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Detection and prevention of blowout in a lean premixed gas-turbine model combustor using dynamical system theory

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Fig. 1	182		
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Abstract

We propose a novel online method of detecting lean blowout (LBO) in a laboratory-scale premixed-type gas-turbine model combustor from the viewpoint of dynamical system theory that enables us to quantify the complexity in combustion instability. The multiscale entropy method and nonlinear forecasting method, which have not been previously adopted for the analysis of combustion instability, demonstrate that combustion instability close to LBO exhibits much more complicated stochastic dynamics than fractional Brownian motion. The permutation entropy, which can be estimated by quantifying the rank order pattern of components in phase space vectors constructed

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from the short time-series data of pressure fluctuations, is used as a control variable to prevent LBO. The online method proposed in this work has the potential to control the complex combustion state close to LBO.

Keywords: gas-turbine combustor, combustion instability, nonlinear dynamics

1. Introduction

Lean premixed combustion is a highly promising method that can effectively reduce the nitrogen oxide (NO_x) emission from gas-turbine engines of all sizes while maintaining combustion efficiency. However, it is commonly affected by two main types of combustion instability: thermoacoustic combustion oscillations and lean blowout (LBO). The former instability, which is caused by closed-loop coupling between unsteady heat release fluctuations, acoustic velocity and pressure field, gives rise to consequence damage in practical combustors. A comprehensive overview of the former instability in several types of combustor is presented in review article[1] and books[2, 3]. Regarding flame-acoustic interaction issues, the physical mechanisms responsible for the instability, particularly, in a swirl-stabilized premixed combustor, have been proposed by Lawn and Polifke[4, 5]. A review on combustion dynamics and its control methods has also been given by Candel[6]. The latter

instability is also problematic and has to be avoided to prevent severe safety problems and high operational costs in gas-turbine engines. Broad safety and reliability margins away from actual LBO limits are considered for the operation of current gas-turbine combustor engines owing to the difficulty in detecting the precursors of LBO. Most approaches to the detection or correlation of LBO limits are build on the idea that LBO is attributable to the competition between characteristic time scales of flow residence and chemical kinetics [7]. At the same time, a sophisticated approach to detecting LBO precursors by a signal-monitoring technique is of practical importance for regulating the complex combustion state under low equivalence ratios to achieve the low emission of NO_x .

Nonlinear time series analysis based on dynamical system theory brings about significant information for revealing the dynamical properties of aperiodic time series data obtained in a broad array of experiments and numerical simulations covering the fields of medicine, biology, electronics, mechanical engineering and chemical engineering [8]. It can be used for the quantification of many important physical invariants, including the fractal dimension, Lyapunov exponent and entropies, as well as for technological applications such as nonlinear forecasting and noise reduction. In fact, it has recently

been exploited in the context of combustion and flame front instabilities (e.g., a thermal pulse combustor [9–11], a ducted premixed combustor [12–15], a diffusion flame subjected to acoustic forcing [16], a diffusion flame with radiative heat loss [17], [18] and swirling flames [19–21]) with emphasis on the treatment of aperiodic evolution in dynamics. Some of these works have been particularly the characterization of complex combustion dynamics, focusing on the self-similar structure quantified by the well-known correlation dimension [9–12, 14, 19, 20], and the orbital instability in a phase space using the largest Lyapunov exponent [9, 10, 12, 18]. Meanwhile, the recurrence plot, which is a two-dimensional representation suitable for extracting hidden order and disorder in patterned structures, has been extensively used for treating intermittent thermoacoustic combustion oscillations [13], [22]. In parallel with these works, we have also applied different types of nonlinear time series analysis (the translation error [23] to quantify the degree of parallelism of nearby trajectories in a phase space, the permutation entropy [24] to estimate the degree of randomness estimated from a sequence of ranks in the values of time series, and nonlinear forecasting to extract the short-term predictability and long-term predictability characteristics of orbital instability using an extended version of the Sugihara-May algorithm [23] and a radial

basis function network [24]), to the complex dynamics of combustion instabilities in a lean premixed gas-turbine combustor. It has mainly been reported in our previous works [23], [24] that the dynamical behavior of combustion instability undergoes a significant transition from stochastic motion in close proximity to LBO to periodic oscillations caused by thermoacoustic combustion oscillations through low-dimensional chaos with increasing equivalence ratio. These works [23], [24] also include a significant amount of discussion in terms of practical application in addition to the suitability of nonlinear time series analysis for the characterization of complex dynamics in a lean premixed gas-turbine combustor. However, no attempts have been made in previous studies [9–24] to propose an online method of detecting an unstable combustion state to suitably control a combustion system in experiments, focusing on the concept of dynamical system theory. We have recently revealed the feasibility of conducting online control of the combustion state in a laboratory-scale lean premixed gas-turbine model combustor using the translation error as a first step [25]. Note that the possible detection method of impending instability in two types of laboratory-scale turbulent combustors have been proposed by obtaining the significant changes in nonlinear measure in terms of Reynolds number [26], but has not led to the online

detector and the subsequent actual control of combustion instability.

