A method for large group decision-making based on evaluation information provided by participators from multiple groups

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Abstract

Large group decision-making (LGDM) is a special group decision-making (GDM) problem, in which a large number of persons take part in decision process, while research concerning this issue is still relative scarce. The objective of this paper is to develop a method to solve the LGDM problem, in which a large number of persons from multiple groups take part in the decision process and express their personal evaluations on the alternatives according to the pre-established identifier set. In the method, the percentage distribution on evaluations of each group concerning each alternative is determined. The decision weight of each group concerning each alternative is obtained by aggregating the subjective weight, which is provided by the organizer, and the objective weight, determined according to the level of consensus among participators' evaluations. According to the percentage distributions and decision weights, the dominance degrees on pairwise comparisons of alternatives are calculated, and a ranking of alternatives can be determined using the PROMETHEE II method. Finally, an example is given to illustrate the use of the proposed method.

Keywords: Large group decision-making (LGDM); Percentage distribution; Weight; Ranking

1. Introduction

Large group decision-making (LGDM) is a very common human activity, which refers to the selection of the best option from a set of feasible alternatives according to the opinions of a large number of persons. In reality, since decision-making results for some problems relate

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to the benefits of a large number of stakeholders, it is necessary for a large number of persons from multiple groups of stakeholders to take part in the decision process [1-5]. For example, a large number of teachers from different schools in a university are allowed to take part in the decision process of reforming the promotion system, a large number of citizens from different professional groups are invited to provide their opinions on the adjustment of subway fares and so on. Thus, the LGDM with the participators from multiple groups is a special group decision-making (GDM) and is different from the traditional GDM that fewer persons take part in the decision process. The distinct characteristics of the LGDM are a large number of participators, multiple groups and a mass of decision data provided by the participators. Therefore, how to solve the LGDM problem is a valuable research topic.

Till now, studies on LGDM are still relatively scarce. Only limited literatures that directly or indirectly relate to LGDM can be found. In most of the existing literatures, the participators are first regarded as independent individuals. Then, for the convenience of processing the huge amount of evaluations or preferences provided by participators, the large group of participators is classified into multiple groups according to participators' evaluations or preferences. Further, based on the results of clustering analysis, two kinds of analyses are conducted. The one is to improve the consensus of participators by identifying and managing the noncooperative behaviors of participators, i.e., consensus reaching process (CRP) in LGDM. The other is to determine the ranking of alternatives by aggregating the opinions of multiple groups, i.e., the LGDM methods. Simultaneously, several studies on LGDM support systems can be found. Thus, the related literatures can be grouped into four categories, i.e., cluster methods in LGDM [6-9], CRP in LGDM [10, 11], LGDM methods [12-16] and LGDM support systems [17-20]. A detailed review of the related work is given in Section 2.

Prior studies have made significant contributions to LGDM analysis. But, it is necessary to point out that the participators would not be regarded as independent individuals in some practical LGDM. This is because, in some practical LGDM, the organizer or decision-maker would consider using stratified sampling to let a large number of persons from multiple groups take part in the decision process [21, 22] since the decision results relate to the benefits of the multiple groups of persons. In this situation, the participators would not be regarded as independent individuals since the evaluations or preferences provided by participators from a group represent the benefits of this group. Thus, the decision analysis should be conducted based on the evaluations or preferences provided by participators from each group. Therefore, it is necessary to develop a new method for solving the LGDM problems with participators from multiple groups.

The objective of this paper is to develop a new method for LGDM based on evaluation information provided by participators from multiple groups. In the method, a large number of participators are asked to provide their evaluations on alternatives using pre-established different identifiers, thus the form of percentage distributions is used to describe the evaluation of each group concerning each alternative. The idea of the method reflects as the following four parts:

- 1) According to the evaluations provided by participators from each group, the percentage distribution on evaluations of each group concerning each alternative is determined;
- 2) The decision weight of each group concerning each alternative is determined by aggregating the subjective weight and the objective weight, where the subjective weight is provided by the organizer and the objective weight is determined according to the consensus of evaluations provided by participators from each group;
- 3) A collective percentage distribution on evaluations concerning each alternative is calculated by aggregating the decision weights and percentage distributions of all the groups;
- 4) To compare any two alternatives using evaluation information in the form of percentage distributions, the dominance degree of one alternative over the other is calculated. On this basis, a dominance degree matrix on pairwise comparisons of alternatives can be constructed. According to dominance degree matrix, a ranking of alternatives is determined using the PROMETHEE II method.

The rest of this paper is arranged as follows. Section 2 provides an overview of the related work on LGDM. Section 3 describes the LGDM problem with participators from multiple groups. Section 4 gives a framework for solving the LGDM problem and presents a LGDM method according to the framework. In Section 5, an example is given to illustrate the use of the proposed method. Finally, Section 6 summarizes and highlights the major contributions of this paper.

2. Related work on LGDM

Till now, studies on LGDM are still relatively scarce. The limited existing studies on LGDM can be classified into four categories, i.e., cluster methods in LGDM, CRP in LGDM, LGDM methods, and LGDM support systems. In the following, a brief literature review will be given with respect to the four categories.

- 1) *Cluster methods in LGDM*. For the convenience of processing the huge amount of

evaluations or preferences provided by participators, the participators in LGDM are usually classified into several groups according to their evaluations or preferences. Thus, several specific cluster methods for classifying participators in LGDM have been developed. For example, Zahir [6] studied the LGDM problem, in which each participator expresses his/her opinion using a preference vector of an extended AHP. To guarantee the precondition for applying the AHP that the individuals in a group should be homogeneous, Zahir [6] proposed a cluster method to classify the participators in LGDM into several homogeneous groups. In the method, a convenient similarity measure between two vectors is first defined. Then, a cluster algorithm is developed based on the similarity measure. On the basis of Zahir's study, Bolloju [7] proposed a method to identify homogeneous groups of participators based on similarities in preferences. In the method, utility function of each participator is estimated according to the AHP model, and unified utility functions are produced through content analysis. Then, based on similarities in preferences, the unified utility functions are classified into several subgroups. Therefore, the utility function of each subgroup can be estimated by aggregating the utility functions of participators in the subgroup. Liu et al. [8] studied a LGDM problem, in which each participator expresses his/her evaluation on alternative concerning each attribute by an interval-valued intuitionistic fuzzy number (IVIFN). To classify the participators into several subgroups, a classification model was proposed. First, the IVIFNs are transformed into single-valued numbers by continuous interval data ordered weighted averaging (C-OWA) operator and basic unit-interval monotonic function. Then, the partial binary tree DEA-DA cyclic classification model is adopted to classify the participators. Tapia-Rosero et al. [9] studied a LGDM problem, in which the opinion provided by each participator is represented by a membership function. Tapia-Rosero et al. supposed that similarly shaped membership functions reflect similar participator opinions. Thus, for detecting similar opinions in LGDM, Tapia-Rosero et al. [9] proposed a cluster method for grouping the membership functions. In the method, to reduce the complexity of handling membership functions, symbolic notations are used to depict each membership function. Then, a shape similarity measure of two membership functions is defined using the symbolic notations. Further, based on the shape similarity measure, similar-shaped membership functions are grouped by agglomerative hierarchical clustering technique.

