Fractal Dimensions of Metropolitan Area Road Networks and the Impacts on the Urban

Built Environment

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1   **Abstract**

2   The fractal dimension of road network emerges as a measure of the complexity of road transport
3   infrastructures. In this study, we measured fractal dimensions of both the geometric form (i.e., the layout of
4   the roads) and structure hierarchy (i.e., the connections among roads) of the major road networks in the
5   largest 95 U.S. metro areas. We explained the causes of the variances in these fractal dimensions,
6   especially the one for structure hierarchy. Further, we hypothesized the impacts of these fractal dimensions
7   on the urban built environment and validated our hypotheses using path analysis. We found that a larger
8   geometric fractal dimension (D_g) shows a more uniform distribution of roads over the metro area, which
9   provides the accessibility to suburban areas and incentives to low-density development. A larger structural
10   fractal dimension (D_s) indicates the highly-connected roads (e.g., highways) tend to join to other highly-
11 connected roads so that most roads can be reached by a small number of neighboring roads (i.e., the small-
12 world phenomenon). As D_s increases and the small-world effect become more significant, daily vehicle miles
13 traveled per capita (DVMT/Cap) decline. However, D_s should be kept low in order to reduce the DVMT/Cap
14 as population size increases. We consider that the low D_s can contribute to more mixed, polycentric and
15 more uniform on an urban area-wide basis. Overall, higher D_g and D_s of the major road network in a metro
16 area leads to higher per capita carbon emissions of transport, and lower quality of life as population
17 increases. In the end, we conclude that fractal dimensions can provide valuable insight into the nature of the
18 transportation land use nexus.

19   **Keywords**

20   Road Network, Fractal Analysis, Path Analysis, Sustainable Urban Development, Carbon Footprint

1. Introduction

Recent literature has shown that the road transport system is a fractal object, whose form and structure is both essentially irregular, scale invariant and self-similar (Kalapala et al., 2006; Lu and Tang, 2004; Zhang and Li, 2012). Fig. 1 displays the geometric topology and structural architecture of the metro Atlanta road network including freeways, arterials, and collectors. In Fig. 1b of the structural architecture, nodes represent the roads and the links between nodes means the corresponding roads are connected. A fractal dimension D is an index for characterizing fractal patterns by measuring the change in detail in response to the change in geographic scales (Mandelbrot, 1983). There are two types of D measures that have been developed for road networks. One is geometric D (D_g), which can distinguish how more or less uniformly road infrastructure is distributed in 2-dimensional space (Thomas and Frankhauser, 2013). The other measure is structural D (D_s), which was first computed by Zhang et al to determine the fractal architecture of road networks in terms of how roads are connected with each other over a range of scales (Zhang and Li, 2012).

Fig.1. Geometrical and structural representation of road network in the metro Atlanta: (a) geometrical form; (b) structural representation (i.e., connectivity).

The D_g and D_s provides an overall measure for the complexity of the road network. The value of D_g is a measure of the space filling capacity of road networks in the two-dimensional space (Batty and Xie, 1996). If the space filling capability is high, the road network tends to occupy the space more uniformly within a given boundary in the two-dimensional space (Thomas and Frankhauser, 2013). Accordingly, the

D_g of the road network can be large with an upper limit of two-dimensional (Chen, 2012). Instead, when the space filling capability is low, the distribution of the road network becomes less uniform and the D_g of the road network is small with a lower limit of one. A set of models including diffusion-limited aggregation (DLA) and dielectric breakdown model (DBM) have been developed to explain the difference in D_g caused by the space filling capacity (Meakin, 1983; Niemeyer et al., 1984). In terms of the fractal architecture of networks, Song et al. proposed a mechanism for the growth of fractal complex networks (e.g., metabolic network of *Escherichia coli*) (Song et al., 2006). Built upon the findings from Song et al., we will explain the variance of D_s caused by the difference in the mechanism of how we assemble the road network.

