An analytical model of asymmetric rolling of unbounded clad sheets with shear effects

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Abstract: Based on the slab method analysis, a new solution for asymmetrical rolling of unbonded clad sheet is presented in this paper. The non-uniformity of the shear stresses and the uniformity of the normal stresses at the vertical sides of each slab across the portion of the deforming material all over the plastic region are considered. The effects of the process parameters such as reduction, work rolls radii ratio and work rolls speed ratio on process outputs are investigated. In addition, the asymmetrical clad sheet unbonded before rolling process is simulated wholly using ABAQUS/explicit software. Comparisons of analytical results from present model with those from experimental studies of the other investigators and the finite element simulation have showed good agreements.

Keywords: Asymmetrical rolling; clad sheet; Pressure distribution; Slab method; FEM.

Nomenclature

F	Rolling force of the asymmetrical clad sheet rolling
$F_1, F_2, F_3,$	Rolling forces in zone <i>I, II, III</i> and <i>IV</i> , respectively
F_4	
h	Variable sheet thickness at the roll gap
h_1, h_2	Upper and lower element heights with respect to horizontal axis, respectively
h_i, h_o	Sheet thicknesses at the entry and the exit of the roll gap, respectively
m_1, m_2	Friction factors at the upper and the lower rolls respectively
m_3	Friction factor at the interface of the clad sheet
k_1, k_2	Mean yield shear stresses of the upper and the lower sheets, respectively
L	Contact length
p	Vertical stress at the roll gap
p_1, p_2	Rolling pressures at the upper and the lower interfaces, respectively
p_3	Contact pressure between the clad sheet
r	Reduction in thickness (%)
R_1, R_2	Radii of the upper and the lower work rolls, respectively
R_{eq}	Equivalent work roll radius
T	Total rolling torque of the asymmetrical clad sheet rolling
T_1, T_2	Rolling torques of the upper and the lower rolls, respectively
V_A	Ratio of the lower to the upper work roll speeds
x_{n1}, x_{n2}	Positions of the upper and the lower neutral points, respectively

x_b	Position of the bonding point
σ_1, σ_2	Normal stresses at the upper and the lower slab element in the roll gap
τ_1, τ_2	Frictional shear stresses at the upper and the lower interfaces, respectively
τ_3	Shear stress at the interface of the clad sheet
$\bar{\tau}_1, \bar{\tau}_2$	Average shear stresses along the vertical side of the upper and the lower slab element, respectively
θ_1, θ_2	Variable angles of contact for the upper and the lower rolls, respectively
$\sigma_{yp1},$	Flow stresses of the upper and the lower sheets, respectively
σ_{yp2}	

1. Introduction

Due to some unique features such as high strength, high resistivity against corrosion, and high electrical conductivity, the usages of laminated sheets have been risen. There are many different methods for mass production of laminated sheets in which the asymmetrical rolling is one of the efficient processes. In the asymmetrical clad sheet rolling, the work rolls radii, speeds, surface roughness and thickness of the sheets may be different. Compared to symmetrical process, the rolling force, pressure, and torque can be decreased in the asymmetrical process and the energy can be saved.

Up to now, a number of studies have been done in the field of the asymmetrical sheet and clad sheet rolling processes. A mathematical model for analysis of the asymmetrical clad sheet rolling process based on the stream function and the upper bound methods were done by Hwang and Chen [1]. Lee and Lee [2] used the slab method and experimental analysis to study roll bonding of silver clad phosphor bronze sheets. Some experimental studies were done by Johnson and Needham [3] to investigate the asymmetrical sheet rolling.

Finite element method (FEM) analysis is one of the powerful methods to determine the stress-strain distribution in the metal forming processes. But, a lot of time and cost are required to carry out this method. FEM was used to evaluate the effects of shear yield stress ratio and thickness ratio on the stress, strain, and rolling force in clad sheet rolling process [4-5].

The slab method is a proper method can be used to analyze metal forming processes like symmetrical and asymmetrical sheet and clad sheet rolling. Via this method, the rolling force and torque can be predicted rapidly and a large amount of solution time can be saved. Studies on the asymmetrical rolling process can be categorized into two groups, the asymmetrical sheet rolling and the asymmetrical clad sheet rolling.

Although some researchers have used the slab method to investigate the asymmetrical sheet and clad sheet rolling processes, but this method with considering the non-uniform distribution of shear stress acting on a vertical slab across the section of the deforming material has never been applied to clad sheet rolling process.

An analytical model for analysis of relationship between frictional coefficient and frictional factor in asymmetrical sheet rolling was proposed by Tzou [6]. Based on that analysis, the frictional factor can be obtained rapidly and easily when the frictional coefficient is known. In author's previous study [7], an analytical solution for ring rolling process was presented according to the slab method theory in which the non-uniformity of the normal and shear stresses across the section of the deforming material throughout the plastic zone was considered. Taking into consideration the strain-hardening effects, Gudur et al. [8] attempted to analyze the asymmetrical rolling process in order to estimate the curvature of the rolled sheet. Using the slab method, the analysis of the asymmetrical sheet rolling process was carried out by Yong et al. [9]. They investigated the influence of asymmetrical rolling factors on deformation area and rolling pressure by dividing the deformation area into three zones. An analytical solution was proposed by Tzou and Huang [10] to investigate the minimum thickness for the asymmetrical PV cold rolling of sheet. They considered two analytical models with the

same and the different frictional coefficients at the upper and lower rolls. In addition, another model was proposed by Tzou and Huang [11] to analyze the minimum thickness for asymmetrical hot-and-cold PV rolling by considering the constant shear friction.

