Influence of fluid displacement patterns on seismic velocity during
supercritical CO₂ injection: Simulation study for evaluation of the
relationship between seismic velocity and CO₂ saturation

Hirotsu Yamabe^{a,+}, Takeshi Tsuji^{b, c,*}, Yunfeng Liang^a, Toshifumi Matsuoka^a

^a Graduate School of Engineering, Kyoto University, Kyoto 615-8540, Japan

^b International Institute for Carbon-Neutral Energy Research (I2CNER), Kyushu University,
Fukuoka 819-0395, Japan

^c Faculty of Engineering, Kyushu University, Fukuoka 819-0395, Japan

+Present address: Schlumberger Information Solutions, Tokyo 104-0028, Japan.

*Corresponding author: Tel: +81-92-802-6875. Fax: +81-92-802-6875.

E-mail address: tsuji@i2cner.kyushu-u.ac.jp

Abstract

Monitoring of injected subsurface carbon dioxide (CO_2) is essential for safe CO_2 capture and storage. Quantitative monitoring requires knowledge of the relationship between CO_2 saturation and P-wave velocity (V_p). V_p response to CO_2 saturation is not unique and depends on the CO_2 distribution within rock pores. This study evaluated the influence of CO_2 distributions on V_p - CO_2 saturation relationships. We conducted two computational studies with different injection pressures, using a two-phase lattice Boltzmann method for CO_2 injection simulation and wave propagation simulation with a finite difference approach for evaluation of V_p change. The change of capillary number associated with various injection pressures affected the CO_2 displacement patterns. Viscous fingering was typical at high capillary numbers, whereas both viscous and capillary fingering were observed at low capillary numbers. We identified a difference in V_p - CO_2 saturation relationships in these two cases; i.e., lower V_p was observed in the high capillary number case than at low capillary number at the same saturation. The difference in V_p response to CO_2 saturation is caused by CO_2 distribution features. We evaluated V_p with consideration of the pore-scale CO_2 distribution. This study demonstrates that capillary number at each reservoir location (e.g., distance from injection well) should be considered to accurately estimate CO_2 saturation from seismic velocity.

Keywords: Lattice Boltzmann; capillary number; saturation; dynamic wave simulation; seismic velocity; CO_2 distribution features

1. Introduction

Monitoring of injected subsurface carbon dioxide (CO₂) is essential for elucidating subsurface CO₂ behavior and thereby ensuring the safety of CO₂ capture and storage (CCS). There are three objectives of such monitoring: (1) detection (whether CO₂ is present); (2) location (where the CO₂ exists); (3) quantification (how much CO₂ exists). Quantitative CO₂ information is required for CCS monitoring, because such information enables prediction of the risk of CO₂ leakage from storage reservoirs, the efficiency of CO₂ injection to be increased, and the risk of injection-induced seismicity to be reduced. The development of reliable monitoring methods therefore contributes to public understanding and to the promotion of CCS projects globally.

Because the physical properties of rocks are controlled by pore geometry and fluid states in pore spaces, determination of changes in rock properties by geophysical monitoring facilitates understanding of CO₂ behavior in reservoirs. Several seismic and electrical surveys have been conducted for CO₂ monitoring purposes at CCS sites, including Sleipner in the North Sea and Nagaoka, Japan (Arts et al., 2004; Spetzler et al., 2008; Chadwick et al., 2009; Saito et al., 2006; Onishi et al., 2009). These geophysical surveys successfully detected subsurface CO₂ plumes (i.e., the spatial distribution of injected CO₂). Seismic surveys are especially well suited for detecting and locating CO₂ distributions within injection reservoirs, but it is difficult to quantify the amount of subsurface CO₂ (i.e., CO₂ saturation) from seismic data. To estimate CO₂ saturation, the relationship between the saturation and P-wave velocity (V_p) derived from seismic data must be determined. This relationship is difficult to be quantified because the response of V_p to CO₂ saturation is not unique, and is also influenced by the CO₂ distribution in the pore spaces of rock (Knight et al., 1998; Toms et al., 2006; Lebedev et al., 2009). Hence, the response of seismic velocity to CO₂ saturation must be characterized by considering the distribution of CO₂ in pore spaces.