The main objective of this study is to propose a more sophisticated and novel online method of detecting LBO precursors based on the concept of dynamical system theory. We discuss the applicability of permutation entropy, which is a coarse-grained version of the Kolmogorov-Sinai (KS) entropy, as a promising online detector from a viewpoint of practical application. On the basis of the extracted complexity, we aim to achieve the suitable control of the combustion state in order to prevent LBO using the permutation entropy as a control variable. In addition to proposing the novel online method of detecting combustion instability, we also conduct an investigation of the complexity of combustion instability from the viewpoint of multiscale entropy and nonlinear forecasting.

2. Experimental apparatus and method of detecting combustion instability

Our experimental system is identical to that used in our recent study [25], and mainly consists of mass flow controllers, a nozzle, an axial swirler and a combustion chamber. Methane gas is used as the main fuel in this study. Air is issued at a volume flow rate (mean axial flow velocity at inlet of combus-

tion chamber) of 127 L/min(5.0 m/s). Note that the experimental conditions in this study are different from those in Ref. [25]. The equivalence ratio ϕ is decreased from 0.75 to 0.49, the value at which thermoacoustic combustion oscillations change to LBO. The pressure fluctuations p' , an important physical quantity that represents the dynamics of the combustion state, are obtained using a pressure transducer (JTEKT Products, PD104K-10kPa) located on the wall of the combustion chamber. Similarly to in our recent study [25], the sampling frequency of p' is set to 5 kHz.

The permutation entropy, which can be estimated by quantifying the rank order pattern of components in phase space vectors constructed from time series data, is an invariant measure of the complexity of dynamics [27], and has been shown to ensure good performance in quantifying the complexity of combustion [24] and flame front dynamics [17]. In this study, we use the permutation entropy to detect the significant changes in the combustion state close to LBO. On the basis of Takens' embedding theorem [28], the time-delayed coordinates expressed as $\mathbf{p}'(t_i) = (p'(t_i), p'(t_i + \tau), \dots, p'(t_i + (D-1)\tau))$ are first introduced to construct the phase space. Here $i = 0, 1, \dots, n$ (n is the number of data in the time series), $\mathbf{p}'(t_i)$ are the phase space vectors, $p'(t_i)$ is the pressure fluctuation at time t_i , D is the embedding dimension,

that is, the dimension of the phase space and τ is the time lag. We index all possible permutations ($D!$ permutations) of rank order in the components of the constructed phase space vectors as π . Note that D and τ are set to 6 and 1 for the estimation of the permutation entropy, respectively. Each permutation represents a coarse-grained pattern of the dynamical behavior. By counting the number of realizations of permutations for all vectors, denoted as $q(\pi)$, the relative frequency for each permutation $p_e(\pi)$ is obtained as $p_e(\pi) = q(\pi)/(N - (D - 1)\tau)$. Following the definition of the Shannon entropy, the permutation entropy H_p is then calculated as $H_p = - \sum_{\pi} p_e(\pi) \log_2 p_e(\pi)$.

The permutation entropy h_p normalized by the maximum permutation entropy ($= \log_2 D!$) is finally obtained as

$$h_p = \frac{- \sum_{\pi} p_e(\pi) \log_2 p_e(\pi)}{\log_2 D!}, \quad (1)$$

where the lower bound of $h_p (= 0)$ corresponds to a monotonically increasing or decreasing process, while the upper bound ($h_p = 1$) corresponds to a completely random process.

