Consensus in Innovation Contest Categorisation by means of Fuzzy Partitions

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Abstract

Consensus decision-making is fuzzy by nature, yet fuzzy consensus decision-making in a medium to large number of decisions is not widely used since it demands additional information that requires extra decision-maker effort. Consensus decision-making rests on properly measured agreement. This paper proposes a fuzzy measure of agreement through fuzzy kappa based on fuzzy partitions. These fuzzy partitions enable decision-makers to assess their decisions with a degree of confidence. A fuzzy partition is built for each decision-maker considering his/her confidence degrees when categorising a set of alternatives or solutions. This enables decision-makers to more easily capture the fuzzy nature of the decision. In addition, this paper presents a real-life experiment based on an innovation contest to show the feasibility of using confidence degrees in real-life applications compared to traditional consensus decision-making. The results suggest that the use of confidence degrees improves the level of agreement in the consensus decision-making process through fuzzy kappa coefficients, and it also improves the level of agreement in the consensus decision-making process.

Keywords: Fuzzy partitions, Fuzzy sets, fuzzy kappa, Cohen's kappa, innovation contest, open innovation, consensus decision-making, crowdsourcing

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1. Introduction

A key challenge for organisations is to maintain their competitiveness through innovation. Critical to innovation is the fuzzy-front end of the new product development process (Klink and Athaide, 2006). Idea generation and selection
5 of the most innovative ideas to pursue is at the core of this front-end process (Toubia, 2006). To increase the chances of identifying promising new products, processes, and strategic or improvement ideas, individuals from a large variety of sources are solicited for these ideas (e.g. users, employees, manufacturers, suppliers, regulators, government agencies, non-government organisations, and
10 general members of society) (Chesbrough, 2006; Von Hippel, 1989).

These requests for innovative ideas are often part of what are referred to as innovation crowdsourcing events (Leimeister et al., 2009; Majchrzak and Malhotra, 2013; Surowiecki, 2005). In these events, an announcement is made by a sponsoring organisation to a variety of sources (via advertisements, postings
15 on list servers, or invitations). The announcement describes the sponsoring organisation; the problem, issue, or question that the organisation is raising; information about the problem; a request for innovative ideas; a website where contributions can be made; and a timeframe (from three days to three months) during which the event will occur. Such events are increasingly used for so-
20 liciting new research & development (R&D) ideas as part of a paradigm shift referred to as open innovation (Chesbrough, 2006). Government (Administration, 2015) and non-government organisations (of the Red Cross, 2015), as well as private sector companies (e.g., (King and Lakhani, 2011; Malhotra and Majchrzak, 2014)) request ideas ranging from new business strategies (Malhotra
25 and Majchrzak, 2014) to how to provide water supplies (OpenIDEO, 2015). These innovation crowdsourcing events are known as innovation contests or community-based innovation contests (Bullinger et al., 2010; Leimeister et al., 2009; Majchrzak and Malhotra, 2013; Piller and Walcher, 2006).

In a successful innovation initiative, be it crowdsourced or more traditional

30 methods of customer focus groups, a large number of new ideas are generated
 (Diehl and Stroebe, 1987; Neyer et al., 2009; Simonton, 1999). In fact, if the
 innovation initiative is very successful, it may generate so many ideas that the
 selection of the most promising ones becomes very difficult or costly (Santos
 and Spann, 2011). An innovation initiative requires identifying both innovative
 35 ideas and interactions (e.g. posts in an innovation contest). The first step is to
 select a set of categorised innovative ideas from a large number of ideas found
 in one or more participant interactions. The chief innovation officers (CIOs)
 select innovative ideas using a criteria including novelty, competitive advantage,
 feasibility, and fit with existing competencies (Cooper and De Brentani, 1984).
 40 The second step is to identify posts based on the innovative ideas selected to
 properly motivate participants (Füller et al., 2012). This identification is, in
 general, done by a group of decision-makers or R&D managers who categorise
 posts as matching the set of CIO-selected ideas (Lamastra, 2009). An alternative
 approach is for CIOs to select the innovative ideas and posts: however, this
 45 approach requires more time from CIOs (and whose time is scarce) (Toubia,
 2006).

Categorising posts based on innovative ideas is inherently subjective (Piller
 and Walcher, 2006) since “the process of decision-making, in particular of a
 group type, is centered on humans, coming with their inherent subjectivity, im-
 50 precision and vagueness in the articulation of opinions” (Herrera-Viedma et al.,
 2014, p. 5). In the case of an innovation contest, the use of confidence degrees
 (i.e. a parameter that captures the certainty of the decision-maker or R&D
 manager) enables us to capture the fuzzy nature of the process and so make it
 closer to reality. Despite an increase in effort by decision-makers being required
 55 when using confidence degrees, their use enables harnessing information about
 uncertainty from each decision-maker in order to better obtain a soft consen-
 sus. Confidence degrees are obtained by means of a set of linguistic labels.
 The use of these linguistic labels enables us to generate a fuzzy partition per
 decision-maker. Finally, the coincidence matrix is assessed to determine the
 60 shared memberships of each post. The soft consensus degree is then computed

using a fuzzy kappa (Dou et al., 2007).