A number of experimental studies, simulations and analyses of field data have examined the relationship between geophysical properties and CO₂ saturation (Nakatsuka et al., 2010, Shi et al., 2011; Alemu et al., 2013). In experimental studies, macroscopic features of CO₂ distribution and saturation can be obtained using X-ray-computed tomography (CT) (Alemu et al., 2013). This technology enables measurement of both time-lapse geophysical properties and saturation during injection of CO₂. However, the microscopic fluid state cannot be detected, because resolution of the conventionally used CT scan is a few millimeters at best, much larger than the size of pores. As another approach, however, recent progress in computer technology has enabled exploration of digital rock physics (Keehm et al., 2001; Knackstedt et al., 2009). Various rock physical properties can be computed by numerical simulation with a digital rock, which is the numerically modeled geometry of a rock core obtained by micro-focus X-ray CT scanning. Despite such efforts, there are still difficulties associated with quantitative CO₂ monitoring.

Understanding the flow of multiphase fluids such as CO₂ and water in porous media has been a subject of great interest over a wide range of scientific and engineering disciplines, because such knowledge is applicable to printing, dense nonaqueous phase liquid remediation, enhanced oil recovery, and CCS. Multiphase flow is complicated by the large number of governing factors. These include interfacial tension, viscosity, density, wetting properties, pore geometry and its heterogeneity, and flow speed. There have been a number of numerical and experimental studies toward understanding these complexities (Lenormand et al., 1983; Homsy, 1987; Weitz et al., 1987; Lenormand and Zarcone 1989; Cottin et al., 2010; Yamabe et al., 2015; Jiang and Tsuji, 2015). When CO₂ is injected into water-saturated rock, drainage displacement usually occurs because the pore surface is usually water-wet. Lenormand et al. (1988) discussed displacement mechanisms in the drainage process with two dimensionless numbers. These are the capillary number (Ca), defined as the ratio of the viscous force to

capillary force ($Ca = \mu_{in} U_{in} / \sigma$ where μ_{in} , U_{in} and σ are the viscosity, velocity of the injected fluid and interfacial tension, respectively), and the viscosity ratio (M), defined as the ratio of non-wetting fluid viscosity to wetting fluid viscosity (in this paper, the ratio of CO₂ viscosity to water viscosity). When considering CO₂ injection into water-saturated media, $M < 1$ is expected because of the lower viscosity of CO₂. In this case, two types of displacement patterns are anticipated, viscous fingering and capillary fingering (Lenormand et al., 1988). When the capillary number is large, tree-like fingers of the invading fluid lead to an unstable displacement, which is called viscous fingering. The fingers can also be observed at low capillary numbers, but the pattern is different from viscous fingering. The fingers grow in all directions, even backward (toward the inlet); this pattern is capillary fingering. When the viscosity ratio or capillary number has an intermediate value, two displacement patterns occur simultaneously, and the resultant pattern is called crossover (Ferer et al., 2004).

These distribution patterns influence geophysical properties of relevance to monitoring. Evaluation of the effect of displacement patterns on geophysical properties is essential for quantitative estimation of CO₂ saturation. The objective of the present work is to provide a quantitative description of CO₂ displacement in porous media and to evaluate the effect of CO₂ displacement on the relationship between CO₂ saturation and V_p via simulation. We used two numerical methods: multiphase lattice Boltzmann (LB) for CO₂ injection simulation, and finite difference (FD) for dynamic simulation of wave propagation on a rotated staggered grid. We applied a synthetic granular rock model to simulate porous media. Multiphase LB simulations provided fluid distributions during injection of CO₂ (Yamabe et al., 2015). We used each solid and fluid distributions derived from the LB simulation to estimate seismic velocity with the FD approach. We executed two case studies with different injection pressures (resulting in changes of capillary numbers), which in turn altered patterns of CO₂ displacement. We then addressed

the displacement patterns and their effects on the relationship between V_p and CO_2 saturation, which gave crucial information for geophysical monitoring of injected CO_2 .

