Anti-Diffusive-Like-Behavior in Semi-Analytic Radiative Shocks via Multigroup S_n Transport with constant cross sections

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Abstract

Semi-analytic lab-frame radiative shock solutions have been presented recently, wherein the radiation is modeled with either grey (frequency independent) nonequilibrium-diffusion or grey S_n -transport. As a first step toward incorporating frequency dependence, we retain grey cross-sections such that the frequency dependence of the problem is strictly due to the Planck function. By using multigroup frequency integration the frequency-dependent radiation-transport equation may be solved, and group-dependent radiation temperatures may be defined for each group. Our main result is: When a Zel'dovich temperature spike exists in a radiative shock solution, there exists a transition group g_T , such that for all frequency groups below g_T the group-dependent radiation temperatures are spatially monotonic, and for all frequency groups above g_T the group-dependent radiation temperatures are spatially non-monotonic. We present numerical evidence of our claim and make no claim as to the monotonicity of group g_T .

Keywords: -adiative shock, semi-analytic, multigroup, Planck function, anti-diffusive

1. Introduction

The interest to solve frequency-dependent radiative-shock problems has existed in the astrophysics community for decades [1, 2, 3, 4, 5, 6]. As computational methods and abilities continue to evolve, it remains pertinent to focus on an understanding of the fundamental physical elements of the problem by either analytic or semi-analytic methods [7, 8, 9, 10, 11], which may also serve as code-verification tests, but these works have neglected frequency dependence, and many are based on simplified radiation-diffusion models. The full transport theory of radiation defines the radiation flux as being proportional to the negative gradient of the radiation pressure, but Fick's Law of Diffusion for radiation defines the radiation flux as being proportional to the negative gradient of the radiation energy density; the variable Eddington factor defines the proportional relation between the radiation pressure and the radiation energy density, and in diffusion models is typically assumed to be $1/3$ everywhere (the P_1 or Eddington approximation). McClarren and Drake [10], analyt-

ically solved the grey comoving-frame radiation-transport equation for a simplified radiative-shock structure, and defined anti-diffusion as occurring when the spatial derivatives of the radiation pressure and the radiation energy densities have different signs, which signals when diffusion models will give a distinctly incorrect result, hence "anti-diffusion". Previous semi-analytic radiative-shock solutions have been presented by Lowrie and Rauenzahn [8] for frequency-independent (grey) equilibrium-diffusion, by Lowrie and Edwards [9] for grey nonequilibrium-diffusion, and Ferguson, Morel, and Lowrie [11] for grey S_n -transport. The S_n -transport method allows the variable Eddington factor (VEF) to be determined; in one dimension S_2 is equivalent to diffusion and also setting the VEF to $1/3$. The latter work confirmed the existence of anti-diffusion, and that it is associated with a local maximum in the radiation energy density, and also when the variable Eddington factor takes values less than $1/3$.

We extend the work of Ferguson, Morel and Lowrie [11] to solve the frequency-dependent lab-frame S_n radiation-transport equation by multi-

group integrating the frequency domain. Multi-
 group frequency integration decomposes the fre-
 quency domain into smaller groups, which when
 summed over recreate the complete frequency do-
 main. This allows us to investigate whether group-
 dependent radiation energy densities exhibit local
 maxima, for both diffusion and S_n -transport mod-
 els ($n > 2$). When the radiation energy density
 exhibits anti-diffusion, from an S_n -transport solu-
 tion, we show that some of the group-dependent ra-
 diation energy densities also exhibit anti-diffusion.
 While the radiation energy density from a diffu-
 sion solution might not exhibit anti-diffusion, we
 also show that some of the group-dependent ra-
 diation energy densities appear to exhibit a local
 maximum. Since these local maximum are
 not associated with anti-diffusion of the radia-
 tion energy density, we label it “anti-diffusive-
 like-behavior” (ADLB). By solving the frequency-
 dependent radiation-transport equation, but re-
 stricting ourselves to grey material cross sections,
 we are able to explain the ADLB of the group-
 dependent radiation energy densities in terms of
 the multigroup frequency-integrated Planck func-
 tion. This work represents the initial step to in-
 corporate frequency dependence into semi-analytic
 radiative-shock solutions.