2) *CRP in LGDM*. Usually, the result of group decision problem obtained by selection process would not be accepted by all participators in the group since some participators might consider that their opinions have not been sufficiently considered [23, 24]. Thus, to achieve a solution accepted by the whole group, consensus reaching process (CRP) has been paid great attention as part of the decision process. CRP is regarded as an iterative procedure aimed at seeking a high degree of agreement amongst participators' preferences before making a group decision [23, 24]. In the last few years, the studies on CRP in LGDM have attracted the attention of some scholars. For example, Palomares et al. [10] proposed a consensus model to detect and manage noncooperative behaviors in LGDM. In their study, a fuzzy clustering-based scheme is used to facilitate the detection of noncooperating individuals or subgroups. And, based on self-organizing maps, a visual analysis tool of the overall CRP is given, which facilitates the monitoring of the process performance across the time. On the basis of Palomares et al.'s study, Quesada et al. [11] proposed a method for managing participators' behavior in CRP in LGDM. In the method, a weighting approach incorporating uninorm aggregation operator is given which assigns different importance weights to participators according to their overall behavior across the CRP. Quesada et al.'s method [11] overcomes the shortage of Palomares et al.'s method [10] that the participators' weights cannot be increased again, even though they change their mind and decide to adopt a more cooperating attitude.

3) *LGDM methods*. In addition, some scholars have proposed several LGDM methods, i.e., the methods for ranking alternatives or selecting the most desirable alternative according to the opinions of a large number of participators. In these studies, the large group of participators is classified into several homogeneous groups according to participators' preferences. Further, based on the results of clustering analysis, the ranking of alternatives is determined by aggregating the preferences of multiple groups. For example, Srdjevic [12] proposed a method for LGDM to solve the decision problem in water management. In the method, participators are classified into different subgroups according to their professions. Then, AHP is used to obtain the ranking of alternatives concerning each group. Further, the social choice voting procedure is used to determine the final ranking of alternatives by

aggregating the rankings of alternatives of all groups. Xu et al. [13] suggested a method for LGDM. In the method, weights of participators are determined by clustering analysis, and then utility of each alternative is determined by aggregating the weights and preferences of participators. Alonso et al. [14] proposed a fuzzy GDM model for large groups of individuals. In their study, a simple clustering technique and a kind of trust network are used to simplify the large group of participators into a smaller group with several spokespersons, and the induced ordered weighted averaging (IOWA) operator is used to aggregate the preferences of spokespersons and to determine a ranking of alternatives. Liu et al. [15] proposed a LGDM method with IVIFNs. In the method, IVIFN samples are first transformed into single-valued samples, and the values and weights of “latent variables for decision-making” are estimated by the partial least squares path modeling. Further, the ranking of alternatives is determined by combining the estimated values and weight vector. Also, Liu et al. [16] proposed a method for determining the decision weights of experts in LGDM analysis. In the method, a minimized variance model is constructed to determine the weight of each expert in the subgroup and an entropy weight model is given to determine the decision weight of each subgroup. The final decision weight of each expert is determined by aggregate the above two types of weights. Then, the ranking of alternatives can be determined by aggregating experts’ evaluations and final decision weights.

4) *LGDM support systems*. The participators of a LGDM may attend the decision process and express their personal opinions at different time and different place. An effective information system or decision support system is necessary to support the participations of participators. Thus, the studies on LGDM support system have attracted the attention of some scholars and several literatures on LGDM support system can be found. For example, Turoff et al. [17] early proposed the concept of social decision support system (SDSS), which is regarded as a LGDM support system in social choice process. In their study, the structure of the SDSS is discussed, the system requirement is analyzed, and an example process model for the SDSS is given. On the basis of Turoff et al.’s study [17], Rodriguez [18] proposed a theoretical model for designing problem-solving systems for societal-scale collectives. In the study, a theoretical framework for designing human collective-intelligence system is first

given, and simulation technology is used to examine the effectiveness of the proposed system under different structures of social-networks. On this basis, Rodriguez [19] extended the theoretical model to the environment with multi-relational networks and grammar-based particle swarms. Carvalho et al. [20] developed a system to support decisions in large-scale groups. The users can log in the system as one of the three roles, i.e., creator, participator or moderator. If the user logs in the system as a creator, then he/she can formulate a new problem; if the user logs in as a participator, then he/she can provide personal opinions on the problem provided by creators; if the user logs in as a moderator, then he/she can receive and summarize the opinions given by participators. To facilitate the LGDM, the system also allows users to define the decision rules.

The existing studies have made significant contributions to LGDM analysis. In most of the existing studies, the participators are regarded as independent individuals. However, in some practical LGDM problems, the participators would not be regarded as independent individuals since the evaluations or preferences provided by participators from a group represent the benefits of this group. Although Srdjevic's [12] method addressed the problem that persons from multiple groups take part in the LGDM processes, maybe it is difficult to solve some practical LGDM problems using Srdjevic's method. This is because, in Srdjevic's method, each participator is asked to express his/her preference information about alternatives using a fuzzy preference relation. But, it is difficult to maintain consistency of the fuzzy preference relation if the decision problem is complicated and the participators lack knowledge, or many alternatives need to be compared [25]. If the number of participators is large, it may be tedious and time-consuming to check the consistency of each fuzzy preference relation and repair those inconsistency ones [25]. Therefore, it is necessary to develop a new method for solving the LGDM problems with participators from multiple groups.