2. Numerical Methods

2.1 Rock model and simulation conditions

We used a synthetic granular rock model to do the LB simulation with a solid–fluid boundary condition (Yamabe et al., 2015). This model is constructed by random packing with a number of spherical grains (Mavko et al., 2003). We assumed the grain-size distribution to be “well–sorted” with a Gaussian spectral shape (Tucker, 2001). The most important parameter in that distribution is the “sorting” parameter (Folk and Ward, 1957; Jerram, 2001), defined as follows.

$$Sorting = \frac{\phi_{84} - \phi_{16}}{4} + \frac{\phi_{95} - \phi_5}{6.6}, \quad (1)$$

where ϕ_k is grain size in phi units (ϕ) at the k^{th} percentage frequency in a cumulative frequency plot. The ϕ is the logarithmic transformation of the Udden–Wentworth scale $\phi = \log_2 d$, where d is grain size in millimeters (Blair and McPherson, 1999). The sorting parameter represents spread of the grain-size distribution (i.e., standard deviation). The simulation studies were conducted for a granular rock model with sorting parameter 0.50 and dimensions $100 \times 100 \times 100$ lattice units representing a total lateral length of 1.0×10^{-3} m along each direction. Figure 1 illustrates a rock model and the geometry of cross sections.

2.2 Lattice Boltzmann

The LB method (McNamara and Zanetti, 1988) has been explored as an alternative numerical method for simulating viscous fluid flow in complicated porous media, because the

method can easily handle complex solid boundaries and is suitable for parallel computation (Chen and Doolen, 1998; Aidun and Clausen, 2010). For a more detailed description of the LB method, see its reviews by Rothman and Zaleski (1997), Succi (2001), and Wolf-Gladrow (2000). A number of LB models applicable to a wide range of engineering problems have been proposed. In particular, great attention has been paid to multi-component/multiphase LB models in the discipline of resource engineering. For multiphase flow simulation, fluid-fluid interaction is introduced to the general LB scheme for phase separation and the imposition of interfacial tension. Various models of phase separation have been developed since the 1990s. All such models can be separated into four categories: color-fluid (Gunstensen et al., 1991), Shan-Chen (Shan and Chen, 1993), free-energy (Swift et al., 1996), and mean-field (He et al., 1998). A number of reviews have discussed the advantages and drawbacks of each multiphase model (Succi, 2001; Huang et al., 2011; Yang and Boek, 2013). In this work, we used a color-fluid model because it requires fewer grids for representing interfaces, the result being a reduction of computational cost. The color-fluid model has a potentially serious drawback, however, in that there are spurious fluid velocities at an interface. Such a spurious velocities disturb the fluid flow when that flow is very slow. We applied a modification suggested by Latva-kokko and Rothmann (2005) to reduce the magnitude of these spurious velocities.

Fluid properties used in LB simulations are shown in Table 1 (Chiquet et al., 2007; Ouyang, 2011). Values in the table simulate the properties of CO₂ and water at 13.8 MPa (2000 psi) and 50 °C (similar to conditions at Nagaoka, the pilot CCS site in Japan; Mito et al., 2008). For computational stability, several parameters differed from realistic values. The density of CO₂ was set to that of water (994 kg/m³), although actual density is 668 kg/m³, and viscosities of both fluids were configured to be 10 times realistic values. These assumptions were made because the multiphase LB model used in this study cannot treat density contrasts and small viscosities. Despite these unrealistic assumptions, the simulation mimics CO₂ injection into

porous media because gravitational effects can be neglected and the ratio of viscosities is fixed. The displacement of water by CO₂ was evaluated, and the solid surface was configured to be fully hydrophilic, so fluid displacement was pure drainage (Jiang et al., 2014). CO₂ dissolution in water and geometric deformations caused by pore pressure changes were neglected.

