Statistical Models of Global Langmuir Mixing

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Abstract

The effects of Langmuir mixing on the surface ocean mixing may be parameterized by applying an enhancement factor which depends on wave, wind, and ocean state to the turbulent velocity scale in the *K*-Profile Parameterization. Diagnosing the appropriate enhancement factor online in global climate simulations is readily achieved by coupling with a prognostic wave model, but with significant computational and code development expenses. In this paper, two alternatives that do not require a prognostic wave model, (i) a monthly mean enhancement factor climatology, and (ii) an approximation to the enhancement factor based on the empirical wave spectra, are explored and tested in a global climate model. Both appear to reproduce the Langmuir mixing effects as estimated using a prognostic wave model, with nearly identical and substantial improvements in the simulated mixed layer depth and intermediate water ventilation over control simulations, but significantly less computational cost. Simpler approaches, such as ignoring Langmuir mixing altogether or setting a globally constant Langmuir number, are found to be deficient. Thus, the consequences of Stokes depth and misaligned wind and waves are important.

Keywords: Langmuir mixing, Statistical modeling, KPP, Climate model

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1. Introduction

The effects of ocean surface waves, particularly the Langmuir turbulence induced by the interaction between the mean flow and the Stokes drift (Craik and Leibovich, 1976; McWilliams et al., 1997; Ardhuin et al., 2008; Teixeira, 2011) or equivalently the Stokes shear force (Suzuki and Fox-Kemper, 2016), are important in enhancing the vertical mixing within the ocean surface boundary layer and potentially contribute to the deepening of the ocean mixed layer (Li et al., 1995; Sullivan et al., 2007; Grant and Belcher, 2009; Kukulka et al., 2009; Huang and Qiao, 2010; Cavaleri et al., 2012; Harcourt, 2013; Fan and Griffies, 2014; Li et al., 2016), though some observations suggest that the direct penetration of Langmuir circulation is confined within the upper half of the mixed layer (Weller and Price, 1988; Thorpe et al., 2003). Neglect of the Langmuir turbulence induced mixing, or Langmuir mixing, in most climate models is partially responsible for the persistent shallow mixed layer biases in the Southern Ocean and elsewhere (Belcher et al., 2012; D’Asaro et al., 2014).

Improvements in the simulated mixed layer depth (MLD), deep water ventilation and ocean subsurface temperature are found in Li et al. (2016, hereafter LW16) when Langmuir mixing is parameterized in the National Center for Atmospheric Research (NCAR) Community Earth System Model (CESM) following the scalings found to encapsulate Large Eddy Simulations (Van Roekel et al., 2012). In LW16, a prognostic wave model, WAVEWATCH III (Tolman, 2009), is coupled with CESM to provide the necessary wave information for the Langmuir mixing parameterization. However, running such a prognostic wave model in a climate model is computationally expensive, even with a relatively coarse resolution wave model. With WAVEWATCH III running on a $3.2^\circ \times 4^\circ$ latitude-longitude grid with 25 frequency and 24 directional bins, LW16 report 36% and 28% increase in computational cost for ocean-wave only and fully coupled simulations, respectively, in the low-resolution CESM with nominal 3° resolution ocean model and T31 atmosphere (Shields et al., 2012). Depending on the choice of other component models in the coupled climate system,

31 running WAVEWATCH III on the same coarse resolution in CESM with the
 32 nominal 1° resolution ocean model and a higher-resolution atmosphere increases
 33 the computational cost by 3-10% (Table 1). Modern global wave hindcasts and
 34 projections typically use 1° spatial resolution or higher (e.g., [Rascle et al., 2008](#);
 35 [Rascle and Ardhuin, 2013](#); [Hemer et al., 2013](#); [Breivik et al., 2015](#)), which is at
 36 least 50 times more computationally expensive. Therefore, this project explores
 37 computationally cheaper possibilities. Statistical modeling of Langmuir mixing
 38 effects—with varying degrees of sophistication—is a natural step, along with more
 39 efficient numerical approaches (e.g., [Webb, 2013](#)).

