On the Uplink Outage Throughput Capacity of Hybrid Wireless Networks with Massive MIMO

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Abstract

In this paper, we investigate theoretical transmission capacity limit of the uplink hybrid wireless network under infrastructure mode. Massive MIMO technology is assumed to be equipped on the base station to further increase the whole network throughput. Multi-user MIMO is preferred over Point-to-Point MIMO to achieve improved scalability and simplify UE design. Another perspective of this paper is to include the fading effect on capacity. Under favorable propagation condition, Massive MIMO greatly mitigates small scale fading effect between each user and base station antenna. Then closed-form outage capacity over large scale fading channel is derived in both low SNR and high SNR scenarios.

Index Terms : Hybrid wireless networks, Massive MIMO, Infrastructure, Fading, Uplink, Outage throughput capacity.

1 Introduction and Motivation

Hybrid wireless network becomes a hot research topic as the fast evolution of wireless communication systems. It is known that the wireless ad-hoc network(WANET) is a wireless network without backbone support. The decentralized feature makes ad-hoc network flexible for certain applications. It allows nodes joining the network dynamically without paying the cost of complex infrastructure. However, the drawbacks of pure ad-hoc networks are previous: less connection stability and signal strength issues for long range communication yield to lower throughput than infrastructure supported networks. Therefore, placing several wired interconnected base stations in ad-hoc network makes sense in terms of improving the whole network performance. Such infrastructure supported ad-hoc network is called hybrid wireless network [1] which takes advantages of both ad-hoc and infrastructure networks. The network topology is shown in Figure 1.

Figure 1: Hybrid wireless network.

Throughput capacity is the key element to analysis the performance of hybrid networks. Channel capacity, indicates how much data could be reliably transmitted over communication channel, eventually depends on what channel model applies. For example, the capacity of AWGN channel is essentially different from fading channels. For AWGN channel, transmitter could send out data at a positive rate while having desired very small error probability. Fading channel can't guarantee to achieve this as long as the probability that channel in deep fading is non-zero. In slow fading scenario, outage capacity is always considered as it assumes that at a given rate R at which transmitter encodes data, whatever coding scheme used, the error probability cannot be made arbitrarily small. In other words, reliable communication occurs when the random channel gain is strong enough to support the desired rate R . Otherwise, outage happens when capacity drops below R in deep fading situation. For fast fading scenario, channel cannot remain constant over symbol coherent period. Since codeword span over several coherent period, we can use block-fading model that consider several parallel sub-channels fade independently. The capacity can be assigned with positive value by coding over a large number of coherence time intervals in order to average independent fading of channels [2].

A lot of research works has been done. Gupta and Kumar [3] initiatively studied scaling law of a random ad-hoc wireless network. When nodes are randomly placed in the network and they randomly choose a destination, the per-node capacity is shown to be $\Theta(\frac{W}{\sqrt{n \log n}})$ as the number of nodes n tends to infinity, where n is the number of nodes and W (the same below) is the transmitting rate each node is capable of using a fixed range. In [4], with b base stations and n nodes settings, the author proves that in order to achieve infrastructure gain, b should grow at least faster than $\sqrt{\frac{n}{\log n}}$ and the maximum throughput scales as $\Theta(bW)$ which increases linearly with the number of base stations. The scaling law of data transmission limit of hybrid wireless networks is studied in [5] [6]. Squared cell model is assumed with b base stations and n wireless nodes and fading environment

is considered. The per-node outage throughput capacity over Nakagami- m fading channel scales as $O\left(\log\left[\left(\epsilon^{\frac{1}{m}}\right)^{\frac{b}{n}} \frac{n}{b}\right] W_1\right)$ under ad-hoc mode and $\Theta\left(\frac{b}{n} \log\left(\epsilon^{\frac{1}{m}} \frac{n}{b}\right) W_2\right)$ under infrastructure mode, where W_1 is the bandwidth shared by ad-hoc transmission and uplink infrastructure transmission and W_2 is bandwidth assigned for downlink infrastructure transmission.

In terms of Massive MIMO implementation under hybrid wireless networks, channel capacity could be very different from previous scenario. Massive MIMO capacity is studied in [7] and [8]. In this paper, considering using Massive MIMO for hybrid wireless network infrastructure to further improve system performance, we focus on studying how the uplink outage capacity scales with number of base stations b , number of users n , as well as large number of antennas M on the base station. Our analysis relies on deriving the outage capacity in closed-form expression including the variables aforementioned.