3. Description of the problem

The problem concerned in this paper is how to select the desirable alternative(s) from a set of feasible alternatives according to the evaluations provided by a large number of participators from multiple groups. The LGDM problem is shown in Fig. 1. The following notations are used to denote the sets and variables in the LGDM problem, which are used throughout this paper.

- $A = \{A_1, A_2, \dots, A_n\}$: the set of n alternatives, where A_j denotes the j th alternative, $j = 1, 2, \dots, n$.
- $G = \{G_1, G_2, \dots, G_m\}$: the set of m groups, where G_i denotes the i th group that takes part in the decision process, $i = 1, 2, \dots, m$.
- $Q = (q_1, q_2, \dots, q_m)$: the vector of numbers of participators with respect to set G , where q_i denotes the number of participators from group G_i and is a great number, $i = 1, 2, \dots, m$. Usually, Q is determined according to the framework of stratified sampling.

Fig. 1. LGDM with participators from multiple groups

- $S = \{S^1, S^2, \dots, S^t\}$: the set of t evaluation identifiers, where S^l denotes the l th evaluation identifier, $l = 1, 2, \dots, t$. Generally, set S is total ordered, i.e., $S^l \succ S^{l'}$ for $l > l'$, $l, l' \in \{1, 2, \dots, t\}$. In this paper, set S may be a set of scores or linguistic terms. For example, if set S is a score set, then it may be $S = \{1, 2, 3, 4, 5\}$; if S is a linguistic term set, then it may be $S = \{\text{Very Poor}, \text{Poor}, \text{Middle}, \text{Good}, \text{Very Good}\}$ [26-29].

- $D_i = [d_{ij}^h]_{n \times q_i}$: the evaluation matrix provided by participators from group G_i , where d_{ij}^h

denotes the evaluation on alternative A_j provided by the h th participator from group G_i by selecting one identifier from set S , $d_{ij}^h \in S$, $i=1,2,\dots,m$, $j=1,2,\dots,n$, $h=1,2,\dots,q_i$. Here, for the convenience of analysis, if set S is the linguistic term set, then the linguistic terms provided by participators will be transformed into the form of scores. For example, linguistic terms "Very Poor", "Poor", "Middle", "Good", and "Very Good" will be transformed into the corresponding scores "1", "2", "3", "4", and "5", respectively.

The problem concerned in this paper is how to rank alternatives or select the most desirable alternative(s) among the set A .

4. The method for LGDM

In this section, we give a method for LGDM with participators from multiple groups. First, a framework of the proposed method is given. Then, according to the framework, the method is described. The method is consisted of the four parts: 1) determine the percentage distributions on evaluations of each group concerning different alternatives; 2) determine the decision weight of each group concerning each alternative; 3) construct the collective percentage distribution on evaluations concerning each alternative; and 4) rank alternatives or select the most desirable alternative(s).

4.1. Framework overview

To solve the problem mentioned above, a method for LGDM is proposed. The framework of the method is shown in Fig. 2, where m groups and n alternatives are considered. It can be seen from Fig. 2 that the method includes four parts. Brief description of each part is given below.

1) *Determine the percentage distributions on evaluations of each group concerning different alternatives.* According to the evaluations provided by participators from each group concerning each alternative, the number of participators who use each identifier to evaluate the alternative is counted. Thus, the distribution of numbers of participators with respect to the all identifiers can be obtained, i.e., the percentage distribution on evaluations of each

group concerning each alternative can be formed.

Fig. 2. The framework of the LGDM method proposed in this paper

2) *Determine the decision weight of each group concerning each alternative.* The subjective weights of multiple group are first determined by the assignment of the organizer or the existing procedures, such as AHP [30]; then the consensus of participators from each group concerning each alternative is measured by calculating the variance of the obtained percentage distribution, and the objective weight of each group concerning each alternative is determined by normalizing the obtained consensus; further, the decision weight of each group concerning each alternative can be determined by aggregating the subjective weight and the objective weight.

3) *Construct the collective percentage distribution on evaluations concerning each alternative.* A collective percentage distribution on evaluations concerning each alternative is determined by aggregating the percentage distributions and the decision weights of all groups.

4) *Rank alternatives or select the most desirable alternative(s).* The dominance degrees on pairwise comparisons of alternatives are calculated according to the obtained collective percentage distributions, and then a ranking of alternatives is determined using the PROMETHEE II method.

4.2. Method

According to the framework proposed in Section 4.1, a detailed description of the proposed LGDM method will be given below.

4.2.1. Determine the percentage distributions on evaluations of each group concerning different alternatives

First, according to the evaluation information provided by participators from group G_i , i.e., evaluation matrix $D_i = [d_{ij}^h]_{n \times q_i}$, $i = 1, 2, \dots, m$, an indication matrix $I_i = [I_{ij}^h]_{n \times q_i}$ can be constructed, where I_{ij}^h is a vector, i.e., $I_{ij}^h = (I_{ij}^{h1}, I_{ij}^{h2}, \dots, I_{ij}^{ht})$. The element I_{ij}^{hl} in vector I_{ij}^h is a 0-1 variable, i.e., $I_{ij}^{hl} = 1$ if $d_{ij}^h = S^l$; $I_{ij}^{hl} = 0$ otherwise, $j = 1, 2, \dots, n$, $h = 1, 2, \dots, q_i$, $l = 1, 2, \dots, t$.

Then, let q_{ij}^l denote the number of participators from group G_i who use evaluation identifier S^l to express the evaluation on alternative A_j . According to the indication matrix $I_i = [I_{ij}^h]_{n \times q_i}$, q_{ij}^l can be calculated by

$$q_{ij}^l = \sum_{h=1}^{q_i} I_{ij}^{hl}, \quad i = 1, 2, \dots, m, \quad j = 1, 2, \dots, n, \quad l = 1, 2, \dots, t. \quad (1)$$

Further, let p_{ij}^l denote the percentage of participators from group G_i who use evaluation identifier S^l to express the evaluation on alternative A_j . According to q_{ij}^l , p_{ij}^l can be obtained by

$$p_{ij}^l = q_{ij}^l / q_i, \quad i = 1, 2, \dots, m, \quad j = 1, 2, \dots, n, \quad l = 1, 2, \dots, t. \quad (2)$$

Obviously, $\sum_{l=1}^t p_{ij}^l = 1$, $0 \leq p_{ij}^l \leq 1$, $i = 1, 2, \dots, m$, $j = 1, 2, \dots, n$, $l = 1, 2, \dots, t$.

