable models. Sixteen models are selected as the most probable models using the Euclidean distance while one model is selected using the Hausdorff distance.

The major orientation of the 1st and 2nd CO₂ plumes selected using the Euclidean distance is almost isotropic, and they belong to the heterogeneity model characterized by short NS direction continuity shown in Table 1

Table 1. This is because the dissimilarity measured using the Euclidean distance is mostly affected by how many grid blocks are saturated within the observed CO₂ plume rather than the overall shape of the simulated CO₂ plumes.

The 1st most similar CO₂ plume selected using the Hausdorff distance is slightly different from the observed CO₂ plume in terms of size, but they have similar shape. The extent can be matched by adjusting a threshold value for observed or simulated CO₂ saturation maps. As shown in Figure 10, the extent and the shape of the CO₂ plume selected using the Hausdorff distance are matched using a smaller threshold value ($=0.01$) for the simulated CO₂ saturations. However, the CO₂ plumes selected using the Euclidean distance cannot be matched to the observed CO₂ plume although the threshold value is adjusted because their overall shape is different.

Figure 10 CO₂ plume ($\text{CO}_2 \text{ saturation} \geq 0.01$) selected using the Hausdorff distance for the 2D case #2. The axes represent distance in km.

Table 8 shows the three most similar CO₂ plumes selected through the ranking process using SCA. The results shown in Table 8 are identical to those shown in Table 7 for the Euclidean distance and the Hausdorff distance. As presented in 2D Case #1, the ranking process using SCA took about 20% of the total computational cost of the ranking process without using SCA.

Table 8 Three most similar simulated CO₂ plumes and their dissimilarities obtained through the ranking process using SCA for the 2D case #2. The number below each figure is the numeric value of dissimilarity between the observed and simulated CO₂ plumes. The red perimeter represents the observed CO₂ plume shown in Figure 9. The axes represent distance in km.

Rank	1	2	3
Euclidean distance	32.22	33.27	33.44
Hausdorff distance	1063.01	1503.33	1552.42

5.2. 3D Observed CO₂ Plume

The aquifer structure shown in Figure 11 has $201 \times 201 \times 10$ grid cells with $\Delta x = 5\text{m}$, $\Delta y = 5\text{m}$, and $\Delta z = 2\text{m}$. CO₂ is injected at the middle of the aquifer model at 18,690.8 kg/day for 1 year. The boundary is connected to an infinite-acting aquifer. The relative permeability curves are shown in Figure 7 and capillary pressure is ignored.

Figure 11 Structure of the aquifer for the 3D application example and the location of the CO₂ injector. The axes represent distance in km. The z-axis is exaggerated by a factor of 20.

Figure 12 shows the facies distribution and the observed CO₂ plume for the reference model. In Figure 12(a), the features in yellow are the sandstone channels embedded in mudstone (clear). The major orientation of fluvial channels is north-south, and their thickness and width are $5 \cdot \Delta z$ and $15 \cdot \Delta x$, respectively. The fluvial model is built using the TiGenerator in SGeMS (Boucher et al., 2010). k_h of sandstone and mudstone are 300 md and 1 md respectively, and the porosities are 0.2 and 0.1, respectively. The ratio of k_v to k_h is assumed to be 0.1. The observed CO₂ plume shown in Figure 12(b) is obtained by simulating flow in the reference model using CMG-GEM and then applying a CO₂ saturation threshold of 0.1.

(a) Facies

(b) 3D view of observed CO₂ plume

(c) Areal view of observed CO₂ plume

(d) Side view from south of observed CO₂ plume

Figure 12 Facies and CO₂ plume simulated in the reference model for the 3D case. The axes represent distance in km. The z-axis is exaggerated by a factor of 20.

The CO₂ plume shown in Figure 12(b) through Figure 12(d) is used as observed data. Simulated CO₂ plumes are not converted to 2D images because the observed CO₂ plume is given as 3D images. In Figure 12(c), it is observed that the injected CO₂ trifurcates in three directions from the injector located at the middle of the aquifer model to the north-west, south-west, and south-east directions. The flow in the south-eastern direction is over a shorter length than in the other directions. In Figure 12(d), the injected CO₂ migrates up and then migrates horizontally along the top of the aquifer.

Figure 13 shows the data used for simulating the prior models. The training image shown in Figure 13(a) is generated using the TiGenerator with the parameters that are

used to build the true model. However, because of the stochastic nature of the TiGenerator, the training image for the prior models exhibits different characteristics from the reference. The probability of sandstone shown in Figure 13(b) is obtained by filtering the true facies shown in Figure 12(a) with a moving average filter with window dimensions $15*\Delta x$, $15*\Delta y$, and $5*\Delta z$. The probability of shale is calculated by subtracting the probability of sandstone from one. The true facies at the injector $(101,101,1) \sim (101,101,10)$ is used as hard data. k_h , k_v , and ϕ of sandstone and mudstone are assumed known.