The rest of this paper is organized as follows: In Section 2, we introduce hybrid wireless network model and Massive MIMO implementation. In Section 3, we provide some preliminary knowledge for future derivation. In Section 4, we derive uplink throughput outage capacity under infrastructure transmission. In Section 5, we draw the conclusion.

2 Network Modeling

2.1 Hybrid Wireless Network Model

The following hybrid network model is considered throughout this paper:

1. The network consists of n nodes and b base stations totally.
2. The whole area has size n and is evenly divided into b hexagonal cells with unit density. Thus cell has distance $c = \sqrt{\frac{2\sqrt{3}}{3} \frac{n}{b}}$ between parallel sides.
3. Each cell contains only one base station which is placed in the center. There exists $K = \Theta(\frac{n}{b})$

users uniform circularly placed surrounded the base station within each cell. Each node in the same cell has the same distance d_k to the base station. Nodes placement are shown in Figure 3.

4. We assume all base stations are wired interconnected to form an infrastructure with unlimited bandwidth. Base stations neither consume nor generate data and they only serve as traffic relays compared to nodes.
5. For the purpose of studying the scaling law of number of base stations and nodes in the network, $b = o(\frac{n}{\log n})$ is assumed that b increases at a slower rate than n . Study[4] shows that in a probabilistic routing strategy, a node chooses between infrastructure and ad-hoc mode according to some probability. For example, if b grows slower than $\sqrt{\frac{n}{\log n}}$, the maximum throughput capacity behaves asymptotically same as pure ad-hoc networks which means no real benefit for using infrastructure. If b grows faster than $\sqrt{\frac{n}{\log n}}$, maximum capacity increases linearly with number of base stations b . Therefore, investment on base station should be at least $\sqrt{\frac{n}{\log n}}$ to achieve infrastructure capacity gain. In this case, our assumption should be enough to have infrastructure gain.

2.2 Massive MIMO in Hybrid Network

Massive MIMO technology appears as the result of increasing demand for wireless throughput. Current wireless broadband standard like long-term evolution (LTE) allows up to eight antennas at the base station. With more transmitting/receiving antennas equipped, more degrees of freedom can be obtained by the propagation channel either for multiplexing or diversity gain. In this case, Massive MIMO could break the limitation of number of equipped antennas on the base station side. Large antenna array with physically smaller antennas is installed on the base station. The

Figure 2: Hexagon Cell with distance c between parallel sides

magnitude of number of antennas could be more than one hundred. In [9], how much antennas would be sufficient for Massive MIMO is studied. Figure 4 shows the Massive MIMO base station architecture.

It is intrinsically to combine Massive MIMO with hybrid network in the sense that hybrid network uses infrastructure to enhance throughput performance. The following details describe Massive MIMO implementation in hybrid network:

1. Multi-user MIMO is preferred in this work compared to Point-to-Point MIMO for its significant simplification of the terminal device. Each user terminal is assumed to have only one antenna.
2. The base station located in each cell has an array of M antennas. The antennas are sufficient to simultaneously serve all K (K is described in hybrid wireless network model) users in the cell. The number of antennas greatly exceeds the number of terminals $M \gg K$.

Figure 3: Nodes are uniform circularly placed in each cell with distance d_k to the base station

3. In uplink transmission, the following model are assumed per channel use[8]:

$$\mathbf{x}_u = \sqrt{\rho_u} \mathbf{G} \mathbf{q}_u + \mathbf{w}_u \quad (1)$$

Where $\mathbf{q}_u$ is a $K \times 1$ vector consists of QAM symbols transmitted by K users, $\mathbf{x}_u$ is the antenna array received vector of size $M \times 1$. $\mathbf{w}_u$ is a $M \times 1$ vector of received noise whose entries are independent and identical distributed (i.i.d.) complex Gaussian random variables with zero mean and unit variance. ρ_u is proportional to SNR. Each terminal is constrained to have an expected power of one,

$$E\{|q_{uk}|^2\} = 1, k = 1, \dots, K.$$

$\mathbf{G}$ is $M \times K$ propagation channel matrix, which is composed of small-scale fading and large scale fading factors:

$$\mathbf{G} = \mathbf{H} \mathbf{D}_\beta^{1/2} \quad (2)$$

In the equation above, Matrix $\mathbf{H}$ accounts for small-scale fading whose dimension is $M \times K$.