We conducted two case studies, “Case 1” and “Case 2,” by varying injection pressure. In Case 1, that pressure was higher than in Case 2, but other parameters were the same. The difference of injection pressure caused changes in fluid velocity and capillary number. Injection pressures in each case were configured so that the average capillary number was 1.00×10^{-3} in Case 1 and 4.69×10^{-5} in Case 2. The capillary number was calculated using average fluid velocity until the CO₂ front reached the outlet of digital rock, because that velocity changed rapidly after percolation. Fluid velocity U_{CO_2} was calculated from the CO₂ flux, $Q_{CO_2} = dV_{CO_2}/dt$, where V_{CO_2} is CO₂ volume at each time step. U_{CO_2} was obtained by dividing the CO₂ flux by cross-sectional area of the rock model, $U_{CO_2} = Q_{CO_2}/A$, where A is cross-sectional area of porous rock. These approaches are similar to Yamabe et al. (2015), although the reservoir conditions used in this study (i.e., capillary number) are different from the previous study.

2.3 Finite difference algorithm on rotated staggered grid

The LB simulation output the water-CO₂-solid distribution at each time step. The distribution of each fluid/solid determines the effective geophysical properties. However, it is difficult to obtain elastic properties by a theoretical approach because of the complex distribution of each phase. Recently, FD algorithm for simulating dynamic wave propagation has been applied to fluid-saturated digital rock with realistic pore geometry (Saenger et al. 2000; Saenger et al. 2004; Ciz et al. 2005; Saenger et al., 2011; Madonna et al., 2012). To estimate changes of V_P during CO₂ displacement, the approach suggested by Saenger et al. (2004) was

adopted. In this approach, the FD algorithm on a rotated staggered grid is used for simulating dynamic wave propagation (Saenger et al., 2000).

Elastic properties of each phase are shown in Table 1 (Mavko et al., 2003). Properties of quartz are used for those of solid phase. Given that average velocity of the rock model is 3,000 m/s and that the wave central frequency is 2 MHz, the wavelength becomes 1.5×10^{-3} m, which is larger than the sample length. The size of the original digital rock model is thus too small to evaluate V_p accurately. We synthetically enlarged the rock sample by replicating the original geometry (here it is called “box”) and deposited them. To make the boundary of deposited boxes smooth, replicated boxes had “mirrored” geometry (Figure 2). We set one mirrored box in the direction perpendicular to wave propagation (x- and y-direction) and three boxes parallel to that propagation (z-direction). Therefore, the simulated geometry was $200 \times 200 \times 400$. This model size for wave propagation is 4×10^{-3} m, which is ~ 2.7 times longer than the wavelength of 2 MHz source signal. The mirrored sample was embedded in a homogeneous solid region (i.e., buffer) whose elastic properties were the same as the solid phase (Figure 2).

At the top of the homogeneous region, we induced a Ricker wavelet (plane wave) with central frequency of 2 MHz. We further set “nonreflecting” layers (in the z-direction) to avoid wave reflection at the boundary of the computational domain (Cerjan et al., 1985). Periodic boundary conditions were configured in the x and y directions. Plane receivers were placed at the top and bottom of the mirrored rock geometry (Figure 2). Received signals were averaged over each plane. V_p of the digital rock sample was calculated by time delay of the peak amplitude observed at the two receivers. These processes are similar to numerical settings suggested by Saenger et al. (2011).