40 Statistical descriptions of the ocean surface waves are common practice in
 41 both research and operations ([Bouws, 1998](#)). Due to their apparently random
 42 phases, the statistical description of ocean surface waves begins with a power
 43 spectrum of surface height displacements, based on the random-phase/amplitude
 44 model ([Pierson et al., 1955](#); [Holthuijsen, 2007](#)). Empirical shapes fitted to wave
 45 spectra are proposed based on the wave measurements and expected dynamical
 46 scalings, e.g., the Phillips spectrum ([Phillips, 1958](#)) for the high frequency
 47 part of the spectrum, the Pierson-Moskowitz spectrum ([Pierson and Moskowitz,](#)
 48 [1964](#)) for fully developed wind-waves and the JONSWAP spectrum ([Hasselmann](#)
 49 [et al., 1973](#)) and modified versions (e.g., [Donelan et al., 1985](#)) for growing wind
 50 waves. The peak frequency is often assumed to be related to the wind speed
 51 as empirically estimated from observations (e.g., [Bouws, 1998](#)). Other wave pa-
 52 rameters can then be derived from the empirical wave spectrum, including the
 53 Stokes drift ([Kenyon, 1969](#); [McWilliams and Restrepo, 1999](#); [Webb and Fox-](#)
 54 [Kemper, 2011](#); [Breivik et al., 2014, 2016](#)), which is of the greatest interest here.
 55 These references assume a one-dimensional wave spectrum without directional
 56 spreading or crossing swell. These effects can be estimated and accounted for by
 57 making some assumptions on the directional spreading distribution and using
 58 observations ([Donelan et al., 1985](#); [Feddersen, 2004](#); [Webb and Fox-Kemper,](#)
 59 [2015](#)).

60 For the purpose of parameterizing Langmuir mixing, empirical wave spectra
 61 are particularly useful. The Stokes drift depends mostly on the high frequency

62 part of the wave spectrum, which is reasonably well described under common
 63 conditions by the empirical spectra (Webb and Fox-Kemper, 2011, 2015). As
 64 shown by Breivik et al. (2016), the Phillips spectrum reasonably approximates
 65 the Stokes drift profile, which is a key ingredient of the LES scaling (Harcourt
 66 and D’Asaro, 2008; Van Roekel et al., 2012). Since high frequency wind waves
 67 are strongly and rapidly coupled with the local wind, predictions to the Stokes
 68 drift profiles and thereby the Langmuir mixing effects can be made from only
 69 the local wind field using the empirical spectra. Of course, the wind and full
 70 wave spectrum, including swell, are normally not in equilibrium, especially in
 71 regions away from the storm tracks (Hanley et al., 2010). Swell are usually mis-
 72 aligned with the local wind and may affect Langmuir turbulence (Van Roekel
 73 et al., 2012; McWilliams et al., 2014). Nevertheless, the mean effect of the mul-
 74 tidirectional waves on the Stokes drift can be estimated empirically (Webb and
 75 Fox-Kemper, 2015) and thereby taken into account. In this paper an approx-
 76 imation to the Langmuir mixing effects based on the Phillips spectrum with
 77 directional spreading will be assessed.

78 Another way to statistically represent the effects of the Langmuir mixing
 79 is simply replacing the *prognostic wave model* with a *data model*, or climato-
 80 logical lookup table, as the source of the wave information. This technique is
 81 commonly used to stand in for climate system components in uncoupled climate
 82 simulations and model evaluations, e.g., the Coordinated Ocean-ice Reference
 83 Experiments (CORE) for global ocean-ice simulations (Griffies et al., 2009). A
 84 global wave climatology can be derived from global wave hindcasts forced by
 85 global wind reanalyses, such as Raschle et al. (2008); Raschle and Ardhuin (2013).
 86 However, as discussed in LW16 and as will be briefly reviewed in Section 2, the
 87 effects of Langmuir mixing are represented by an enhancement factor, which
 88 is a nonlinear function of the wave parameters (e.g., Stokes drift) as well as
 89 other parameters (e.g., surface friction velocity, boundary layer depth, etc.).
 90 Deriving enhancement factors from a wave climatology neglects some necessary
 91 cross-correlations among the wave parameters (Myrhaug, 2013, 2015). There-
 92 fore, a climatology of the enhancement factor itself, which is estimated from

93 an ocean-wave coupled simulation, is assessed here. The feasibility of this ap-
 94 proach is supported by observational evidence that the surface wave induced
 95 enhancement of vertical mixing within the ocean surface boundary layer does
 96 not significantly correlate with other parameters on short timescales of days
 97 to weeks (D’Asaro, 2001; Tseng and D’Asaro, 2004). Remaining issues in this
 98 approach, to be addressed here, are how stable such a climatological estimate
 99 is under climate change, what temporal and spatial resolution is required for
 100 good performance of the climatology, and whether the feedback from climate
 101 variables onto the enhancement factor are significant.

