Missing chaos in global climate change data interpreting?

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Abstract

The main problem of ecological data modelling is their interpretation and its correct understanding. This problem cannot be solved solely by a big data collection. To sufficiently understand ecosystems we need to know how these processes behave and how they respond to internal and external factors. Similarly, we need to know the behavior of processes that are involved in the climate system and the biosphere of the earth. In order to characterize precisely the behavior of individual elements and ecosystems we need to use deterministic, stochastic and chaotic behavior. Unfortunately, the chaotic part of systems is typically completely ignored in almost all approaches. Ignoring of chaotic part leads to many biased outcomes. To overcome this gap we model chaotic system behavior by random iterated function system which provides a generic guidelines for such data management. This also allows to replicate a complexity and chaos of ecosystem.

Keywords:

stochasticity, determinism, entropy, chaos, wetland ecosystem, Kullback-Leibler (KL) divergence

1. Introduction

In the past several decades scientific effort has been focused on studying and understanding global climate changes. The effect of climatic changes has become more and more visible and in many regions of the world these changes are represented by increasing of weather extremes (see [5, 7, 18]). In September 2013, the IPCC's fifth assessment stated more conservatively: "Time series of cyclone indices, such as power dissipation, aggregate compound of tropical cyclone frequency, duration, and intensity measuring the total wind energy of tropical cyclones have, since the late 1970s, been showing increasing trends in the North Atlantic and trends increasing with less intensity in the Western North Pacific. We probably have a large enough data collection but the interpretation of longer-term trends is once again constrained by data quality concerns (for additional information, especially in the context of the typhoon "Haiyan", see e.g. [32]).

It is not sufficient to only have "big data" for a correct interpretation of ecological and climatic processes. We need to know how different processes behave and how they respond to input

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from the outside. The knowledge of process behavior is crucial and important for a correct interpretation since the large data sets will not necessarily increase statistical learning, accuracy and understanding.

The studied biosphere has usually been divided into smaller sectors (ecosystems) due to thematic and methodology purposes. The behavior and variability of such smaller sectors of the biosphere might be different in comparison to the complete biosphere as a heterogeneous and complex system. For these reasons, difficulty in ecological data interpretation exists on different and larger scales.

If we have ecosystems, such as systems with repeated subsystem patterns, the ecosystem will function like a carpet with repeated patterns. The pattern of such a carpet will not disappear by cutting the carpet into a smaller pieces (e.g. Sierpinski carpet [29]). What does this mean in an ecological point of view? How is this possible in an ecosystem? The reason is that the parts of the ecosystem pattern and structures are generated by a stochastic system, whereas the other parts are generated by a deterministic and chaotic system. As a deterministic system (see [21]) we understand a system that is completely predictable, i.e. knowing the state of the system in time t will allow us to accurately predict the state of the system in time t_1 ($t_1 > t$).

As a stochastic phenomenon we understand a system in which probabilities are assigned to its events. As a chaotic system we understand a system that has a free choice of future behavior, i.e. the system may choose from several prospective next states and the process of choice is neither deterministic nor stochastic. Chaos can naturally be defined in dynamical systems, and it relates to a level of freedom of the system in order to choose several ways of evolution.

By "Evolution ways" we understand a set of the ways, which would lead to a stable state defined as the attractor. We assume that coexisting attractors are not in competition among themselves. The way which is selected does not have to be a right way, because all ways are equal. Ecological systems can reach stable equilibrium, but this equilibrium is not a finite state in different time scales. Finding of a right way (attractor changing) caused by a high complexity of ecological systems with many of interactions occurred in different times and structure scales.

Chaos is well visible in various branches of ecology, since the role of chaotic dynamics in natural processes (including population dynamics; see [33]). The latter had been the subject of hot debates, and we hope the approach, which we developed, can positively contribute to this dispute.

The human activities as well as the animal species are important for the biosphere within different ecosystems interacting among each other in many diverse ways and levels of hierarchies (see [1]). Principally we cannot separate humans from this system. Our abstraction is only limited in this way despite the big effort of trying to understand the holistic truth more or less objectively and possibly correctly.