Time variations of p' , the three-dimensional phase space $(p'(t_i), p'(t_i - \tau), p'(t_i - 2\tau))$ and h_p for $\phi = 0.61$ are shown in Fig. 1. h_p is estimated for

the phase space constructed from short time-series data of p' and is obtained every 100 ms in this study. In other words, the library data of the phase space consists of 100 ms time series data of p' . At $t = 11.0 - 11.2$ s, $\mathbf{p}'(t_i)$ exhibit limit cycles due to the regular fluctuations with large amplitudes generated by thermoacoustic combustion oscillations. They fill the core of the phase space at $t = 20.8 - 21.0$ s owing to the appearance of irregular fluctuating pressures with small amplitudes, indicating the degeneration of the deterministic nature. Note that the physical mechanism at the onset of the intermittent switching between two states was phenomenally discussed in our recent study [25]. The important point to note in Fig. 1 is that when p' drastically changes with the onset of intermittency, h_p also changes simultaneously. The response of the permutation entropy to the drastic change in pressure fluctuations indicates that similar to the translation error [25], the permutation entropy can be used to detect a drastic change in the combustion state.

3. Results and discussion

The pressure spectrum of p' is shown in Fig. 2 for different ϕ . Thermoacoustic combustion oscillations with dominant frequency $f \sim 185$ Hz are

clearly observed for $0.75 \geq \phi \geq 0.61$. This instability mode corresponds to the 1/4 acoustic mode in the longitudinal direction of the combustor. Upon decreasing ϕ to 0.50, close to the LBO limit, the dominant peak of the pressure spectrum is notably diminished, and the distribution of the pressure spectrum starts to consist of a broad frequency band owing to the occurrence of bursts with a small amplitude. Quantification of the complexity in combustion instability, in particular close to LBO, has been the focus of considerable attention, and in this section we undertake an in-depth study of the characteristics of the pressure fluctuations close to LBO by making use of two sophisticated methods of nonlinear time series analysis: the multiscale entropy method to quantify the complexity in coarse-grained dynamics and the nonlinear forecasting method to extract the short-term predictability and long-term predictability characteristics of the dynamics.

Multiscale entropy, which was first proposed by Costa et al. [29], can quantify the degree of complexity in dynamical behavior at a multiple temporal scale using a coarse-graining approach. In this method, we first construct the time series of p' divided by a time scaling factor $\{q_j^{(s)}\}_{j=1}^{n/s}$, defined by

$$q_j^{(s)} = \frac{1}{s} \sum_{t_i=(j-1)s+1}^{js} p'(t_i), \quad (2)$$

where s is the scaling factor, $1 \leq j \leq n/s$. Note that $\{q_j^{(s=1)}\}_{j=1}^n$ corresponds to the original time series data. The multiscale entropy S_E is estimated as a function of s for each D -dimensional phase space consisting of coarse-grained time series of p' :

$$S_E = \ln \frac{\sum_{i=1}^{N-D} \Theta_i^D}{\sum_{i=1}^{N-D} \Theta_i^{D+1}}, \quad (3)$$

where Θ is the Heaviside function defined as

$$\Theta = \begin{cases} 1, & \|\mathbf{q}^D(t_j) - \mathbf{q}^D(t_l)\|_{max} \leq r \\ 0, & \|\mathbf{q}^D(t_j) - \mathbf{q}^D(t_l)\|_{max} > r. \end{cases} \quad (4)$$

Here, $\|\mathbf{q}^D(t_j) - \mathbf{q}^D(t_l)\|_{max}$ represents the maximum distance between vectors $\mathbf{q}^D(t_j)$ and $\mathbf{q}^D(t_l)$ in D -dimensional phase space, $\mathbf{q}^D(t_j) = (q^{(s)}(t_j), q^{(s)}(t_j + \tau), \dots, q^{(s)}(t_j + (D-1)\tau))$ and r is the percentage of the time series data lying within one standard deviation of p' . Note that D and τ are set to 2 and 1 for the estimation of the multiscale entropy, respectively. Similarly to in a previous study [29], r is set to 0.15. For low (high) values of s , S_E represents the degree of complexity in a high (low)-frequency region.