2. Governing Equations

In this section we present the steady-state, nondi-
 mensional equations of radiation transport and ra-
 diation hydrodynamics. We follow the nondimen-
 sionalization presented by Lowrie and Edwards [9]
 and Ferguson, Morel and Lowrie [11] throughout;
 variables containing a tilde-~ carry the appropri-
 ate dimension. We add frequency-dependent vari-
 ables to their nondimensionalization, present the
 frequency-dependent radiation-transport equation
 and introduce multigroup frequency-integration of
 this equation and the frequency-dependent radi-
 ation variables. The frequency-integrated (grey)
 equation and variables are used to define the ra-
 diation variables, and derive the radiation sources,
 that are used in the conservation equations of radi-
 ation hydrodynamics. In steady-state, these conser-
 vation equations reduce to a system of two ordinary
 differential equations that can be solved to produce
 radiative-shock solutions.

2.1. Frequency-dependent nondimensionalization

We add to the nondimensional definitions of Fer-
 guson, Morel and Lowrie [11] the nondimensional
 definitions of frequency, the frequency-dependent
 radiation intensity, the frequency-dependent Planck
 function, and we include the material temperature,
 respectively:

$$\nu = \tilde{\nu} \frac{\tilde{h}}{\tilde{k}_B \tilde{T}_0}, \quad (1a)$$

$$I_\nu = \tilde{I}_\nu \frac{\tilde{k}_B}{\tilde{\alpha}_R \tilde{c} \tilde{h} \tilde{T}_0^3}, \quad (1b)$$

$$B_\nu = \tilde{B}_\nu \frac{\tilde{k}_B}{\tilde{\alpha}_R \tilde{c} \tilde{h} \tilde{T}_0^3}, \quad (1c)$$

$$T = \frac{\tilde{T}}{\tilde{T}_0}, \quad (1d)$$

where ν is the frequency, I_ν is the frequency-
 dependent radiation intensity, B_ν is Planck’s func-
 tion, $\tilde{h}$ is Planck’s constant, $\tilde{k}_B$ is Boltzmann’s con-
 stant, $\tilde{\alpha}_R$ is the radiation constant, $\tilde{c}$ is the speed of
 light, $\tilde{T}_0$ is a reference temperature which takes its
 value at the upstream equilibrium pre-shock state,
 $\tilde{T}$ is the material temperature of the fluid, and T
 is the nondimensional material temperature. This
 nondimensionalization (1) was chosen so that I_ν
 and B_ν would have the same nondimensionaliza-
 tion, just as $\tilde{I}_\nu$ and $\tilde{B}_\nu$ have the same dimensions,
 and so that the nondimensional Planck function,
 determined by expressions (1a) and (1c),

$$B_\nu = \frac{2\tilde{k}_B^4}{\tilde{\alpha}_R \tilde{c}^3 \tilde{h}^3} \frac{\nu^3}{e^{\nu/T} - 1}, \quad (2)$$

when frequency integrated would produce a sensible
 nondimensional expression,

$$\begin{aligned} \int_0^\infty B_\nu d\nu &= \frac{1}{\tilde{\alpha}_R \tilde{c} \tilde{T}_0^4} \int_0^\infty \tilde{B}_\nu d\tilde{\nu} \\ &= \frac{1}{4\pi} \frac{\tilde{\alpha}_R \tilde{c} \tilde{T}_0^4}{\tilde{\alpha}_R \tilde{c} \tilde{T}_0^4} = \frac{T^4}{4\pi}. \end{aligned} \quad (3)$$

Setting the derivative of the nondimensional Planck
 function with respect to the nondimensional fre-
 quency to zero produces an expression relating the
 nondimensional peak frequency to the nondimen-
 sional material temperature,

$$\nu_{\text{peak}} = 2.821 T, \quad (4a)$$

which agrees with the standard expression relating the peak frequency to the material temperature

$$\tilde{\nu}_{\text{peak}} = 2.821 \frac{\tilde{k}_B \tilde{T}}{\tilde{h}}, \quad (4b)$$

after using the nondimensionalizations for frequency (1a) and temperature (1d). The nondimensional Planck function admits a functional form of the Rayleigh-Jeans Law for $\nu \ll \nu_{\text{peak}}$,

$$B_\nu = \frac{2\tilde{k}_B^4}{\tilde{\alpha}_R \tilde{c}^3 \tilde{h}^3} \nu^2 T, \quad (5a)$$

and Wien's Law for $\nu \gg \nu_{\text{peak}}$,

$$B_\nu = \frac{2\tilde{k}_B^4}{\tilde{\alpha}_R \tilde{c}^3 \tilde{h}^3} \nu^3 e^{-\nu/T}, \quad (5b)$$

which will define our frequency boundaries.