Figure 4: Massive MIMO base station equipped with large antenna array.

Each entry is the complex small-scale fading coefficient (for example, fading coefficient follows Rayleigh or Rician distribution) between the K th user terminal and the M th antenna on base station. Matrix $\mathbf{D}_\beta^{1/2}$ is a $K \times K$ diagonal matrix. The diagonal entries of $\mathbf{D}_\beta^{1/2}$ are normalized large-scale fading coefficients related to the antenna array and K th user terminal.

4. In downlink transmission, the propagation matrix $\mathbf{G}^T$ is basically the transpose of the uplink one. In this case, for per channel use, the downlink model is:

$$\mathbf{x}_d = \sqrt{\rho_d} \mathbf{G}^T \mathbf{s}_d + \mathbf{w}_d \quad (3)$$

Where $\mathbf{s}_d$ is an $M \times 1$ vector transmitted by M antennas. $\mathbf{x}_d$ is a $K \times 1$ received vector by K user terminals. $\mathbf{G}^T$ is the transposed channel matrix of the uplink. $\mathbf{w}_d$ is the noise vector with $\mathcal{CN}(0, 1)$ entries. ρ_d is the proportional to SNR. The total transmission power of all M

antennas is confined as,

$$E\{\|\mathbf{s}_d\|^2\} = 1$$

2.3 Transmission Modes

Two transmission modes are used in hybrid networks: infrastructure mode and ad-hoc mode.

2.3.1 Infrastructure Mode

In infrastructure mode, the source and destination nodes are located in different cells respectively.

The total transmission can be divided into three phases:

1. Uplink Phase. In uplink phase, K simultaneous scheduled source nodes firstly forward data through wireless channel to the M antenna array on the base station located in the same cell.
2. Transport Phase. This step is also called relay phase. Data is relaid via wired infrastructure to the base station in destination cell.
3. Downlink phase. Finally, data is delivered to K destination nodes through M antenna array.

2.3.2 Ad-hoc Mode

Ad-hoc mode will only be activated when transmission from source node to destination node occurs in the same cell. In other words, no infrastructure participates as assistance. In many literatures, for example[10], multi-hop scheme are resorted to optimize the transmission that nodes only forward data to their nearest neighbors as relay to destination. The merits behind such transmission strategy implies that intra-cell long-range transmission is restricted considering the interference brought by other nodes. In other scenarios[11], the restriction is weakened by allowing nodes transmit data directly to their destination in one-hop fashion. Opportunistic transmission is assumed that there are $\kappa = \Theta(\frac{n}{b})$ nodes whose Euclidean distance is greater than the one between scheduled source

node and destination node, can be simultaneously scheduled as opportunistic nodes which have the same destination as source node.

It is more meaningful to implement Massive MIMO on the base station side for the assumption that BS has unlimited calculation power. In our Mu-MIMO scenario that each user terminal device only has one antenna, it is not practical to have Massive MIMO on the user device for the consideration of keeping UE design simple and reserving device battery life. However, distributed MIMO systems in ad-hoc networks are studied in [12]. Instead of using traditional multi-hop networks with n source-destination pairs, single antenna user cooperate with each other to form clustered transmit and receive groups. The behavior of such transmit and receive clusters could be analyzed with MIMO method. In our case, we don't consider any type of MIMO scenario in ad-hoc mode.

3 Preliminaries

3.1 The Number of Nodes Per Cell

As number of base stations $b = o(\frac{n}{\log n})$ and number of users $n \rightarrow \infty$, there exists $\Theta(\frac{n}{b})$ nodes within each cell. The number of nodes K in each hexagonal cell with distance $c = \sqrt{\frac{2\sqrt{3}}{3} \frac{n}{b}}$ between parallel sides is bounded by $\Theta(\frac{n}{b})$. The similar proof of square cell setup can be found in [13].