Based on the slab method theory, an analytical solution was presented by Hwang and Tzou [12] for asymmetrical sheet rolling process in which the effects of the shear stress and internal moment at the roll gap were considered. Gao et al. [13] studied the asymmetrical cold rolling by considering the differences in the coefficient of friction between rolls and sheets rather than the differences of rolls radii or rotation speeds. According to the slab method, Salimi and Kadkhodaei [14] considered non-uniformity of the normal and shear stresses in deriving the governing equations for analysis of plane strain asymmetrical rolling. Likewise, Zhang et al. [15] presented a new stress field for asymmetrical sheet rolling in which the uniform normal and non-uniform shear stresses on the vertical sides of each slab were considered. In the earlier works on clad sheet rolling, the slab method mainly was applied to evaluate the rolling pressure and force. Furthermore, the slab method was also applied to investigate the plastic deformation of clad sheet, considering constant shear friction and hybrid friction, respectively in [16-17]. An analytical model was presented by Pan et al. [18] for analysis of asymmetrical cold and hot bond rolling of unbonded clad sheet. In their model, they assumed uniform normal stress, but ignored the shear stress distribution in the vertical sides of the slabs. However, the physical condition of the asymmetrical rolling of unbounded clad sheets in which different friction states exist at the interfaces of clad sheet with the upper and the lower rolls and at the interface of two layers necessitate to consider a non-uniform distribution of the shear stress on the vertical sides of the slabs. Therefore,

this postulate is taken into account in this study as a novelty to extend the Pan et al. [18] study and obtain a more accurate model.

In this paper, a solution based on the slab method of analysis is presented for asymmetrical clad sheet rolling process. At first, a mathematical model for the stress field acting on a vertical slab in the roll gap is presented in which the uniformity of the normal stress and non-uniformity of shear stress across the section of the deforming material are considered. After that, imposing the force balance on the slab with differential thickness, one dimensional differential equation for pressure at the roll gap is obtained. Finally, imposing relevant boundary conditions on the differential equation provides an analytical solution for the rolling pressure. In the analytical solution, a variety of factors for asymmetrical clad sheet rolling process such as the friction factor, the friction factor ratio, the roll speed ratio, and the roll radii ratio have been used to investigate the behavior of the process. Based on the present analysis, complete expressions for the asymmetrical clad sheet rolling pressure, force and torque are obtained and the position of the bonding point and neutral points are determined. Eventually, by using ABAQUS/explicit software, the asymmetrical clad sheet rolling process is simulated and the results are used to evaluate the accuracy of present model.

2. Mathematical models

In Fig.1, the schematic illustration of the asymmetrical clad sheet rolling unbonded before rolling is shown. According to this figure, two unbonded metal sheets with uniform unit thickness and a total heights h_i and h_o at the entry and the exit, respectively are

rolled through the asymmetrical rollers. The upper sheet is softer than the lower one and has lower yield stress. The horizontal distance from the exit point in the roll gap is taken as the x-axis and the origin is its intersection with the normal plane passing by centers of two rolls.

The plastic deformation zone at the roll gap can be divided into four different zones. Zone *I* for the unbonded zone ($x_b \leq x \leq L$) in which only the softer sheet, the upper layer, is yielded. When the harder sheet is also yielded, the bonding point (x_b) is generated and after that, the sheets are bonded together completely. In zone *II* ($x_{n1} \leq x \leq x_b$), the peripheral velocities of the upper and the lower rolls are larger than that of the sheets, thus the direction of both the upper and lower frictions on the clad sheet are in the forward direction. Zone *III* ($x_{n2} \leq x \leq x_{n1}$) is a cross shear zone in which the clad sheet velocity is less than that of the lower roll and greater than that of the upper roll. Hence, the frictional stress on the upper surface (between the upper roll and the clad sheet) is backward and on the lower surface (between the lower roll and the clad sheet) is forward. In zone *IV* ($0 \leq x \leq x_{n2}$), the velocity of the clad sheet is higher than the peripheral velocities of the upper and lower rolls and the frictional stress on both sides of the clad sheet are in backward direction. A neutral point on the upper roll (x_{n1}) is the point at which the peripheral speed of the upper roll becomes equal to the clad sheet velocity. Moreover, the point at which the peripheral speed of the lower roll gets equal to the velocity of the clad sheet is the location of the neutral point on the lower roll (x_{n2}).

$$L = \sqrt{R_{eq} \cdot h_i \cdot r} \quad (1)$$

Where R_{eq} is equivalent work roll radius and can be obtained from Eq 2, r is the reduction in thickness and h_i is the total sheet thickness at entry. Subscripts 1 and 2 are designated to quantities at the upper and the lower sheets and rolls, respectively. From simple geometry

$$h_1 = \frac{h_o}{2} + \frac{x^2}{2R_1}, \quad h_2 = \frac{h_o}{2} + \frac{x^2}{2R_2}, \quad h = h_1 + h_2 = h_o + \frac{x^2}{R_{eq}}, \quad R_{eq} = \frac{2R_1R_2}{(R_1+R_2)} \quad (2)$$

$$\tan\theta_1 = \frac{x}{R_1}, \quad \tan\theta_2 = \frac{x}{R_2}, \quad \frac{dh}{dx} = \frac{2x}{R_{eq}}$$

Where h is a variable sheet thickness at the roll gap; h_1 and h_2 are upper and lower element heights with respect to horizontal axis, respectively; θ_1 and θ_2 are variable contact angles; R_1 and R_2 denote the radii of the upper and lower rolls, respectively.

