Comparison of electron width models for fast line profile calculations

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Abstract

The first non-vanishing term in the perturbation expansion of the electron contribution to the line width, commonly used in spectral line broadening by plasmas, was previously expressed in terms of the thermally averaged bremsstrahlung Gaunt factor. The approximations in the derivation, however, suggest that the result is uncertain. The electron width formula is tested with the hydrogen Balmer series and found suspect. Calculations for the He II Lyman series also display similar difficulties. The limitation of this electron width formulation is traced to the absence of an explicit strong collision cutoff beyond which the second-order theory is invalid.

Key words: Stark broadening; electron line widths

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1. Introduction

Spectral lines provide valuable information [1] and Stark-broadening models have been developed [2,3,4] to characterize plasmas [5,6,7]. These models share approximations such as quasi-static ions and impact electrons [1]. Due to the large number of calculations often necessary to analyze spectra, these efforts employ fast algorithms to compute the width contribution from perturbing electrons. One approach expresses the first non-vanishing term in the perturbation expansion of the electron width as the thermally averaged bremsstrahlung Gaunt factor [8]. This formula was successfully tested for transitions involving atomic levels with low principal quantum numbers (PQN) [8]. Contrary to other approaches [2,3,9], the Gaunt factor expression [8] does not include an explicit strong collision cutoff beyond which perturbation theory becomes invalid. Consequently there remained uncertainty whether the formula extends to spectral lines associated with higher PQN's.

Unfortunately, plasma experiments are challenging and data often contain large uncertainties making it difficult to discriminate among models. One notable exception is the high-precision measurements of the hydrogen Balmer series [10]. The purpose here is to compare two electron width models [2,8] for fast line profile calculations at the conditions of the measured spectra. This requires an extension of the O'Brien-Hooper formalism [8] to neutral radiators readily accomplished from their main results. The comparisons show that the connection to bremsstrahlung used in line profile calculations [4,7,8,11,12] is suspect. A second test for the He II Lyman series yields similar conclusions. Possible consequences in the analysis of recent opacity experiments [6] are entertained. The electron width models are also applied to isolated lines in Li-like systems and compared with measurements [13].

2. Electron widths

The line shape function for an ion emitting or absorbing (the radiator) a photon of energy $\hbar\omega$ while immersed in a plasma is given in the 'standard' Stark-broadening theory by [1,9]

$$I(\omega) = \int_0^\infty dFW(F) J(\omega, F) \quad (2.1)$$

where $W(F)$ is the ion electric microfield probability distribution function and (neglecting lower state broadening to simplify notation)

$$J(\omega, F) = -\pi^{-1} \text{Re} \text{Tr}_a \left\{ \vec{d} \cdot \left[i\Delta\omega - i\vec{d} \cdot \vec{F}/\hbar + \phi(\Delta\omega) \right]^{-1} \rho \vec{d} \right\} \quad (2.2)$$

Here, Tr_a denotes a trace over the internal states of the radiator, $\vec{d}$ is the radiator dipole operator, r describe the radiator internal state populations, and the matrix elements of the detuning frequency from line center are given by

$$D W_{mn} = W - W_m + W_n \quad (2.3)$$

with $\hbar\omega_a$ the energy of the radiator internal state a .

The description of electron broadening in f often assumes weak collisions of duration much shorter than the internal radiator state lifetimes involved in the radiative transition [1]. Hence, collisions are treated in perturbation theory retaining only the first non-vanishing term in the electron-radiator interaction simplified to the dipole approximation. The resulting second-order electron width is then given by

$$\phi(\Delta\omega) = -\frac{\vec{d} \cdot \vec{d}}{3\hbar} \int_0^\infty dt e^{i\Delta\omega t} Tr_e \left\{ \vec{F}_e \cdot e^{-iH_e t/\hbar} \rho_e \vec{F}_e e^{iH_e t/\hbar} \right\} \quad (2.4)$$

where Tr_e , ρ_e , and H_e denote a trace, density matrix, and Hamiltonian for an electron gas, respectively, and $\vec{F}_e$ is the electric field produced by the electron gas at the radiator. In the line formulations of interest here, the electron width is rewritten in the form

$$\phi(\Delta\omega) = -\frac{4\pi n_e e^2}{3\hbar} \sqrt{\frac{2m}{\pi T}} \vec{d} \cdot \vec{d} G(\Delta\omega) \quad (2.5)$$

where m , n_e , and T , are the electron mass, number density, and temperature (energy units), respectively, and the G -function is related to the space and time dependent charge-density fluctuations in an electron gas [14]. Two approaches for computing the G -function [2,8] that are commonly used in experimental plasma diagnostics are compared in the present work.