The paper is organised as follows. Section 2 establishes the theoretical framework regarding consensus-decision-making and fuzzy partitions. Section 3 defines the framework and associated methodology used to translate a crisp partition into a fuzzy one by exploiting the degrees of confidence of the decision-makers. In Section 4 a real-case application of the methodology presented is detailed along with research findings and results. Section 5 discusses the implication of selecting different parameters in our methodology. Finally, the conclusions and directions for further research are explained in Section 6.

2. Theoretical framework

In this section, firstly some theoretical background information on the consensus process is presented; secondly, an overview is provided on the use of fuzzy partitions for building a set of fuzzy indices (classically used to compare a given partition with an *a priori* structure, or just another partition) are analysed to compare the fuzzy partitions obtained. Finally, these indices are used to measure agreement and extended for more than two decision-makers.

2.1. Consensus process

Group decision-making (GDM) is a problem-solving activity involving groups of decision-makers that produces a final choice based on the selection among several possibilities (Edwards, 1977). Consensus process is a step considered in some types of GDM technique that focuses on obtaining the maximum agreement between a set of decision-makers (Herrera-Viedma et al., 2007, 2002). In general, GDM requires the majority of individuals to approve a choice, and that the minority agrees with the final choice (Herrera-Viedma et al., 2002; Kaner, 2014). Other group decision-making techniques involve voting-based methods (Ayad and Kamel, 2010), or structured techniques consisting of several rounds using an anonymous summary known as Delphi (Linstone et al., 1975).

Consensus decision-making techniques seek to: 1) empower all participants in the decision process; 2) listen to other participants; 3) challenge the other

90 perspective until reaching a consensus; 4) integrate different viewpoints from
other participants; 5) provide the mechanics to work on the dynamics that are
not rational (such as emotions and intuitions); and 6) allocate different times
to each decision if need be (Bressen, 2007). These properties are better fulfilled
when using confidence degrees as a feedback mechanism since this enables the
95 uncertainty of the decision to be captured and used later in the discussion
(Edwards, 1977).

Traditional approaches to consensus decision-making that do not use confi-
dence degrees are widely used when the subjective categorisation of information
based on subjective criteria (such as novelty) is needed (Saldaña, 2012). This
100 subjectiveness is handled by an inter-rater reliability assessment of the classifica-
tions assigned to the data, where two or more decision-makers analyse the data,
and make various predefined categorisations (Gwet, 2014). A good categori-
sation that overcomes this subjectivity weakness is defined when the decision-
makers have an agreement that is higher than a specific threshold (Landis and
105 Koch, 1977). Traditional approaches assume that each decision-maker has the
same level of decision-making certainty, and so introducing unnecessary error
into the categorising by assuming that each decision-maker is absolutely cer-
tain about the categories assigned. For example, two decision-makers seeking
to categorise emotional intensity in an idea may not be absolutely certain.

110 Decision-making in a fuzzy environment is a decision process in which the
goals, constraints, or the system under control are fuzzy by nature (Bellman and
Zadeh, 1970). However, consensus decision-making for medium to large datasets
has not adopted the use of fuzzy logic when categorising alternatives since adop-
tion demands an additional decision-maker effort (e.g. a post to an online dis-
115 cussion thread would require not only the categorisation of a post to a category,
but a confidence degree for each assigned category) in a vague, complex, pro-
cess that might introduce possible inconsistencies among the decision-makers
(Martínez and Montero, 2007). Some sets of consensus decision-making tech-
niques such as multi-attribute utility analysis use confidence degrees to capture
120 the different decision-maker preferences (Torrance et al., 1982). Multi-attribute

utility analysis has been extended to fuzzy environments using different coefficients (Chen, 2000; Chiclana et al., 2013). This paper proposes capturing and using in posterior discussion one confidence degree per decision instead of multiple confidence degrees.

125 Capturing one confidence degree simplifies the decision-making process by capturing the fuzzy nature of the decision, while adding a minimal additional effort for the decision-makers (e.g. a post with ten categories requires one confidence degree instead of ten confidence degrees). The use of these confidence degrees helps to overcome the uncertainty management challenge (Martínez and
130 Montero, 2007) by enhancing decision-makers with additional information to assess their own decision-maker certainty through a degree of confidence. Moreover, these confidence degrees enable capturing intuition, or a viewpoint that is worth exploring if there is disagreement, by setting a low confidence degree (Bressen, 2007). Thus, the fuzzy approach improves the good properties of consensus decision-making, while enabling normalising to reflect a decision-maker's
135 personality with regard to confidence (i.e. overconfidence, confident, or underconfidence). This is known as a soft consensus approach (Cabrerizo et al., 2009, 2010; Herrera-Viedma et al., 2005, 2007).

2.2. Fuzzy partitions

140 Within the framework of fuzzy sets introduced by Zadeh in 1965, a fuzzy set is characterised by a membership function that associates each element with a real number in the interval $[0, 1]$. A fuzzy partition of a set is then defined by giving the degree of membership or adequacy of each individual to each considered category (classes or clusters). The formal description of a fuzzy
145 partition was introduced by Ruspini (1969). A fuzzy partition of a given set $X = \{x_1, x_2, \dots, x_N\}$ into K clusters C_k , $k \in \{1, \dots, K\}$, is defined by a set of values μ_{ik} , in which each value indicates the membership degree of the alternative i to the category k and satisfies the following conditions:

$$0 \leq \mu_{ik} \leq 1, \forall i, k \text{ and } \forall k \exists i \text{ such that } \mu_{ik} > 0.$$

In general, a fuzzy partition is expressed by a matrix $F_X = (\mu_{ik})$, where in
 150 each row, the marginal membership degree of the alternative x_i to the category
 C_k is represented. In addition, when the constraint $\sum_{k=1}^K \mu_{ik} = 1, \forall i$ is satis-
 fied, the fuzzy partition is termed a fuzzy probabilistic partition. Note that in
 the extreme case in which $\mu_{ik} \in \{0, 1\} \forall i, k$, a crisp partition is obtained. Thus,
 fuzzy probabilistic partitions can be considered as a generalisation of crisp par-
 155 titions, in the same way than fuzzy sets are considered as a generalisation of
 crisp sets.