102 The structure of this paper is as follows. The Langmuir mixing parameteri-
 103 zation scheme from LW16 is briefly reviewed in Section 2. In Sections 3 and 4,
 104 two options to approximate the Langmuir mixing effects which are based on (i)
 105 an enhancement factor climatology and (ii) empirical wave spectra are explored
 106 as computationally cheaper substitutes for the prognostic wave model in LW16.
 107 The results from implementing these two options in CESM are presented in
 108 Section 5, followed by a discussion in Section 6. This paper ends with the main
 109 conclusions in Section 7. Note that the statistical modeling approaches explored
 110 in this paper aim to represent only Langmuir mixing effects. Whether they work
 111 or not in this regard does not imply applicability for other ocean surface wave
 112 effects.

113 **2. Langmuir mixing parameterization and basic simulation**

114 Classical scaling suggests that the bulk vertical turbulent kinetic energy
 115 (VKE) in the ocean surface boundary layer (OSBL), $\langle w^2 \rangle$, scales with the
 116 squared friction velocity, $u^{*2} \equiv |\boldsymbol{\tau}|/\rho_0$, where $\boldsymbol{\tau}$ is the surface wind stress and
 117 ρ_0 is the surface ocean density. The ratio of these two, $\langle w^2 \rangle/u^{*2}$, is suggested
 118 to be about 0.6, based mostly on studies of the atmospheric boundary layer
 119 and laboratory experiments. However, observation (D’Asaro, 2001; Tseng and
 120 D’Asaro, 2004; D’Asaro et al., 2014) and large eddy simulation (LES) based
 121 studies (Skylingstad and Denbo, 1995; McWilliams et al., 1997; Harcourt and

122 D’Asaro, 2008; Grant and Belcher, 2009; Van Roekel et al., 2012), show that
 123 for the OSBL in the presence of surface waves, this ratio can be significantly
 124 greater than the canonical value, due to additional energy extracted from sur-
 125 face waves. This behavior was also confirmed by the rapid distortion theory
 126 (RDT) calculations of Teixeira (2011). As the real OSBL is likely to be in the
 127 regime of parameter space where the Langmuir turbulence competes with both
 128 convection- and shear-driven turbulence in order to energize vertical turbulent
 129 mixing (Li et al., 2005; Belcher et al., 2012), an OSBL model without surface
 130 waves, such as the *K*-Profile Parameterization (KPP, Large et al., 1994), will
 131 likely underestimate vertical mixing within the OSBL.

132 McWilliams and Sullivan (2000) suggest that the extra mixing induced by
 133 the Langmuir turbulence can be parameterized by applying an enhancement
 134 factor, $\mathcal{E}$, to the turbulent velocity scale used in KPP,

$$W = \frac{ku^*}{\phi} \mathcal{E}, \quad (1)$$

135 with $k = 0.4$ the von Kármán constant and ϕ the dimensionless flux profile.
 136 Here W is the velocity scale from which the vertical eddy diffusivity in KPP
 137 is defined. It essentially represents the root mean square value of the vertical
 138 turbulent velocity, and retains its dependence on winds and convection through
 139 u^* and the convective velocity scale w^* (see more detail in Large et al., 1994).
 140 McWilliams and Sullivan (2000) propose a formula for $\mathcal{E}$,

$$\mathcal{E} = \sqrt{1 + 0.08La_t^{-4}}, \quad (2)$$

141 to fit a few LES results, where $La_t = \sqrt{u^*/u_0^S}$ is the turbulent Langmuir number
 142 (McWilliams et al., 1997) and $u_0^S = |\mathbf{u}^S(0)|$ the magnitude of the surface Stokes
 143 drift velocity. By incorporating WAVEWATCH III into CESM, LW16 find that
 144 (2), as well as a modified formula by Smyth et al. (2002), generally introduce too
 145 much mixing in the extratropical regions in CESM, though sensitivity differs in
 146 other models (Fan and Griffies, 2014).