The equivalent stress fields acting on vertical elements at zones *I, II, III* and *IV* are shown in Fig.2. This figure includes the uniformity of the normal stresses in all vertical elements. But, the shear stresses on the vertical sides of the differential elements vary from $\tau_1 = m_1k_1$ at the upper surface to $\tau_2 = m_2k_2$ at the lower surface. For the upper sheet in zones *I* and *II* and the lower sheet in zone *IV*, the shear stresses obtain their minimum values zero at points in between the vertical side and the shear stresses are linearly distributed from the upper and lower surfaces to these points. But, in zones *III* and *IV* of the upper sheet and zones *I, II* and *III* of the lower sheet, the shear stresses are linearly distributed from the upper surface to the lower surface and at the center, it becomes equal to mean value of its surfaces.

Fig. 2 Stress fields acting on a vertical elements at (a) zone I (b) zone II (c) zone III (d)

zone IV

2.1 Clad sheet rolling pressure

2.1.1 Zone I ($x_b \leq x \leq L$)

In zone I, the harder sheet (layer2) is not yielded which means; $p + \sigma_2 < 2k_2$, but the softer sheet (layer1) is yielded with the yield criterion; $p + \sigma_1 = 2k_1$ where $k_1 = \frac{\sigma_{yp1}}{\sqrt{3}}$,

$k_2 = \frac{\sigma_{yp2}}{\sqrt{3}}$ and σ_{yp2} are the mean yield stresses of the upper and lower sheets, respectively.

Based on the stress field of Fig. 2 (a), the horizontal and vertical force equilibriums for the softer sheet lead to the following relations;

$$\frac{d(h_1\sigma_1)}{dx} + p_1\tan\theta_1 - p_3\tan\theta_3 - (\tau_1 + \tau_3) = 0 \quad (3)$$

$$p = p_1 + \tau_1 - \tau_3 \frac{d(h_1)}{dx} = p_3 - \tau_3\tan\theta_3 \quad (4)$$

where σ and p denote the horizontal and the vertical stresses, respectively; τ_3 and θ_3 are the shear stress and the contact angle at the interface of the clad sheet, respectively. Since the harder sheet has not yielded, the contact angle θ_3 at the interface becomes equal to the contact angle θ_2 between the roll and clad sheet in zone I [18]. While the directions of both the upper and the lower frictions on the clad sheet are forward, the equivalent shear stress at zone I is defined as $\tau_{e1} = m_1k_1 + m_3k_1$. The distributions of shear stress on the vertical elements in zone I, for the upper layer is shown in Fig. 3. According to Fig. 3, the shear stresses are equal to τ_1 and τ_3 at the upper and lower interfaces of the upper sheet, respectively and distributed linearly towards the both sides.

Combining Eqs 3 with 4 and using the Von-Mises yield criterion as well as the geometrical and frictional conditions, the following relation is obtained

$$\frac{dp}{dx} = \frac{4k_1}{(x^2 + R_{eq}(h_o - h_{i2}))}x - \frac{S_1}{(x^2 + R_{eq}(h_o - h_{i2}))}x^2 - \frac{R_{eq}\tau_{e1}}{(x^2 + R_{eq}(h_o - h_{i2}))} \quad (5)$$

Solving the resulted differential equation, the rolling pressure distribution in zone I, p_I , can be obtained as follows

$$p_I = -S_1x + \frac{H_1}{2}\ln(x^2 + U_1) + C_{d1}\omega_1 + B_1^* \quad (6)$$

Fig. 3. Distribution the shear stress on the vertical elements of zone *I* for the upper layer

where B_1^* is a constant of integration which will be derived later and

$$S_1 = \left(\frac{\tau_1}{R_1^2} + \frac{\tau_3}{R_2^2} + \frac{2\bar{\tau}_{I1}}{R_1 R_{eq}} \right), \quad H_1 = 4k_1, \quad U_1 = R_{eq}(h_o - h_{i2})$$

$$L_1 = \tau_{e1} R_{eq}, \quad C_{d1} = \left(\frac{S_1 U_1 - L_1}{\sqrt{U_1}} \right), \quad \omega_1 = \tan^{-1} \frac{x}{\sqrt{U_1}} \quad (7)$$

$$\bar{\tau}_{I1} = \frac{1}{4}(\tau_1 - \tau_3)$$

Since the softer sheet is yielded, $\tau_3 = m_3 k_1$, and k_1 is the mean yield shear stress of the softer sheet. In view of the fact that the normal stress at entry side of the roll gap is equal to σ_{i1} , the boundary condition at the entry is

$$\sigma_1 = \sigma_{i1} \quad \text{or} \quad p_i = 2k_1 - \sigma_{i1} \quad \text{at} \quad x = L \quad (\text{or} \quad \omega = \omega_{i1}) \quad (8)$$

Applying Eq 8 into 6, the constant B_1^* can be obtained as follows

$$B_1^* = 2k_1 - \sigma_{i1} - \left(-S_1 L + \frac{H_1}{2} \ln(L^2 + U_1) + C_d \omega_{i1} \right) \quad (9)$$

Given that the harder sheet in zone I has not yielded, its thickness does not change, i.e.