Thus, according to $p_{ij}^1, p_{ij}^2, \dots, p_{ij}^t$, the percentage distribution on evaluations of group G_i concerning alternative A_j is determined, i.e.,

$$P_{ij}(x) = \begin{cases} p_{ij}^1, & x = S^1, \\ p_{ij}^2, & x = S^2, \\ \vdots & \vdots \\ p_{ij}^t, & x = S^t, \end{cases} \quad i = 1, 2, \dots, m, \quad j = 1, 2, \dots, n.$$

In this paper, for convenience, the percentage distribution is expressed in the form of vectors, i.e., $P_{ij} = (p_{ij}^1, p_{ij}^2, \dots, p_{ij}^t)$, $i = 1, 2, \dots, m$, $j = 1, 2, \dots, n$.

4.2.2. Determine the decision weight of each group concerning each alternative

In LGDM analysis, the evaluations provided by participators from multiple groups and the decision weights of multiple groups need to be combined to form a collective evaluation with respect to each alternative. On the determination of the decision weight of each group, the preference of the organizer concerning multiple groups and the consensus of evaluations provided by participators from each group should be considered. On the one hand, since the participators from multiple groups have different knowledge, experience or benefit demand, organizer may pay different attention to the evaluations provided by participators from multiple groups. Thus, the organizer can assign different weights or important degrees to the evaluations provided by multiple groups of participators according to the need of practical LGDM. These weights can be regarded as subjective weights. On the other hand, since the number of participators from each group is large and there exist differences among the evaluations provided by the participators, the consensus of evaluations provided by the participators from each group needs to be paid attention. The higher consensus will imply the existence of common knowledge or same benefit demand of the participators from a group. Thus, the consensus of evaluations can be regarded as the objective weight of each group, and the objective weight of the group should be greater if the evaluations provided by participators from a group have a higher consensus. Therefore, before integrating the evaluation information of multiple groups and selecting the most desirable alternative(s), it is necessary to determine the decision weight of each group by aggregating the subjective weight and objective weight. In the following, the calculation process of the decision weights of the multiple groups is given.

Let $w^{sub} = (w_1^{sub}, w_2^{sub}, \dots, w_m^{sub})$ denote the vector of subjective weights of groups, where

w_i^{sub} denotes the subjective weight of group G_i , such that $\sum_{i=1}^m w_i^{sub} = 1$ and $0 \leq w_i^{sub} \leq 1$,

$i=1,2,\dots,m$. Here, the subjective weights are given by the organizer according to the importance of multiple groups. If the organizer considers the fairness among participators from multiple groups, then the decision weight of group G_i can be determined according to

the number of participators, i.e., $w_i^{sub} = q_i / \sum_{k=1}^m q_k$, $i=1,2,\dots,m$. Particularly, if the organizer

has not enough confidence to directly assign the subjective weights of multiple groups, then he/she can invite some related experts to provide their preferences on the importance of multiple groups by the existing procedures, such as AHP [30], and the subjective weights can be determined by aggregating experts' preferences.

Further, the objective weights of the multiple groups can be determined according to the consensus analysis of evaluations provided by participators from each group. Here, for the convenience of calculation, let $B = \{B^1, B^2, \dots, B^t\}$ be a score set corresponding to $S = \{S^1, S^2, \dots, S^t\}$. If S is a score set, then we know $B^l = S^l$, $l=1,2,\dots,t$; if S is a linguistic term set, then B^l denotes the transformed score with regard to linguistic term S^l , $l=1,2,\dots,t$. Accordingly, the percentage distribution $P_{ij} = (p_{ij}^1, p_{ij}^2, \dots, p_{ij}^t)$ with regard to set $S = \{S^1, S^2, \dots, S^t\}$ is transformed into the one with regard to set $B = \{B^1, B^2, \dots, B^t\}$, i.e.,

$$P_{ij}(x) = \begin{cases} p_{ij}^1, & x = B^1, \\ p_{ij}^2, & x = B^2, \\ \vdots & \vdots \\ p_{ij}^t, & x = B^t, \end{cases} \quad i=1,2,\dots,m, \quad j=1,2,\dots,n.$$

Let $Var(P_{ij})$ denote the variance of percentage distribution $P_{ij} = (p_{ij}^1, p_{ij}^2, \dots, p_{ij}^t)$, then $Var(P_{ij})$ can be calculated by

$$Var(P_{ij}) = \sum_{l=1}^t p_{ij}^l (B^l - \sum_{k=1}^t p_{ij}^k B^k)^2, \quad i=1,2,\dots,m, \quad j=1,2,\dots,n. \quad (3)$$

Let $Var(P^*)$ denote the theoretical greatest variance among all possible percentage

distributions with regard to the score set $B = \{B^1, B^2, \dots, B^t\}$, where P^* is a special percentage distribution, i.e.,

$$P^*(x) = \begin{cases} 0.5, & x = B^1, \\ 0, & x = B^2, \\ \vdots & \vdots \\ 0, & x = B^{t-1}, \\ 0.5, & x = B^t. \end{cases}$$

Then, $Var(P^*)$ can be calculated by

$$Var(P^*) = 0.5[B^1 - (0.5B^1 + 0.5B^t)]^2 + 0.5[B^t - (0.5B^1 + 0.5B^t)]^2. \quad (4)$$

Let CI_{ij} denote the consensus of evaluations provided by participators from group G_i concerning alternative A_j , then CI_{ij} is given by

$$CI_{ij} = 1 - \sqrt{\frac{Var(P_{ij})}{Var(P^*)}}, \quad i=1,2,\dots,m, j=1,2,\dots,n. \quad (5)$$

Clearly, by Eqs. (3)-(5), we have the following properties:

- 1) $0 \leq CI_{ij} \leq 1$;
- 2) $CI_{ij} = 1$ if and only if there exist $p_{ij}^g = 1$ and $\sum_{l=1}^t p_{ij}^l = 1$, where $g \in \{1, 2, \dots, t\}$;
- 3) $CI_{ij} = 0$ if and only if $P_{ij} = P^*$.