The variation in S_E is shown in Fig. 3 as a function of s for different ϕ . S_E for a stochastic process is also estimated in this study on the basis of the

finding obtained in our previous study [24] that the dynamical behavior of combustion instability close to LBO can be ascribed to fractional Brownian motion dominated by an antipersistent process. It is well known that the degree of complexity of fractional Brownian motion can be determined by the Hurst exponent H , which ranges from 0 to 0.5 for an antipersistent process. For thermoacoustic combustion oscillations at $\phi = 0.70$, S_E is high ($1 \leq s \leq 15$), indicating that the degree of complexity is high in high-frequency regions. The interesting point to note here is that S_E is nearly zero at $s = 27$. This means that the complexity of p' is minimum at $s = 27$. The dominant frequency, $f \sim 185\text{Hz}$, obtained in Fig. 2 is in good agreement with the ratio of the sampling frequency to the scaling factor ($= 5000/27$), which clearly shows that the strongest periodicity of p' at $\phi = 0.70$ appears when $s = 27$. In this sense, the multiscale entropy can be used to extract the periodic mode of thermoacoustic combustion oscillations without using power spectrum analysis. At $\phi = 0.50$, which is close to LBO, S_E is extremely high compared with that for thermoacoustic combustion oscillations at $\phi = 0.70$. The dynamical behavior of p' exhibits a much more complicated stochastic process than fractional Brownian motion, and is most complicated at $s = 9$ (frequency region containing 556Hz). These results show that the multiscale

entropy method is valid for quantifying the complexity of dynamics close to LBO.

One of the authors has recently reported that the nonlinear forecasting method taking into account the updating of library data is useful for predicting the short-term complex temporal behavior of flame front instability induced by radiative heat loss [18] and the intermittent chaos in magneto-hydrodynamic instability [30]. In a numerical study on spatially extended systems, Gotoda et al. [31] proposed a new nonlinear forecasting methodology to distinguish a pure deterministic process from a stochastic process. An important point of this methodology [31] is that stochastic phenomena such as fractional Brownian motion have short-term predictability and long-term unpredictability, while the incremental process exhibits only short-term unpredictability. The correlation coefficient between the actual and predicted increments process of the time series data is nearly zero, regardless of the prediction time, if the dynamical behavior is governed by a stochastic process [31]. In this method, the time series of p' close to LBO is initially divided into two intervals: one is used as library data and the other is used as reference data for comparison with the predicted time series data of p' . We next find the vectors $\mathbf{p}'(t_k)$ ($k = 1, 2, \dots, K$) neighboring the vector $\mathbf{p}'(t_p)$ from all the

points in the phase space constructed by the library data. Here, $\mathbf{p}'(t_p)$ is the final point of the trajectory in the phase space. Note that $p'(t_i)$ is expressed as $\mathbf{p}'(t_i) = (p'(t_i), p'(t_i - \tau), p'(t_i - 2\tau), \dots, p'(t_i - (D - 1)\tau))$ when applying the nonlinear forecasting method. where τ is estimated by making use of the mutual information. $p'(t_k + T\Delta t)$ represents the future value corresponding to $p'(t_k)$ after T time steps. We then obtain the predicted value $p'(t_p + T\Delta t)$ corresponding to $p'(t_p)$ after T time steps from Eq. (5).

$$\hat{p}'(t_p + T\Delta t) = \frac{\sum_{k=1}^K \exp(-\|\mathbf{p}'(t_p) - \mathbf{p}'(t_k)\|) \cdot p'(t_k + T\Delta t)}{\sum_{k=1}^K \exp(-\|\mathbf{p}'(t_p) - \mathbf{p}'(t_k)\|)} \quad (5)$$

Similarly to in a recent study [25], K is set to 1000. In this method, the library data is updated while keeping the size of the updated library data constant. We estimate the correlation coefficient C between the actual and predicted pressure fluctuations p' in terms of the duration t_p of the actual temporal fluctuations of p' added to the library data. Note that t_p corresponds to prediction time. In this work, values of p' of up to $t = 10$ s are used as the library data and those after $t = 10$ s are used as the reference data.

The variation in C for $\phi = 0.50$, close to LBO, is shown in Fig. 4 as a

function of t_p . C at $t_p = 2 \times 10^{-4}$ s is 0.85, which means that the one-step-ahead prediction of p' close to LBO is feasible with relatively high accuracy. A significant decrease in C is clearly observed with increasing t_p , which is due to long-term unpredictability. In contrast, C for the incremental process $\Delta p' (= p'(t_{i+1}) - p'(t_i))$ is almost zero regardless of t_p , indicating that the prediction of the incremental process is unfeasible. This is a distinctive feature of stochastic processes. Nair et al. [26] reported that the aperiodic pressure fluctuations in combustion noise represents high-dimensional chaos using Kaplan and Glass method [32] known as the prototype of the translation error [25] in combination with random shuffling (RS) surrogate data method. The RS surrogate data method is in general used for rejecting the existence of white noise, but at least cannot reject the possible existence of different kinds of stochastic process such as Brownian/fractional Brownian motions. The results obtained by the multiscale entropy and nonlinear forecasting methods demonstrate that combustion instability close to LBO observed in this study exhibits stochastic behavior, indicating the presence of much more complicated dynamics than fractional Brownian motion dominated by an antipersistent process. As mentioned in Sec. I, the prevention of LBO by using the permutation entropy as an online detector is attempted

for the purpose of examining its possible practical application as follows.