3. Numerical Results

In the LB simulation, CO₂ was injected into a synthetic granular medium saturated with water. The injection direction was defined as the x direction. The simulation continued until the distributions of CO₂ reached equilibrium. Figure 3 shows the CO₂ distribution from the multiphase LB simulation in two cases. A clear difference of CO₂ distribution between the cases is evident. There were also large differences in saturation characteristics of the equilibrium state of each case; equilibrium saturations in Cases 1 and 2 were 95.1% and 56.0%, respectively. Differences of the distributions and equilibrium saturation are discussed in the next section.

The LB simulations yielded distributions of CO₂ and water. V_p changes during CO₂ injection were estimated by wave propagation simulation with the FD algorithm. Figure 4 shows simulated V_p -CO₂ saturations in Cases 1 and 2. Because V_p of CO₂ is lower relative to that of water, an increase in CO₂ saturation reduces V_p . Plots of the two cases were almost the same at low saturation. However, with this saturation higher than 30%, V_p in each case was clearly different. V_p was higher in Case 2 (low injection pressure or low capillary number) than in Case 1 (high injection pressure or high capillary number), even with the same CO₂ saturation. It is believed that the difference of V_p response to CO₂ saturation is caused by variation of the CO₂ distributions.

4. Discussion

4.1 Displacement patterns and saturations

The simulation results (Figure 3) show characteristics of viscous and capillary fingering. The invading CO₂ flows via several fingers toward the outlet, especially in Case 1 (viscous fingering). In Case 2, the front exhibits fingering but with fewer fingers than in Case 1. Figure 3 (Case 2-d) shows that fingers of the front in Case 2 moved not only in the direction of injection but also toward the inlet (backward). Movement of a finger toward the inlet is very characteristic of capillary fingering. Nevertheless, the front simultaneously moved toward the

outlet, even though the pores were narrower. Therefore, there was both viscous and capillary fingering in Case 2, and the behavior in that case corresponds to the crossover region between viscous and capillary fingering (Yamabe et al., 2015).

A number of studies in the literature have demonstrated that equilibrium saturation is much higher when viscous fingering ($S_{viscous}$) is dominant compared with when there is crossover of capillary and viscous fingering ($S_{crossover}$) (Lenormand et al., 1988; Zhang et al., 2011; Wang et al., 2012), i.e., $S_{viscous} \gg S_{crossover}$. The simulated equilibrium saturation demonstrated that viscous fingering was dominant in Case 1 (higher saturation), and that Case 2 (lower saturation) corresponds to the region of aforementioned crossover. More discussion about saturation and displacement mechanisms is conducted in Yamabe et al. (2015).

4.2 V_P response to CO_2 saturation

Since change of V_P during fluid displacement is dependent not only on fluid saturation but also on its distribution, the displacement pattern discussed above affects V_P . Therefore, for geophysical monitoring, we should link the CO_2 distribution (or behavior) and V_P . From the standpoint of rock physics, fluid distribution patterns are often classified into two types, patchy and uniform saturation (Mavko and Mukerji, 1998; Johnson, 2001). When the CO_2 patch is larger than the fluid diffusion length (Mavko and Mukerji, 1998), the fluid pressure in pore cannot reach equilibrium state. In this case (i.e., patchy saturation), V_P is higher. In contrast, when the patch size is smaller than the length, the fluid is equilibrated (i.e., uniform saturation), resulting in lower seismic velocity. However, the fluid diffusion length in our system is a little larger than the model size (Figure 2), and the patch size is similar to our model. Thus, it is difficult to accurately consider this mechanism in our simulation.

The seismic velocity in heterogeneous media is also influenced by scattering phenomena, which is characterized by scale of heterogeneity (e.g., patch size of CO_2 ; Figure 5a). If

heterogeneous size (patch size) in porous rock is larger relative to the wavelength, the seismic velocity is higher (Mavko et al., 2003). Aki and Richards (1980) classified scattering phenomena on the diagram with two dimensionless numbers (Figure 5b): ka and kL , where k is wave number, a is heterogeneity size, and L is path length. The parameter D_{wave} in Figure 5b is called wave parameter, defined as $D_{wave} = 4L/ka^2$. In the present studies, heterogeneity size of two-phase fluids, a , can be estimated on the order of 10^{-3} m, and L is also the same order. Since $k \approx 4200$ (1/m), $L \approx a \approx 10^{-3}$ (m), $D_{wave} \approx 1$ and $ka \approx kL \approx 4$, which means scattering domain is on the boundary of Ray theory and effective medium where V_p is strongly affected by fluid distribution heterogeneity (Figure 5). Since the frequency of the wave source was not varied in this research, these saturation states can be addressed by CO_2 patch size.