Usually, the greater the consensus CI_{ij} is, the greater the important degree of evaluations of group G_i will be, since the greater CI_{ij} implies that participators from group G_i have common knowledge or the same benefit demand with respect to alternative A_j . Thus, the objective weight of group G_i concerning alternative A_j can be determined according to the consensus CI_{ij} . If let w_{ij}^{ob} denote the objective weight of group G_i concerning alternative A_j , then w_{ij}^{ob} can be obtained by the following Eq. (6).

$$w_{ij}^{ob} = \begin{cases} CI_{ij} / \sum_{k=1}^m CI_{kj} & \text{if } \sum_{k=1}^m CI_{kj} > 0 \\ w_i^{sub} & \text{if } \sum_{k=1}^m CI_{kj} = 0 \end{cases}, \quad i=1,2,\dots,m, j=1,2,\dots,n. \quad (6)$$

Clearly, $\sum_{i=1}^m w_{ij}^{ob} = 1$, $0 \leq w_{ij}^{ob} \leq 1$, $i=1,2,\dots,m$, $j=1,2,\dots,n$. In Eq. (6), if $\sum_{k=1}^m CI_{kj} = 0$, then we let $w_{ij}^{ob} = w_i^{sub}$. This implies that the subjective weights are only considered when determining the final decision weights.

Let w_{ij} denote the decision weight of group G_i concerning alternative A_j , then w_{ij} can be determined by aggregating the subjective weight w_i^{sub} and the objective weight w_{ij}^{ob} , i.e.,

$$w_{ij} = \alpha w_{ij}^{ob} + \beta w_i^{sub}, \quad i=1,2,\dots,m, j=1,2,\dots,n, \quad (7)$$

where α and β are coefficients, $0 \leq \alpha, \beta \leq 1$, $\alpha + \beta = 1$. Particularly, $\alpha = 0$ (or $\beta = 1$) implies that subjective weights are only considered in LGDM analysis, and $\alpha = 1$ (or $\beta = 0$) imply that objective weights are only considered. Apparently, we know $0 \leq w_{ij} \leq 1$, $i=1,2,\dots,m$, $j=1,2,\dots,n$, and $\sum_{i=1}^m w_{ij} = 1$, $j=1,2,\dots,n$. Here, other aggregation operators, such as the weighted geometric averaging operator [31], the t-norm operators [32] and the uninorm operators [33], can be selected to aggregate the subjective and objective weights according to the practical situation.

4.2.3. Construct the collective percentage distributions on evaluations concerning alternatives

To rank alternatives, it is necessary to aggregate the percentage distributions and decision weights of the multiple groups to obtain a collective percentage distribution on evaluations concerning each alternative. Let $P_j = (p_j^1, p_j^2, \dots, p_j^t)$ denote the collective percentage distribution on evaluations concerning alternative A_j , then p_j^l can be obtained by the following Eq. (8), i.e.,

$$p_j^l = \sum_{i=1}^m w_{ij} p_{ij}^l = w_{1j} p_{1j}^l + w_{2j} p_{2j}^l + \dots + w_{mj} p_{mj}^l, \quad j=1,2,\dots,n, \quad l=1,2,\dots,t. \quad (8)$$

For the collective percentage distribution $P_j = (p_j^1, p_j^2, \dots, p_j^t)$, we have the following theorems.

Theorem 1. $\sum_{l=1}^t p_j^l = 1$.

The proof of [Theorem 1](#) is provided in [Appendix A](#).

Theorem 2. If subjective weights are only considered in LGDM analysis (i.e., $\alpha = 0$) and are

given by $w_i^{sub} = q_i / \sum_{k=1}^m q_k$, $i=1,2,\dots,m$, then we have $p_j^l = \sum_{i=1}^m q_{ij}^l / \sum_{i=1}^m q_i$.

The proof of [Theorem 2](#) is provided in [Appendix B](#).

4.2.4. Rank alternatives or select the most desirable alternative(s)

First, the dominance degrees on pairwise comparisons of alternatives are calculated according to the obtained collective percentage distributions. Consider arbitrary two alternatives A_j and A_k . The collective percentage distributions on evaluations concerning alternatives A_j and A_k are $P_j = (p_j^1, p_j^2, \dots, p_j^t)$ and $P_k = (p_k^1, p_k^2, \dots, p_k^t)$, respectively. Let r_{jk} denote the dominance degree of $P_j = (p_j^1, p_j^2, \dots, p_j^t)$ over $P_k = (p_k^1, p_k^2, \dots, p_k^t)$, i.e., the dominance degree of A_j over A_k . According to the study of Liu et al. [\[34\]](#), r_{jk} can be calculated by

$$r_{jk} = \sum_{l=1}^t \sum_{h=1}^l p_j^l p_k^h - 0.5 \sum_{l=1}^t p_j^l p_k^l, \quad j, k=1,2,\dots,n, \quad (9)$$

and accordingly, the dominance degree of A_k over A_j , i.e., r_{kj} , can be calculated by

$$r_{kj} = \sum_{l=1}^t \sum_{h=l}^t p_j^l p_k^h - 0.5 \sum_{l=1}^t p_j^l p_k^l, \quad j, k=1,2,\dots,n. \quad (10)$$

According to the r_{jk} and r_{kj} , $j, k=1,2,\dots,n$, a dominance degree matrix on pairwise comparisons of alternatives can be constructed, i.e.,

$$R = [r_{jk}]_{n \times n} = \begin{matrix} & \begin{matrix} A_1 & A_2 & \cdots & A_n \end{matrix} \\ \begin{matrix} A_1 \\ A_2 \\ \vdots \\ A_n \end{matrix} & \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ r_{21} & r_{22} & \cdots & r_{2n} \\ \vdots & \vdots & & \vdots \\ r_{n1} & r_{n2} & \cdots & r_{nn} \end{bmatrix} \end{matrix}.$$

Clearly, in matrix $R = [r_{jk}]_{n \times n}$, $r_{jk} + r_{kj} = 1$, $r_{jj} = 0.5$ and $0 \leq r_{jk}, r_{kj} \leq 1$, $j, k = 1, 2, \dots, n$. In fact, the dominance degree matrix $R = [r_{jk}]_{n \times n}$ is also in the form of fuzzy preference relations [25, 35].