The standard deviation of pressure fluctuations p'_{rms} and the permutation entropy h_p are shown in Fig. 5 as a function of equivalence ratio ϕ . Note that the changes in the dynamical behavior of p' in terms of the equivalence ratio presented in this study are similar to those obtained in our recent study [25], and the dynamical behavior becomes complex as the combustion state approaches LBO. A gradual decrease in p'_{rms} with decreasing ϕ is observed for thermoacoustic combustion oscillations. p'_{rms} abruptly decreases at $\phi = 0.61$ owing to the appearance of intermittent pressure fluctuations. For $\phi \leq 0.60$, p'_{rms} remains nearly unchanged, showing the difficulty of capturing changes in the combustion state close to LBO, similarity to in our recent study [25]. In contrast, h_p monotonically increases with decreasing ϕ even for $\phi \leq 0.60$, which indicates that the permutation entropy can capture significant changes in the complex combustion state close to LBO. This also indicates that the detection of LBO precursors is feasible by setting a threshold for the permutation entropy. The prevention of LBO for (I) power reduction and (II) transient processes [33] has been conducted relatively recently for an active control system by power spectrum analysis of temperature fluctuations inside a combustor. A closed-loop feedback control system is introduced in

this study to take into account the above two processes by adjusting the secondary fuel supply while using the permutation entropy as the control variable.

As an example of power reduction, the time variations in both the volume flow rate of the secondary fuel $Q_{\text{CH}_4, \text{secondary}}$ and the equivalence ratio ϕ with decreasing volume flow rate of the main fuel $Q_{\text{CH}_4, \text{main}}$ are shown in Fig. 6. Note that only $Q_{\text{CH}_4, \text{main}}$ is decreased until LBO occurs. Similarly to in our recent study [25], the control of the combustion state is attempted by correlating $Q_{\text{CH}_4, \text{secondary}}$ with the difference between $h_{p, \text{thre}}$ and h_p ($Q_{\text{CH}_4, \text{secondary}} = \alpha(h_p - h_{p, \text{thre}})$). Here, α is a coefficient and $h_{p, \text{thre}}$ is the threshold of h_p used for the control regulator. In this study, $h_{p, \text{thre}}$ and α are set to 0.8 and 0.5, respectively. LBO is clearly prevented even when $Q_{\text{CH}_4, \text{main}}$ is decreased to 6.54 L/min. The results obtained for the transient process are shown in Fig. 7. $Q_{\text{CH}_4, \text{main}}$ is decreased in order to approach LBO, but LBO is prevented by adding the secondary fuel. The secondary fuel supply system returns to the initial setting after LBO is prevented by increasing in $Q_{\text{CH}_4, \text{main}}$. Although our study is limited to a laboratory-scale gas-turbine model combustor, these results indicate that the proposed detection method using the permutation entropy, which involves the online update

of a small part of the library, is applicable to combustion control and has potential use for the prevention of LBO without knowing the actual LBO limit. As was shown in Fig. 5, the rms value of pressure fluctuations remains nearly unchanged at equivalence ratios close to the lean blowout, while a significant change in the permutation entropy is clearly observed. This indicates that the permutation entropy is feasible for use as a better online detector than the rms value for our experimental system. However, we need to show the superiority of the permutation entropy to the rms value for different types of gas-turbine model combustors as Nair et al. [26] attempted to estimate a nonlinear measure for several types of turbulent combustors. At present, our system is capable of controlling the combustion state up to $h_p = 0.8$, but the limitation of our fuel flow control system makes combustion control close to the LBO limit more difficult. The implementation of combustion control under a broad range of experimental conditions is required in our future study, and it is also necessary to reduce the response time of current control system. Finally, we emphasize that the following two points are significantly different between this study and our previous study [25].

- (1) The previous study [25] focused on whether or not the intermittent behavior of combustion instability is low-dimensional chaos using recurrence

plots and nonlinear forecasting. In contrast, this study focuses on characterizing the dynamical behavior of combustion state close to lean blowout using multiscale entropy and nonlinear forecasting, and reveals that the dynamical behavior is governed by a stochastic process such as fractional Brownian motion. This is an important and original point that has not been reported in [25].