3.2 Fenton-Wilkinson Method

The summation of several independent log-normal random variables can be approximated by another log-normal random variable with appropriately chosen parameters[14],

$$I = \sum_{k=1}^{N_I} 10^{\Omega_{k(dBm)}/10} \approx 10^{Z_{(dBm)}/10} = \hat{I} \quad (4)$$

where $\Omega_{k(dBm)}$ are Gaussian random variables with means $\mu_{\Omega_{k(dBm)}}$ and variances $\sigma_{\Omega_k}^2$, $Z_{(dBm)}$ is a Gaussian random variable with mean $\mu_{Z_{(dBm)}}$ and variance σ_Z^2 , so that,

$$\Omega_k = 10^{\Omega_{k(dBm)}/10}$$

$$Z = 10^{Z_{dBm}/10}$$

where Ω_k and Z are the log-normal random variables. The problem is to determine $\mu_{Z_{(dBm)}}$ and σ_Z^2 in terms of $\mu_{\Omega_{k(dBm)}}$ and $\sigma_{\Omega_k}^2$.

The mean $\mu_{Z_{(dBm)}}$ and variance σ_Z^2 can be obtained by matching the first two moments of the approximation $\hat{I}$. For space limitation, we omit the proof of the derivation.

$$\mu_{\hat{Z}} = \frac{\sigma_{\hat{\Omega}}^2 - \sigma_{\hat{Z}}^2}{2} + \ln \left(\sum_{k=1}^{N_I} e^{\mu_{\hat{\Omega}_k}} \right) \quad (5)$$

$$\sigma_{\hat{Z}}^2 = \ln \left(\left(e^{\sigma_{\hat{\Omega}}^2} - 1 \right) \frac{\sum_{k=1}^{N_I} e^{2\mu_{\hat{\Omega}_k}}}{\left(\sum_{k=1}^{N_I} e^{\mu_{\hat{\Omega}_k}} \right)^2} + 1 \right) \quad (6)$$

Note that nature logarithms are used here for convenient purpose.

$$\Omega_k = 10^{\Omega_{k(dBm)}/10} = e^{\xi \Omega_{k(dBm)}} = e^{\hat{\Omega}_k}$$

where $\xi = (\ln 10)/10 = 0.23026$ and $\hat{\Omega}_k = \xi \Omega_{k(dBm)}$. Also $\mu_{\hat{\Omega}_k} = \xi \mu_{\Omega_{k(dBm)}}$ and $\sigma_{\hat{\Omega}_k}^2 = \xi^2 \sigma_{\Omega_k}^2$.

The same applies to Z where $\hat{Z} = \xi Z_{(dBm)}$. Since standard deviation of log-normal shadowing is largely independent of the radio path length, identical variances are often assumed that for $\hat{\Omega}_k, k = 1, \dots, N_I$, $\sigma_{\hat{\Omega}_k}^2 = \sigma_{\hat{\Omega}}^2$. Finally,

$$\mu_{Z_{(dBm)}} = \xi^{-1} \mu_{\hat{Z}} \quad (7)$$

$$\sigma_Z^2 = \xi^{-2} \sigma_{\hat{Z}}^2 \quad (8)$$

4 Uplink Outage Throughput Capacity Under Infrastructure Transmission Mode

In this section, we focus on investigation of uplink Massive MIMO performance in terms of outage capacity under infrastructure mode in hybrid wireless network. With very large antenna array on the base station, a lot of things become different from the traditional MIMO system point of view.

One of the key characters that base station antennas greatly exceeds the number of user terminals is called favorable propagation[15]. The vector value based propagation matrix are asymptotically orthogonal as $M \gg K$. The propagation matrix $\mathbf{G}$ is described in equation (2) which contains both small scale fading matrix $\mathbf{H}$ and large scale fading diagonal matrix $\mathbf{D}_\beta^{1/2}$. Each column-vector of $\mathbf{H}$ correlates to small scale fading between the k th terminal and the M base station antennas. When $M \gg K$, column-vectors in matrix $\mathbf{H}$ become long and are asymptotically pairwise orthogonal according to random matrix theory[9]. The propagation matrix thus can be written as:

$$\left(\frac{\mathbf{G}^H \mathbf{G}}{M}\right)_{M \gg K} = \mathbf{D}_\beta^{1/2} \left(\frac{\mathbf{H}^H \mathbf{H}}{M}\right)_{M \gg K} \mathbf{D}_\beta^{1/2} \approx \mathbf{D}_\beta \quad (9)$$

From equation (9) we can observe that long propagation matrix makes small scale fading effect greatly mitigated under favorable propagation while large scale fading coefficient dominates.