PROMETHEE II was developed by Brans in 1982 [36]. It is regarded as one of the most well-known outranking methods and has been widely applied to solve practical decision making problems [37]. PROMETHEE II includes the five parts [37-40]: 1) determine deviations based on pairwise comparisons of alternatives concerning each criterion; 2) construct a relevant preference function for each criterion; 3) calculate global preference index; 4) calculate positive and negative outranking flows for each alternative; and 5) calculate the net outranking flow of each alternative and determine a complete ranking of alternatives. Compared with other outranking methods, PROMETHEE II is a simple ranking method in conception and application [41]. In this paper, since the information on pairwise comparisons of alternatives, i.e., the dominance degree matrix $R = [r_{jk}]_{n \times n}$, is similar to the one used in PROMETHEE II, PROMETHEE II can be used to rank alternatives or select the most desirable alternative(s).

Let $\Phi^+(A_j)$ be the dominance degree (or positive preference flow) which is a measure that alternative A_j is dominating the other alternatives, and $\Phi^-(A_j)$ be the non-dominance degree (or negative preference flow) which is a measure that alternative A_j is dominated by the other alternatives. $\Phi^+(A_j)$ and $\Phi^-(A_j)$ can be respectively calculated by the following formulas:

$$\Phi^+(A_j) = \frac{1}{n} \sum_{k=1}^n r_{jk}, \quad j=1, 2, \dots, n, \quad (11)$$

$$\Phi^-(A_j) = \frac{1}{n} \sum_{k=1}^n r_{kj}, \quad j=1,2,\dots,n. \quad (12)$$

Let $\Phi(A_j)$ be the relative dominance degree (or the net outranking flow) which measures the difference between dominance and non-dominance degrees of alternative A_j . By Eqs. (11) and (12), $\Phi(A_j)$ can be calculated by the following formula:

$$\Phi(A_j) = \Phi^+(A_j) - \Phi^-(A_j), \quad j=1,2,\dots,n. \quad (13)$$

$\Phi(A_j)$ can be regarded as the ranking value of alternative A_j . Obviously, the greater $\Phi(A_j)$ is, the better alternative A_j will be. Therefore, according to the relative dominance degrees $\Phi(A_1), \Phi(A_2), \dots, \Phi(A_n)$, we can determine a ranking of all the alternatives or select the most desirable alternative(s).

In summary, the procedure for the above four parts of the LGDM method is given below.

Step 1. Determine percentage distribution $P_{ij} = (p_{ij}^1, p_{ij}^2, \dots, p_{ij}^t)$ using Eqs. (1) and (2),

$$i=1,2,\dots,m, \quad j=1,2,\dots,n.$$

Step 2. Determine subjective weight w_i^{sub} by the assignment of the organizer or the

existing procedures, $i=1,2,\dots,m$, and calculate objective weight w_{ij}^{ob} using Eqs.

(3)-(6), $i=1,2,\dots,m, \quad j=1,2,\dots,n$. Further, determine decision weight w_{ij}

using Eq. (7), $i=1,2,\dots,m, \quad j=1,2,\dots,n$.

Step 3. Determine collective percentage distribution P_j using Eq. (8), $j=1,2,\dots,n$.

Step 4. Construct dominance degree matrix $R = [r_{jk}]_{n \times n}$ using Eqs. (9) and (10), and

calculate relative dominance degree $\Phi(A_j)$ using Eqs. (11)-(13), $j=1,2,\dots,n$.

Further, determine a ranking of alternatives according to the obtained relative dominance degrees.

5. Illustrative example

In this section, an example on teacher appointment system reformation in a university is used to illustrate the use of the proposed method.

University B is a famous university focusing on finance, economics and management in China. It has 7 schools, i.e., School of Finance (G_1), School of Economics (G_2), School of Accounting (G_3), School of Statistics (G_4), School of International Trade (G_5), School of Public Administration (G_6) and School of Public Finance and Taxation (G_7). The vector of numbers of full-time teachers in the seven schools is $Q = (112, 125, 142, 116, 89, 104, 115)$. To further promote the capabilities on teaching, academic research and social service, University B wants to reform the current teacher appointment system. For this, University B investigated the teacher appointment systems of peer universities from home and abroad, and then the three candidate teacher appointment systems, i.e., the alternatives A_1 , A_2 and A_3 , are designed. To select a desirable alternative, all the teachers are asked to take part in the decision process based on a web-based system. In the system, detailed introduction of the three alternatives is given. Each teacher can login in the system and provide his/her personal preference, i.e., each teacher select one score from the pre-established score set $S = \{1, 2, 3, 4, 5\}$ (1: the worst; 5: the best) to express his/her personal evaluation concerning each alternative. The evaluation matrices provided by teachers from the 7 schools are $D_1 = [d_{1j}^h]_{3 \times 112}$, $D_2 = [d_{2j}^h]_{3 \times 125}$, $D_3 = [d_{3j}^h]_{3 \times 142}$, $D_4 = [d_{4j}^h]_{3 \times 116}$, $D_5 = [d_{5j}^h]_{3 \times 89}$, $D_6 = [d_{6j}^h]_{3 \times 104}$ and $D_7 = [d_{7j}^h]_{3 \times 115}$, where $d_{ij}^h \in \{1, 2, 3, 4, 5\}$, $i = 1, 2, \dots, 7$, $j = 1, 2, 3$, $h = 1, 2, \dots, q_i$. The details of the matrices D_i are omitted, $i = 1, 2, \dots, 7$. To solve the problem, the method proposed in this paper is used. According to the framework shown in Fig. 2 and the procedure given in Section 4, the solution process and computation results are illustrated as follows.

Step 1. According to the evaluation matrices $D_i = [d_{ij}^h]_{3 \times q_i}$, $i = 1, 2, \dots, 7$, the percentage distribution on evaluations of each school concerning each alternative is determined using Eqs. (1) and (2), shown as in Table 1.

Step 2. The subjective weight vector of the schools provided by the organizer of the

decision activity is $w^{sub}=(0.14, 0.16, 0.18, 0.14, 0.11, 0.13, 0.14)$. According to the percentage distributions on evaluations concerning the alternatives, the consensuses of evaluations of each school concerning the three alternatives are calculated using Eqs. (3)-(5), the computation results are shown in Table 2. On this basis, the objective weight of each school concerning each alternative is calculated using Eq. (6), and the computation results are shown in Table 3. Further, the decision weight of each school concerning each alternative is calculated using Eq. (7) (here $\alpha = \beta = 0.5$), the computation results are shown in Table 4.