The usefulness of D_g and D_s in planning more sustainable road networks and cities depends on the connections between the two fractal dimensions of road networks and the urban built environment. The exploration between fractal dimensions of the road system and urban growth started in 90s, encouraged by Batty and Longley (Batty and Longley, 1994). Thomas and Frankhauser explained the meaning of fractal dimension for built-up areas and the geometrical form of road networks (Thomas and Frankhauser, 2013). The lower ratio of fractal dimensions between built-up areas and the geometrical form of road networks in a certain area indicates where roads are available but the land remains underdeveloped. Abundo et al. investigated how urban population density and growth influences and is influenced by road network infrastructure (Abundo et al., 2013). Results show that cities on more developed continents tend to exhibit higher D_g road networks. This implies that road network infrastructures in more developed cities branches over more orders of magnitude than that in less developed cities. In addition, before the development of D_s of the road network, most studies only investigated the relationship between D_g and urban form (Lu and Tang, 2004; Thomas and Frankhauser, 2013). We will explicitly explain the linkages between D_s and the urban built environment.

In this study, we first made the hypotheses of the impacts of D_g and D_s on the urban built environment and employed path analysis to validate these hypotheses. The path analysis is a useful statistical technique for testing and estimating potentially causal relations (Etminani-Ghasrodashti and Ardeshiri, 2015; Ewing et al., 2014; Nasri and Zhang, 2014). The path analysis allows us to explore the direct and indirect paths between fractal dimensions and the urban built environment. Due to data sparsity, the measures of the urban built environment in this study include urban population density (UPD), per

capita daily vehicle miles traveled (DVMT/Cap), annual per capita carbon emissions of transport (TCE/Cap/year), and the quality of life (QoL). We consider that lower TEC/Cap/year and higher QoL are more sustainable. If the effects of D_g and D_s on the urban built environment are statistically significant, we might offer another perspective from D_g and D_s to redesign road infrastructures and improve the urban built environment.

The paper is organized as follows: Section 2 is a brief summary of the theories to interpret D_g and D_s , hypotheses of the impacts of D_g and D_s on the urban built environment; Section 3 describes the data and methods for calculating the D_g and D_s of road networks and the construction of path analysis; Section 4 represents the results of D_g , D_s and path analysis; Section 5 and 6 is discussion and conclusion, respectively.

2. Theories and Hypotheses

2.1. The Theory to Explain the Change in D_g Value of Road Networks

There are multiple mathematical models including DLA and DBM to understand the process that leads to the emergence of geometrical fractal patterns. The DLA describes the process how particles undergoing a random walk cluster together to form aggregates of particles. The DBM is a stochastic model to simulate dielectric breakdown. Considering the similarity between road expansion and the branching of discharge channels, the DBM is more suitable to understand the space filling capability. In the DBM, the space filling capability is defined as the dependence on local electric field. The low dependence means a high space filling capability of the electrons. If the dependence is low, then the discharge tends to breakdown the area more uniformly. As a result, the D_g of the branches tends to be large. Instead, if the dependence is high, the distribution of branches is less uniform and the D_g tends to be small. We can borrow the same underlying factor of dielectric breakdown to understand the geometric form of road infrastructure systems. The space filling capability of road infrastructures implies some dependences on local conditions, which might be caused by land use constraints, policies and economic development.

2.2. Hypotheses of the Impacts of D_g on the Urban Built Environment

A larger D_g represents a more uniformly distributed network. Because of the increasing supply of roads which provides accessibility to suburban areas and incentives to low-density development, we hypothesize that the UPD will decline as D_g increases. However, the negative impact of D_g on UPD can be mitigated as population increase and fill up the metro area. Meanwhile, although the supply of road

infrastructures in the suburban areas increases in the case of a large D_g , housing and population density may not reach the same level as it does in more central urban areas and therefore the incentive for suburban shopping, foods, and other recreational facilities remains comparatively low. Accordingly, the quality of life in the metro area may be low.