Figure 13 Training image and probability of sandstone for simulating prior models using SNESIM in the 3D case. The axes represent distance in km. The z-axis is exaggerated by a factor of 20.

200 models honoring the prior data are generated using the single normal equation simulation (SNESIM) (Strebelle, 2002) of SGeMS. The first row of Table 9 shows facies in three models out of that suite. The middle row of Table 9 shows the regions where the CO_2 saturation is greater than 0.01. That threshold value ($=0.1$) for CO_2 saturation is used in order to determine if a grid block is saturated with CO_2 . The simulated 3D CO_2 saturation maps are converted to binary images by applying the same threshold.

Table 9 Three facies models generated using SNESIM, and the corresponding CO₂ saturations simulated using CMG-GEM. The unit of the axes are in kilometers. The z-axis is exaggerated by a factor of 20.

The dissimilarity between the observed and simulated 3D CO₂ plumes in the 200 prior models are measured using the Euclidean distance and the Hausdorff distance. Then the most probable models using each measure are selected and compared.

The models whose dissimilarity is less than 110 % of the smallest dissimilarity are selected as the most probable ones. Four models are selected using the Euclidean distance and three using the Hausdorff distance. Table 10 shows the CO₂ plumes in the three selected models where the red lines represent the observed CO₂ plume shown in Figure 12(b) to Figure 12(d).

Table 10 Areal and side views from south of three most similar simulated CO₂ plumes obtained using the Euclidean distance and the Hausdorff distance for the 3D case. The number below each figure is the numeric value of dissimilarity between the observed and simulated CO₂ plumes. The red perimeter represents the observed CO₂ plume shown in Figure 12. The axes represent distance in km. The z-axis in the side views is exaggerated by a factor of 20.

Rank	1	2	3
Euclidean distance	49.59	51.89	52.96
Hausdorff distance	55.93	58.31	58.32

In Table 10, the three most similar CO₂ plumes selected using the Euclidean are well confined within the boundary of the observed CO₂ plume. However, the most probable models selected using the Euclidean distance do not reflect the south-eastern flow. This is because the south-eastern flow in any prior model is not the same to that of

the observed CO₂ plume. The prior models should be calibrated so that they have the south-eastern CO₂ flow. However, we do not conduct the calibration process because the objective of this example is to demonstrate which measure is more accurate.

The difference in the vertical extent looks large due to the factor of 20 exaggeration of the vertical axis. In reality, the difference between the vertical thicknesses of the observed and simulated CO₂ plumes is small compared to the match in the horizontal extent. However, the Euclidean distance does not discriminate between the horizontal and vertical extent of the CO₂ plume and consequently the models selected using the Hausdorff distance are ranked the lowest by the Euclidean distance despite the excellent match in the horizontal extent of the plume. In this regard, unless prior models have CO₂ plumes that are almost identical to the observed CO₂ plume, the Euclidean distance fails to find models that yield plumes similar to the observed CO₂ plume.

Contrary to the Euclidean distance, the three most similar CO₂ plumes selected using the Hausdorff distance have the similar horizontal flows to the observed CO₂ plume. The Hausdorff distance seems to fail to find a model with vertical migration similar to the observed CO₂ plume. However, this is understandable because the vertical thickness of the aquifer is much smaller than the horizontal extent of the aquifer and consequently, the mismatch is mostly affected by the horizontal migration.

Lüth et al. (2015) pointed out that a CO₂ plume footprint area is sensitive to a threshold for seismic amplitudes. Likewise, the area or the volume of a simulated CO₂ plume varies according to a threshold for CO₂ saturations. Because the two thresholds are limits for different physical properties, they are likely to have uncertainty in matching the thresholds. Thus a dissimilarity measure of CO₂ plumes should find simulated CO₂ plumes similar to an observed one regardless of the uncertainty associated with the thresholds for seismic amplitudes or flow simulation results. Table 10 shows the results

for a CO₂ saturation threshold of 0.1 while Table 11 shows the results for a CO₂ saturation threshold of 0.25 where the observed CO₂ plume shown in Figure 12 is used as the observation. The Hausdorff distance selected the same most probable models regardless of the threshold values. In this regard, the Hausdorff distance can accurately measure the shape dissimilarity between CO₂ plumes despite the uncertainty associated with thresholds.