147 Based on the LES studies by Harcourt and D’Asaro (2008) and Van Roekel

et al. (2012), LW16 propose a formula,

$$\mathcal{E} = |\cos \alpha| \sqrt{1 + (1.5La_{\text{SL,proj}})^{-2} + (5.4La_{\text{SL,proj}})^{-4}}. \quad (3)$$

The surface layer averaged and projected Langmuir number,

$$La_{\text{SL,proj}} = \sqrt{\frac{u^* \cos(\alpha)}{|\langle \mathbf{u}^{\text{S}} \rangle_{\text{SL}}| \cos(\theta_{\text{ww}} - \alpha)}}, \quad (4)$$

is used to account for both the effects of the Stokes depth and the wind-wave misalignment. Here, θ_{ww} is the angle between wind and waves, and $\langle \mathbf{u}^{\text{S}} \rangle_{\text{SL}}$ the surface layer averaged Stokes drift,

$$\langle \mathbf{u}^{\text{S}} \rangle_{\text{SL}} = \frac{1}{H_{\text{SL}}} \int_{-H_{\text{SL}}}^0 \mathbf{u}^{\text{S}}(z) dz, \quad (5)$$

with $\mathbf{u}^{\text{S}}(z)$ the Stokes drift profile and H_{SL} the surface layer depth, taken here as proportional to the critical Richardson number boundary layer depth calculated by KPP: $H_{\text{SL}} = 0.2H_{\text{BL}}$. The angle between wind and Langmuir cells α is estimated based on the law of the wall (Van Roekel et al., 2012),

$$\alpha \approx \tan^{-1} \left[\frac{\sin(\theta_{\text{ww}})}{\frac{u^*}{u_0^{\text{S}} k} \ln(|H_{\text{BL}}/z_1|) + \cos(\theta_{\text{ww}})} \right], \quad (6)$$

with z_1 the onset depth of the law of the wall, which is taken as four times the significant wave height (Thorpe, 2007).

Note that the enhancement factor applied to the turbulent velocity scale is the only modification to KPP here. The particular coefficient choices in (3) are trained against the LES results with and without waves (Van Roekel et al., 2012; Harcourt and D’Asaro, 2008). No retuning of coefficients is allowed to improve the climate model after values are set to match the LES results. Notable alternative approaches, such as paying more attention to the wave-free state by retuning the parameters in KPP to reproduce the LES results without wave forcing (Reichl et al., 2016) and changing the KPP boundary layer depth in LW16 to simulate the wave-driven entrainment, can be used readily in the framework developed here. However, a more comprehensive set of LES is required to justify these approaches and the examination of these approaches

170 is beyond the scope of the present work, which focuses solely on changing the
171 source of wave information without changes to parameterization settings.

172 Results from two sets of simulations will be discussed in this paper: the
173 Ocean-Wave only simulations and the Atmosphere-Ocean-Land-Ice-Wave fully
174 coupled simulations. Both consist of a control simulation without Langmuir mix-
175 ing parameterization (CTRL) and simulations with Langmuir mixing enhance-
176 ment factor from various sources: a prognostic wave model, WAVEWATCH III
177 (WW3), the Data Wave climatology (DWAV) and the Theory Wave estimate
178 (TWAV). All simulations are summarized in Table 2 and will be detailed in the
179 following three sections.

180 In the Ocean-Wave only simulations, the ocean model was running on a
181 nominal 1° resolution grid under CORE-II inter-annual forcing (62 years forc-
182 ing cycle). A suffix, FULL, is used for the simulation with the Data Wave
183 climatology to distinguish it from the four sensitivity test simulations detailed
184 in Section 5.2. All the simulations with the Langmuir mixing parameteriza-
185 tion were branched from the control simulation at the beginning of the model
186 year 195 (corresponding to the forcing year 1956) and run throughout a fourth
187 CORE-II inter-annual forcing cycle to the model year 248 (forcing year 2009).
188 The data from the last 50 years of the simulation (corresponding to forcing year
189 1960 to 2009) is analyzed here.