2.3. The Theory to Explain the Change in D_s value of Road Networks

Song et al. proposed a mechanism for the growth of fractal complex networks (e.g., metabolic network of *Escherichia coli*) (Song et al., 2006). If the highly connected nodes are directly connected with other high degree nodes to formalize a complex large network (Type I), a small-world network emerges (the emergence of a great hub as shown in Fig. 2a) but without fractal topology. Conversely, if the highly connected nodes are connected indirectly through the connection with low degree nodes (Type II), then a network with fractal topology but not the small-world effect does appear (Fig. 2b). If the highly connected nodes are attached to either high degree nodes directly (Type I) or indirectly through low degree nodes (Type II) with a certain probability to formalize a complex large network, the network has both small-world effect and fractal topology (Fig. 2c). Based on the above mechanisms to develop a large network, the fractal dimension of the network increases if the probability p declines for linking highly connected nodes connected to low connected degree nodes (Type II). When $p = 0$, the network is non-fractal. That is, the high fractal dimension network has more significant small-world properties than networks with a low fractal dimension.

Fig.2. Mechanisms of forming a small-world network and a fractal-like network: a) direct connections between higher degree nodes lead to the small-world effect without fractal topology (Type I); b) indirect connections between higher degree nodes lead to a fractal topology but without small-world effect (Type II); c) the network has both small-world effect and fractal property with the combination of Type I and Type II with probability $p=0.5$.

2.4. Hypotheses of the Impacts of D_s on the Urban Built Environment

Based on the interpretation of D_s and the connectivity of road network, we made two hypotheses for the effect of D_s on DVMT/Cap. On the one hand, given the characteristic of small-world effect, DVMT/Cap will decline as D_s increases because of the existence of highly connected roads (e.g., high volume freeways) that have a larger number of connections with more roads. This advantage of small-world networks is consistent with a previous study by Latora and Marchiori that proved that the small-world network is an efficient form of transportation (Latora and Marchiori, 2001). On the other hand, as the road network relies more on a few high volume freeways for transporting, most roads become less important as D_s increases. These roads only provide incentives to low-density development as population increases. High density and commercial development occurs more likely around the intersections of highways. Instead, when D_s is smaller and more low degree roads (e.g., arterials and collectors) plays

important roles of connecting highways, more land along these low-degree roads become valuable for high density and commercial development. These low-degree roads are not directly connected with each other and correspondingly incentivize the development of multiple commercial centers as population increases. The decentralization of commercial centers will lead to a reduction in DVMT/Cap. Overall, the first hypothesis is made based on the efficiency of small-world network itself in transporting. The second hypothesis is made based on the dynamics of land use and transportation nexus. D_s have both positive and negative impacts on DVMT/Cap and the net impact depends on population size.

3. Data and Methodologies

3.1. Data

We collected road network, population, urban area, economic development, and traffic data (i.e., lines of lanes, daily vehicle miles traveled, and transportation carbon emissions) for the 95 U.S. metropolitan areas. Road networks are required for calculating D_g and D_s . The data of road networks was available for year 2005 from the ArcGIS resources (<http://resources.arcgis.com/en/help/main/10.1/index.html#/001z00000039000000>). The road network composed of freeways, arterials, and collectors. We did not include local streets in the network analysis because current algorithms is inefficient to consider local streets for calculating the D_s . To build the path model, the 2010 metro population, gross metropolitan products, urban population and areas were obtained from U.S. Census. We chose the 2011 traffic data available from 2012 Urban Mobility Report given the fact that total transport carbon emissions are not available in previous Urban Mobility reports (Schrang et al., 2012). QoL is measured in this research by the Well-Being Index from Gallup. The index measures Americans' perceptions of their lives and their daily experiences of life evaluation, emotional health, physical health, healthy behavior, work environment and basic access (Gallup and Healthways, 2013). The Well-Being index is calculated on a scale of 0 to 100, which zero represents the lowest possible well-being and 100 represents the highest possible well-being. The summary of variables in the path model is provided in Table 1. In this study, we explored the longer term impacts of D_g and D_s in 2005 on the urban built environment after 6 years. We consider that there are delayed responses to the properties of road networks. For example, real estate developers need years to build new homes and offices regarding to the investment

or upgrading of road networks. In general, it takes about five to six years before the structural change of the urban built environment can be felt (Cervero, 2003; Nasri and Zhang, 2014).

Table 1. Variables included in the path model.