The multigroup method typically bounds the semi-infinite frequency domain $\nu \in [0, \infty)$ into a smaller subset $\nu \in [\nu_{\min}, \nu_{\max}]$, and discretizes this subset into G finite groups so that $g \in \mathbb{Z}[0, G-1]$ represents one group whose frequency domain is $\nu \in [\nu_g, \nu_{g+1}]$ such that $\cup_g [\nu_g, \nu_{g+1}] = [\nu_{\min}, \nu_{\max}]$. We choose to further impose that $\nu_{\min}$ and $\nu_{\max}$ should be determined via the Rayleigh-Jeans Law (5a) and Wien's Law (5b), respectively, such that $B_{\nu_{\min}} = 10^{-20} = B_{\nu_{\max}}$, in order to retain the correct shape of the Planck function. The value 10^{-20} was chosen so that the resulting values of $\nu_{\min}$ and $\nu_{\max}$ would reasonably estimate the frequency-integrated Planck function

$$\int_{\nu_{\min}}^{\nu_{\max}} B_\nu d\nu \approx \int_0^\infty B_\nu d\nu. \quad (6)$$

Of course, the values for $\nu_{\min}$ and $\nu_{\max}$ may be significantly shifted higher or lower without changing the integrated value of the nondimensional Planck function, but this will distort the shape of the Planck function. While this may not be expected to cause a problem, we show evidence in Section 3 that the shape of the Planck function may control the shape of the radiative shock, especially the radiation temperature near the Zel'dovich spike.

2.2. The equations of radiation transport

We now present the 1-D, steady-state, lab-frame, purely absorbing frequency-dependent, S_n -transport radiation equation ([12, eqn. 93.4]),

$$\begin{aligned} \mu_m \partial_x I_{\nu,m} &= \sigma_t (B_\nu - I_{\nu,m}) \\ &+ \mu_m \beta [\sigma_t (I_{\nu,m} + 2B_\nu) + I_{\nu,m} \nu \partial_\nu \sigma_t - \nu \partial_\nu (\sigma_t B_\nu)] \\ &+ \sigma_t \beta^2 \left(B_\nu - \frac{1}{4\pi} (2\mathcal{E}_\nu + \mathcal{P}_\nu) \right), \end{aligned} \quad (7)$$

where the subscript- m on μ_m and $I_{\nu,m}$ represents the discretization of the angular coordinates with $m \in \mathbb{Z}[1, N]$ (the S_n method), $\partial_{(\cdot)}$ represents a derivative with respect to $(\cdot)$, σ_t is the material absorption cross section or total cross section since we have chosen to neglect scattering, $\beta \equiv u/\mathcal{C}$ where u is the material velocity, $\mathcal{C} \equiv \tilde{c}/\tilde{a}_0$ and $\tilde{a}_0$ is the reference sound speed of the material, $\mathcal{E}_\nu$, $\mathcal{F}_\nu$ and $\mathcal{P}_\nu$ are the frequency-dependent zeroth, first and second angle-integrated angular moments of $I_{\nu,m}$, respectively,

$$\mathcal{E}_\nu = 2\pi \int_{-1}^1 I_\nu d\mu \approx 2\pi \sum_{m=1}^N w_m I_{\nu,m}, \quad (8a)$$

$$\mathcal{F}_\nu = 2\pi \int_{-1}^1 \mu I_\nu d\mu \approx 2\pi \sum_{m=1}^N w_m \mu I_{\nu,m}, \quad (8b)$$

$$\mathcal{P}_\nu = 2\pi \int_{-1}^1 \mu^2 I_\nu d\mu \approx 2\pi \sum_{m=1}^N w_m \mu_m^2 I_{\nu,m}, \quad (8c)$$