To define a fuzzy degree of compatibility or similarity between fuzzy par-
 titions, t-norms and t-conorms (denoted with a superscript ‘*’) have extended
 intersection and union operators to fuzzy sets theory (Dubois and Prade, 2004).
 160 The following t-norms and t-conorms are broadly used:

- MinMax:

- Min: $M(y_1, \dots, y_n) = \min\{y_1, \dots, y_n\}$
- Max: $M^*(y_1, \dots, y_n) = \max\{y_1, \dots, y_n\}$

- Probabilistic product:

- $\prod(y_1, \dots, y_n) = y_1 \cdot \dots \cdot y_n$
- $\prod^*(y_1, \dots, y_n) = 1 - \prod_{i=1}^n (1 - y_i)$

- Lukasiewicz:

- $W(y_1, \dots, y_n) = \max\{1 - n + \sum_{i=1}^n y_i, 0\}$
- $W^*(y_1, \dots, y_n) = \min\{\sum_{i=1}^n y_i, 1\}$

170 2.3. Indices to compare crisp partitions

The most classic indices to compare two partitions are described below. The
 Rand index (Rand, 1971), and most of the following indices, are based on pair-
 wise comparisons of data alternatives. The original crisp version of these indices
 handles two crisp partition matrices (P_X and Q_X) of the same set of alternatives

175 X . Both partitions can “categorise” data alternatives into a different number of clusters. The following sets should be defined to introduce these indices:

- S_1 is the subset of tuples $\{x, y\} \in X$ paired both in P_X and in Q_X ;
- S_2 is the subset of tuples $\{x, y\} \in X$ paired in P_X but not paired in Q_X ;
- S_3 is the subset of tuples $\{x, y\} \in X$ not paired in P_X but paired in Q_X ;
- 180 • S_4 is the subset of tuples $\{x, y\} \in X$ neither paired in P_X nor in Q_X .

Let the following numbers be the cardinal of the previously defined sets:

$$a = |S_1|, b = |S_2|, c = |S_3|, d = |S_4|. \quad (1)$$

Note that $\{S_1, S_2, S_3, S_4\}$ is a partition of the set consisting of the subsets of two elements of X and that $a + b + c + d = |X|$. On the one hand, $S_1 \cup S_4$ are the concordant pairs between P_X and Q_X (agreements), while on the other
 185 hand $S_2 \cup S_3$ are the discordant pairs (disagreements). Bearing this in mind, the Rand index is defined as the ratio between the concordant pairs and the total number of pairs. One of the main criticisms against the Rand index is that it is not *corrected by chance*, that is to say, its expected value is not zero when comparing two random partitions (Cohen, 1968). The adjusted Rand index is
 190 corrected by assuming that the number of clusters to be compared is the same in both partitions (Hubert and Arabie, 1985).

Another common criticism of the Rand index is that it gives the same importance to both agreement terms a and d , thus making no difference between pairs of alternatives that are joined or separated in both partitions. Moreover,
 195 term d may dominate the other terms a , b and c . The removal of this term d results in the so-called Jaccard coefficient (Halkidi et al., 2001). It can be interpreted as the proportion of good pairs with respect to non-neutral pairs. The absence of term d is also exhibited in the so-called Fowlkes-Mallows index (Fowlkes and Mallows, 1983), Minkowski measure (Jiang et al., 2004) and the
 200 ω_τ statistic (Jain et al., 1988).

The main concern of these related indices is that they do not perform properly when the clusters to be analysed are imbalanced (i.e. the probability of one cluster vastly exceeds others) (Cohen, 1968). To solve this, Cohen’s kappa coefficient measuring the similarity between two partitions (i.e. P and Q) was
205 defined as follows:

$$\omega_\kappa(P, Q) = \kappa = \frac{p_o - p_e}{1 - p_e}, \quad (2)$$

where p_o is the relative coincidence observed between partition P and Q and p_e is the hypothetical probability of chance coincidences, using the observed categorisations to compute the probabilities of each partition randomly choosing each category. If partitions completely match, then $\kappa = 1$. However, if the
210 categorisations differ completely, then $\kappa = 0$. The computation of both terms p_o and p_e is detailed in Dou et al. (2007).

2.4. Measuring Agreement: Indices to compare fuzzy partitions

The design of suitable measures for comparing fuzzy partitions is an important topic of research in several multidisciplinary areas (Kukar, 2003; Osei-Bryson, 2010) such as clustering. Some of the previously introduced indices in
215 Subsection 2.3 have been extended into fuzzy versions: fuzzy Rand index and others (Campello, 2007) and fuzzy kappa index (Dou et al., 2007).