Variables description	Data source	Mean	Standard deviation
Natural log of urban population	US census	13.85	0.94
Natural log of UPD	US census, Urban mobility report	7.80	0.36
Ratio of urban population to metro population	US census	0.77	0.13
Natural log of urban lane density (mile/square mile)	Urban mobility report	1.87	0.33
Natural log of DVMT/Cap (miles/day/person)	Urban mobility report	2.93	0.21
Ratio of auto commuters to urban population	US census, Urban mobility report	0.54	0.07
Natural log of TCE/cap/year (pounds CO ₂ /year/person)	Urban mobility report	8.87	0.24
Natural log of per capita gross metropolitan products (\$/person/year)	US census	11.05	0.31
QoL: quality of life	Gallup	67.15	1.68

3.2. Overview of the Analysis Steps and Methods

The analysis started with the measure of geometric fractal dimension (D_g) using box-counting algorithm. To calculate the structural fractal dimension (D_s), we first converted the geometric form of road network to its structural form. We used the Maximum Excluded Mass Burning (MEMB) algorithm developed by Song et al. to compute the D_s of the road network (Song et al., 2006). We also measured the small-world property and network connectivity of the structural networks to verify the mechanism for the growth of the fractal structure. In the last step, we constructed a path model to test the relationships of D_g and D_s with the urban built environment.

3.2.1. Box-Counting Algorithm for Computation of Geometric Fractal Dimension (D_g)

Box counting is a method of measuring fractal dimension of geometric objects. We used a Matlab box counting algorithm to compute the geometric fractal dimension D_g of road network (Moisy, 2008). We begin by rendering the metro data image to a 1790×2522 logical array (i.e., it contains only 0 and 1). The algorithm overlaid the grids with the box of size R to cover the image. The box sizes are power of two, i.e., $R = 1, 2, 4 \dots 4096$. For each sized grid, we counted the number of boxes N with a road in it. If the object is a fractal, then it will follow a power-law $N = c \cdot R^{-D_g}$, where c is constant and D_g less than 2.

3.2.2. Conversion of Road Network from Geometric Form to Structural Representation

In the geometric form of road network, each road segment is connected with two or more road segments. We first connect two neighboring road segments to form a long natural road within the threshold of 60° deflection angle (Zhang and Li, 2012). The formation of natural roads is based on the Gestalt principle of good continuity, which states that from a human perspective the segments that continue on a visual line should be grouped together. The process goes on until the deflection angle is greater than 60 degrees and is repeated until all road segments are visited. The process is an approximation of how a road network is self-organized to form a complex transport hierarchy. Further, we used the roads as nodes and the connections as the links to represent the structural topology of the road network. This abstract representation demonstrates a structure of how roads are connected with each other.

3.2.3. MEMB for Computation of Structural Fractal Dimension (D_s)

The MEMB algorithm follows the same philosophy of the box-counting approach. For each box of size l , we need to count the minimum number of boxes (N_s) that covers all the nodes in the graph. The box size is defined as the number of steps between two nodes and therefore, the maximum distance between two nodes cannot exceed to the size of the box. If the graph is a fractal, then we would observe a power-law $N_s = c \cdot l^{-D_s}$, where c is constant. More details of the algorithm can be referred to the previous research (Song et al., 2006).

3.2.4. Verification of the Mechanism for the Growth of the Fractal Structure

We verified the theory to explain the variance in D_s based on the 95 structural networks of real road infrastructures. We measured the small-world property of the road structural network using clustering coefficient (CC) (Eqs. 1 and 2). However, we cannot directly compare the CCs of two road structural networks because of the difference in number of nodes and some unknown random effects. Therefore, we randomly rewired 20% of the links in the original road structural network and accordingly the small-world effect is reduced by breaking links between most-connected nodes (Kitsak et al., 2007). The loss should be more significant for the road structural network with larger structural fractal dimension (D_s) (Zhang and Li, 2012).

$$C_i = \frac{2\{e_{pg}\}}{k_i(k_i - 1)} \quad (1)$$

$$CC = \frac{1}{n} \sum_{i=1}^n C_i \quad (2)$$

where, k_i is the number of neighbors (p, q ...) of node i; e_{pq} is the number of links among neighbor p, q of node i; n is the number of nodes.