2.1 Lee model

The second-order width can be written in terms of an integral over the momentum transfer in the electron-radiator collision [2,14]. Assuming $\hbar\Delta\omega \ll T$ yields

$$\phi(\Delta\omega) \approx \frac{2e^2 T}{3\pi\hbar\Delta\omega} \vec{r}_a \cdot \vec{r}_a \int_0^{k_m} dk k^2 \text{Im} \varepsilon^{-1}(k, \Delta\omega) \quad (2.1.1)$$

leading to

$$G(DW) = \int_0^k \frac{dk}{k} \exp \left\{ -\frac{1}{2k^2 I_e^2} \frac{DW^2}{W_p^2} \right\} \frac{1}{|e(DW, k)|^2} \quad (2.1.2)$$

where λ_e and ω_p are the electron Debye length and plasma frequency,

$$\lambda_e^2 = \frac{T}{4\rho n_e e^2} \quad (2.1.3)$$

$$\omega_p^2 = \frac{4\rho n_e e^2}{m} \quad (2.1.4)$$

e is the elementary unit of electric charge, and $\epsilon(\omega, k)$ is the dielectric function for a homogeneous electron gas. Thus, the latter neglects the net radiator charge effects on the perturbing electrons. Nevertheless, it is expected to be valid for charged radiators [9].

The cutoff k_m , which limits the momentum transfer in a collision and avoids the divergence of the integral, can be estimated by considering the partial wave expansion of the full electron-radiator interaction [15,16], which contains the spherical Bessel function $j_\ell(kr_a)$ [17] with r_a the radial position of the bound electron. In most cases of interest $kr_a \ll 1$ and the lowest order non-vanishing term yields the dipole approximation. Thus, a reasonable upper limit is given by k values that are cutoff by fast oscillations in $j_\ell(kr_a)$ [15],

$$\frac{\hbar^2 k_m^2}{2m} \approx E_j \quad (2.1.5)$$

where E_j is the binding energy of the radiator j^{th} level. For hydrogenic systems,

$$k_m^2 a_o^2 \gg \frac{Z^2}{n^2} \quad (2.1.6)$$

with Ze the nuclear charge, n the level PQN, and a_o the Bohr radius. Further approximations replace the dielectric function by its $D\omega \rightarrow 0$ and ∞ limits to get [2]:

$$G_o = \frac{1}{2} \left[\ln \left(1 + k_m^2 \lambda_e^2 \right) - \frac{k_m^2 \lambda_e^2}{1 + k_m^2 \lambda_e^2} \right] \quad (2.1.7)$$

$$G_\infty(D\omega) = \frac{1}{2} E_1 \left(\frac{1}{2 k_m^2 \lambda_e^2} \frac{D\omega^2}{\omega_p^2} \right) \quad (2.1.8)$$

with $E_1(z)$ the exponential integral [17]. Finally, Lee takes [2]

$$G_L(D\omega) = S_n + \text{Min}[G_o, G_\infty(D\omega)] \quad (2.1.9)$$

where neglected strong collisions are accounted by $S_2 = 1.5$, $S_3 = 1.0$, $S_4 = 0.75$, $S_5 = 0.5$ and $S_n = 0.4$ for $n > 5$ [9,18]. For large detuning,

$$G_L(DW \rightarrow \infty) = S_n + k_m^2 / e^2 \frac{W_p^2}{DW^2} \exp\left(-\frac{1}{2k_m^2 / e^2} \frac{DW^2}{W_p^2}\right) \quad (2.1.10)$$

Thus, a correction is necessary to reproduce the quasi-static limit [2]. The Griem et al. [9] and Calisti et al. [3,18] models are similar to Lee's [2] and need not be considered separately.