Figure 4 demonstrates that V_p in Case 1 (higher capillary number) was slower than that in Case 2 (lower capillary number) when CO_2 saturation exceeded 30%. The difference of CO_2 distribution in the two cases is shown in Figure 6 (at almost the same saturation in Case 1 and 2). We see that CO_2 in Case 2 concentrates on the inlet side (left side in the figure) compared with Case 1. The tendency of concentration of CO_2 distribution is discussed in Yamabe et al. (2015). The high concentration of CO_2 distribution in Case 2 (low capillary number) means large CO_2 patch (large size of heterogeneity), which leads to higher seismic velocity. The simulated relationship of V_p - CO_2 saturation reflects these tendencies toward CO_2 patch size.

To confirm that the response of V_p to CO_2 saturation reflects the pore-scale CO_2 distribution, we did further simulations. We prepared rock models in which water and CO_2 were distributed with very small size. In the models, both fluids were synthetically located on each grid by random numbers. We prepared these “randomly-distributed models” with CO_2 saturations 5, 10, 20,..., 90%. The fluid distribution state in the model with 50% CO_2 saturation is shown in Figure 7. It is observed from Figure 7 that the bubble size of each fluid was much smaller than those in Cases 1 and 2 (LB results). In this case, heterogeneity size, a , can be

considered as 10^{-5} (m) and $ka \approx 4 \times 10^{-2}$, scattering domain of Aki and Richards (1980) is on effective medium theory. Thus the seismic velocity in randomly-distributed models should be lower than LB simulated fluid distribution. Simulated V_p response to CO_2 saturation in the randomly-distributed models is shown in Figure 4 (red symbols). As expected, V_p in randomly-distributed media was much lower than those from the LB simulations. These results demonstrate that the FD approach for digital rock model provides a successful means to estimate V_p by considering the fluid distribution state.

Compared with randomly-saturated media, the CO_2 phase in LB the results forms larger bubbles. This is partially because the injection process in both Case 1 and 2 was drainage. Under drainage conditions, the injected fluid continuously invades pore spaces, producing the large CO_2 cluster. This tendency under drainage has been reported using field log data (Azuma et al., 2013). Even though the differences in Case 1 and 2 is smaller than that from randomly-distributed models (Figure 4), there is a detectable difference of V_p response to CO_2 saturation, which is caused by differences in displacement mechanisms, and this difference cannot be neglected.

Although our simulation study was conducted at the millimeter scale, we could upscale the implications derived from this study to field-scale seismic monitoring data. The large capillary numbers are expected close to the injection well, and these numbers decrease with distance from the well. The expected distribution state far from the well thus becomes more patchy-saturated. This indicates that the sensitivity of V_p to CO_2 saturation for same rock varies with distance from the well. Whereas, the sensitivity of V_p to CO_2 saturation is also dependent on the relationship between CO_2 bubble (or plume) size and frequency. Although we usually use lower frequency sources in seismic and logging experiments (several 10 – 100 Hz), the CO_2 behavior difference occurs even in larger-scale. Indeed, the scattering phenomena of seismic wave due to size of heterogeneity (Aki and Richards, 1980) have been considered in field-scale reflection

seismic surveys. Because CO₂ behavior controlled by capillary number influences the sensitivity of field-scale data to the CO₂ saturation, the capillary number for each reservoir location must be carefully considered when estimating CO₂ saturation from V_p.