(2) The translation error [25] is an important invariant measure derived from the viewpoint of orbital instability in phase space. In contrast, the permutation entropy derived from the viewpoint of entropy represents the degree of randomness in the coarse-grained dynamical behavior, whose physical meaning is different from the translation error. The main advantages of the permutation entropy are that the number of parameters required for the estimation is small and that the estimation is robust in the presence of noise compared with the translation error. This is very important when considering a detector applicable to actual combustors. On this basis, in this study, we newly propose the permutation entropy as a more sophisticated detector than the translation error [25].

4. Conclusions

We have proposed a novel online method of detecting lean blowout (LBO) in a laboratory-scale premixed-type gas-turbine model combustor based on the concept of dynamical system theory that enables us to quantify the complexity in combustion instability. The multiscale entropy method and the nonlinear forecasting methodology recently proposed by Gotoda et al. [31] demonstrate that combustion instability close to LBO exhibits stochastic behavior, indicating the presence of much more complicated dynamics than fractional Brownian motion dominated by an antipersistent process. The permutation entropy, which can be estimated by quantifying the rank order pattern of components in phase space vectors constructed from the short time-series data of pressure fluctuations, is utilized to detect the significant changes in the combustion state at a low equivalence ratio. Our results show that the permutation entropy, which involves the online update of a small part of the library, has potential use as a control variable for preventing LBO.

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5. List of Figure Captions

- Fig.1: Time variation of pressure fluctuations p' , three dimensional phase space $(p'(t_i), p'(t_i - \tau), p'(t_i - 2\tau))$ and permutation entropy h_p for equivalent ratio $\phi = 0.61$.
- Fig.2: Pressure spectrum of pressure fluctuations p' for different equivalence ratio ϕ .
- Fig.3: Variation in S_E as a function of scaling factor s for different equivalence ratios ϕ .
- Fig.4: Variation in correlation coefficient C between the actual and predicted pressure fluctuations p' (or the incremental process $\Delta p' (= p'(t_{i+1}) - p'(t_i))$) as a function of the duration t_p of the actual temporal fluctuations of p' (or $\Delta p'$) added to the library data for equivalence ratio $\phi = 0.50$.
- Fig.5: Standard deviation of pressure fluctuations p'_{rms} and permutation entropy h_p as functions of equivalence ratio ϕ .
- Fig.6: Time variations of equivalence ratio ϕ and volume flow rate of secondary fuel $Q_{CH_4, secondary}$ with decreasing volume flow rate of main

fuel $Q_{\text{CH}_4, \text{main}}$ for power reduction.

- Fig.7: Time variations of equivalence ratio ϕ and volume flow rate of secondary fuel $Q_{\text{CH}_4, \text{secondary}}$ with decreasing volume flow rate of main fuel $Q_{\text{CH}_4, \text{main}}$ for transient process.

Figure 1: Time variation of pressure fluctuations p' , three dimensional phase space $(p'(t_i), p'(t_i - \tau), p'(t_i - 2\tau))$ and permutation Entropy h_p for equivalent ratio $\phi = 0.61$.

Figure 2: Pressure spectrum of pressure fluctuations p' for different equivalence ratio ϕ .

Figure 3: Variation in S_E as a function of scaling factor s for different equivalence ratios ϕ .

Figure 4: Variation in correlation coefficient C between the actual and predicted pressure fluctuations p' (or the incremental process $\Delta p' (= p'(t_{i+1}) - p'(t_i))$) as a function of the duration t_p of the actual temporal fluctuations of p' (or $\Delta p'$) added to the library data for equivalence ratio $\phi = 0.50$.

Figure 5: Standard deviation of pressure fluctuations p'_{rms} and permutation entropy h_p as functions of equivalence ratio ϕ .

Figure 6: Time variations of equivalence ratio ϕ and volume flow rate of secondary fuel $Q_{\text{CH}_4, \text{secondary}}$ with decreasing volume flow rate of main fuel $Q_{\text{CH}_4, \text{main}}$ for power reduction.

Figure 7: Time variations of equivalence ratio ϕ and volume flow rate of secondary fuel $Q_{\text{CH}_4, \text{secondary}}$ with decreasing volume flow rate of main fuel $Q_{\text{CH}_4, \text{main}}$ for transient process.