190 The configurations of the fully coupled simulations are the same as described
191 in LW16, except with higher resolution. Here a roughly 2° grid is used for the
192 atmosphere and the land model, whereas a nominal 1° grid is used for the sea
193 ice and the ocean model. Simulations with the pre-industrial forcing scenario
194 and the 20th century transient scenario are prefixed by PI and 20C, respectively.
195 In the former scenario, the greenhouse gas concentrations were held fixed at the
196 pre-industrial level, and simulations with Langmuir mixing parameterization
197 were branched from the control simulation at the beginning of the model year
198 21 and run for 80 years. Whereas in the latter scenario, the greenhouse gas
199 concentrations history since 1850 were applied. For both scenarios the data
200 from the last 30 years of the simulations are analyzed.

201 3. The Data Wave Model

202 3.1. Enhancement factor statistics

203 As the enhancement factor, $\mathcal{E}$, is the only wave parameter in our simple
 204 Langmuir mixing parameterization, a deeper look at its statistics is warranted.
 205 The essential hypothesis to be tested here is that a *data wave model*, i.e., a
 206 climatology of $\mathcal{E}$, can reproduce the Langmuir mixing effects introduced by the
 207 computationally expensive *prognostic wave model*.

208 A practical data wave model for Langmuir mixing requires that (i) the
 209 ocean model should not be too sensitive to high-frequency variations of $\mathcal{E}$, or
 210 to covariances of $\mathcal{E}$ and other variables (u^* , MLD, etc.) that occur on short
 211 timescales, and (ii) the low-frequency $\mathcal{E}$ climatology should be robust under cli-
 212 mate change (i.e., changes in forcing such as wind and temperature). Of course,
 213 high-frequency and climate-induced changes to u^* , temperature, etc., are *ex-*
 214 *pected*, but the data wave model will pass this variability along unhindered—it
 215 relies only on stability of the enhancement factor. Since monthly mean statistics
 216 are typically examined in climate models as they average over synoptic weather,
 217 a month is taken as a reasonable timescale dividing the high- and low-frequency.
 218 Conveniently, a month also lies between the zero-crossing and integral timescales
 219 of the Stokes drift autocorrelation¹ (Haney et al., 2015).

220 The first requirement may result from either an extremely fast linear re-
 221 sponse or an extremely slow response that is not necessarily linear. By design,
 222 $\mathcal{E}$ applied to the turbulent velocity scale in KPP instantaneously changes the
 223 boundary layer depth. Tests varying the sampling frequency of $\mathcal{E}$ from three
 224 hours to a month show that the monthly mean ocean model response is not sen-
 225 sitive to high frequency variations of $\mathcal{E}$. On the other hand, observations show
 226 no significant correlation between the amount of surface wave induced enhance-
 227 ment of vertical mixing and other wind or wave properties on timescales of days

¹The zero-crossing timescale is defined as the time lag at which the autocorrelation function first makes a zero-crossing. The integral timescale is defined as $\tau = \int_0^\infty c(t)dt$, where $c(t)$ is the autocorrelation function with lag t (Tennekes and Lumley, 1972).

228 to weeks (D’Asaro, 2001; Tseng and D’Asaro, 2004). One may speculate that
 229 this is a result of the relatively slow evolution of major shifts in the boundary
 230 layer depth or seasonal regime and the fact that faster changes needed are well
 231 approximated by changes to u^* or convective forcing without the need to also
 232 vary the enhancement factor.

233 The second requirement is tested by analyzing the low frequency statistics
 234 of $\mathcal{E}$ from an ocean-wave coupled simulation. This simulation, denoted as WW3
 235 hereafter, was forced by the inter-annually varying Coordinated Ocean-ice Ref-
 236 erence Experiments phase II atmospheric datasets (CORE-II, Large and Yeager,
 237 2009). The coupled system is virtually the same as described in LW16, and the
 238 Langmuir mixing parameterization scheme corresponds to the case VR12-MA
 239 therein, as outlined in Section 2. The only difference is that here $\mathcal{E}$ is calcu-
 240 lated in the wave model, and is the only wave variable passed on to the ocean
 241 model. This code modification does not significantly influence results. How-
 242 ever, under this code algorithm a data wave model is easily implemented, as
 243 the statistics of $\mathcal{E}$ simply replace the prognostic wave model $\mathcal{E}$. The nonlinear
 244 relations in (3)—(6) make it impossible to reconstruct the $\mathcal{E}$ climatology from
 245 separate climatologies of La_t , α , H_{BL} and the spectral significant wave height,
 246 H_{m0} , because these variables are significantly related (Myrhaug, 2013, 2015).