The extension of Cohen’s kappa preserves the formula given in (2) for computing Cohen’s kappa statistic by taking into account the fuzziness of the partition in the calculation of the proportions p_o and p_e . The computation of such
220 terms needs the selection of a t-norm in order to aggregate the membership degrees of a certain alternative to a category:

$$\kappa_\tau = \frac{p_{o_\tau} - p_{e_\tau}}{1 - p_{e_\tau}}, \quad (3)$$

where τ is a t-norm (see Subsection 2.2) and p_{o_τ} and p_{e_τ} stand for the observed and expected agreement, respectively, computed by employing the same t-norm
225 τ . The values of p_{o_τ} and p_{e_τ} are defined by Dou et al. (2007) (see Subsection

3.4).

3. A Fuzzy Methodology to Measure Agreement in Classification

This section presents a fuzzy methodology to measure agreement based on confidence degrees and decision-maker preferences for two decision-makers. In Figure 1 the different steps of this methodology are highlighted. Initially, we present how to transform from a given crisp partition, in which each decision-maker has categorised each alternative into a unique category (i.e. “3.1” on Figure 1 and in Subsection 3.1), to a fuzzy partition by taking into account the confidence degree stated by the decision-maker in each classification (i.e. “3.2” on Figure 1 and in Subsection 3.2). This section describes the use of fuzzy indices to define a fuzzy membership matrix: the coincidence matrix between decision-makers (i.e. “3.3” on Figure 1 and in Subsection 3.3). This coincidence matrix is used to measure the agreement or consensus degree (i.e. “3.4” on Figure 1 and in Subsection 3.4). This consensus degree, as well as the decision-maker preferences and confidence degrees are used as feedback. For this reason, this framework can be classified as a feedback mechanism and consensus degree based on distance between the decision-makers according to the taxonomy given by Palomares et al. (2014). Finally, the methodology can be extended to more than two decision-makers following the steps defined in Subsection 3.5.

Let us assume a decision-making problem in which two decision-makers $\mathcal{E}_1$ and $\mathcal{E}_2$ analyse a set of alternatives $X = \{x_1, \dots, x_i, \dots, x_N\}$ to classify or assign them on a set of categories or classes $C = \{c_1, \dots, c_k, \dots, c_K\}$ (i.e. each decision-maker $\mathcal{E}$ states his/her selections through his/her preferences associated to a crisp membership degree $\delta_{ik}^{\mathcal{E}} \in \{0, 1\}$, $\forall i \in \{1, \dots, N\}$ and $k \in \{1, \dots, K\}$). In addition, decision-maker $\mathcal{E}$ associates with each alternative a confidence degree $D_i^{\mathcal{E}}$, $\forall i \in \{1, \dots, N\}$, to express the level of confidence that he/she has when assigning a category to this alternative. The values of this confidence degree are given by a linguistic ordinal term set $S = \{s_0, \dots, s_g\}$ whose cardinality or granularity is $\#S = g + 1$, being $D_i^{\mathcal{E}} \in S$. Therefore, an N -dimensional vector

Figure 1: Scheme of the two-decision-maker soft consensus agreement proposed. Number indicates the section in which each step is detailed.

255 $D^{\mathcal{E}} = \{D_1^{\mathcal{E}}, \dots, D_N^{\mathcal{E}}\}$ is associated with decision-maker $\mathcal{E}$.

3.1. Stating a crisp partition

Algorithm 1 enables us to define, for each decision-maker $\mathcal{E}_1$ and $\mathcal{E}_2$, his/her crisp membership matrix according to the initial decision-maker preferences.

```

1  Considering a decision-maker  $\mathcal{E}$ , the crisp matrix  $(\delta_{i,k}^{\mathcal{E}})$  corresponding to
   decision-maker  $\mathcal{E}$ 's opinions is computed as follows:
2  foreach alternative  $x_i$  do
3      foreach category  $c_k$  do
4          if decision-maker  $\mathcal{E}$  has selected category  $c_k$  then
5               $\delta_{ik}^{\mathcal{E}} = 1$ ;
6          else
7               $\delta_{ik}^{\mathcal{E}} = 0$ ;
8          end
9      end
10 end

```

Algorithm 1: Defining the crisp partition

260 **Example 1.** Let us assume that two averagely confident decision-makers $\mathcal{E} = \{\mathcal{E}_1, \mathcal{E}_2\}$ provide their decision-maker preference on classifying a set of four alternatives $X = \{x_1, x_2, x_3, x_4\}$ in three categories $\mathcal{C} = \{A, B, C\}$. They

use the linguistic term set $S = \{Strongly\ uncertain(0), Uncertain(1), Partially\ uncertain(2), Neither\ certain\ nor\ uncertain(3), Partially\ certain(4), Certain(5), Strongly\ certain(6)\}$ to express their confidence, as depicted in Table 1.