Table 11 Areal and side views from south of three most similar simulated CO₂ plumes obtained using the Euclidean distance and the Hausdorff distance for a different threshold (≈ 0.25) applied for simulated CO₂ saturations in the 3D case. The number below each figure is the numeric value of dissimilarity between the observed and simulated CO₂ plumes. The red perimeter represents the observed CO₂ plume shown in Figure 12. The axes represent distance in km. The z-axis in the side views is exaggerated by a factor of 20.

Rank	1	2	3
Euclidean distance	51.45	53.32	53.36
Hausdorff distance	55.86	55.93	61.04

Table 12 shows the areal and side views of the three most similar CO₂ plumes selected through the ranking process using SCA. The CO₂ plumes selected using the ranking process using SCA are the same to those selected using the ranking process

without using SCA as shown in Table 10 and Table 12. 20% of the 200 models were sampled based on the shape dissimilarity between the observed CO₂ plume and the SCA approximations, and then only the 40 sampled models were simulated using CMG-GEM. It took about 2500 seconds and 4 seconds to run CMG-GEM and SCA for a model, respectively. $200 * 2500 / 3600 = 138.89$ hours and $(200 * 4 + 20 * 2500) / 3600 = 14.11$ hours were spent for the ranking process without using SCA and the ranking process using SCA, respectively. Even though about 80% of the total computational cost of the ranking process without using SCA was saved, the ranking process using SCA presented the same results.

Table 12 Areal and side views from south of three most similar simulated CO₂ plumes obtained through the ranking process using SCA for the 3D case. The number below each figure is the numeric value of dissimilarity between the observed and simulated CO₂ plumes. The red perimeter represents the observed CO₂ plume shown in Figure 12. The axes represent distance in km. The z-axis in the side views is exaggerated by a factor of 20.

Rank	1	2	3
Euclidean distance	49.58	51.89	55.24
Hausdorff distance	55.93	58.31	58.32

5.3. Sleipner L9 model

The industrial-scale CO₂ injection has been operated at the Sleipner gas field in the North Sea by Statoil and partners (Chadwick and Noy, 2015). The Sleipner L9 model

is a benchmark model for the uppermost CO₂ unit released by Statoil in 2011. The log₁₀ horizontal permeability and porosity of the Sleipner L9 model shown in Figure 14 are correlated to the depth of grid cells. The horizontal permeability and porosity range from 1692 md to 1977 md, and 0.3519 to 0.3595, respectively.

Figure 14 Log₁₀ horizontal permeability and porosity of the Sleipner L9 model provided by Statoil. The axes represent distance in km.

CO₂ was injected to the Sleipner gas field since 1999, and Figure 15 shows simulated and observed CO₂ plumes for 2008. In Figure 15, the green area is computed for the geologic model shown in Figure 14 using CMG-GEM. The red line represents the CO-water contact obtained from 4-dimensional seismic surveys (Cavanagh, 2013). The simulated plume migrates shorter to the north direction, and longer to the other directions than the observed CO₂ plume. We attempt to obtain better match to the observed CO₂ plume by lowering permeability in the east, west, and south zones out of the observed CO₂ plume. The permeability of the Utsira sand is known to be greater than 1000 md (Chadwick and Noy, 2010; Singh et al., 2010; Zweigel et al., 2004). There is no evidence for low permeability of the Utsira sand, but we generate multiple geologic models that have the low permeability zones to test the practical value of our approach.

Figure 15 CO₂ plume ($S_{CO_2} \geq 0.1$) simulated for the geologic model shown in Figure 14 using CMG-GEM and observed CO₂-water contact at 2008. The green and red represent simulated and observed, respectively. The axes represent distance in km.

Permeability and porosity are assumed to have a log normal distribution and a normal distribution, respectively. Figure 16 shows the probability density function of the $\log_{10}$ horizontal permeability and porosity. Zhu et al. (2015) and Chadwick and Noy (2010) assumed strong contrasts between the east-west and the north-south permeabilities to reproduce the long plume along the north-south direction. Instead of anisotropy of horizontal permeability, as given in Table 13, we assume short and long correlation lengths for the east-west and the north-south directions, respectively. The correlation between $\log_{10}$ horizontal permeability and porosity is assumed to be 0.7.

(a) $\log_{10}$ horizontal permeability

(b) Porosity

Figure 16 Probability density functions of $\log_{10}$ horizontal permeability and porosity

Table 13 Statistical parameters of the multiple geologic models for the Sleipner L9 model

	Log ₁₀ horizontal permeability (md)	Porosity (fraction)
Mean	2	0.25
Standard deviation	0.6	0.05
Minimum	0.4545	0.1212
Maximum	3.5455	0.3788
Major correlated direction	North-south	North-south
Major correlation length (km)	2	2
Minor correlation length (km)	0.5	0.5

Conditioning data for $\log_{10}$ horizontal permeability and porosity are randomly selected within the observed CO₂ plume and the simulated CO₂ plume (green) out of the observed CO₂ plume (red) shown in Figure 15. High permeability and porosity are randomly chosen in the observed CO₂ plume while low permeability and porosity are randomly chosen in the simulated CO₂ plume out of the observed CO₂ plume. Table 14 shows the permeability and porosity of two models among 200 models that are generated using the sequential Gaussian simulation and the sequential Gaussian co-simulation in SGeMS (Remy et al., 2011).