$h_2 = h_{i2}$ and $\frac{dh_2}{dx} = 0$. Based on the stress field of Fig. 2 (a), the horizontal and vertical force equilibriums for the harder sheet lead to the following relation

$$\frac{d\sigma_2}{dx} = \frac{\tau_2 - \tau_3}{h_{i2}} \left(\frac{x^2}{R_2^2} + 1 \right) \quad (10)$$

Solving the differential Eq10, the horizontal stress of layer 2 in zone I, σ_2 can be obtained as follows

$$\sigma_2 = A_1 x^3 + D_1 x + B_q^* \quad (11)$$

where

$$A_1 = \frac{D_1}{3R_2^2}, \quad D_1 = \frac{\tau_2 - \tau_3}{h_{i2}} \quad (12)$$

And B_q^* is a constant of integration which can be derived using the boundary

condition $\sigma_2 = \sigma_{i2}$ at $x = L$ (or $\omega = \omega_{i1}$) as follows

$$B_q^* = \sigma_{i2} - (A_1 L^3 + D_1 L) \quad (13)$$

When the harder sheet starts to yield at x_b , the yield criterion for this layer becomes $p +$

$\sigma_2 = 2k_2$. Hence, the bonding point (x_b) can be obtained from the following equation

$$\left(-S_1 x_b + \frac{H_1}{2} \ln(x_b^2 + U_1) + C_d \omega_{b1} + B_1^* \right) + (A_1 x_b^3 + D_1 x_b + B_q^*) = 2k_2 \quad (14)$$

where

$$\omega_b = \tan^{-1} \frac{x_b}{\sqrt{U_1}} \quad (15)$$

x_b can easily be obtained by numerical solution.

2.1.2 Zone II ($x_{n1} \leq x \leq x_b$)

In zone II, the velocities of the upper and the lower rolls are larger than the velocity of the sheet, thus the direction of both the upper and lower frictions on the clad sheet in zone II are forward i.e. $\tau_{e2} = m_1 k_1 + m_2 k_2$. In Fig. 4, the shear stress distributions on the vertical elements in zone II are shown.

(b)

Fig.4. Distribution the shear stress on the vertical elements of zone II for the (a) upper layer (b) lower layer

Moreover, sheets have been bonded completely in this zone, but the thickness ratios of clad sheet at the entry and exit are dissimilar, namely $\beta_i = \frac{h_{i1}}{h_1}$ and $\beta_o = \frac{h_{b1}}{h_b} = \frac{h_{o1}}{h_o}$, respectively.

According to the stress field of Fig. 2 (b), the horizontal and vertical force equilibriums for layers results to the following relations, respectively

$$\begin{aligned} \frac{d(h_1 \sigma_1)}{dx} + (\tan \theta_1 - \tan \theta_3) p - (\tan^2 \theta_1 + 1) \tau_1 - (\tan^2 \theta_3 + 1) \tau_3 \\ + \bar{\tau}_{II1} \frac{d(h_1)}{dx} \tan \theta_1 = 0 \end{aligned} \quad (16)$$

$$\frac{d(h_2 \sigma_2)}{dx} + (\tan \theta_2 + \tan \theta_3) p - (\tan^2 \theta_2 + 1) \tau_2 + (\tan^2 \theta_3 + 1) \tau_3 \quad (17)$$

$$+\bar{\tau}_{II2} \frac{d(h_2)}{dx} \tan \theta_2 = 0$$

where

$$\bar{\tau}_{II1} = \frac{1}{4}(\tau_1 - \tau_3) \quad , \quad \bar{\tau}_{II2} = \frac{1}{2}(\tau_2 + \tau_3) \quad (18)$$

With combining Eq 16 with Eq17 and considering the geometrical conditions, the rolling pressure distribution in zoneII can be obtained as follows

$$p_{II} = -S_2 x + H \ln(x^2 + U) + C_{d2} \omega + B_2^* \quad (19)$$

where

$$S_2 = \left(\frac{\tau_1}{R_1^2} + \frac{\tau_2}{R_2^2} + \frac{\bar{\tau}_{II1}}{R_1^2} + \frac{\bar{\tau}_{II2}}{R_2^2} \right) R_{eq} \quad , \quad H = 2k_e \quad , \quad U = R_{eq} h_o$$

$$C_{d2} = \left(\frac{S_2 U - L_2}{\sqrt{U}} \right) \quad , \quad L_2 = \tau_{e2} R_{eq} \quad , \quad \omega = \tan^{-1} \frac{x}{\sqrt{U}} \quad (20)$$

$$k_e = k_1 \beta_o + k_2 (1 - \beta_o) \quad , \quad h_b = h_o + \frac{x_b^2}{R_{eq}} \quad , \quad h_{b1} = h_b - h_{i2} \quad , \quad h_{o2} = h_o - h_{o1}$$

At bonding point, x_b , the rolling pressures in zone I and II are equal i.e. $p_I = p_{II}$.

Considering this constraint, B_2^* in Eq 19 as a constant of integration can be obtained as follows

$$B_2^* = \left(-S_1 x_b + \frac{H_1}{2} \ln(x_b^2 + U_1) + C_{d1} \omega_b + B_1^* \right) -$$

$$\left(-S_2 x_b + H \ln(x_b^2 + U) + C_{d2} \omega_b \right) \quad (21)$$

2.1.3 Zone III ($x_{n2} \leq x \leq x_{n1}$)

Zone III is a cross shear zone in which the clad sheet velocity is less than that of the lower roll and greater than that of the upper roll. However, the frictional stress on the upper side is backward and on the lower side is forward in zone III, i.e. $\tau_{e3} = -\tau_1 + \tau_2$.