where w_m are the quadrature weightings associated with the S_n method, and the β^2 terms in equation (7) are designed to ensure that the sources of radiation energy and radiation momentum, equations (14) below, go to zero in equilibrium. Frequency-dependent variables may now be frequency integrated over group g as

$$z_g = \int_{\nu_g}^{\nu_{g+1}} z_\nu d\nu, \quad (9)$$

and their frequency-integrated values may be obtained by summing over the G groups,

$$\begin{aligned} z &= \sum_{g=0}^{G-1} z_g = \sum_{g=0}^{G-1} \int_{\nu_g}^{\nu_{g+1}} z_\nu d\nu \\ &= \int_{\nu_{\min}}^{\nu_{\max}} z_\nu d\nu \approx \int_0^\infty z_\nu d\nu. \end{aligned} \quad (10)$$

We chose to assume that the material cross section is independent of frequency so that we can focus on the frequency dependence of the Planck function. Applying multigroup frequency-integration to the radiation-transport equation (7), and performing an integration-by-parts on the frequency derivative containing the Planck function, gives

$$\begin{aligned}\mu_m \partial_x I_{g,m} &= \sigma_t (B_g - I_{g,m}) \\ &+ \sigma_t \mu_m \beta \left(I_{g,m} + 3B_g - [\nu B_\nu]_{\nu_g}^{\nu_{g+1}} \right) \\ &+ \sigma_t \beta^2 \left(B_g - \frac{1}{4\pi} (2\mathcal{E}_g + \mathcal{P}_g) \right). \quad (11)\end{aligned}$$

The solution of equation (11) for $I_{g,m}$ yields the solutions of the group-dependent radiation quantities, $\mathcal{E}_g$, $\mathcal{F}_g$ and $\mathcal{P}_g$, as well as the grey radiation quantities, $\mathcal{E}$, $\mathcal{F}$ and $\mathcal{P}$, and the variable Eddington factor (VEF), f ,

$$\mathcal{E} = \sum_{g=0}^{G-1} \mathcal{E}_g = \sum_{g=0}^{G-1} \sum_{m=1}^N w_m I_{g,m}, \quad (12a)$$

$$\mathcal{F} = \sum_{g=0}^{G-1} \mathcal{F}_g = \sum_{g=0}^{G-1} \sum_{m=1}^N w_m \mu_m I_{g,m}, \quad (12b)$$

$$\mathcal{P} = \sum_{g=0}^{G-1} \mathcal{P}_g = \sum_{g=0}^{G-1} \sum_{m=1}^N w_m \mu_m^2 I_{g,m}, \quad (12c)$$

$$f = \frac{\mathcal{P}}{\mathcal{E}}. \quad (12d)$$

If equation (11) is similarly group-summed we obtain the grey S_n radiation-transport equation

$$\begin{aligned}\mu_m \partial_x I_m &= \sigma_t \left[-I_m + \frac{1}{4\pi} T^4 + \beta \mu \left(I_m + \frac{3}{4\pi} T^4 \right) \right. \\ &\left. + \frac{1}{4\pi} \beta^2 (T^4 - 2\mathcal{E} - \mathcal{P}) \right] \equiv Q_m. \quad (13)\end{aligned}$$

The zeroth and first angle-integrated angular moments of the grey radiation-transport equation (13) represent the sources of radiation energy, S_{re} , and radiation momentum, S_{rp} , respectively,

$$\begin{aligned}S_{\text{re}} &\equiv 2\pi \sum_{m=1}^N w_m Q_m = \partial_x \mathcal{F} \\ &= \sigma_t [T^4 - \mathcal{E} + \beta \mathcal{F} + \beta^2 (T^4 - 2\mathcal{E} - \mathcal{P})], \quad (14a)\end{aligned}$$

$$\begin{aligned}S_{\text{rp}} &\equiv 2\pi \sum_{m=1}^N w_m \mu_m Q_m = \partial_x \mathcal{P} \\ &= \sigma_t [-\mathcal{F} + \beta (T^4 + \mathcal{P})]. \quad (14b)\end{aligned}$$