Table 1: Example for decision-maker preferences and confidence degrees for three alternatives

	Decision-maker #1			Decision-maker #2		
	Preference	Confidence	Degree	Preference	Confidence	Degree
x_1	A		3	A		5
x_2	C		0	C		0
x_3	C		2	B		5
x_4	A		1	C		2

265 In this first step we transform this data to a crisp partition obtaining the results detailed in Table 2.

Table 2: Transformation example for crisp partitions for the two decision-makers, four alternatives, and three categories

	DM #1				DM #2		
	A	B	C		A	B	C
x_1	1	0	0	x_1	1	0	0
x_2	0	0	1	x_2	0	0	1
x_3	0	0	1	x_3	0	1	0
x_4	1	0	0	x_3	0	0	1

3.2. Fuzzification

To make use of the extra information provided by the N -dimensional vector $D^{\mathcal{E}}$ given by decision-maker $\mathcal{E}$, this subsection introduces a methodology to
270 translate the crisp partition defined by $(\delta_{ik}^{\mathcal{E}})$ to a fuzzy one defined by $(\mu_{ik}^{\mathcal{E}})$. This new fuzzy partition is defined from the confidence degree vector $D^{\mathcal{E}}$, and the crisp membership values, $(\delta_{ik}^{\mathcal{E}})$, and satisfies the following conditions:

1. $\mu_{ik}^{\mathcal{E}} \in [0, 1], \forall i, k;$
2. $\sum_{k=1}^K \mu_{ik}^{\mathcal{E}} = 1, \forall i;$
- 275 3. $\mu_{ik}^{\mathcal{E}} = 1 \iff \delta_{ik}^{\mathcal{E}} = 1$ and $D_i^{\mathcal{E}} = \max(S) = s_g;$
4. $\mu_{ik}^{\mathcal{E}} = \frac{1}{K} \iff D_i^{\mathcal{E}} = \min(S) = s_0;$

5. An interpolation function f is considered to define μ_{ik} when $D_i^\varepsilon \neq s_0$ and $D_i^\varepsilon \neq s_g$.

Three different functions (i.e. linear, logarithmic, and exponential) have been considered to obtain the interpolation in condition #5. Figure 2 displays on the
 280 ordinates axis the new membership degree of the category originally chosen for the alternative, according to the confidence degree shown in the abscesses axis.

Figure 2: Example of the fuzzy membership degree of a category that had a crisp membership degree of 1. Logarithmic, linear and exponential functions are depicted by taking $K = 3$ and $\#S = 7$.

The fuzzification is a two-step process. The first step is to compute the maximum value for each alternative using one of the three interpolation functions
 285 introduced. Given an alternative x_i $i \in \{1, \dots, N\}$ and that $k^* \in \{1, \dots, K\}$ such that $\delta_{ik^*}^\varepsilon = 1$. Then, μ_{ik^*} is defined according to Table 3.

Table 3: Three different functions of fuzzification		
	μ_{ik^*}	Decision-maker assumption
Linear	$\frac{D_i}{\#S-1} \cdot \frac{K-1}{K} + \frac{1}{K}$	Average confidence
Exponential	$\frac{\log(1+D_i)}{\log(\#S)} \cdot \frac{K-1}{K} + \frac{1}{K}$	Underconfident
Logarithmic	$\frac{e^{D_i}}{e^{\#S-1}} \cdot \frac{K-1}{K} + \frac{1}{K}$	Overconfident

The second step is to compute the fuzzy membership degrees of the other categories, $\mu_{i\bar{k}^*}$. They are equally defined to satisfy condition #2 as follows:

$$\mu_{ik} = \frac{1 - \mu_{ik^*}}{K - 1}, \forall k \neq k^* \quad (4)$$

The indices defined in Subsection 2.2 are usually used to measure the compatibility between two partitions of the same set of alternatives. Applying the methodology detailed in the previous Subsection 3.2, any crisp partition with an associated vector of degree of confidence can be easily transformed into a fuzzy partition. The process for fuzzifying opinions made by a decision-maker $\mathcal{E}$ on selecting from a set C of K alternatives is detailed in Algorithm 2.

```

1 foreach alternative  $x_i$  do
2   Fuzzify the vector  $\delta_{i*}$  into  $\mu_{i*}$ :
3   foreach category  $c_k$  do
4     if  $\delta_{ik} = 1$  then
5       Compute the new value of the chosen category  $\mu_{ik*}$ :
6       - Linear:
7          $\mu_{ik*} = \frac{D_i}{\#S-1} \cdot \frac{K-1}{K} + \frac{1}{K}$ ;
8       - Logarithmic:
9          $\mu_{ik*} = \frac{\log(1+D_i)}{\log(\#S)} \cdot \frac{K-1}{K} + \frac{1}{K}$ ;
10      - Exponential:
11         $\mu_{ik*} = \frac{e^{D_i}}{e^{\#S-1}} \cdot \frac{K-1}{K} + \frac{1}{K}$ ;
12      else
13        Split the rest for the other unchosen categories  $\mu_{ik^*}$ :
14         $\mu_{ik^*} = \frac{1-\mu_{ik*}}{K-1}$ ;
15      end
16    end
17 end

```

Algorithm 2: Fuzzification

Example 2. Using the previous example where the crisp partitions were stated, their fuzzification is done based on the confidence degrees and linear interpolation (i.e. given by the normal confidence of the decision-makers).

Let us take the membership degree stated decision-maker $\mathcal{E}_1$ regarding the first alternative and first category $\delta_{11} = 1$, with confidence $D_1 = 3$. Given that $K = 3$, $\mu_{1k*} = \frac{D_1}{\#S-1} \cdot \frac{K-1}{K} + \frac{1}{K} = \frac{3}{7} \cdot \frac{3-1}{3} + \frac{1}{3} = 0.667$. The others values $\mu_{1k^*} = \frac{1-\mu_{1k*}}{K-1} = \frac{0.667}{3-1} = 0.167$. Table 4 depicts the values obtained from this example.