We also measured network connectivity using average betweenness of the roads (i.e., ratio of the number of shortest paths between any two nodes that passing the node of interest to the number of shortest path between any two nodes other than the node of interest; see Eqs. 3-5). The road network with a small value of D_s should have a higher connectivity than that with a high D_s (Kitsak et al., 2007). This is because more low degree roads serve as the connections between highly connected roads in the low D_s structural network.

$$B_i = \sum_{i \neq j \neq k} \frac{N_{jk}(i)}{N_{jk}} \quad (3)$$

$$Normal(B_i) = \frac{B_i - \min(B)}{\max(B) - \min(B)} \quad (4)$$

$$B_{avg} = \frac{\sum_{n=1}^N Normal(B_i)}{N} \quad (5)$$

where, N_{jk} is the number of shortest paths from node j to node k; $N_{jk}(i)$ is the number of those paths that pass through i; N is total number of nodes.

3.2.5. Path Development and Statistical Analysis

The path analysis is an extension of multiple regression analysis to test and verify the multilevel cause-effect relationships. It allows for the complex interdependency among variables to approximate the interactions in a complex system. As compared to single regression analysis, the path analysis allows the partition of net effects into direct and indirect associations. The path analysis was firstly used by psychologists and later it becomes a popular tool used by ecologists to study the dynamics of ecosystems. However, the application of path analysis to study the interactions between transportation and the built environment is limited, mainly focusing on the study of travel behaviors. Nasri and Zhang developed a path model to study the relationship between macro-level urban form, regional mobility and travel behaviors (Nasri and Zhang, 2014). The analysis suggests that the overall form of urban areas plays a more important

role than the pattern at neighborhood scale in influencing the travel demand. Ewing et al. developed a path model to quantify the elasticity values of vehicle miles traveled with respect to different urban variables in US urbanized areas (Ewing et al., 2014).

We built and refined a theoretical model which reflects the interrelationships among fractal dimensions, land use, traffic, and other socio-economic factors (Table 2). The connections among variables other than those related to D_g and D_s have been discussed or verified (Cervero and Murakami, 2010; Lin and Yang, 2009; Nasri and Zhang, 2014; Silva et al., 2012). Adding these variables can help explore the impacts of D_g and D_s on transportation and land use. But it should be noted that our path model only captured a handful of factors that influence the urban built environment and our goal of the path analysis is to test the hypotheses for the impacts of D_g and D_s . On a methodological note, we used the observations of 95 metropolitan areas to run our path analysis. The total sample size is not large but we guaranteed more than five observations per parameter (i.e., path coefficients and error terms) (Bentler and Chou, 1987). Meanwhile, we did not miss any path that is statistically significant (have non-zero values). The final path model best captured the interrelationships of the variables we had in this study to explore impacts of D_g and D_s on transportation and land use.

We used the STATA 13 software and robust maximum likelihood to avoid the assumption of multivariate normality of endogenous variables (which is equivalent to the dependent variable in linear regression). The standardized root mean squared residual is 0.04, which is smaller than recommended 0.08 (Hu and Bentler, 1999). The estimated total effects of UPD, density of roadway lane miles and per capita gross metropolitan product on DVMT/Cap is in the line with previous studies which have quantified the influence of built environment on vehicle miles traveled (Cervero and Murakami, 2010). The relationships in the path analysis are statistically significant and as such are amenable to test the relationships between fractal dimensions of road networks and the urban built environment.

Table 2. The paths and hypotheses included in path analysis.