Equation (14b) can be rearranged to solve for the grey radiation flux

$$\mathcal{F} = -\frac{1}{\sigma_t} \partial_x \mathcal{P} + \beta (T^4 + \mathcal{P}). \quad (15)$$

2.3. The equations of radiation hydrodynamics

The 1-D, nondimensional, steady-state, radiation-hydrodynamic equations of mass conservation, momentum conservation, and energy conservation, respectively, are

$$\partial_x (\rho u) = 0, \quad (16a)$$

$$\partial_x (\rho u^2 + p) = -\mathbb{P} S_{\text{rp}}, \quad (16b)$$

$$\partial_x \left[u \left(\frac{1}{2} \rho u^2 + \rho e + p \right) \right] = -\mathbb{P} C S_{\text{re}}, \quad (16c)$$

whose solutions also satisfy the steady-state material internal-energy equation

$$\rho u \partial_x e + p \partial_x u = -\mathbb{P} C (S_{\text{re}} - \beta S_{\text{rp}}), \quad (16d)$$

where ρ is the material density, u is the material velocity, e is the material specific internal-energy, p is the material pressure, $\mathbb{P}$ is a measure of the radiation effects on flow dynamics and is proportional to the ratio of radiation pressure to material pressure and alternatively, the radiation energy to material energy [9] given by

$$\mathbb{P} \equiv \frac{\tilde{\alpha}_R \tilde{T}_0^4}{\tilde{\rho}_0 \tilde{a}_0^2}, \quad (17)$$

where $\tilde{\rho}_0$ is a reference density which takes its value at the upstream pre-shock equilibrium state. To reduce this system of equations, we introduce the ideal-gas equation-of-state

$$p = \frac{\rho T}{\gamma}, \quad (18a)$$

$$e = \frac{T}{\gamma(\gamma - 1)}, \quad (18b)$$

where $\gamma = 5/3$ is the adiabatic index of a nonrelativistic monatomic gas. In the same manner as Ferguson, Morel, and Lowrie [11], a system of two reduced ordinary differential equations (ODEs) can be derived from equations (14) and (16), and it is assumed that the VEF is known,

$$\begin{aligned}\partial_x \mathcal{P} &= \frac{\sigma_t \mathcal{M}_0}{\mathbb{P} C} \left[\frac{T - 1}{\gamma - 1} + \frac{\mathcal{M}_0^2}{2\rho^2} (1 - \rho^2) \right. \\ &\left. + \mathbb{P} \left(\frac{\mathcal{P} + T^4}{\rho} - \frac{4}{3} \right) \right], \quad (19a)\end{aligned}$$

$$\partial_{\mathcal{M}} x = \frac{-2\rho T}{\mathcal{M}(2T\partial_x \rho + \rho\partial_x T)}, \quad (19b)$$

where $\partial_x \rho$ can be derived from the momentum-conservation equation (16b), $\partial_x T$ can be derived from the material internal-energy-conservation equation (16d), $\mathcal{M} \equiv u/\sqrt{T}$ is the local Mach number, and $\mathcal{M}_0$ is the initial Mach number.

Given the VEF, the simultaneous solution of equations (19) produce the radiative-shock solutions, including $\mathcal{E}$, $\mathcal{F}$, and $\mathcal{P}$. Initially, the VEF is assumed to be 1/3 everywhere, and a nonequilibrium-diffusion shock is produced; in 1-D, nonequilibrium diffusion is equivalent to S_2 . This allows for the solution of $I_{g,m}$ in the multigroup S_n radiation-transport equation (11), and a new VEF (12d) as well as updated values for the group-dependent and grey radiation variables (12). The new VEF is used to update the radiative-shock solution. Once a satisfactory solution for the radiation energy density has been determined, the radiation temperature, $\theta \equiv \mathcal{E}^{1/4}$, and the group-dependent radiation temperatures, $\theta_g \equiv \mathcal{E}_g^{1/4}$, may be determined and compared with the material temperature, T .