Table 4: Example of fuzzy partitions for two decision-makers in three alternatives

	Decision-maker #1				Decision-maker #2		
	A	B	C		A	B	C
x_1	0.667	0.167	0.167	x_1	0.889	0.056	0.056
x_2	0.333	0.333	0.333	x_2	0.333	0.333	0.333
x_3	0.222	0.222	0.556	x_3	0.056	0.889	0.056
x_4	0.444	0.278	0.278	x_4	0.238	0.222	0.556

3.3. Coincidence matrix

We adopted the definition of fuzzy coincidence by (Herrera et al., 1997, p. 311) as a “fuzzy set defined on the set of decision-maker pairs and characterised by closeness observed among their respective opinions”. For this reason, fuzzy kappa from Dou et al. (2007) is adapted to process values in a coincidence matrix in an approach similar to the “soft coincidence among preferences” (Kacprzyk and Fedrizzi, 1988; Cabrerizo et al., 2009; Herrera-Viedma et al., 2014). This approach obtains the coincidence by means of a similarity measured among decision-makers (Herrera-Viedma et al., 2014). This coincidence matrix is defined according to one of the t-norms considered in Section 2.2. This enables us to give different coincidence meanings based on strict coincidence (i.e. min for the t-norm) or less strict coincidence (i.e. Lukasiewicz or product as a t-norm). Algorithm 3 details the computation of coincidence matrix for any t-norm.

```

1 foreach alternative  $x_i$  do
2   Compute  $fMatrix_{K \times K}$  as follows:
3   - Row  $k$  is the t-norm applied to element  $\mu_{ik}^{\mathcal{E}_1}$  and  $\mu_{i*}^{\mathcal{E}_2}$ :
4      $fMatrix_{k,*} = T(\mu_{ik}^{\mathcal{E}_1}, \mu_{i*}^{\mathcal{E}_2})$ ;
5   - Column  $k$  is the t-norm applied to  $\mu_{i*}^{\mathcal{E}_1}$  and  $\mu_{ik}^{\mathcal{E}_2}$ :
6      $fMatrix_{*,k} = T(\mu_{i*}^{\mathcal{E}_1}, \mu_{ik}^{\mathcal{E}_2})$ ;
7 end

```

Algorithm 3: Computing coincidence matrix for t-norm

Example 3. Let us compute the coincidence matrix of the previous example assuming a product t-norm. Table 5 depicts the values obtained for the coincidence matrix.

Table 5: Example of coincidence matrix between experts $\mathcal{E}_1$ and $\mathcal{E}_2$ regarding alternative x_1

	A	B	C			A	B	C
A	$0.667 \cdot 0.889$	$0.667 \cdot 0.056$	$0.667 \cdot 0.056$		A	0.593	0.037	0.037
B	$0.167 \cdot 0.889$	$0.167 \cdot 0.056$	$0.167 \cdot 0.056$	=	B	0.148	0.009	0.009
C	$0.167 \cdot 0.889$	$0.167 \cdot 0.056$	$0.167 \cdot 0.056$		C	0.148	0.009	0.009

3.4. Soft consensus degree

320 Soft consensus degrees are based on the coincidence matrix (Herrera et al., 1997; Herrera-Viedma et al., 2014). Our proposed measure is based on the fuzzy kappa index introduced in Subsection 2.4. It adds robustness since it takes into account agreement occurring by chance. Algorithm 4 details the computation of fuzzy kappa index implemented according to the definitions given by Dou et al. (2007).

```

1 foreach alternative  $x_i$  do
2   Compute coincidence matrix  $fMatrix_{K \times K}$ 
3   Let  $f_{o,p}$  be the sum of the elements of the diagonal of  $fMatrix$ :
4      $f_{o,p} = \sum_{i=1}^K fMatrix_{ii}$ ;
5   Compute  $fMatrix2_{K \times K}$  as follows:
6      $fMatrix2 = fMatrix * fMatrix^t$ ;
7   Let  $f_{e,p}$  be the sum of the elements of the superior triangle of
      $fMatrix2$ :
8      $f_{e,p} = \sum_{i=1, i < j}^K fMatrix_{ij}$ ;
9 end
10 Accumulate all  $f_{o,p}$  and  $f_{e,p}$  and divide them between the number of
    alternatives:
11    $p_{o\tau} = \frac{1}{M} \sum_{p=1}^N f_{o,p}$ ;
12    $p_{e\tau} = \frac{1}{M} \sum_{p=1}^N f_{e,p}$ ;
13 Compute Cohen's kappa as:
14    $\kappa_\tau = \frac{p_{o\tau} - p_{e\tau}}{1 - p_{e\tau}}$ ;

```

Algorithm 4: Computing fuzzy kappa

Example 4. To conclude with the previous example, let us compute κ_τ following Algorithm 4. The first step is to compute the partial $f_{o,p}$ and $f_{e,p}$
 $f_{o,1} = 0.593 + 0.009 + 0.009 = 0.612$, $f_{e,1} = 0.037 \cdot 0.148 + 0.037 \cdot 0.148 + 0.009 \cdot$
 $0.009 = 0.011$, $f_{o,2} = 0.333$, $f_{e,2} = 0.037$, $f_{o,3} = 0.241$, $f_{e,3} = 0.009$, $f_{o,4} = 0.322$

330 and $f_{e,4} = 0.032$. The expected probability $p_{e\tau} = \frac{0.011+0.037+0.009+0.032}{4} =$
 0.022 and observed probability $p_{o\tau} = \frac{0.612+0.333+0.241+0.322}{4} = 0.377$ are then
 computed. Finally, fuzzy kappa or the soft consensus degree is obtained $\kappa_\tau =$
 $\frac{0.377-0.022}{1-0.022} = 0.363$.