2. Chaos versus stochastics in methane emissions from a wetland ecosystem

Methane is the third-largest contributor to the greenhouse effect after water vapor and carbon dioxide (see [11]). Methane is naturally released by wetland ecosystems ([4]) and rice paddies (see [6]). It is a product of anaerobic decomposition processes of the organic matter cycles mainly in water-saturated soils (see [23]). Methane production can be described as a dissipative process of entropy when a highly organized organic structure is decomposed to basic simple compounds. The methane releasing process from the soil and plant stands is highly determined by its production. Methane production depends on the occurrence and activity of self-organized methanogenic bacteria, the amount of decomposable organic matter and suitable anaerobic conditions. Methane release from deeper anaerobic soil layers is influenced by many processes, including methane oxidation by

methanotrophic bacteria ([28]) as well as the way it escapes from the soil environment and starts its interaction with the atmosphere.

Yet, there is a lot of randomness in the process of methane flux from a bacterial point of view as the methane is produced via a biological processing into an external bacteria space. The emission of methane or other greenhouse gases from bare soils and plant stands is a result of several partial processes continuously taking place within the soils (see [26]). Soil processes are usually strictly determined by physical and environmental parameters ([20]) and their behavior has been classified in an appropriate scale, either as deterministic or stochastic ([1]).

Methane emissions are typically modelled via trend process fitting, which makes the ratio of stochasticity and chaos present in the system questionable. In [16], [17] we have modelled methane emissions from a nature sedge-grass marsh in the temperate zone of Europe (The Czech Republic). The trend component was estimated and the noise components decomposed in standard and non-standard emissions. Here, we interpreted it as a stochasticity (standard emission) and chaos. In [30] we have again justified that an anomalous emission (chaos) is significantly present. Thus, we can conclude that the process of methane release from the soil is both chaotic and stochastic with deterministic modelled trend. [24] consider the release of chemical exergy associated with emissions to be a source of degradation and chaos creation. Here emission flows possess exergy (energy necessary for reach of equilibrium) and hence have the potential to cause instability to the ecosystem.

The relation between a stochastic and chaotic model is ([16]):

- (1) The parameters typically associated with chaos are dimension, rate of information generated (entropy), and the Lyapunov spectrum.
- (2) Entropies, as a measure of self-organization in our case, correspond to the so-called heavy tail parameters.

What is source of the detected chaos? We assumed that the main sources of the chaos are various interactions among ecosystem components. The processes of methane release from the ecosystem to the atmosphere occur simultaneously by different ways. Normal emissions relates to stochasticity, non-specific ways of methane release relate to a chaotic behavior of the system.

We used in [27] the Kullback Leibler information divergence, also called cross-entropy, for chaos assessment of the methane emission. Such entropy already includes the amount of chaos ([19]). It also reflects the amount of dissimilarity from one distribution to another, which is useful for possible hypothesis testing. We found a convenient analytical approach to empirically assess how much of the chaos detected is measurable via tail entropy through non-specific ways of releasing methane. We constructed a graphical tool to check for t of the models and justified the presence of chaos in the system. Chaos can be also reconstructed by iterated function systems with probabilities generating tail behavior using the Pareto distribution, which is developed in section 3. The emissions of methane can partially be explained by a deterministic model, having explanatory variables, such as the water level, soil properties, degree of anaerobic conditions. However this explanation cannot be complete due to the detected chaotic part. The complexities of methane emissions from wetlands and numerous dependencies and interactions support the opinion that it is a complicated and chaotic system.

3. Reconstructing chaos from Pareto: Iterated function system

The iterated function systems (IFSs) were developed by [13] as a method of constructing fractals through the use of a set of specific transformations (functions) of an image upon itself. Practically,

it is made up of the recursive application of a set of simple (affine) mappings to a set of initial points. A contraction is a transformation that reduces the distance between every pair of points. There are two important types of contractions: a similarity, which reduces all distances by the same number $0 \leq s < 1$ e.g. $f(x, y) = (sx, sy)$ and an affinity, which reduces distances by different proportions in different directions e.g. $f(x, y) = (s_1 x, s_2 y)$.