Path		Hypotheses	Reference
Start	End		
D_g	Natural log of UPD	UPD will decline as D_g increases.	This study
D_g * natural log of urban population	Natural log of UPD	The negative impacts of D_g on UPD can be mitigated as urban population increases.	This study
Natural log of urban lane density	Natural log of UPD	UPD will increase as urban lane density increases.	(Wade et al., 1999)

Ratio of urban population to the metro population	Natural log of DVMT/Cap	More urban population will contribute to a lower DVMT/Cap.	(U.S Department of Transportation, 2009)
D_s	Natural log of DVMT/Cap	DVMT/Cap will decline as D_s increases.	This study
D_s * natural log of urban population	Natural log of DVMT/Cap	The growth of urban population will offset the positive impacts of D_s on reducing DVMT/Cap.	This study
Natural log of urban lane density	Natural log of DVMT/Cap	The growth of urban lane density will increase DVMT/Cap.	(Ewing et al., 2014)
Natural log of per capita gross metropolitan products	Natural log of DVMT/Cap	Higher per capita gross metropolitan products will contribute to higher DVMT/Cap.	(Cervero and Murakami, 2010; Ewing et al., 2014)
Natural log of UPD	Natural log of DVMT/Cap	Higher UPD will contribute to lower DVMT/Cap.	(Cervero and Murakami, 2010; Ewing et al., 2014)
Natural log of DVMT/Cap	Natural log of TCE/Cap/year	Higher DVMT/Cap will contribute to higher TCE/Cap/year.	(Cervero and Murakami, 2010)
D_g	QoL	Higher D_g will lower the QoL.	This study
Natural log of TCE/cap/year	QoL	Higher TCE/Cap/year will lower the QoL.	(Delmelle et al., 2013; Schneider, 2013)
Ratio of auto commuters to urban population	QoL	Higher usage of private vehicles can contribute to a higher QoL; but the adverse impacts of vehicles (e.g., air pollution, congestions, longer trips and stresses of driving) can reduce the QoL. The impact of vehicle usage on the QoL may not be significant.	(Mullen, 2015)
Natural log of per capita gross metropolitan products	QoL	Higher per capita gross metropolitan products will promote the QoL.	(Easterlin, 1995; Easterlin and Angelescu, 2012)

4. Results

4.1. Fractal Property of Road Networks

The power relationship between the number of boxes and the box size indicates scale invariance (or fractal property) of the road network (Brown et al., 2002). Power plots for metropolitan Atlanta are shown as an example in Fig. 3. We found that the road networks of the largest 95 U.S. metro areas are fractals in terms of both geometrical topology and structural hierarchy.

Fig. 3. The power plot of the road network in metropolitan Atlanta: the power relationship between the number of boxes and the box size shows scale invariance (or fractal property) of the geometrical and structural road networks. a) $D_g=1.61$; and b) $D_s=3.36$.

4.1.1. Geometrical Fractal Dimension (D_g)

The D_g values obtained for our 95 metro area road networks were found to range in values from 1.2 to 1.7 (See *Appendix A*. Table A.1). The Boston-Cambridge-Newton, MA-NH Metro area shows the largest D_g (1.68) and the Boise City-Nampa, ID Metro area shows the smallest D_g (1.26). As shown in Fig. 4, we found that the smaller the D_g , the more dramatically the density of road network decreases from the center of the urban area to suburban and more rural areas. Conversely, as D_g increases the density of the road network in suburban areas is getting closer to that in more central urban areas. That is, a larger D_g represents a more uniformly distributed network.

Fig.4. D_g and corresponding road networks: a road network with large D_g fills up the entire metropolitan area.

4.1.2. Structural Fractal Dimension (D_s)

The D_s values for the road networks of the 95 metro areas analyzed range from 2.2 to 3.6 (See Appendix A. Table A.1). The Milwaukee-Waukesha-West Allis, WI Metro area has the road network with the largest D_s (3.64) and the Salt Lake City, UT Metro area has the road network with the smallest D_s (2.08).

We calculated the ratio of CCs between the original and rewired road structural networks to quantify the loss of small-world effects and found the positive correlation between loss of small-world effect and D_s (See Fig.5). The average betweenness of the roads decreases as D_s increases (see Fig. 6). Therefore, combining the theoretical growth mechanism of small-world and fractal network with the empirical findings in real road structural networks, we concluded that the larger value of D_s is associated with more significant small-world effect. The value of D_s can help distinguish the difference of how roads are connected with each other.

Fig. 5. Statistically significant correlation between the loss of small-world effect due to random rewiring and D_s of road structural networks: CC is clustering coefficient (see Eqs. 1 and 2).

Fig. 6. Statistically significant correlation between B_{avg} and D_s of road structural networks: B is average betweenness of nodes in the network.