3.5. Extending soft consensus degree to several decision-makers

335 In this subsection, a extension of the soft consensus degree from two to sev-
 eral decision-makers who each assign a set of alternatives into several categories
 by means of a fuzzy partition is proposed. Herrera-Viedma et al. (2014) identify
 two approaches to measure agreement for three or more decision-makers accord-
 ing to reference domain (i.e. decision-makers or alternatives). The first type
 340 is based on a fuzzy partition per decision-maker (Kacprzyk and Fedrizzi, 1988;
 Carlsson et al., 1992), while the second type is based on a fuzzy partition per
 alternative (Alonso et al., 2013; Cabrerizo et al., 2010; Palomares et al., 2014).
 This approach can be applied to our proposed two-decision-maker soft consen-
 sus degree (as shown below) since the index introduced in (2) takes into account
 345 the different distribution of each category that Cohen's kappa normalises (Co-
 hen, 1968). Thus, we consider a decision-maker-based approach to extend the
 considered consensus degree described in three steps:

Step 1: Soft consensus degree between a pair of decision-makers, i and k :

$$cde_{ik} = \kappa_\tau.$$

350 **Step 2:** Average of consensus degrees for each decision-maker: $ca_i = \sum_{k=1, k \neq i}^R \frac{cde_{ik}}{R-1}$,
 where R stands for the number of decision-makers.

Step 3: Aggregated soft consensus degree for the group of decision-makers:

$$cgd = \sum_{i=1}^R \frac{ca_i}{R}.$$

355 Most real-life situations measuring consensus are based on the preferences
 of several decision-makers, instead of two decision-makers' preferences. This
 extension allows us to measure consensus degree by taking into account the
 different categories' occurrences.

4. Use Case in Innovation Contest

4.1. Problem definition

360 Several online communities in five topics created a set of ideas in an innovation contest. Among these ideas, innovative ideas or categories were identified from each innovation contest by chief innovation officers. From now on, the initially posted ideas are named as posts or alternatives, and innovative ideas are ideas or categories. The R&D managers or decision-makers then classified
365 the community posts based on decision-maker preferences and the confidence degrees using our proposed methodology for two decision-makers. This process is depicted in Figure 3.

Figure 3: Use case definition

4.2. Data

Data was obtained from an innovation contest operated by the United States
370 Agency for International Development (USAID), a U.S. federal agency for funding developing countries. USAID offered to the development public the opportunity to suggest development programme ideas using a 3-day innovation contest event. There were several different topics that people could focus on, including, for example, empowering women. This event was promoted by partnering
375 with several organisations throughout the world, and 1279 individuals from 135 countries participated, generating 5268 posts across five topics.

A real-life experiment, in which the posts from five topics were categorised using the proposed methodology based on one confidence degree per post, was conducted. A jury of staff members from the USAID strategy unit synthesised

the novel ideas after reading all the posts in the challenges (Ferguson, 2010). Two students per topic were then hired by the research team, trained to become decision-makers, and tasked with categorising each post into ideas, while assessing how confident they were in their decision in a scale ranging from zero to six. The first 100 posts (i.e the first batch) were used as decision-maker training and the decision-makers reached consensus after independently categorising and defining the confidence degree of each post. This set of decision-makers then independently read, categorised, and assessed their confidence degree for the second batch (i.e. a set of posts in a topic), with disagreements discussed using the confidence degree to grasp the other decision-maker’s uncertainty. The process was repeated for the next batch until no batches were left to categorise and define. Table 6 provides information on how many batches, posts, and the average number of words.

Table 6: Statistics of the posts and contests

Contest	# Posts per contest	Ideas	Batch	# Posts per batch	# Words per post
Empowering Women and Girls	1618	7	1	100	106
			2	200	134
			3	300	142
			4	1018	120
Pursuing Grand Challenges	591	4	1	100	215
			2	200	160
			3	291	137
Fostering Science, Technology and Innovation	768	6	1	100	119
			2	200	112
			3	468	115
Inspiring a New Generation	1515	6	1	100	136
			2	200	122
			3	300	150
			4	915	126
Building Stronger Partnerships	776	6	1	100	176
			2	200	162
			3	476	142

4.3. Results

Each decision-maker assessed the idea and level of confidence for each post
 395 for a topic. Three different ways of measuring agreement were computed to
 assess the effect of possible decisions on the fuzzification process using product
 t-norm for the fuzzy kappa (see (3)). Moreover, crisp kappa was calculated in
 order to have a threshold measure. Table 7 details the results obtained.