247 To evaluate the robustness under changes in the mean wind forcing, the $\mathcal{E}$
 248 statistics from the last 10 years of a prognostic wave model simulation (corre-
 249 sponding to forcing year 2000-2009, denoted as period P_2) are compared with
 250 that from the first 10 years (corresponding forcing year 1960-1969, denoted as
 251 period P_1). Table 3 shows the changes in the 10-m wind, the friction velocity,
 252 the significant wave height, the surface Stokes drift magnitude and the enhance-
 253 ment factor, respectively, in different regions. The differences between these
 254 two periods are measured by two methods, the normalized mean difference,
 255
$$\text{NMD} = \frac{\overline{x(P_2)} - \overline{x(P_1)}}{\overline{x(P_1)}} \times 100\%,$$
 and the normalized root mean square difference,
 256
$$\text{NRMSD} = \sqrt{\left(\frac{\overline{x(P_2)} - \overline{x(P_1)}}{\overline{x(P_1)}} \right)^2} \times 100\%,$$
 in which $\overline{(\)}$ represents the spatial and tem-
 257 poral average over 10 years and x represents any of the wave variables. As

258 seen in Table 3, during the 50-year analysis window, the global 10-year mean
 259 wind increases by 2.5%, correspondingly, significant wave height 3.9%, Stokes
 260 drift 3.8%. However, the mean enhancement factor is virtually unchanged.
 261 This difference among wave statistics in response to the changing wind forcing
 262 is particularly significant for the Southern Hemisphere extra-tropical regions
 263 (consistent with Hemer et al., 2013). The normalized root mean square differ-
 264 ence shows a similar lack of sensitivity in enhancement factor when compared
 265 to the other wave and boundary layer statistics. The enhancement factor is
 266 more stable than other wave statistics under changes in the mean wind forcing
 267 and presumably also under projected climate change (e.g., Hemer et al., 2013;
 268 Fan et al., 2014). This result is partially expected because $\mathcal{E}$ is a dimensionless
 269 number describing a relation between the Langmuir mixing and the wind forc-
 270 ing, rather than a wave state that depends directly on the forcing, although it
 271 is certainly possible for dimensionless numbers to change as much as, or more
 272 than, their constituents. While the enhancement factor is found on the fly from
 273 a combination of more variable physical parameters, such as u^* , apparently the
 274 majority of the variability in wave forcing is captured without variation in the
 275 enhancement factor. To the extent that such a climatology is based purely on
 276 basin geometry, large-scale wind patterns (e.g., storm tracks), and seasonality,
 277 it should be robust in the face of climate change—indeed it is only imperceptibly
 278 changed over 50 years of intensive late 20th century climate change. While im-
 279 portant covariances in wave parameters make $\mathcal{E}$ stable, $\mathcal{E}$ itself has no important
 280 covariances with the other variables. Therefore, a data model for the Langmuir
 281 mixing effects is possible.

282 3.2. An enhancement factor climatology

283 A monthly $\mathcal{E}$ climatology is created from the simulation WW3, by taking
 284 the monthly average of the 6-hourly output of $\mathcal{E}$ over the last 10 years of the
 285 simulation (2000-2009). One could take the monthly average over a different
 286 10-year time window or make the average window longer, but the differences in
 287 the resulting $\mathcal{E}$ climatology are trivial. Grid points with sea ice fraction greater

288 than 0.05 for longer than half the average window are eliminated from the
 289 average. Consistently, the same ice fraction threshold is used when applying the
 290 enhancement factor in the ocean model to make sure the Langmuir mixing effects
 291 only being applied to ice free regions. Note that this sea ice threshold is much
 292 smaller than that used in WAVEWATCH III (0.67, with another smaller value
 293 of 0.33 for adding ice to obstructions). One may infer from this large range that
 294 there is considerable uncertainty about the effects of Langmuir turbulence near
 295 the sea ice margin. Physically, surface waves are attenuated in the marginal ice
 296 zone due to viscous damping and multiple wave scattering by floe edges (Squire
 297 and Moore, 1980; Bennetts and Squire, 2012), and a wave-ice interaction model
 298 is required to represent these effects (See, e.g., Williams et al., 2013).