The sum rate capacity of uplink MU-MIMO can be obtained from [2] assuming receiver knows the channel:

$$C_{sum_ul} = \log_2 \det(\mathbf{I}_K + \rho_u \mathbf{G}^H \mathbf{G}) \quad (10)$$

by applying favorable propagation $(\mathbf{G}^H \mathbf{G})_{M \gg K} \approx M \mathbf{D}_\beta$, we get

$$C_{sum_ul M \gg K} \approx \log_2 \det(\mathbf{I}_K + M \rho_u \mathbf{D}_\beta) \quad (11)$$

$\mathbf{D}_\beta$ is a diagonal matrix that $\mathbf{D}_\beta = \text{diag}\{\beta_1, \beta_2, \dots, \beta_K\}$, equation (11) can be rewritten as:

$$C_{sum_ul M \gg K} = \sum_{k=1}^K \log_2(1 + M \rho_u \beta_k) \quad (12)$$

Equation (12) indicates that the sum-rate could be represented as individual rate correlated to each terminal with little interference. This yields an simplification of linear decoding process by using matched-filter at the BS.

We already observed that large-scale fading is a dominant factor in capacity equation. To further derive the outage capacity of the uplink channel, we assume each large-scale fading coefficient β_k has the following expression[16]:

$$\beta_k = \phi d_k^{-\alpha} \zeta_k \quad (13)$$

Where ϕ is a constant related to the antenna gain and carrier frequency, d_k is the distance between the base station and k th terminal. α is the path loss exponent. ζ_k is the log-normal shadowing with $10 \log_{10} \zeta_k \sim \mathcal{N}(0, \sigma_k^2)$. Thus we apply this path loss and shadowing model to equation (12),

$$C_{sum_ulM \gg K} = \sum_{k=1}^K \log_2 (1 + M \rho_u \phi d_k^{-\alpha} \zeta_k) \quad (14)$$

We assume that all user terminal devices transmit at a sum rate $R_{bit}/s/Hz$, the related outage probability is

$$P_{out} = Pr \left\{ \sum_{k=1}^K \log_2 (1 + M \rho_u \phi d_k^{-\alpha} \zeta_k) < R \right\} \quad (15)$$

To solve the equation above, we then explore the outage capacity in both high SNR and low SNR scenarios.

4.1 Low SNR

At low SNR, we employing $\ln(1+x) \approx x$ and obtain the following,

$$P_{out} = Pr \left\{ \log_2^e M \rho_u \phi \sum_{k=1}^K (d_k^{-\alpha} \zeta_k) < R \right\} \quad (16)$$

For nodes are uniform circularly placed, the path loss $d_k^{-\alpha}$ corresponds to each node is fixed during solving the equation. Since ζ_k is log-normal distributed with $10 \log_{10} \zeta_k \sim \mathcal{N}(0, \sigma_k^2)$, we

could derive $d_k^{-\alpha}\zeta_k$ is also log-normal distributed with $10\log_{10}(d_k^{-\alpha}\zeta_k) \sim \mathcal{N}(\mu_k, \sigma_k^2)$ where $\mu_k = 10\log d_k^{-\alpha}$.

Applying Fenton-Wilkinson method we approximate the summation of log-normal distributed random variables $d_k^{-\alpha}\zeta_k$ with a new log-normal random variable, say χ that $10\log_{10}\chi \sim \mathcal{N}(\mu_\chi, \sigma_\chi^2)$.

The new parameter of χ is chosen by,

$$\mu_\chi = \xi^{-1} \left(\frac{\xi^2}{2} (\sigma_k^2 - \sigma_\chi^2) + \ln \left(\sum_{k=1}^K e^{\xi\mu_k} \right) \right) \quad (17)$$

$$\sigma_\chi^2 = \xi^{-2} \ln \left(\left(e^{\xi^2\sigma_k^2} - 1 \right) \frac{\sum_{k=1}^K e^{2\xi\mu_k}}{\left(\sum_{k=1}^K e^{\xi\mu_k} \right)^2} + 1 \right) \quad (18)$$

Then the outage probability is then,

$$\begin{aligned} P_{out} &= Pr \{ \log_2^e M \rho_u \phi \chi < R \} \\ &= Pr \left\{ \chi < \frac{R}{\log_2^e M \rho_u \phi} \right\} \end{aligned} \quad (19)$$

The cumulative distribution function(CDF) of the log-normal distributed random variable χ is given by

$$F(x) = \frac{1}{2} \operatorname{erfc} \left(-\frac{\ln x - \mu_\chi}{\sigma_\chi \sqrt{2}} \right) \quad (20)$$

where $\operatorname{erfc}(x)$ is the complementary error function.