5. Conclusions

We simulated V_p change caused by CO₂ displacement starting with a water-saturated synthetic rock model, via the LB method and dynamic wave simulation with an FD approach on a rotated staggered grid. We conducted two case studies by varying injection pressure so as to produce capillary numbers of 1.00×10^{-3} (Case 1) and 4.69×10^{-5} (Case 2). These conditions impacted the displacement pattern of CO₂. Viscous fingering was the dominant displacement pattern in the high capillary number case, whereas crossover of viscous and capillary fingering was observed in the lower capillary number case. This demonstrates that decreases in capillary number altered the dominant fingering pattern, from viscous fingering to a crossover of viscous and capillary fingering.

Applying dynamic wave simulation with the FD approach to LB results (Cases 1 and 2) and randomly-distributed media, we successfully evaluated the relationship of V_p-CO₂ saturation, which is the most important relationship in geophysical monitoring of CCS projects. The resultant relationships from both cases in LB simulation showed similar trend, however, there was a detectable difference in V_p response that cannot be neglected. We conclude that this difference is caused by different CO₂/water displacement mechanisms. Viscous fingering (high capillary number) generating smaller size of CO₂ patch yields relatively lower velocity, whereas capillary fingering (low capillary number) leads to higher velocity. The capillary numbers vary with distance from the injection well. Low capillary numbers are expected far from the well and high numbers near the well. The capillary number for each reservoir location must be considered when estimating CO₂ saturation from V_p.

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Figure Captions

Fig. 1. Constructed rock model with sorting parameter = 0.5 (a). White color shows solid grains in rock model. Panels (b-1), (b-2), (b-3) and (b-4) show cross-sections at $x=20$, 40, 60 and 80, respectively.

Fig. 2. Process of mirroring and embedding in homogeneous solid region. Geometry obtained by LB simulation is mirrored and deposited.

Fig. 3. CO₂ distribution (red parts) in granular rock model (solid grains are white) obtained by LB simulation. The water phase is not visualized. CO₂ is injected along x-direction. Top five panels (Case 1a–e) show results of Case 1 (high injection pressure), and bottom three panels (Case 2a–e) those of Case 2 (low injection pressure). Snapshots are ordered chronologically from left to right. Rightmost panels (Case 1e and 2e) show equilibrated state.

Fig. 4. Relationship between V_p and CO₂ saturation, calculated by wave propagation simulations. Circles depict results of Case 1 and diamonds those of Case 2. Red symbols show results of random-distributed models.

Fig. 5.(a) Scale dependent (or frequency dependent) scattering domains and its effect on seismic velocity. Note that ds is diameter of scattering heterogeneity (Mavko et al., 2003). (b) Aki and Richards' diagram showing the domain of various scattering mechanisms (Aki and Richards, 1980). Blue and orange zones are our simulation conditions.

Fig. 6. Comparison of CO₂ distribution in both cases, with similar saturation (55.5% in Case 1, 56.0% in Case 2). Right graph shows average CO₂ saturation in y-z plane vs. distance from inlet (x-coordinate). Solid curve represents saturation profile in Case 1 and dashed line that in Case 2.

Fig. 7. CO₂ distribution in random-distributed model with CO₂ saturation = 50% (left panel). Right panel shows cross-section at $x = 50$.

Table 1

Density, viscosity, interfacial tension (IFT), and bulk and shear moduli in simulation. For stable computation, CO₂ density and viscosity of both fluids were assigned unrealistic values (realistic values listed in parentheses).

	Water	CO ₂	Solid (quartz)
Density (kg/m ³)	994	994 (668)	2650
Viscosity (cP)	5.51 (0.551)	0.521 (0.0521)	-
IFT (mN/m)	35.0		-
Bulk modulus	2.25	0.0493	36.0
Shear modulus	0	0	44.0

Figure 1

Figure 2

Figure 3

Figure 4

Figure 5

Figure 6

Figure 7