Generally, iterated function systems or IFSs are methods for constructing fractals; the resulting constructions are always self-similar. IFS fractals can be of any number of dimensions, but are commonly computed and drawn in 2D. The fractal is made up of the union of several copies of itself, each copy being transformed by a function (hence "function system"). The canonical example is the Sierpinski gasket. The shape of an IFS fractal is made up of several possibly-overlapping smaller copies of itself, each of which is also made up of copies of itself, ad infinitum. This is the source of its self-similar fractal nature.

Formally, a random IFS consists of an Euclidean space (or a complete metric space in general) together with a finite set of mappings

$$\{\mathbb{R}^m, f_i, p_i\}_{i=1}^n \quad (1)$$

where p_i is a probability corresponding to the i -th map, obviously $\sum_{i=1}^n p_i = 1$. It is known that if f_i are contractions, see [13], then there exists a unique nonempty bounded closed set $A \subset \mathbb{R}^m$ (set-attractor) satisfying

$$A = \bigcup_{i=1}^n f_i(A)$$

and a unique probability (measure) μ (unique measure-attractor), supported on A , satisfying the invariance equation $\mu(\cdot) = \sum_{i=1}^n p_i \mu(f_i^{-1}(\cdot))$. Recently it was shown how, for a given continuous probability measures on $\mathbb{R}$, find an IFS having μ as its unique measure-attractor, see [31]. Let μ be a probability measure on $\mathbb{R}$ and F its distribution. One can simply define the generalized inverse distribution function by

$$F^{-1}(z) = \inf_{x \in \mathbb{R}} \{F(x) \geq z\}, \quad 0 \leq z \leq 1.$$

It is true, see [31], that if $Z \sim U(0, 1)$, then $F^{-1}(Z)$ is a μ -distributed random variable, here U is a random variable uniformly distributed on the interval $(0, 1)$. This basic property helps us to simulate from an arbitrary distribution on $\mathbb{R}$, by simulating uniform random numbers on the unit interval. Moreover, if μ is continuous then $F(F^{-1}(z)) = z$ for $0 < z < 1$. This is a crucial property for the following result, again see [31].

Theorem 1 (Stenflo). *A continuous distribution, μ , on $\mathbb{R}$ with distribution function, F , is the measure-attractor of the IFS with monotone maps $f_i(x) = (F^{-1} \circ u_i \circ F)(x)$, for any x with $F(x) > 0$, and probabilities $p_i = \frac{1}{n}$, where $u_i(z) = \frac{z}{n} + \frac{i-1}{n}$, $0 \leq z \leq 1$, $i = 1, 2, \dots, n$, for any $n \geq 2$.*

Entropy (measure of disorder and randomness of a system) is an important measure of disorder (for statistical aspects see [27]): The (differential) entropy of the Pareto distribution can be found without difficulty (see [35])

$$E_{par} = \ln\left(\frac{x_m}{\alpha}\right) + \frac{1}{\alpha} + 1, \quad x \in [x_m, \infty).$$

Clearly $\lim_{\alpha \rightarrow 0^+} E_{par} = +\infty$, which implies high uncertainty for small (positive) α and fixed x_m . Our problem is to illustrate possible chaos generation by iteration of Pareto-based functions. This justifies our approach in [27], since methane emissions are aggregated in a similar way. Accordingly, we considered Pareto distribution with probability density function

$$h(x) = \begin{cases} \alpha x_m^\alpha x^{-(\alpha+1)}, & x \geq x_m \\ 0, & x < x_m \end{cases},$$

where $x_m > 0$ is a threshold and $\alpha > 0$ is a shape parameter, which is known as the tail index. Using Theorem 1 we obtain f_i in the following form

$$f_i(x) = \begin{cases} \frac{x_m n^{\frac{1}{\alpha}}}{(n-i + (\frac{x_m}{x})^\alpha)^{\frac{1}{\alpha}}}, & x \geq x_m \\ \frac{x_m n^{\frac{1}{\alpha}}}{(n-i+1)^{\frac{1}{\alpha}}}, & x < x_m \end{cases}, \quad i = 1, \dots, n.$$