With the CDF of χ , we have the following

$$\begin{aligned} P_{out} &= F(T) \\ &= \frac{1}{2} \operatorname{erfc} \left(-\frac{\ln T - \mu_\chi}{\sigma_\chi \sqrt{2}} \right), T = \frac{R}{\log_2^e M \rho_u \phi} \end{aligned} \quad (21)$$

Unfortunately there's no closed-form expression of $\operatorname{erfc}(x)$ in terms of elementary functions. According to [17], an tight exponential upper bounds and a pure exponential approximation for the complementary error function are derived

$$\text{erfc}(x) \leq \frac{1}{2}e^{-2x^2} + \frac{1}{2}e^{-x^2} \leq e^{-x^2}, x > 0 \quad (22)$$

$$\text{erfc}(x) \approx \frac{1}{6}e^{-x^2} + \frac{1}{2}e^{-\frac{4}{3}x^2}, x > 0 \quad (23)$$

Applying the approximation (23) to equation(21), we got

$$P_{out} = \frac{1}{2} \left(\frac{1}{6}T' + \frac{1}{2} (T')^{\frac{4}{3}} \right), \quad (24)$$

where

$$T' = e^{-\left(-\frac{\ln T - \mu_\chi}{\sigma_\chi \sqrt{2}}\right)^2} \quad (25)$$

The outage capacity is obtained by solving $P_{out} = \epsilon$. However, solving the non-linear equation by applying newton's method does not turn out to obtain physically meaningful root. Therefore, we try to approximate the complementary error function using the tight upper bound (22) to get a linear equation.

$$P_{out} \leq \frac{1}{2} \left(\frac{1}{2} (T')^2 + \frac{1}{2} T' \right) = \epsilon \quad (26)$$

where

$$T' = \frac{\sqrt{1 + 16\epsilon} - 1}{2} \quad (27)$$

Equation (27) is satisfied when $\epsilon < \frac{1}{2}$ Combining (25) and (27) we got

$$C_\epsilon = \log_2^e M \rho_u \phi \exp \left\{ \mu_\chi - \sigma_\chi \sqrt{2 \ln \left(\frac{2}{\sqrt{1 + 16\epsilon} - 1} \right)} \right\} \quad (28)$$

From equation (28) we can see that the capacity is related to μ_χ and σ_χ which are defined in (17) and (18).

Since the distance $c = \sqrt{\frac{2\sqrt{3}}{3} \frac{n}{b}}$ between the parallel sides of a cell is bounded by $\Theta(\sqrt{\frac{n}{b}})$ as the system increases. With uniform placement, the distance d_k between the k th user in one cell and the base station is proportional to c and it is bounded by $\Theta(\sqrt{\frac{n}{b}})$ as well. Usually the path loss

exponent α has range from 1.6 to 5 based on specific path loss model. Here we assume free space environment that $\alpha = 2$. Since $\mu_k = 10 \log d_k^{-\alpha}$, then we obtain $\mu_k = \Theta(\log \frac{b}{n})$. We also know that number of nodes in a cell K is bounded by $\Theta(\frac{n}{b})$. From equation (18) we can obtain that

$$\sigma_\chi^2 \approx \ln\left(\frac{1}{K} + 1\right) = \Theta\left(\log\left(\frac{b}{n} + 1\right)\right). \quad (29)$$

Since we have $\mu_k = \Theta(\log \frac{b}{n})$, the term $\ln\left(\sum_{k=1}^K e^{\xi\mu_k}\right)$ in equation (17) tends to zero. Then $\mu_\chi \approx -\frac{\xi}{2}\sigma_\chi^2$. We can also derive that

$$\mu_\chi \approx \Theta\left(\log\left(\frac{n}{b}\right)\right). \quad (30)$$

Then we make the conclusion that the transmission rate under low SNR for Massive MIMO uplink is:

$$R_{ul}^{low} = O\left(M \left(\left(\frac{n}{b}\right) e^{-\sqrt{\log(\frac{b}{n}+1)\delta_\epsilon}}\right)\right) \quad bit/s/Hz \quad (31)$$

Where $\delta_\epsilon = 2 \ln\left(\frac{2}{\sqrt{1+16\epsilon}-1}\right)$.