4.2. Summary of Path Analysis

4.2.1. Effectson UPD

From the result of path analysis (Fig. 7), we find a direct negative association between D_g and UPD (path coefficient $B = -4.35$). The negative impact of D_g on UPD is mitigated by the population given a positive interaction effect between D_g and population in UPD ($B = 0.19$). Results of path analysis also show other factors that influence the UPD. Density of urban roadway lane miles also has a significant and positive association with UPD ($B = 0.59$). High density of lane miles can be understood as an incentive to high-density development by providing as many people as possible the accessibility to products, services and business. The path model explained 73% of the variance in UPD, indicated by the R^2 value (0.73).

Fig. 7. Results of path analysis: all the path coefficients are significant ($p < 0.05$) except the link from Ratio of auto commuters to urban population to QoL. Summary of statistics is available in the *Appendix A* (See Tables A.2-5).

4.2.2. Effects on DVMT/Cap

The direct negative effect of D_s on DVMT/Cap is revealed by the path shown in Fig. 7. A larger D_s is associated with fewer DVMT/Cap ($B = -0.35$). However, this significant negative direct effect is offset somewhat by the interaction between D_s and urban population ($B = 0.026$), yielding a net positive effect of

D_s on DVMT/Cap for urban populations larger than 700,000 ($-0.35D_s + 0.026D_s * \ln(\text{urban population}) = 0$, in which urban population = 701,894). We also find a negative influence of UPD on DVMT/Cap. A 100% increase in UPD is associated with 59% decline in DVMT/Cap ($B = -0.59$). Higher UPD provides incentives for compact development and public transportation, which could reduce vehicle miles traveled. High density of roadway lanes is associated with large DVMT/Cap ($B = 0.51$). The increase in road supply can stimulate the travel demand. People tend to drive longer in a metro area with higher per capita gross metropolitan products ($B = 0.18$). Commuting to workplaces and driving to fulfill personal needs is positively correlated with economic growth (Ecola and Wachs, 2012). A higher ratio of people living in urban areas to total population of the entire metro area is associated with reduced DVMT/Cap ($B = -0.42$). Households living in rural areas tends to have higher DVMT/Cap (U.S Department of Transportation, 2009). The path model explained 64% of the variance in the DVMT/Cap, indicated by the R^2 value (0.64).

4.2.3. Effects on TCE/Cap/year

We find that DVMT/Cap directly affects the TCE/Cap/year. Specifically, a 1% increase in DVMT/Cap is associated with a 1.05% increase in TCE/Cap/year ($B = 1.05$). The path model explained 83% of the variance in TCE/Cap, indicated by the R^2 value (0.83).

According to the paths show in Fig. 7, D_g affects TCE/Cap/year indirectly through the impacts on UPD and DVMT/Cap. An increase in D_g will lead to a decline in UPD and further an increase in DVMT/Cap. As a result, the TCE/Cap/year will increase. Population size of a metro area can mitigate the negative impact of D_g on TCE/Cap/year. D_s indirectly affects TCE/Cap/year through the impacts on DVMT/Cap. Population size determines the negative or positive impacts of D_s on TCE/Cap/year. When population size is small, the effect of D_s alone is dominant. An increase in D_s will lead to a decrease in DVMT/Cap and consequently a decrease in TCE/Cap/year. However, when population size is large, the interactive effect of D_s and urban population overwhelms that of D_s alone. An increase in D_s will lead to an increase in DVMT/Cap and an increase in TCE/Cap/year. We plot D_s versus TCE/Cap/year for metro areas with more than 5,000,000 populations. As shown in Fig. 8, the D_s and TCE/Cap/year of metro areas with more than 5,000,000 populations is positively correlated.

Fig.8. The impact of D_s on TCE/Cap/year based on metro areas with more than 5,000,000 populations.