Notice that one can show that f_i , $i = 1, \dots, n$, are Lipschitz continuous with constant $Lip_n = n^{\frac{1}{\alpha}}(n-i+1)^{-\frac{(\alpha+1)}{\alpha}}$, but they are not contractive in general. In practice software that implements IFS only requires the whole system to be contractive on average, see [8] and [2]. There are several definitions and their modifications. We have used so-called global log average contractive condition $E[\ln Lip_{1,\dots,n}; p] = \sum_{i=1}^n p_i \ln Lip_i < 0$. For specific values of parameters it means that $\sum_{j=1}^n \frac{\ln Lip_j}{2} < 0$ should hold. From Table 1 is obvious that for increasing α we need less iteration function. Our

Table 1: Sufficiency for existence of attractor for given α .

α	0.5	0.9	1.1	1.3	1.5	2
n	12	3	2	2	2	2

configuration was chosen as follows: $n = 3$, $p_1 = p_2 = p_3 = \frac{1}{3}$, $\alpha = 1.3$ and $x_m = 1$, whereas uniformity of distribution follows from Theorem 1. This represents a reliable choice of parameters, see e.g. [27], where authors modelled CH_4 flux "emissions" by the infinite moving average process. Compositions of functions f_1, f_2, f_3 were computed randomly with probabilities p_1, p_2, p_3 and this was replicated 1000 times (due to time complexity). We obtain observations at given value $x = 3$, the histogram of which (compared to real density) can be seen in Figure 7. Densities with perturbed parameters α and x_m are plotted in Figures 1-8. From this, we can see the self-similarity of chaos (and heavy tails, respectively), since iteration of density of the tail of emission creates heavy-tailed pattern density.

The Lyapunov exponents were calculated using definition 1 and $N=1200$ as it is explained in above Table 2. Lyapunov exponents provide a way to identify the qualitative dynamics of a system. Since the maximum exponent will dominate, this limit is practically useful only for finding the largest exponent. Chaos can be defined as the divergence between neighboring trajectories and the presence of a positive exponent could be considered as the diagnostics of chaos. However, the Lyapunov exponent itself merely quantifies the degree of "sensitivity to initial conditions" (i.e., local instability in a state space). But chaos arises when there is an attractor in bounded phase space

Figure 1: Approximation of Pareto density with $\alpha = 0.5$ and $x_m = 0.5$.

and a positive Lyapunov coefficient. The maximal Lyapunov exponent can be computed using [34] as given in the following definition.

Definition 1.

$$\lambda(x_1) = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{i=1}^N \frac{|x_i|}{|x_1|}, \quad (2)$$

where $x_{i+1} = f_{k_i}(x_i)$, $i \geq 1$, x_1 is a starting point and $\{k_i\}_{i=1}^N$ is the sequence of indexes of functions from IFS (1).

In Table 2 we computed approximation of Lyapunov exponents for specific values of α, x_m . For given α the dimension (n) of IFS (1) was chosen according to results in Table 1 for IFS (1) where we chosen $p_1 = p_2 = \dots = p_n = 1/n$, i.e. discrete uniform distribution. The reason for choice of discrete uniform distribution is the maximum entropy principle (of probability theory), which is equivalent in our setup with Jeynes non-informative prior (of statistics and information theory) and its interpretation for ecological science is complexity of learning the mixing probabilities of individual components in dynamical model of ecosystem. The other important remark is that Lyapunov exponent (2) is only limit approximation of real chaotic behavior of our system which typically needs only $N = 20 * 60 = 1200$. The interpretation for $N = 1200$ is frequency of data evaluation (in seconds) of emission measurement cycle in chamber, i.e. each single experiment run has taken about 20 min (closing time of the chamber).