Furthermore, the shear stress state for a slab in zone *III* is illustrated in Fig.5 in which the variations of the shear stress on the vertical side of the elements for both upper and lower layers are identical.

Fig.5. Shear stress distribution on the vertical element of zone *III*

Taking into consideration the stress field of Fig. 2 (c), and combining the horizontal and vertical force equilibriums for the element and solving the subsequent differential equation, the rolling pressure distribution in zone *III* can be obtained as follows

$$p_{III} = -S_3 x + H \ln(x^2 + U) + C_{d3} \omega + B_3^* \quad (22)$$

where

$$S_3 = \left(-\frac{\tau_1}{R_1^2} + \frac{\tau_2}{R_2^2} + \frac{\bar{\tau}_{III1}}{R_1^2} + \frac{\bar{\tau}_{III2}}{R_2^2} \right) R_{eq}, \quad C_{d3} = \left(\frac{S_3 U - L_3}{\sqrt{U}} \right), \quad L_3 = \tau_{e3} R_{eq} \quad (23)$$

$$\bar{\tau}_{III1} = \frac{1}{2}(\tau_1 + \tau_3), \quad \bar{\tau}_{III2} = \frac{1}{2}(\tau_2 + \tau_3)$$

In the upper neutral point ($x=x_{n1}$), there are the same rolling pressures in zone II and III, ($p_{II} = p_{III}$). Consequently, the constant of integration (B_3^*) can be obtained as follows

$$B_3^* = (-S_2 x_{n1} + C_{d2} \omega_{n1} + B_2^*) - (-S_3 x_{n1} + C_{d3} \omega_{n1}) \quad (24)$$

where

$$\omega_{n1} = \tan^{-1} \frac{x_{n1}}{\sqrt{U}} \quad (25)$$

2.1.4 Zone IV ($0 \leq x \leq x_{n2}$)

In zone IV, the velocity of the clad sheet is higher than that of the upper and lower rolls; hence the frictional stresses on both side of the clad sheet are backward i.e. $\tau_{e4} = -(\tau_1 + \tau_2)$. The shear stress distributions on the vertical element in zone IV are shown in Fig.6.

(a)

(b)

Fig.6. Distribution the shear stress on the vertical element of zone IV for the (a) upper layer (b) lower layer

Based on the stress field of Fig. 2 (d), using the horizontal and vertical force equilibriums and solving the consequent differential equation, the rolling pressure distribution in zone IV can be obtained as follows

$$p_{IV} = -S_4 x + H \ln(x^2 + U) + C_{d4} \omega + B_4^* \quad (26)$$

where

$$S_4 = \left(-\frac{\tau_1}{R_1^2} - \frac{\tau_2}{R_2^2} + \frac{\bar{\tau}_{IV1}}{R_1^2} + \frac{\bar{\tau}_{IV2}}{R_2^2} \right) R_{eq} \quad , \quad C_{d4} = \left(\frac{S_4 U - L_4}{\sqrt{U}} \right) \quad , \quad L_4 = \tau_{e4} R_{eq} \quad (27)$$

$$\bar{\tau}_{IV1} = \frac{1}{2}(\tau_1 + \tau_3) \quad , \quad \bar{\tau}_{IV2} = \frac{1}{4}(\tau_2 - \tau_3)$$

The boundary condition at the exit is

$$\sigma = \sigma_0 \quad \text{or} \quad p_o = 2k_e - \sigma_0 \quad \text{at} \quad x = 0 \quad (\text{or} \quad \omega = 0) \quad (28)$$

Applying the Eq28 on to Eq27, the constant B_4^* can be obtained as follows

$$B_4^* = 2k_e(1 - \ln U) - \sigma_0 \quad (29)$$

p_I , p_{II} , p_{III} and p_{IV} are the roll pressures at the zones I , II , III and IV , respectively. The specific shear stresses in four zones can be obtained as follows

$$\begin{aligned} (\tau_3)_I &= m_3 k_1 \text{ zone } I \\ (\tau_3)_{II} &= \frac{\varphi_{22} - \gamma_2 + \tau_1 \emptyset \varphi_2 + 2\tau_2 \emptyset \varphi_1}{\alpha_2 - \varphi_2 \emptyset - 2\varphi_1 \emptyset} \text{ zone } II \\ (\tau_3)_{III} &= \frac{\varphi_{32} - \gamma_2 + 2\tau_1 \emptyset \varphi_2 + 2\tau_2 \emptyset \varphi_1}{\alpha_2 - 2\varphi_2 \emptyset - 2\varphi_1 \emptyset} \text{ zone } III \\ (\tau_3)_{IV} &= \frac{\varphi_{42} - \gamma_2 + 2\tau_1 \emptyset \varphi_2 - \tau_2 \emptyset \varphi_1}{\alpha_2 - \varphi_2 \emptyset - 2\varphi_1 \emptyset} \text{ zone } IV \end{aligned} \quad (30)$$