Table 14 Permeability and porosity of two models among 200 models. The axes represent distance in km.

The flow of injected CO₂ is simulated for the 200 models using CMG-GEM. Because the observed plume shown in Figure 15 is given as a 2D image, the simulated CO₂ plumes are converted to 2D images as described in Figure 4 and Figure 5. The 200 simulated plumes are compared to the observed plume using the Euclidean distance and the Hausdorff distance.

Table 15 shows the three most similar simulated CO₂ plumes in terms of the Euclidean distance and the Hausdorff distance. The CO₂ plumes selected using the Euclidean distance are almost confined to the observed CO₂ plume, but they do not have the long migration to the north-south direction. The CO₂ plumes selected using the Hausdorff distance migrate out of the observed frontal contact in the south zone and they have the large Euclidean distances to the observed CO₂ plume. However, the CO₂ plumes

selected using the Hausdorff distance are more similar to the observed CO₂ plume than the those selected using the Euclidean distance in terms of overall shape.

Table 16 shows the log₁₀ horizontal permeability of the most probable model corresponding to the most similar CO₂ plume selected using the Hausdorff distance. In Table 16, the CO₂ migration has stronger resemblance to the permeable zones in the top layers (3rd, 8th, 13th) than the bottom layers (26th, 31st, 36th). This is because buoyant CO₂ flow is strongly affected by heterogeneity of top layers.

The total computation cost of running the 200 models is about 160 hours, but 20% of 160 hours are needed to select the most probable models using SCA. 20% of the 200 models are preselected by measuring dissimilarity between the observed CO₂ plume and 200 SCA approximations using the Euclidean distance or the Hausdorff distance. The final most probable models are selected based on the CMG-GEM results among the 40 preselected models. The final selection results obtained using SCA are identical to those shown in Table 15. However, 80% of the computation cost of the ranking process without SCA is saved by preselecting the 40 models using SCA that are expected to have plume migrations similar to the observed CO₂ plume.

Table 15 Three most similar simulated CO₂ plume obtained using the Euclidean distance and the Hausdorff distance for the Sleipner L9 model. The number below each figure is the numeric values of dissimilarity between the observed and simulated CO₂ plumes. The red perimeter represents the observed CO₂ plume shown in Figure 15. The axes represent distance in km.

Rank	1	2	3
Euclidean distance	<p>21.7256</p>	<p>21.7256</p>	<p>21.8403</p>
Hausdorff distance	<p>385.7297</p>	<p>422.6053</p>	<p>469.6207</p>

Table 16 Log₁₀ horizontal permeability of the most probable model corresponding to the most similar CO₂ plume selected using the Hausdorff distance. The axes represent distance in km.

6. CONCLUSIONS

The objective of this study is to find a set of the most probable models that honor the spatial characteristics of an observed CO₂ plume that is described as a binary image. Quantitative CO₂ saturation images are converted to binary images to compare the shape

of the CO₂ plumes that have considerable uncertainty in the CO₂ saturations. In order to select models whose CO₂ plumes are similar to an observed CO₂ plume, a measure that computes the shape dissimilarity between the observed CO₂ plume and prior ones is necessary. The measure should be sensitive to rotation, scale, and location because they make different spatial characteristics of CO₂ plumes.

The Euclidean distance is the most common way to measure shape dissimilarity between CO₂ plumes sensitively to rotation, scale, and location. Contrary to the Euclidean distance, the Hausdorff distance distinguishes the spatial distribution of the difference between CO₂ plumes. The Hausdorff distance selected more visually and spatially similar CO₂ plumes to the observed CO₂ plumes than the Euclidean distance for the 2D and 3D cases. About 80% of the computational cost of the ranking process was saved by prescreening the models using SCA although the ranking processes using SCA and without using SCA selected the same models.

In this study, threshold values for seismic amplitudes and CO₂ saturations should be chosen carefully because CO₂ plumes are described as binary images. However, the model selection process using the Hausdorff distance and the SCA method for assessing flow characteristics is relatively immune to uncertainties associated with the imaging of the CO₂ plume. The method uses the uncertain image of a CO₂ plume and consistently selects a subset of models that reflect the spatial characteristics of the observed CO₂ plume.

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