The subroutines to compute Stark profiles of one- and two- electron systems with G_L were incorporated into widely used codes for designing and analyzing experiments [19,20]. In addition, the subroutines were generously made available for other applications [21].

2.2 O'Brien-Hooper model

An alternative expression for the G -function is given by the thermal average [8]

$$G_{OH}(DW, Z_{net}) = \frac{\rho}{\sqrt{3}} \int_0^\infty \frac{dE}{T} e^{-E/T} g(E, DW, Z_{net}) \quad (2.2.1)$$

where $g(E, DW, Z_{net})$ is the bremsstrahlung Gaunt factor for absorption of a photon with energy $\hbar\Delta\omega$ by an electron with initial energy E scattering in the Coulomb potential produced by the net radiator charge, $Z_{net}e$. It can be expressed in terms of the radial integrals [8,22,23,24]

$$\int_0^\infty dr \psi_\ell(r, E) \psi_{\ell \pm 1}(r, E + \hbar\Delta\omega) \quad (2.2.2)$$

for partial wave ℓ and boundary conditions for the perturbing electron wave function

$$y_\ell(r \rightarrow 0, E) \propto r^{\ell+1} \quad (2.2.3a)$$

$$y_\ell(r \rightarrow \infty, E) \propto \frac{\sin(qr + d_\ell)}{qr} \quad (2.2.3b)$$

with wave vector $\hbar^2 q^2 / 2m = E$ and d_ℓ the phase shift.

In the case of neutral radiators $y_\ell(r, E) = j_\ell(qr)$, the spherical Bessel function in the partial wave expansion of plane waves. Thus, the Gaunt factor reduces to the Born approximation [24] with thermal average [25]

$$\begin{aligned} G(\Delta\omega; Z_{net} = 0) &= G_{OH}^{Born}(\Delta\omega) \\ &= e^{\hbar\omega/2T} K_0(\hbar\Delta\omega/2T) \end{aligned} \quad (2.2.4)$$

where $K_o(z)$ is the modified Bessel function of the second kind [17]. For large detuning

$$G_{OH}^{Born}(DW \rightarrow \infty) \propto DW^{-1/2} \quad (2.2.5)$$

reproducing the Holtsmark quasi-static limit [2]. The G -function expression in Eq. (2.2.1) describes ideal electrons interacting with the net radiator charge, however, electron-screening effects can be approximated by $G_{OH}(DW = w_p)$ for $DW \leq w_p$ [26,27]; thus, avoiding the well-known infrared divergence [28]. There is no correction for strong collisions in the O'Brien-Hooper formulation.

3. Results

In this Section the hydrogen Balmer spectrum is computed with G_{OH} and G_L , which requires the extension of the O'Brien-Hooper formalism to neutral radiators above. The line widths near line center for the He II Lyman series are compared. The Mg $He\gamma$ line profiles, used to characterize plasmas in opacity measurements [6], provide an example of systematic errors from these electron width formulas. Finally, the widths of the 3s-3p isolated lines of Li-like systems, which present well-known challenges for Stark-broadening theory [13], are considered.

3.1 H Balmer series

The Balmer emission spectrum was measured with high precision in a wall-stabilized arc [10] providing a test for Stark broadening theories. A spectrum calculation using the line shape codes from Lee [2] was in good agreement with measurements [21] and is reproduced in Fig. 1. Level populations were computed [29] as well as the radiative recombination into the hydrogen L -shell and emission from the $H + e^-$ system to compare with the experimental spectrum.