Lyapunov exponent and entropy are good disorder measures. As given in this methodology, we can well reconstruct chaos from emission measurements of ecosystem. This confirms that we face at least three integral parts of ecosystem, namely stochastic, deterministic and chaotic. To interpret Table 2 we can conclude that the more heavier tails (smaller α), the more chaotical behavior is observed in ecosystem (this is visible from positive Lyapunov exponents λ). Also we can see the effect of threshold, i.e. the higher x_m the higher chance to get chaos (i.e. positive λ). This reflects the ecological fact that higher threshold brings to regions of nonstandard diffusion, e.g. caused by

Figure 2: Approximation of Pareto density $\alpha = 0.9$ and $x_m = 0.5$.

Table 2: Approximation of $\lambda(x_1)$ (without replications).

α/x_m	0.5	0.9	1.1	1.3	1.5	2	3
0.5	0.2274	0.8156	0.9792	1.1673	1.3946	1.5424	2.1402
0.9	-0.6822	-0.0467	0.0992	0.2379	0.4329	0.6862	1.1376
1.1	-0.9001	-0.2263	-0.0175	0.0446	0.1516	0.4834	0.9415
1.3	-1.0465	-0.4240	-0.1966	-0.0616	0.0710	0.4108	0.7577
2	-1.3377	-0.7098	-0.5192	-0.3815	-0.2012	0.1134	0.4922
3	-1.4663	-0.8687	-0.6668	-0.5094	-0.3397	-0.1059	0.3316

ebullition of methane. We can see in Table 2 that at the boundary between stable and unstable regimes the Lyapunov exponent oscillates around zero (highlighted by blue color).

To interpret the Lyapunov exponent from the point of view of methane emission we can use a similar way as in [10]. Therein the notion of exponential divergence can be quantified using the concept of a Lyapunov exponent. There are as many Lyapunov exponents as there are variables in the phase space (in differential or difference equation models). The attractor is chaotic if (and only if) at least one of the Lyapunov exponents is positive. The definition of Lyapunov exponents and thus the chaos can be extended to include noisy systems, which is of big importance in order to describe the natural populations. Under this definition, the Lyapunov exponent λ tells us whether the endogenous part of the mixed deterministic/stochastic system amplifies or dampens over time the effects of exogenous perturbations. A chaotic system ($\lambda > 0$) is a *noise amplifier*: the effects of perturbations are compounded and cannot be ignored in predicting the future state of the system. A nonchaotic system ($\lambda < 0$) is a *"noise muffler"*: the effects of a perturbation are transient and asymptotically have no effect on the systems long term dynamics. In the case of Methane emissions we can say that higher thresholds in Table 2 bring us to more chaotical parts of emission, caused by e.g. ebullition (see e.g. [14]), which is hard to be long-term predicted. On the other hand lower thresholds are containing more non-chaotic systems, which are easier predictable

Figure 3: Approximation of Pareto density with $\alpha = 0.5$ and $x_m = 1$.

either statistically (e.g. confidence intervals because of finite variance) or even deterministically (deterministic diffusion).

4. Conclusions and discussion on chaos and global climatic changes

Methane emission as a chaotic process, together with the emission of other greenhouse gases (evidently also processes with important chaotic parts), changed the physical properties and increased the chaos of the atmosphere with a reverse effect on ecosystems at a large scale (see [15]). The global climate change is usually presented as a physical system ([9]), with the only restricted interaction among ecosystems of the biosphere. This approach puts the biosphere outside of the described physical systems. This situation is supported by an understandable effort for simple modelling of the climate (philosophically along the line of Occam's razor and statistical parsimony). However from a holistic point of view we shall not separate the biosphere or other components from global climate processes so that heavy interactions are possibly not eliminated.