where

$$\begin{aligned} \varphi_1 &= \frac{2\beta_o x}{R_{eq}} \quad , \quad \varphi_2 = \frac{2(1 - \beta_o)x}{R_{eq}} \quad , \quad \emptyset = \frac{x^2}{4R_1^2} \\ \varphi_{22} &= m_2 k_2 \left(\frac{x^2}{R_2^2} + 1 \right) \varphi_1 - m_1 k_1 \left(\frac{x^2}{R_1^2} + 1 \right) \varphi_2 \\ \gamma_2 &= \frac{4\beta_o(1 - \beta_o)x^2}{R_{eq}^2} (2k_2 - 2k_1) \quad , \quad \alpha_2 = \frac{2x}{R_{eq}} \left(\left(\frac{1}{R_1} - \frac{2\beta_o}{R_{eq}} \right)^2 x^2 + 1 \right) \\ \varphi_{32} &= - \left(m_2 k_2 \left(\frac{x^2}{R_2^2} + 1 \right) \varphi_1 + m_1 k_1 \left(\frac{x^2}{R_1^2} + 1 \right) \varphi_2 \right) \\ \varphi_{42} &= -m_2 k_2 \left(\frac{x^2}{R_2^2} + 1 \right) \varphi_1 + m_1 k_1 \left(\frac{x^2}{R_1^2} + 1 \right) \varphi_2 \end{aligned} \quad (31)$$

2.2 Determination of the neutral points

Position of the neutral points, x_{n1} and x_{n2} can be found by using the following constraints

At $x = x_{n1}$, the rolling pressure in zone II and III are equal, i.e., $p_{II} = p_{III}$

$$-S_2x + H \ln(x^2 + U) + C_{d2} \omega + B_2^* = -S_3x + H \ln(x^2 + U) + C_{d3} \omega + B_3^* \quad (32)$$

Moreover, at $x = x_{n2}$, the rolling pressure in zone III and IV are equal, i.e., $p_{III} = p_{IV}$

$$-S_3x + H \ln(x^2 + U) + C_{d3} \omega + B_3^* = -S_4x + H \ln(x^2 + U) + C_{d4} \omega + B_4^* \quad (33)$$

Combining Eqs. (32) and (33), the following relation is obtained

$$(S_3 - S_2)x_{n1} + (C_{d2} - C_{d3})\omega_{n1} + B_2^* - (S_3 - S_4)x_{n2} - (C_{d4} - C_{d3})\omega_{n2} - B_4^* = 0 \quad (34)$$

Using the volume constancy relation of the material, the positions of the upper and lower neutral points have the following relationship[16]

$$x_{n1} = \sqrt{V_A x_{n2}^2 + (V_A - 1) \frac{h_o}{R_A}} \quad (35)$$

where

$$V_A = \frac{V_2}{V_1}, \quad R_A = \frac{1}{R_{eq}} - \frac{h_o}{2R_{eq}^2} \quad (36)$$

Through Eqs 34 and 35, positions of the neutral points x_{n1} and x_{n2} can be obtained.

2.3 rolling force

Since the contact angle of the rolls and that of the sheets is assumed to be small, the rolling force is given by

$$F = \int_{x_b}^L p_I dx + \int_{x_{n1}}^{x_b} p_{II} dx + \int_{x_{n2}}^{x_{n1}} p_{III} dx + \int_0^{x_{n2}} p_{IV} dx = F_I + F_{II} + F_{III} + F_{IV} \quad (37)$$

Where p_I, p_{II}, p_{III} and p_{IV} are the roll pressures at zones I, II, III and IV and x_{n1} and x_{n2} are the upper and lower neutral points, respectively.

2.4 rolling torque

The rolling torque per unit width can be calculated by integrating the moment of the frictional shear force along the arc length of contact around the roll axis. Therefore, the rolling torque for the upper roll can be expressed as

$$\begin{aligned} T_1 &= R_1 \left(+ \int_{x_b}^L m_1 k_1 dx + \int_{x_{n1}}^{x_b} m_1 k_1 dx - \int_{x_{n2}}^{x_{n1}} m_1 k_1 dx - \int_0^{x_{n2}} m_1 k_1 dx \right) \\ &= R_1 m_1 k_1 (L - 2x_{n1}) \end{aligned} \quad (38)$$

and the rolling torque for the lower roll can be expressed as

$$\begin{aligned} T_2 &= R_2 \left(+ \int_{x_b}^L m_2 k_2 dx + \int_{x_{n1}}^{x_b} m_2 k_2 dx + \int_{x_{n2}}^{x_{n1}} m_2 k_2 dx - \int_0^{x_{n2}} m_2 k_2 dx \right) \\ &= R_2 m_2 k_2 (L - 2x_{n2}) \end{aligned} \quad (39)$$

Therefore, total rolling torque can be obtained as follows

$$T = T_1 + T_2 \quad (40)$$

3. Finite element Simulation

To verify the analytical model, finite element simulation of asymmetrical clad sheet unbonded before rolling was carried out using ABAQUS/explicit software. Due to the asymmetrical nature of the clad sheet rolling process, the size and/or rotational speeds of both rolls may be different. Moreover, with assuming plane strain condition, the process is simulated in two dimensions. The rolls are modeled as discrete rigid bodies and the sheets are assumed as elastic perfect plastic materials.