From equation (31) we can see that in low SNR scenario, uplink outage capacity scales linearly with number of base station antennas M , which means that increasing number of antenna will always increase throughput capacity. Figure 5 below shows uplink outage capacity with different M settings when outage probability $\epsilon = 0.001$.

4.1.1 High SNR

In high SNR scenario, we approximate $\log(1+x) = \log(x)$ to equation (15),

$$P_{out} = Pr\left\{\sum_{k=1}^K \log_2(M\rho_u\phi d_k^{-\alpha}\zeta_k) < R\right\} \quad (32)$$

From the previous section we have seen that ζ_k is log-normal distributed with $10\log_{10}(\zeta_k) \sim \mathcal{N}(0, \sigma_k^2)$. We define another random variable $L = \log_2 M\rho_u\phi d_k^{-\alpha}\zeta_k$ that $L \sim \mathcal{N}(\mu_L, \sigma_L^2)$ and

$$\mu_L = \log_2(M\rho_u\phi d_k^{-\alpha}) \quad (33)$$

Figure 5: Uplink outage throughput capacity under low SNR scenario ($\epsilon = 0.001$).

$$\sigma_L^2 = \left(\frac{\sigma_k}{10 \log_{10} 2}\right)^2 \quad (34)$$

The summation of independent normal distributed random variables is also normal distributed.

That is $\hat{L} = \sum_{k=1}^K L$, $\hat{L} \sim \mathcal{N}(\mu_{\hat{L}}, \sigma_{\hat{L}}^2)$ with $\mu_{\hat{L}} = K\mu_L$, $\sigma_{\hat{L}}^2 = K\sigma_L^2$.

Then we can rewrite equation (32)

$$\begin{aligned} P_{out} &= Pr \left\{ \sum_{k=1}^K L < R \right\} \\ &= Pr \left\{ \hat{L} < R \right\} \\ &= \frac{1}{2} \text{erfc} \left(-\frac{R - \mu_{\hat{L}}}{\sigma_{\hat{L}} \sqrt{2}} \right) = \epsilon \end{aligned} \quad (35)$$

Apply approximation (22)

$$C_\epsilon = \mu_{\hat{L}} - \sigma_{\hat{L}} \sqrt{2 \ln \left(\frac{2}{\sqrt{1 + 16\epsilon} - 1} \right)}, \epsilon < \frac{1}{2} \quad (36)$$

By analysis of (33) and (34), we can get $\mu_{\hat{L}} = \Theta(\frac{n}{b} \log(M \frac{b}{n}))$ and $\sigma_{\hat{L}} = \Theta(\sqrt{\frac{n}{b}})$. So the transmission rate under high SNR for Massive MIMO uplink is:

$$R_{ul}^{high} = O\left(\frac{n}{b} \log(M \frac{b}{n}) - \sqrt{\frac{n}{b} \delta_\epsilon}\right) \quad \text{bit/s/Hz} \quad (37)$$

Where $\delta_\epsilon = 2 \ln\left(\frac{2}{\sqrt{1+16\epsilon}-1}\right)$.

Under high SNR scenario, the uplink outage throughput capacity yield logarithmic increase with number of base station antennas M . From Figure 6 we can see that doubling M from 25 to 50 almost have the same capacity gain when doubling M from 50 to 100.

Figure 6: Uplink outage throughput capacity under high SNR scenario ($\epsilon = 0.001$).

5 Conclusions

In this paper, with Multi-user Massive MIMO implementation in the hybrid wireless network, we derived the theoretical uplink outage throughput capacity for infrastructure mode. Under favorable propagation condition, small scale fading is averaged out according to asymptotic orthogonality of

the propagation matrix with increased base station antenna M . Large scale fading on throughput capacity is examined for both high SNR and low SNR cases. Closed-form outage throughput capacity is derived. We proved that at low SNR, the infrastructure mode uplink outage capacity is $O\left(M\left(\left(\frac{n}{b}\right)e^{-\sqrt{\log(\frac{b}{n}+1)\delta_\epsilon}}\right)\right)$ bit/s/Hz. At high SNR, the infrastructure uplink outage capacity is $O\left(\frac{n}{b}\log(M\frac{b}{n}) - \sqrt{\frac{n}{b}\delta_\epsilon}\right)$ bit/s/Hz. Increasing number of antennas at low SNR scenario yields better capacity gain compared to the one at high SNR.

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