Global climatic changes are causationally determined, but the significance of a direct human effect is still being discussed. Human society likes to accept commercial thoughts with 100% probability and also tries to find 100% of significance for direct human effects on the climate change. However, we should be aware of the fact that statistical significance maintains only the stochastic part of the system.

The stochastic, deterministic, and chaotic parts of the ecological systems are in coherent and symbiotic relation and they cannot simply be separated. In the case of methane emission from wetlands as in our example (see Figure 9), the most deterministic part of the data comes from the behavior of methanogenic bacteria, the stochastic part comes from a variation of environmental factors (temperature, humidity) and the chaotic part comes from interactions between all underlying factors as well as the self organization of many systems with different scales, e.g. bacteria.

In relation to the IPCC's fifth assessment we cannot believe that we can decrease the amount of chaos in the data by simply increasing the amount and quality of data. Humans are trying to assess nature and natural phenomena in a relative way, by defying several models and measures. However,

Figure 4: Approximation of Pareto density $\alpha = 0.9$ and $x_m = 1$.

our perception and conceptual thinking cannot provide a perfect picture of natural phenomena. Even if we reduce our concern to social phenomena, we will observe chaotic behavior (see e.g. Zipf's law, [36]). Thus nature, including some instances of social behavior and social systems, is highly complex, and the only prediction you can make is that it is unpredictable.

Chaos theory looks at this unpredictability of nature and tries to make sense of it. As Seth Borenstein pointed out ([3]), top scientists from a variety of fields say they are about as certain that global warming is a real, man-made threat as they are that cigarettes kill. That is an issue because scientists from around the world gathered in Stockholm for a meeting of a U.N. panel on climate change, and they released a report saying it is extremely likely - which they define in footnotes as 95% certain - that humans are principally to be blamed for the temperature increases since 1951. One climate scientist involved said that the panel may even boost the figure to virtually certain and 99% for some places. There is another group stating that if something is not 100% clear, we just cannot do anything.

With the methane emission example we outlined the problem of deceptiveness regarding a complete certainty of the climate change. More care should be taken regarding the interpretations and the use of various observations of the climate change as well as in their methodologies and approaches. The difficulty of interpretation can be demonstrated by the fact that the decrease in human-induced methane emissions in the 1990s was only transitory, see [22]. The Kyoto Protocol aimed to reduce the emissions of methane. We should be aware of deterministic, stochastic and chaotic parts of physical and biological systems. The consistency of the modeler's abstraction and natural systems behavior should be thoroughly studied and interpreted. One reason is the complex way of interaction that generates a certain amount of chaos in the system. The amount and possible measuring of huge data sets does not necessarily increase the information, since deterministic, stochastic and chaotic parts are present. Moreover, we can conclude that natural systems contain chaotic behavior because of their large complexity and interactivity (see also [33] and [25]). . As outlined in this paper, we think that chaos is a necessary and natural part of the ecosystem behavior. Thus we doubt the possibilities of a 100% judgment (a 100% paradigm of expectations)

Figure 5: Approximation of Pareto density with $\alpha = 0.9$ and $x_m = 3$.

that the climate change has been caused solely by human effect. The currently reached consensus of 95-98% to justify a human impact on the global climate change is probably the maximal attainable threshold.

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Figure 6: Approximation of Pareto density $\alpha = 1.3$ and $x_m = 3$.

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Figure 7: Approximation of Pareto density with $\alpha = 1.3$ and $x_m = 1$.

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Figure 8: Approximation of Pareto density $\alpha = 1.3$ and $x_m = 0.5$.

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Figure 9: Methane emission in 2014 as the illustrative example of chaos importance measured in the sedge-grass marsh wetland ecosystem (Czech Republic, Central Europe). Emission data is separated into three categories according to their behavior. Deterministic (red) behavior is only restricted to the narrow confidence interval of the polynomial trend line. Stochastic (green) behavior is in the continuous area bordered with prediction (dashed) lines. Blue points exceeding prediction borders are anomalous with chaotic behavior. Black box on the x-axis indicates the length of the growing period.

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