Aluminum as material of the upper layer and copper as material of the lower layer with the material properties given in Table 1 were considered in the simulation. Shear yield

stress of the material was also calculated based on the Von Mises yield criterion, i.e.

$$k = 0.577 \sigma_y.$$

Table.1 Material properties of aluminum and copper in the FE simulation

Layer	Materials	poisson's ratio ν	Modulus of Elasticity E (GPa)	Density ρ (kg/m ³)
Upper	Al	0.36	68	2698
Lower	Cu 99%	0.343	110	7924

From 2D CPE4R element type, 638 elements with 580 nodes and 1278 elements with 1116 nodes are used to mesh the upper and the lower layers, respectively. Schematic illustration of the mesh model for clad sheet in the roll gap is shown in Fig. 7. Number of elements was checked to be appropriate by applying the mesh sensitivity test.

Because of the non-linearity behavior of the process, the dynamic/explicit analysis was taken into account. The mass scaling strategy was considered to perform the convergence of the problem. Two rolls were only allowed to rotate around their centers and constrained in the other directions. There is no constraint on the clad sheet while it is rolled between two rolls. The simulation shows that throughout the process, the thickness of the sheets are continuously reduced and their length are regularly increased.

Fig. 7 Schematic illustration of the FE mesh model

4. Results and Discussion

From the presented mathematical model, the effective factors on the clad sheet rolling process are investigated. The effect of various shear yield stress ratios of sheets (k_2/k_1) on the rolling pressure $p/2k_1$ and the rolling tensile stress $\sigma/2k_1$ distributions through the contact length are shown in Fig. 8. Obviously, when the shear yield stresses of both sheets are equal i.e. $k_2 = k_1$, there is no bonding point. It is seen that with increasing the shear yield stress ratio, the rolling pressure and tensile stress increase as well. As the (k_2/k_1) increase, the generation of bonding point is postponed and its position moves toward the output. Moreover, with increasing (k_2/k_1) , the magnitude of the specific shear stress at the interface of layer (τ_3/k_1) increases. Since the directions of the frictional stresses at the upper and the lower surfaces are opposite, τ_3/k_1 has almost a greater magnitude in zone III.

(a)

(b)

Fig.8 (a) Specific rolling pressure and the rolling tensile stress(b) the specific shear stress at the interface of layer for the various shear yield stress ratios

$$R_1 = 100 \text{ mm}, R_2 = 110 \text{ mm}, h_i = 3 \text{ mm}, m_1 = m_2 = 0.5, m_3 = 1$$

$$r = 20\%, V_2/V_1 = 1.1, \beta_i = 0.3, \sigma_o = \sigma_{i1} = \sigma_{i2} = 0, k_1 = 98.1 \text{ Mpa}$$

In Fig.9, distributions of rolling pressure $p/2k_1$ and rolling tensile stress $\sigma/2k_1$ along the contact length with respect to the various friction factors ratios m_2/m_1 (m_1 is fixed) are observed. It is seen that $p/2k_1$ and $\sigma/2k_1$ increase with increasing the friction factors ratios. When $m_2/m_1 = 1$ and 1.5, the specific rolling pressure at the lower neutral point is

larger than that one for the upper neutral point. But for $m_2/m_1 = 0.5$, different condition exists. In addition, with increasing the friction factor ratios (m_2/m_1), the magnitude of the specific shear stress at the interface of layer τ_3/k_1 increases. Moreover, τ_3/k_1 has the greatest magnitude in zone III.

(a)

(b)

Fig.9 (a) Specific rolling pressure and the rolling tensile stress (b) the specific shear stress at the interface of layer for the various friction factors ratios

$$R_1 = 100 \text{ mm}, R_2 = 110 \text{ mm}, h_i = 3 \text{ mm}, m_1 = 0.5, m_3 = 1, k_1 = 98.1 \text{ Mpa}$$

$$r = 20\%, k_2/k_1 = 1.5, V_2/V_1 = 1.1, \beta_i = 0.3, \sigma_o = \sigma_{i1} = \sigma_{i2} = 0$$

Fig. 10 shows the effect of the various speeds ratios (V_2/V_1) on the rolling pressure $p/2K_1$, the rolling tensile stress $\sigma/2k_1$ and the specific shear stress at the interface of layers τ_3/k_1 through the contact length. There is no cross shear zone when the rolls speeds are equal. But, the length of cross shear zone increases with increasing the speeds ratio. Obviously, the rolling pressure increases when the speeds ratio decreases. It is obvious that the rolling pressure decreases in the asymmetrical process which confirms the advantages of this process with respect to symmetrical rolling process.

Fig.10 (a) Specific rolling pressure and the rolling tensile stress (b) the specific shear stress at the interface of layer for the various speeds ratios

$$R_1 = 100 \text{ mm} , R_2 = 110 \text{ mm} , h_i = 3 \text{ mm} , m_1 = m_2 = 0.5 , m_3 = 1 ,$$

$$r = 20\% , k_2/k_1 = 1.5 , \beta_i = 0.3 , \sigma_o = \sigma_{i1} = \sigma_{i2} = 0 , k_1 = 98.1 \text{ Mpa}$$

The effect of various roll radii ratios $R_2/R_1, R_1$ is assumed to be fixed, on rolling pressure $p/2k_1$ and tensile stress $\sigma/2k_1$ along the contact length are shown in Fig. 11. Clearly, as R_2/R_1 increases, $p/2k_1$ and $\sigma/2k_1$ increase too. In addition, the contact length as well as the distance between the neutral points increase i.e. the length of cross shear zone increases. Instead, increasing the roll radii ratio has no influence on the specific shear stress at the interface of the layers.