Table 1.

The percentage distribution on evaluations of each school concerning each alternative

Schools	Percentage distributions		
G_1	$P_{11} = (0.018, 0.045, 0.116, 0.357, 0.464)$	$P_{12} = (0.509, 0.366, 0.062, 0.036, 0.027)$	$P_{13} = (0.036, 0.098, 0.223, 0.429, 0.214)$
G_2	$P_{21} = (0.096, 0.16, 0.24, 0.288, 0.216)$	$P_{22} = (0.176, 0.256, 0.24, 0.2, 0.128)$	$P_{23} = (0.112, 0.256, 0.264, 0.272, 0.096)$
G_3	$P_{31} = (0.007, 0.028, 0.197, 0.282, 0.486)$	$P_{32} = (0.127, 0.155, 0.247, 0.239, 0.232)$	$P_{33} = (0.373, 0.232, 0.113, 0.127, 0.155)$
G_4	$P_{41} = (0.104, 0.224, 0.259, 0.241, 0.172)$	$P_{42} = (0.086, 0.155, 0.31, 0.233, 0.216)$	$P_{43} = (0.397, 0.224, 0.181, 0.129, 0.069)$
G_5	$P_{51} = (0.337, 0.292, 0.18, 0.135, 0.056)$	$P_{52} = (0.079, 0.135, 0.202, 0.247, 0.337)$	$P_{53} = (0.067, 0.146, 0.191, 0.236, 0.34)$
G_6	$P_{61} = (0.346, 0.308, 0.192, 0.096, 0.058)$	$P_{62} = (0.125, 0.144, 0.192, 0.25, 0.289)$	$P_{63} = (0.058, 0.154, 0.211, 0.231, 0.346)$
G_7	$P_{71} = (0.174, 0.33, 0.33, 0.105, 0.061)$	$P_{72} = (0.061, 0.139, 0.191, 0.279, 0.33)$	$P_{73} = (0.07, 0.096, 0.191, 0.226, 0.417)$

Table 2.

The consensuses of evaluations of each school concerning the three alternatives

Schools	Alternatives		
	A_1	A_2	A_3
G_1	0.532	0.534	0.487
G_2	0.375	0.358	0.417
G_3	0.549	0.340	0.262
G_4	0.379	0.391	0.357
G_5	0.391	0.358	0.361
G_6	0.409	0.318	0.373
G_7	0.459	0.383	0.370

Table 3.

The objective weight of each school concerning each alternative

Schools	Alternatives		
	A_1	A_2	A_3
G_1	0.172	0.199	0.185
G_2	0.121	0.133	0.159
G_3	0.177	0.127	0.100
G_4	0.122	0.146	0.136
G_5	0.127	0.133	0.137
G_6	0.132	0.119	0.142
G_7	0.149	0.143	0.141

Table 4.

The decision weight of each school concerning each alternative

Schools	Alternatives		
	A_1	A_2	A_3
G_1	0.156	0.170	0.163
G_2	0.141	0.147	0.159
G_3	0.179	0.153	0.140
G_4	0.131	0.143	0.138
G_5	0.118	0.122	0.124
G_6	0.131	0.124	0.136
G_7	0.144	0.141	0.140

Step 4. Based on percentage distributions P_1 , P_2 , and P_3 , the dominance degree matrix on pairwise comparisons of alternatives can be constructed using Eqs. (9) and (10), i.e.,

$$R = \begin{matrix} & A_1 & A_2 & A_3 \\ \begin{matrix} A_1 \\ A_2 \\ A_3 \end{matrix} & \begin{bmatrix} 0.5 & 0.529 & 0.5 \\ 0.471 & 0.5 & 0.472 \\ 0.5 & 0.528 & 0.5 \end{bmatrix} \end{matrix}.$$

Based on matrix R , dominance degree ($\Phi^+(A_i)$), non-dominance degree ($\Phi^-(A_i)$) and relative dominance degree ($\Phi(A_i)$) of each alternative can be calculated using Eqs. (11)-(13), respectively, and the calculation results are: $\Phi^+(A_1)=0.51$, $\Phi^+(A_2)=0.481$, $\Phi^+(A_3)=0.509$, $\Phi^-(A_1)=0.49$, $\Phi^-(A_2)=0.519$, $\Phi^-(A_3)=0.491$, $\Phi(A_1)=0.02$, $\Phi(A_2)=-0.038$, $\Phi(A_3)=0.018$. According to the obtained relative dominance degrees, a ranking of the alternatives can be determined, i.e., $A_1 \succ A_3 \succ A_2$. Therefore, A_1 is the most desirable teacher appointment system.

In addition, a sensitive analysis is conducted to investigate the influence of coefficients α and β on the ranking of the alternatives, and the computation results are presented in Table 5. It can be seen from Table 5 that the different values of α and β have influence on the ranking of the alternatives.

Table 5.

Rankings of the alternatives with different values of α and β .

α and β	Rankings of alternatives
$\alpha=0, \beta=1$	$A_1 \succ A_3 \succ A_2$
$\alpha=0.1, \beta=0.9$	$A_1 \succ A_3 \succ A_2$
$\alpha=0.2, \beta=0.8$	$A_1 \succ A_3 \succ A_2$
$\alpha=0.3, \beta=0.7$	$A_1 \succ A_3 \succ A_2$
$\alpha=0.4, \beta=0.6$	$A_1 \succ A_3 \succ A_2$
$\alpha=0.5, \beta=0.5$	$A_1 \succ A_3 \succ A_2$
$\alpha=0.6, \beta=0.4$	$A_3 \succ A_1 \succ A_2$
$\alpha=0.7, \beta=0.3$	$A_3 \succ A_1 \succ A_2$
$\alpha=0.8, \beta=0.2$	$A_3 \succ A_1 \succ A_2$
$\alpha=0.9, \beta=0.1$	$A_3 \succ A_1 \succ A_2$
$\alpha=1, \beta=0$	$A_3 \succ A_1 \succ A_2$

6. Conclusions

This paper has presented a novel method to solve the LGDM problem, in which the evaluation information provided by a large number of participators from multiple groups is used. Using the method, the percentage distributions on evaluations provided by participators from multiple groups concerning alternatives can be determined. According to the percentage distributions and subjective weights of multiple groups, the decision weights of multiple groups concerning alternatives can be obtained. By aggregating the percentage distributions and decision weights of multiple groups, collective percentage distributions on evaluations concerning alternatives can be obtained. According to the derived collective percentage distributions, the dominance degrees on pairwise comparisons of alternatives are calculated, and a ranking of alternatives can be determined using PROMETHEE II method. The major contributions of this paper are discussed as follows.