The present work does not compare directly with the experimental spectra. Instead, it assumes the profiles with G_L , as shown in Fig. 1, are in good agreement with the measurements. Thus, only the emission bound-bound spectrum is calculated with standard Stark-broadening theory [1] plus Doppler broadening using level populations in thermal equilibrium. The electric microfield distribution in the quasi-static ion approximation uses the APEX model for neutral radiators [30] while the electrons are treated with either G_L or G_{OH}^{Born} explicitly retaining the frequency dependence of the electron broadening. All dipole allowed couplings between initial levels with PQN $n = 3$ through $n = 20$ are included. The final states are restricted to the $n = 2$ levels with coupling between 2s and 2p but without any coupling to the initial levels except for

the radiative transitions, which include all dipole allowed lines from $n = 3, 4 \dots 20$ to $n = 2$. The calculations neglect interference terms [1]. A Hessenberg decomposition algorithm is used to accelerate the integration over the microfield distribution at each detuning frequency [31].

The area normalized Balmer emission bound-bound spectra at conditions comparable to the experiments [10] are displayed in Fig. 2. The plots show significant discrepancies with G_{OH}^{Born} producing broader lines and a faster approach to the Inglis-Teller limit [32]. Thus, these comparisons together with the agreement between calculations using G_L and the measurements suggest G_{OH}^{Born} overestimates the electron widths.

The ratios of the electron widths G_{OH}^{Born}/G_L for the two electron densities in Fig. 2 computed near line center ($DW = W_p$) are given in Table 1. The table shows, consistent with the spectra in Fig. 2, a discrepancy between the electron broadening models increasing with PQN. Note that G_{OH}^{Born} remains fixed independent of the transition; it is G_L that decreases with increasing upper level PQN as a result of the decreasing momentum transfer cutoff in Eq. (2.1.5).

It follows from the $DW \rightarrow \infty$ limit of the G -functions that the discrepancy increases in the line wings. The ratios G_{OH}^{Born}/G_L for the $H\beta$ line at the conditions in Fig. 2 are plotted in Fig. 3 as a function of detuning frequency. The curves in the figure terminate at the estimated value of the Weisskopf frequency, which for a hydrogenic system with nuclear charge Ze is given by [2]

$$\hbar\omega_{kf} \approx \frac{3Z\hbar}{4n^2a_o} \sqrt{\frac{3T}{m}} \quad (3.2.1)$$

and approximates the onset of quasi-static electrons. Both formulas have the $DW^{-1/2}$ behavior for $DW > W_{kf}$ (imposed on G_L) but the approach, as seen in Fig. 3, is significantly different.

3.2 Modified O'Brien-Hooper model

As mentioned above, G_{OH}^{Born} does not contain a strong collision cutoff. It is possible to introduce a minimum impact parameter in the electron-radiator collision following the arguments for k_m , which are equivalent to excluding perturbing electrons from penetrating the radiator. For the sum over partial waves, then [24]

$$\begin{aligned} \ell_{\min} &\gg qR \\ &\gg \frac{3n^2}{2Z} \sqrt{\frac{2a_o E}{e^2}} \end{aligned} \quad (3.2.1)$$

where E and $\hbar q$ are the initial scattering electron energy and momentum, respectively, R estimates the size of the bound electron orbit, and the second line assumes a hydrogenic radiator with nuclear charge Ze and level with PQN n . The modified O'Brien-Hooper G -function is then given by limiting the sum over partial waves to $\ell \leq \ell_{\min}$. That is,

$$\tilde{G}_{OH}^{Born}(\omega) = \frac{\pi}{2} \int_0^\infty \frac{dE}{T} e^{-E/T} \sqrt{\frac{E + \hbar\omega}{E}} \sum_{\ell \geq \ell_{\min}} \ell \Gamma^2(\ell) X^\ell \{A_\ell^2(X) + B_\ell^2(X)\} \quad (3.2.2)$$

where

$$G\left(\ell + \frac{1}{2}\right) A_\ell(z) = {}_2F_1\left(\frac{\ell}{2}, \ell + \frac{1}{2}; \frac{\ell+1}{2}; z\right) \quad (3.2.3)$$

$$G\left(\ell + \frac{3}{2}\right) B_\ell(z) = \frac{\sqrt{z}}{2} {}_2F_1\left(\frac{\ell+1}{2}, \ell + \frac{3}{2}; \frac{\ell+3}{2}; z\right) \quad (3.2.4)$$

with ${}_2F_1(a, b, c; z)$ and $G(z)$ the hypergeometric and Gamma functions [17], and