(a)

(b)

Fig.11 (a) Specific rolling pressure and the rolling tensile stress (b) the specific shear stress at the interface of layer for the various speeds ratios

$$R_1 = 100 \text{ mm} , h_i = 3 \text{ mm} , m_1 = m_2 = 0.5 , m_3 = 1 , k_1 = 98.1 \text{ Mpa}$$

$$r = 20\% , k_2/k_1 = 1.5 , V_2/V_1 = 1.1 , \beta_i = 0.3 , \sigma_o = \sigma_{i1} = \sigma_{i2} = 0$$

Fig. 12 shows the effects of the roll radius ratio (R_2/R_1) on the asymmetrical clad sheet rolling force. It could be seen that the values obtained for rolling force from the present slab analysis have a fine conformity with those from the FEM results. Moreover, it is seen as R_2/R_1 increases, the total rolling force increases too.

Fig. 12 Asymmetrical clad sheet rolling force given the to the roll radius ratio (R_2/R_1).

$$R_1 = 100 \text{ mm}, h_i = 3 \text{ mm}, m_1 = m_2 = 0.5, m_3 = 1, k_1 = 98.1 \text{ Mpa}$$

$$r = 20\%, k_2/k_1 = 1.5, V_2/V_1 = 1.1, \beta_i = 0.3, \sigma_o = \sigma_{i1} = \sigma_{i2} = 0$$

The effect of the roll speed ratio (V_2/V_1) on the asymmetrical clad sheet rolling force is shown in Fig. 13. Comparison between the results of the finite element simulation with those of the present analytical model demonstrates that when the roll speed ratio (V_2/V_1) increases, the rolling force decreases. In fact, having higher roll speed ratio could be considered as an advantage of asymmetrical rolling process.

Fig. 13 Effects of the roll speed ratio (V_2/V_1) on the rolling force

$$R_1 = 100 \text{ mm}, R_2 = 110 \text{ mm}, h_i = 3 \text{ mm}, V_1 = 50 \frac{\text{mm}}{\text{s}}, r = 20\%, \beta_i = 0.3$$

$$k_1 = 98.1 \text{ Mpa}, k_2/k_1 = 1.5, \sigma_o = \sigma_{i1} = \sigma_{i2} = 0, m_1 = m_2 = 0.5, m_3 = 1$$

In Fig. 14, results of the present slab and FE analyses are compared with Pan et al. model [18] and experimental studies [1]. In the experiments of [1], Al and Cu were considered as materials of sheets in the asymmetrical clad sheet rolling. It should be noted that the available experimental data for asymmetric clad sheet rolling, particularly for the rolling force, is very inadequate and more experiments are needed. Although the results of the present model are nearly close to those from Pan model, Fig. 14 shows that the results of the present model are in a better agreement with the FE and experimental ones than those of Pan model. It should be pointed out that although the present model is less accurate than the numerical model, this is an analytical solution which can easily and quickly predict the characteristics in asymmetrical clad sheet rolling and can save computational cost. It can be seen that the rolling force for all models and analyses increases as the reductions of the clad sheet thickness increase. Some assumptions such as the real material behavior, considering the constant friction factors in the interfaces and their exact magnitudes, ignoring the roll deformation, etc. in the simulation process may be considered as reasons to have differences between the results from simulation and experimental studies in Fig. 14.

Fig. 14 Comparison of (a) rolling force (b) clad thickness ratio at exit (β_0) obtained from present slab and FE analyses with Pan et al. model [18] and experimental studies [1]

$$R_1 = 50 \text{ mm}, R_2 = 100 \text{ mm}, h_i = 10 \text{ mm}, V_1 = 9.44 \frac{\text{mm}}{\text{s}}, \frac{V_2}{V_1} = 1$$

$$m_1 = m_2 = 0.5, m_3 = 1, \beta_i = 0.2, \sigma_o = \sigma_{i1} = \sigma_{i2} = 0$$

According to the ABAQUS/explicit simulation, strain distribution in the deforming zone of asymmetrical clad sheet rolling is shown in Fig. 15. In this case study, since the upper layer is softer than the lower one, the higher plastic strain happens in the upper layer which is apparent in Fig. 15.

Fig. 15 Strain distribution in the deforming zone of asymmetrical clad sheet rolling

4. Conclusions

An analytical model for general asymmetrical clad sheet rolling is presented in which the non-uniformity of the shear stresses at the vertical sides of each slab is considered for the first time in this process. This work aims to explore the characteristics of the clad sheet unbonded before rolling by presenting the complete expressions for the asymmetrical clad sheet rolling pressure, force and torque and predicting the position of neutral and bonding points. Moreover, by using ABAQUS/explicit software, the asymmetrical clad sheet rolling process is simulated. Using different shear yield stress ratios, roll speeds ratios, and friction factors ratios, the asymmetrical clad sheet rolling process was investigated by applying the slab method. It was found that with increasing the shear yield stress ratios or the friction factors ratios or decreasing the roll speeds ratio, the specific rolling pressure increases. It is proved that the rolling pressure in the asymmetrical clad sheet rolling process is lower than the symmetrical process which

confirms the advantages of this process with respect to symmetrical rolling process. Moreover, as the shear yield stress ratios or the friction factors ratio decrease, the bonding point is generated sooner in deforming area. Comparison of the present slab method with FEM and experimental results, illustrate the accuracy of the present analysis.

5. Acknowledgements

The authors are grateful for the research support of the Iran National Science Foundation (INSF).

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