First, this paper focuses on a new GDM problem with a lot of practical backgrounds, i.e., the LGDM problem. In the problem, a large number of participators from multiple groups take part in the decision process, and evaluation information provided by the participators from each group is represented in the form of percentage distributions. The use of percentage distributions has several characteristics such as easy description, visualization and convenient information processing.

Second, this paper presents a framework for solving the LGDM problem. The framework is different from the way of traditional GDM analysis. It includes the four parts: 1) determine the percentage distributions on evaluations of each group concerning the alternatives; 2) determine the decision weight of each group concerning each alternative; 3) construct the collective percentage distribution on evaluations concerning each alternative; and 4) rank alternatives or select the most desirable alternative(s). The presented framework lays a good foundation for further conducting studies on LGDM analysis methods.

Third, this paper presents a technology for determining the decision weights of multiple groups based on subjective and objective information. In LGDM analysis, the decision weight of each group needs to be considered, and it needs to be concerned how to determine the decision weight of each group. In this paper, the decision weights of multiple groups are

determined by aggregating the subjective weights and the objective weights, where the subjective weights are provided by organizer of decision-making activity and the objective weights are determined according to the consensus of evaluations provided by participators from multiple groups. In the process of determining the decision weights, it is important to consider the consensus of participators from each group in LGDM analysis.

Fourth, this paper presents an approach to ranking alternatives to select the most desirable alternative(s). Using the approach, the percentage distributions on evaluations of multiple groups concerning alternatives and the decision weights of multiple groups are aggregated to construct the collective percentage distributions concerning alternatives. Based on the obtained collective percentage distributions, a dominance degree matrix is constructed by calculating the dominance degrees on pairwise comparisons of alternatives. Further, a ranking of alternatives can be determined using the PROMETHEE II method.

In terms of future research, the proposed method can be extended to the LGDM problem with multiple criteria, and the DSS based on the proposed method needs to be developed. Also, we will try to extend our method to some real industry applications using the existing technology such as DETECT (DEcision Triggering Event Composer & Tracker) [42, 43].

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Appendix A. Proof of Theorem 1.

By Eq. (8), we know

$$\begin{aligned} p_j^1 &= w_{1j}p_{1j}^1 + w_{2j}p_{2j}^1 + \dots + w_{mj}p_{mj}^1, \\ p_j^2 &= w_{1j}p_{1j}^2 + w_{2j}p_{2j}^2 + \dots + w_{mj}p_{mj}^2, \\ &\dots\dots\dots \\ p_j^t &= w_{1j}p_{1j}^t + w_{2j}p_{2j}^t + \dots + w_{mj}p_{mj}^t. \end{aligned}$$

Thus, we have

$$\begin{aligned}
\sum_{l=1}^t p_j^l &= p_j^1 + p_j^2 + \dots + p_j^t \\
&= (w_{1j}p_{1j}^1 + w_{2j}p_{2j}^1 + \dots + w_{mj}p_{mj}^1) + (w_{1j}p_{1j}^2 + w_{2j}p_{2j}^2 + \dots + w_{mj}p_{mj}^2) \\
&\quad + \dots + (w_{1j}p_{1j}^t + w_{2j}p_{2j}^t + \dots + w_{mj}p_{mj}^t) \\
&= (w_{1j}p_{1j}^1 + w_{1j}p_{1j}^2 + \dots + w_{1j}p_{1j}^t) + (w_{2j}p_{2j}^1 + w_{2j}p_{2j}^2 + \dots + w_{2j}p_{2j}^t) \\
&\quad + \dots + (w_{mj}p_{mj}^1 + w_{mj}p_{mj}^2 + \dots + w_{mj}p_{mj}^t) \\
&= w_{1j}(p_{1j}^1 + p_{1j}^2 + \dots + p_{1j}^t) + w_{2j}(p_{2j}^1 + p_{2j}^2 + \dots + p_{2j}^t) + \dots + w_{mj}(p_{mj}^1 + p_{mj}^2 + \dots + p_{mj}^t).
\end{aligned}$$

Since $\sum_{l=1}^t p_{ij}^l = 1$, $i = 1, 2, \dots, m$, we have $\sum_{l=1}^t p_j^l = w_{1j} + w_{2j} + \dots + w_{mj} = 1$.

Appendix B. Proof of Theorem 2.

If $\alpha = 0$, then we have $w_{ij} = w_i^{sub}$ according to Eq. (7). Further, since $w_i^{sub} = q_i / \sum_{k=1}^m q_k$,

then we have $w_{ij} = q_i / \sum_{k=1}^m q_k$. Thus, by Eq. (8), we have

$$\begin{aligned}
p_j^l &= \sum_{i=1}^m w_{ij} p_{ij}^l \\
&= w_{1j} p_{1j}^l + w_{2j} p_{2j}^l + \dots + w_{mj} p_{mj}^l \\
&= p_{1j}^l q_1 / \sum_{k=1}^m q_k + p_{2j}^l q_2 / \sum_{k=1}^m q_k + \dots + p_{mj}^l q_m / \sum_{k=1}^m q_k.
\end{aligned}$$

Moreover, by Eq. (2), we have $p_{1j}^l = q_{1j}^l / q_1$, $p_{2j}^l = q_{2j}^l / q_2$, ..., $p_{mj}^l = q_{mj}^l / q_m$. Thus, we

$$\text{have } p_j^l = q_{1j}^l / \sum_{k=1}^m q_k + q_{2j}^l / \sum_{k=1}^m q_k + \dots + q_{mj}^l / \sum_{k=1}^m q_k = \sum_{i=1}^m q_{ij}^l / \sum_{i=1}^m q_i.$$

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