$$X = \frac{E}{E + \hbar\omega} \quad (3.2.5)$$

obtained using

$$\int_0^\infty dr j_\ell(q_1 r) j_\ell(q_2 r) = \frac{\rho}{4q_2} \frac{q_1^\ell}{q_2^\ell} \frac{\Gamma(\ell + \frac{1}{2})}{\Gamma(\frac{\ell}{2})} \frac{{}_2F_1\left(\frac{\ell}{2}, \ell + \frac{1}{2}; \frac{\ell+1}{2}; \frac{q_1^2}{q_2^2}\right)}{\Gamma(\frac{\ell}{2}) + \frac{1}{\Gamma(\frac{\ell}{2})} \frac{\Gamma(\ell + \frac{3}{2})}{2}} \quad (3.2.6)$$

Due to the large number of partial waves required to obtain accurate results, it is often more efficient to compute the sum for $\ell < \ell_{\min}$ and subtract the average from $\tilde{G}_{OH}^{Born}$ in Eq. (2.2.4).

The ratios $\tilde{G}_{OH}^{Born}/G_L$ evaluated at $DW = W_p$ are given in Table 1 displaying improved agreement (values closer to unity) with G_L . Also displayed in the table are values for $\ell_{\min}$ calculated at $E = T$ showing that significant contributions from the sum over partial waves are excluded in the modified G -function for high PQN's. The frequency dependent ratio $\tilde{G}_{OH}^{Born}/G_L$ is plotted in Fig. 3 for the $H\beta$ line showing improved agreement with G_L .

The O'Brien-Hooper formulation does not contain any correction for missing strong collisions [8]. If strong collisions are added to G_{OH}^{Born} it increases the disagreement with G_L for all PQN's in Table 1. Adding the strong collision correction to $\tilde{G}_{OH}^{Born}$ yields

$$1.2 \leq (\sigma_n + \tilde{G}_{OH}^{Born}) / G_L \leq 1.8 \quad (3.2.7)$$

with the larger ratios occurring for the larger PQN's. That is, for larger PQN the strong collision correction S_n is considerable but unfortunately uncertain.

3.3 He II Lyman series

The O'Brien-Hooper electron widths were first applied to the He II Lyman- α line [8] and later to more ionized charged radiators [4,7,11,12]. The ratio G_{OH}/G_L computed at $DW = W_p$ for the He II Lyman series at three electron densities are provided in Table 2. The comparisons show 25% agreement between the Lee and O'Brien-Hooper widths for Lyman- α , where the latter were shown [8] to be in excellent agreement with classical paths calculations [33] at the middle density in Table 2. Subsequent measurements of the He II Lyman- α line [34] reported experimental line widths in agreement with the unified theory [35] as well as the classical paths calculations [33]; thus, implying agreement with the O'Brien-Hooper model. As in Table 1, the ratio G_{OH}/G_L in Table 2 increases for higher PQN's as well as with increasing electron density. In this case, adding a strong collision correction to G_{OH} would negatively impact the agreement with experiments as well as increase the ratio G_{OH}/G_L for the larger PQN's.

3.4 Mg He γ □□□□

Line profiles are presented in Fig. 4 for the helium-like Mg $n = 4$ to $n = 1$ transitions used to diagnose plasma conditions in recent opacity experiments [6,36]. These calculations again use standard broadening theory with the APEX low-frequency microfield component for charged radiators [37,38]. The calculations include dipole allowed couplings between the $n = 4$ and 5 levels; the latter were found to impact the line profile. The calculations include Doppler and instrumental broadening where the latter assumes a Gaussian to simulate the spectral resolving power, $l/D \approx 1000$, in the experiments [36,39]. The figure shows the calculation with G_{OH} (used in the opacity experiments [6,36]) broader than with G_L at the same plasma conditions. In

order to match the line wing (often used for density determination rather than the more uncertain line core), the density in the calculation with G_L has to be increased by $\sim 25\%$ as shown in Fig. 4.