299 This monthly $\mathcal{E}$ climatology is illustrated in Fig. 1 and is termed the *Data*
 300 *Wave* model hereafter. The most striking feature of the Data Wave model is
 301 that it has a strong seasonal cycle, which will be shown to have non-negligible
 302 effects. Its value is relatively big in local summer and small in local winter. At
 303 first, this pattern is counterintuitive, as storms are most vigorous in local winter
 304 and so are the waves. However, as will be detailed in Section 6, $\mathcal{E}$ measures the
 305 relative effects of waves to other forcing such as wind and convective forcing.
 306 In winter convection dominates and waves are not expected to have as large
 307 relative effects as in summer (as observations show in Belcher et al., 2012).
 308 This behavior is built into the surface layer averaged Stokes drift (4). Another
 309 important feature is that $\mathcal{E}$ is relatively small as compared with previous studies
 310 (e.g. Fan and Griffies, 2014), in which a fully developed sea and aligned wind
 311 and waves are assumed. For comparison, using the typical value of $La_t = 0.3$
 312 which results from empirical scalings for fully-developed waves (Pierson and
 313 Moskowitz, 1964) and assuming wind and wave alignment, (2) gives $\mathcal{E} = 3.30$
 314 while the equivalent from Van Roekel et al. (2012) gives $\mathcal{E} = 1.52$. The values
 315 in Fig. 1 rarely exceed $\mathcal{E} = 1.5$. As shown in LW16 these assumptions cause too
 316 much mixing.

317 A comparison between the Data Wave and the original 10 years of prognostic
 318 model data from the 6-hourly WW3 data is made in the density-shaded scatter

319 plots of $\mathcal{E}$, u_0^S and the magnitude of the surface layer averaged Stokes drift
 320 $u_{SL}^S = |\langle \mathbf{u}^S \rangle_{SL}|$ in Figs. 2a, 2b and 2c. The yellow, green and blue color represent
 321 the highest 0-30%, 31-60% and 61-90% centered distribution², respectively. The
 322 monthly mean data in the Data Wave climatology is linearly interpolated back to
 323 a 6-hourly frequency to match sample size with the WW3 data. As expected, the
 324 monthly mean removes the extreme values and therefore all the three variables
 325 in the Data Wave climatology have a narrower distribution than the WW3 data.
 326 Despite the spread of the scatter, the distributions for $\mathcal{E}$ and u_0^S are centered on
 327 the one-to-one reference line, indicating no significant bias is introduced to these
 328 variables by the monthly mean. However, the Data Wave overestimates u_{SL}^S ,
 329 presumably due to the fact that the intermittent storm-induced swells affect
 330 u_{SL}^S more than u_0^S .

331 The Data Wave model only approximates $\mathcal{E}$ within the limits of the simu-
 332 lation quality of WW3. Practically, the detailed spatial distribution of $\mathcal{E}$ could
 333 be different from that shown in Fig. 1 if the simulated large scale wind and
 334 wave patterns were different, for example, in the case of fully coupled simu-
 335 lations with a prognostic atmosphere model, where wind is biased, or in the
 336 case of using a better wave model, where the wave physics is improved so that
 337 the simulated Stokes drift is different given the same wind forcing. However,
 338 those deviations are from the model biases in the simulated wind and waves.
 339 This is the main reason of deriving the $\mathcal{E}$ climatology from the ocean-wave only
 340 simulations (in which the wind forcing is based on observations) rather than
 341 fully coupled simulations. Potential improvement on the $\mathcal{E}$ climatology could
 342 probably be achieved by coupling the ocean model to a wave model with higher
 343 resolution and better wave physics, which is beyond the scope of this paper.
 344 An estimate of potential improvement from a high-resolution, data assimilating
 345 wave model will be presented in experiment DWAV-RA13 below.

²Note that the number of scatters within the yellow, green and blue areas are equal (30% of total number of scatters), but the density is highest in the yellow area.