Table 7: Consensus degrees using the product t-norm

Contest	Batch	Crisp	Linear	Logarithmic	Exponential
Building stronger partnerships	2	0.32	0.67	0.74	0.51
	3	0.26	0.79	0.82	0.68
Empowering women and girls	2	0.46	0.74	0.84	0.34
	3	0.45	0.75	0.85	0.37
	4	0.80	0.79	0.87	0.51
Fostering, science, technology, and science	2	0.23	0.82	0.86	0.67
	3	0.46	0.82	0.86	0.66
Pursuing major challenges	2	0.42	0.79	0.85	0.56
	3	0.39	0.73	0.80	0.48

Figure 4: Agreement measurement by type of kappa and innovation contest using probabilistic product as the t-norm

Figure 4 shows that, despite using product t-norm, kappa's estimation order
 400 from each different fuzzification normalisation function (i.e. linear, exponen-

tial, and logarithmic) is as expected. Two overconfident decision-makers show more agreement than an average underconfident decision-maker when using the same confidence degrees. An exponential fuzzification is more accurate than the traditional crisp kappa and other considered fuzzy kappa definitions (i.e. logarithmic and linear).

Consensus is clearly improving over time in all but the smaller contests (i.e. “Pursuing major challenges”). This indicates that the level of agreement improved over time as long as the data set is large enough (> 600 posts). The use of confidence degrees improved the quality and efficiency of the experts’ categorisation by decreasing time for each consecutive batch (as reported one expert). A possible explanation for this decrease in time is that experts do not need to be absolutely certain about their decisions since humans have their own subjectivity (Herrera-Viedma et al., 2014). This proposed method enabled us to capture this uncertainty at a reasonable cost (i.e. confidence degree for patterns instead of categories). Moreover, this confidence degree was offered as a feedback mechanism for those patterns where agreement was not reached, and so providing additional support for the discussion.

5. Discussion

This paper proposes a feasible method to consider the fuzzy nature of decisions when categorising a set of alternatives by means of confidence degrees. The method enables the use of different fuzzyfication functions - three of which are depicted in Table 3 - and the use of different t-norms described in Section 2.2. The selection of these elements has several implications that are discussed. Linear, exponential, and logarithmic functions are considered when associated respectively with normal, underconfident, and overconfident decision-makers. Overconfident decision-makers, whose subjective confidence in their judgement is reliably greater than objective judgement (Pallier et al., 2002), are compensated by using a logarithmic normalisation function during the fuzzification process. Underconfident people, whose subjective confidence in their judgement is

430 reliably lower than the objective judgement, are compensated by using an exponential normalisation function. Finally, people with normal confidence, whose subjective confidence is close to the objective function, are not compensated using a linear normalisation function.

The selection of the t-norm to obtain the shared membership degrees (i.e. coincidence matrix) is relevant. A minimum t-norm, that represents the standard semantics for weak conjunction logic $T(x, y) = \min(x, y)$, overemphasises total agreements among the two decision-makers, and thus reduces the relevance of disagreements. A product t-norm (i.e. $T(x, y) = x \cdot y$) penalises low partial agreement, while boosting high partial agreement. A Lukasiewicz t-norm, that 440 is pointwise smaller than the product t-norm $T(x, y) = \max\{x + y - 1, 0\}$, overpenalises low partial agreement while overboosting high partial agreement.

Several measures exist to assess the level of agreement, ranging from simple ratio agreements to measures that take into account the distribution of each category (Cohen, 1968). The consensus degree defined in this paper introduces 445 the use of a measure that takes into account the distribution of each category that is fuzzy by nature (Dou et al., 2007). Table 8 compares the typical consensus forms of measurement in three dimensions: 1) implementation complexity; 2) assumption of the distribution of categories; and 3) certainty of the decision-maker at each decision. The ratio measurement considers that all categories are equally likely, hence introducing distortion in the measurement. The kappa 450 improves ratio measurement by considering that each category is not equally likely to happen. Finally, fuzzy kappa considers that every idea or category is not equally likely to happen and that the decision-makers are not always certain about their decisions.

455 6. Conclusions and Future Research

The focus of this research was to develop a fuzzy measure of agreement through fuzzy kappa based on fuzzy partitions that consider the fuzzy nature of GDM using one confidence degree for each decision (instead of for each cat-

Table 8: Comparison of consensus degrees

Measure	Formula	Complexity	Distribution of categories	Certainty decision
Ratio	P_o^{ik}	Simple	Equally likely	Constant
kappa	$\kappa = \frac{p_o - p_e}{1 - p_e}$	Medium	Some categories are more likely than others	Constant
Fuzzy kappa	$\kappa_\tau = \frac{p_{o\tau} - p_{e\tau}}{1 - p_{e\tau}}$	Advanced	Some categories are more likely than others	Not constant

egory). This confidence degree is used to define a fuzzy partition based on
460 decision-maker preferences. A coincidence matrix is computed by two or more
fuzzy partitions using a specific t-norm. Finally, the consensus degree is ob-
tained from this coincidence matrix.

A real use case in an innovation contest is considered. The results suggest
that this methodology captures the degree of consensus more accurately, with-
465 out much additional effort. Moreover, the fuzzification of the decision allows
us to model and normalise different types of confidence (i.e. overconfidence,
average confidence and underconfidence). Finally, the assessment of the differ-
ent conceptualisations of t-norm enables us to consider less strict coincidence
models (i.e. probabilistic product t-norm) or stricter models (i.e. min t-norm).

470 The limitations of this research are related to the constraints imposed on
using real-life experiments to assess different parametrisation (i.e. fuzzification
parametrisation and different types of t-norm for the fuzzy kappa). Thus, future
research should estimate acceptable and optimal fuzzy consensus degrees for the
different parametrisations of the kappa index.

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Graphical Abstract