3.5 Isolated lines

Only hydrogenic or helium-like systems were considered in the comparisons above. It is known, however, that the line widths of Li- and Be-like systems display a disturbing disagreement between theoretical and experimental results [13]. These isolated lines are produced by ions with one or two electrons outside a closed K -shell. It is then interesting to compare the line widths using G_L and G_{OH} for these lines.

The examples in Table 3 are for the 3s-3p transitions in N V and Ne VIII. The widths g_L and g_{OH} computed with G_L and G_{OH} , respectively, are compared with experimental values g_{expt} , which have been scaled to $n_e = 10^{18} \text{ cm}^{-3}$ [13]. The table shows good agreement between both g_{OH} and g_L with the experimental values for N V where other electron width models also agree with measurements [13]. On the other hand, both models underestimate g_{expt} for the Ne VIII lines by more than a factor of 2. This disagreement for Ne VIII, however, is not unique to these two models and most calculations, including more rigorous close-coupling methods, also underestimate the width by $\sim 50\%$ [13]. In Table 3 are also displayed calculations with G_L neglecting the strong collisions, denoted by $g_L (S = 0)$. These results indicate that in these cases the strong collision correction adds significantly to the line widths.

4. Conclusion

In spectral line Stark-broadening theory the electron width is often computed with a second-order term in the electron-radiator interaction providing rapid algorithms to generate line profiles. The O'Brien-Hooper model [8], which made contact with inverse bremsstrahlung, was shown to produce larger electron widths for the hydrogen Balmer series compared with those from Lee [2] where the discrepancy increases with principal quantum number of the radiating atomic levels.

Contrary to other second-order electron width models [2,3,9], the O'Brien-Hooper formalism is independent of the radiator internal states. That is, there is no explicit cutoff to avoid strong collisions where the perturbation expansion becomes invalid. It was shown that introducing a

minimum impact parameter in the partial wave sum for the bremsstrahlung Gaunt factor significantly improves agreement with the hydrogen Balmer series results from Lee.

The inferences regarding the O'Brien-Hooper formalism rely on the accuracy of the results from Lee. An electron width formula similar to G_L , however, was found inaccurate for the H_{13} line at high densities due to simultaneous strong collisions [18]. Nevertheless, calculations with G_L yield good agreement with the Balmer series measurements [10]. Therefore inspection of Figs. 1 and 2 indicate that the O'Brien-Hooper formalism overestimates the width of all the lines in these figures.

Although the O'Brien-Hooper model compares well with experiments for the He II Ly α line, the Lyman series displays increasing discrepancies between the O'Brien-Hooper and Lee formulations with increasing principal quantum number. Potential errors in plasma characterization relying on these electron width models for the Mg Hey $\square\square\square\square$ were considered for recent opacity experiments [6].

The two electron width models were tested with isolated 3s-3p transitions of Li-like systems [13]. Both yielded reasonable agreement with other models including more rigorous but computationally intensive close-coupling methods. The agreement is welcome since the fast electron width models are applied to multi-electron ions [3,4]. In these cases the O'Brien-Hooper formula produced somewhat smaller line widths, which are not readily explained by simply adding the strong collision correction in the Lee model.

The behavior of the O'Brien-Hooper formalism for levels with high principal quantum number is not surprising. It was previously noted [8] that the missing strong collision cutoff made uncertain the connection between electron widths and bremsstrahlung. It was argued [8] that for charged radiators close collisions were partially neglected by excluding the bound states in the Coulomb basis set. For neutral radiators, however, there are no bound levels to exclude. Thus, the approximation appears reasonable for lines involving deeply bound levels of charged radiators but weakly bound levels are more likely to experience strong collisions and the cutoff plays a significant role. On the other hand, even for deeply bound states of hydrogen the O'Brien-Hooper formalism fails near line center and becomes worst in the line wings.

Finally, the absence or approximate treatment of strong collisions in second-order models remains one of several open questions in Stark-broadening theory. The uncertainty in these

practical as well as the more rigorous electron width models [13] requires future high precision measurements of line profiles [40,41].