3. Results

The solutions we present are for $\mathcal{M}_0 = 3$, $\mathbb{P} = 10^{-4}$, and $\sigma_t = 577.35$, so that our S_2 solutions will resemble the solutions of Lowrie and Edwards [9]. We bound the frequency domain in such a way that $B_\nu(T_f) \geq 10^{-20}$, where $T_f = 3.662$ is the post-shock equilibrium temperature, which produces the pertinent frequencies: $\nu_{\min} = 4.715 \times 10^{-10}$, $\nu_{\text{peak}} = 10.330$, $\nu_{\max} = 211.329$. The angle- and frequency-integrated Planck function evaluated at T_f using the integration boundaries $[\nu_{\min}, \nu_{\max}]$ gives the value 179.8182; the analytic result is 179.8174, such that the relative error due to using $\nu_{\min}$ and $\nu_{\max}$ is $\sim 4 \times 10^{-6}$.

The radiative-shock solutions presented in the top plot of Figure 1 were generated by solving the $G = 9$, S_2 radiation-transport equation, and those solutions presented in the middle plot were generated by solving the $G = 9$, S_{16} radiation-transport equation. The colors of the group-dependent radiation temperatures correspond to the multigroup frequency bins defined in the bottom plot of Figure 1, wherein $B_\nu(T_f)$ is plotted over the frequency range $[\nu_{\min}, \nu_{\max}]$. The value of ν_{peak} is used as the interface between groups 5 and 6, groups 6-9 are equally log-spaced over the interval $\nu \in [\nu_{\text{peak}}, \nu_1]$, and the same log-spacing to the left of ν_{peak} defines

Figure 1: (Top:) Solutions for the material temperature, T , the radiation temperature, θ , and the group-dependent radiation temperatures, θ_g , for $\mathcal{M}_0 = 3$ using S_2 and 9 groups. (Middle:) Same conditions as the top plot except using S_{16} . (Bottom:) The Planck function evaluated at the spike temperature (dashed), the final-equilibrium post-shock temperature (solid), and the initial-equilibrium pre-shock temperature (dotted), overlaid with the frequency-group bins.

groups 2-5.

In the top plot of Figure 1 the radiation temperatures for groups 1-6 are monotonic, while those for groups 7-9 are nonmonotonic and represent ADLB since they are the solutions from a nonequilibrium-diffusion model. In the middle plot of Figure 1 the radiation temperature exhibits anti-diffusion, the radiation temperatures for groups 1-3 are monotonic, while those for groups 4-9 are nonmonotonic. Since the nonmonotonicity in the top plot from diffusion solutions begins in group 7, and the nonmonotonicity in the middle plot from transport solutions begin in group 4, the nonmonotonicity of the transport solutions in groups 4-6 represent anti-diffusion only, and the nonmonotonicity in groups 7-9 is dominantly from ADLB. Similar ADLB of the group-dependent radiation temperatures is presented in the results of Vaytet et. al. [6].

4. Conclusion

We presented new semi-analytic solutions of the group-dependent radiation temperatures for radiative shocks using a multigroup S_n -transport model, wherein the material cross-section was assumed to be purely absorbing and frequency independent. We have shown that group-dependent radiation temperatures, from a diffusion model, may exhibit anti-diffusive-like-behavior (ADLB) when their group includes frequencies that are sufficiently larger than the peak frequency of the Planck function evaluated at the final-equilibrium temperature. ADLB represents the foundation of our primary result: When a Zel'dovich temperature spike exists in a radiative-shock solution, there exists a transition group g_T , such that for all frequency groups below g_T the group-dependent radiation energy densities are spatially monotonic, and for all frequency groups above g_T the group-dependent radiation energy densities are spatially nonmonotonic. A transition from monotonic to non-monotonic group-dependent radiation temperatures was shown to exist for nonequilibrium diffusion and S_n -transport, and it was seen that the frequency boundary of this behavior is lower for S_n -transport. While anti-diffusion itself is strictly a manifestation of the angular dependence of the radiation field, the results we presented imply that the frequency dependence of the Planck function plays a role in ADLB in radiative shocks, and that the coupled effects of anti-diffusion and ADLB are apparent in the group-dependent radiation

temperatures.

Future work should involve investigating analytic frequency-dependent material cross-sections and determining their effect on radiative shock structures, as well as trying to better understand the structure of the group-dependent radiation temperatures, the frequency dependence of ADLB, and the use of these results as a code-verification tool.

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figure 1a

figure 1c

figure 1b

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