4.2.4. Effects on QoL

The D_g is found to have a strong direct negative effect on QoL ($B = -4.0$). Metro areas with larger TCE/Cap/year tend to have lower QoL because larger TCE/Cap/year is associated with longer driving distances and more congestion ($B = -2.32$). Given the negative effect of D_g on TCE/Cap/year, an increase in D_g can indirectly lower QoL. Similarly, D_s has an indirect impact on TCE/Cap/year, and correspondingly affect the QoL. When population size is small, a larger D_s can lead to an increase in QoL. However, when population size grows bigger, a larger D_s can lead to a decline in QoL. Higher per capita gross metropolitan products is, as expected, associated with higher QoL ($B = 3.26$). However, the use of automobiles (i.e. the ratio of auto commuters to total urban population) does not significantly affect QoL ($B = 4.87$, $P = 0.24$). It indicates the necessity of planning for multiple transport modes to ensure both personal ride comfort and the city-wide urban traffic environment. The path model explained 29% of the variance in QoL, as indicated by the R^2 value (0.29).

5. Discussion

In general, the results of path analysis validated our hypotheses of the positive and negative impacts of D_g and D_s on UPD, DVMT/Cap, TCE/Cap/year and QoL. But the applicability of D_g and D_s is not limited to the explanations of these four variables. For example, the explanation of how D_s affect DVMT/cap could also be applied to understand the traffic congestion. The wasted carbon emission per capita (WCE/Cap) due to congestion was found to be negatively associated with D_s (See *Appendix A*. Table

A.6). However, we also find that population growth has offset to some extent the beneficial effect of a high D_s value on reduced congestion. Due to the dominance of freeways and the cluster of destinations (e.g., workplaces, commercial centers) around freeway intersections, in case of large D_s , population growth substantially increases the chances of freeway congestion. For road networks with small D_s values, freeways are found to be indirectly connected through arterials and collectors, which create a buffer between freeways. The travel destinations are dispersed along the road network. Both can reduce the likelihood of traffic congestion. Overall, it might provide an opportunity to use fractal dimensions for developing planning strategies for more sustainable transportation and land use.

Although we found some direct paths from D_g and D_s to the urban built environment, we still did not verify the process of how D_g and D_s affect the land use and transportation as described in the hypotheses in Section 2. In the future study, we will define variables that can characterize the local dependences or the space filling capability based on land constraints, policies and economic development and test the correlation with D_g . Also, we will have metrics that characterize the mixed land use pattern and test the correlation with D_s . In terms of measures of the emergent land use pattern, we only compared the UPD as one general property among cities. However, we did not measure the impacts of D_g and D_s on spatial difference in density across the city like the research in 2002 (Shen, 2002). The impacts of D_g and D_s on other measures of urban structure such as job-housing balance should be investigated in the future. By adding the aforementioned variables and metrics to our current path model, we might be able to connect policy tools and planning strategies with D_g , D_s as well as the emergent pattern of land use and transportation.

We started this fractal analysis in 2013 by looking at road networks in 2005 and the impacts on the urban built environment in 2011. Since 2012, urban mobility reports are available for the DVMT/Cap and TCE/Cap/year. At the same time, more updated road networks, such as TIGER files, are available to trace the changes in D_g and D_s after 2005. In the future, we will use these panel data to further explore the adaptive evolution of the fractal dimensions of road networks and the urban built environment.

6. Conclusion

In summary, we have shown that the fractal dimension D_g and D_s can offer comprehensive metrics that may afford planners a better understanding of urban system spatial dynamics. Metropolitan area road

networks with a large D_g tend to be more evenly spread over their entire geographic region, while road networks with a small D_g are likely to have decreasing road density when moving from the center to the border of the metro area. The magnitude of D_s was found to reflect a trade-off between small-world and fractal properties of the road structural network. A large D_s means that the metro area's road network demonstrates a significant small-world property. A small D_s means that the road network has a more distinct fractal hierarchy but at the expense of its small-world property. Using path analysis, we further explored and explained the connections of D_g and D_s with the urban built environment. An increase in D_g directly leads to a lower UPD and QoL. An increase in D_s contributes to a decrease in DVMT/Cap when the urban population is small, but when population is large, an increase in D_s can lead to the increase in DVMT/Cap. More investigations of these connections will be needed in the future studies to prove the usefulness of D_g and D_s in planning and design road network infrastructures to create a more sustainable urban environment.

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