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Table 1

Ratio of electron widths at $DW = W_p$ and $T=1\text{eV}$ for the H Balmer series

	$n_e = 10^{16} \text{ cm}^{-3}$		$n_e = 10^{17} \text{ cm}^{-3}$		
n	G_{OH}^{Born} / G_L	$\tilde{G}_{OH}^{Born} / G_L$	G_{OH}^{Born} / G_L	$\tilde{G}_{OH}^{Born} / G_L$	$\ell_{\min}$
3	1.16	0.95	1.20	0.94	3
4	1.35	0.96	1.48	0.94	6
5	1.55	0.96	1.77	0.95	10
6	1.80	1.00	2.18	1.00	14
7	1.97	0.99	2.50	0.97	19
8	2.14	0.98	2.84	0.95	26
9	2.33	0.97	3.23	0.92	32
10	2.52	0.96	3.67	0.90	40
11	2.72	0.95	4.15	0.87	49
12	2.93	0.94	4.70	0.84	58
13	3.16	0.93	5.29	0.81	68
14	3.40	0.92	5.94	0.78	79
15	3.65	0.91	6.63	0.74	91
16	3.92	0.90	7.34	0.69	104
17	4.21	0.88	8.06	0.64	117
18	4.52	0.87	8.76	0.58	131
19	4.85	0.86	9.43	0.53	146
20	5.20	0.84	10.0	0.47	162

Table 2

Ratio G_{OH}/G_L at $DW = W_p$ and $T=4$ eV for the He II Lyman series

n	$n_e = 10^{17} \text{ cm}^{-3}$	$n_e = 3 \cdot 10^{17} \text{ cm}^{-3}$	$n_e = 10^{18} \text{ cm}^{-3}$
2	0.76	0.75	0.75
3	0.93	0.94	0.96
4	1.09	1.11	1.17
5	1.24	1.29	1.38
6	1.42	1.52	1.68
7	1.54	1.67	1.89
8	1.67	1.84	2.14
9	1.80	2.02	2.39
10	1.94	2.20	2.69
11	2.08	2.40	3.01
12	2.23	2.62	3.36
13	2.38	2.84	3.75
14	2.54	3.10	4.18
15	2.72	3.37	4.64
16	2.90	3.65	5.14
17	3.09	3.97	5.66
18	3.30	4.30	6.19
19	3.52	4.66	6.73
20	3.74	5.04	7.26

Table 3

Comparison of electron line widths for the 3s-3p transitions scaled to $n_e = 10^{18} \text{ cm}^{-3}$

N V			
$T [\text{eV}]$	g_{OH}/g_{expt}	g_L/g_{expt}	$g_L(S=0)/g_{\text{expt}}$
6.8	0.86	0.92	0.55
14.8	0.85	0.92	0.59
18	0.87	0.94	0.62
22	0.84	0.89	0.60
24	0.85	0.90	0.56
Ne VIII			
$T [\text{eV}]$	g_{OH}/g_{expt}	g_L/g_{expt}	$g_L(S=0)/g_{\text{expt}}$
30	0.38	0.45	0.31
42	0.39	0.46	0.32

Figure Captions

Fig 1 – Hydrogen Balmer emission spectra: Weise et al. [10] (*dots*) and OPAL with G_L [21] (*solid*).

Fig 2 – Hydrogen Balmer bound-bound spectra computed with G_L (*solid*) and G_{OH}^{Born} (*dash*) for plasma temperature $T = 1eV$ and two electron densities indicated in the figure. The higher density spectra were multiplied by 100 for clarity.

Fig 3 – Ratio of the O'Brien-Hooper to Lee G -functions as a function of detuning frequency for the $H\beta$ line at plasma conditions in Fig. 2: G_{OH}^{Born}/G_L (*heavy solid*) and (*heavy dash*). $\tilde{G}_{OH}^{Born}/G_L$ (*thin solid*) and (*thin dash*). The plots terminate at the Weisskopf frequency.

Fig 4 – The Mg $He\gamma$ line profiles at $T = 182eV$ with G_L (*short-dash*) and G_{OH} (*solid*) at $n_e = 3.1 \cdot 10^{22} cm^{-3}$ plus profile with G_L (*long-dash*) at $n_e = 3.9 \cdot 10^{22} cm^{-3}$.

Figure

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