346 4. The Theory Wave Model

347 While the Data Wave model mimics the mean effects from a prognostic wave
348 model, another approach towards a statistical modeling of the Langmuir mixing
349 effects is to approximate $\mathcal{E}$ from wave theory. The essential question this section
350 addresses is: can we reasonably approximate $\mathcal{E}$ from only the atmospheric and
351 oceanic parameters that are available in the climate model? From (3) - (6), the
352 ingredients to approximate $\mathcal{E}$ are the Stokes drift profile, the boundary layer
353 depth and the surface friction velocity. As the latter two are readily available
354 from the ocean model, only a reasonable approximation to the Stokes drift
355 profile remains. For simplicity Stokes drift from swell will not be explicitly
356 included in the derivation, though the bulk effect is roughly accounted for by a
357 magnitude loss coefficient (see more discussion in Section 4.3 and 4.4). Swell is
358 remotely generated and thus affected by a complex history of wind forcing and
359 wave propagation. Therefore, their accurate representation requires a prognostic
360 wave model.

361 4.1. Approximating the Stokes drift profile

362 The basis of approximating the Stokes drift profile from an empirical wave
363 spectrum is that the Stokes drift scales with the third moment³ of a depth-
364 dependent wave spectrum, which accentuates the higher frequency part of the
365 spectrum near the surface (Webb and Fox-Kemper, 2011). Breivik et al. (2014,
366 2016) show that a reasonable approximation to the full Stokes drift profile could
367 be made by using the Phillips spectrum (Phillips, 1958) with a simple peak cut-
368 off for low frequencies, based on the assumption that the Phillips spectrum
369 approximates the largest contribution to Stokes drift near the surface, the high
370 frequency part of the real spectrum, reasonably well. With the Phillips spec-

³The n th moment of the wave spectrum is defined as $m_n = \int_0^\infty \int_{-\pi}^\pi \omega^n S(\omega, \theta) d\theta d\omega$.

trum,

$$S_{\text{Phil}} = \begin{cases} \alpha g^2 \omega^{-5}, & \omega > \omega_p \\ 0, & \omega \leq \omega_p, \end{cases} \quad (7)$$

in which $\alpha = 0.0083$ is the Phillips' parameter, the Stokes drift profile is (Breivik et al., 2014, 2016)

$$\begin{aligned} \mathbf{u}_{\text{Phil}}^{\text{S}}(z) &= \hat{\mathbf{e}}^{\text{w}} \frac{2}{g} \int_0^\infty \omega^3 S_{\text{Phil}}(\omega) e^{2kz} d\omega \\ &= \hat{\mathbf{e}}^{\text{w}} u_{0,\text{Phil}}^{\text{S}} [T_1(k_p, z) - T_2(k_p, z)], \end{aligned} \quad (8)$$

where

$$\begin{aligned} T_1(k, z) &= e^{2kz}, \\ T_2(k, z) &= \sqrt{2\pi k|z|} \operatorname{erfc}\left(\sqrt{2k|z|}\right). \end{aligned} \quad (9) \quad (10)$$

Here $\hat{\mathbf{e}}^{\text{w}}$ is the (unidirectional) wave direction, for now assumed to be constant with depth, $u_{0,\text{Phil}}^{\text{S}} = |\mathbf{u}_{\text{Phil}}^{\text{S}}(0)| = 2\alpha g/\omega_p$ the surface Stokes drift, $\operatorname{erfc}(\cdot)$ the complementary error function, $k_p = \omega_p^2/g$ the peak wavenumber and $z \leq 0$ the depth. The deep-water wave dispersion relation $\omega^2 = gk$ is used.

While the unidirectional wave approximation is used here with the Phillips spectrum, as most other empirical wave spectra (Webb and Fox-Kemper, 2011), Webb and Fox-Kemper (2015) show that the unidirectional wave approximation tends to overestimate the magnitude of Stokes drift as wave spreading and multidirectional waves tend to prevent part of the wave energy from contributing to the Stokes drift. No swell is accounted for in the Phillips spectrum, and discussion of these effects is postponed until Section 4.3. Directional spreading of wind waves, however, can be approximated following Webb and Fox-Kemper (2015) who employed a directional-Stokes-drift-component derived from the spectral spreading function by Donelan et al. (1985), hereafter DHH,

$$\mathbf{u}_{\text{Phil-s}}^{\text{S}}(z) = \hat{\mathbf{e}}^{\text{w}} \frac{2}{g} \int_0^\infty H_{\text{DHH}}(\omega/\omega_p) \omega^3 S_{\text{Phil}}(\omega) e^{2kz} d\omega, \quad (11)$$

389 where

$$H_{\text{DHH}}(\omega/\omega_p) = \begin{cases} \frac{0.52-3.3(\omega/\omega_p)+8.9(\omega/\omega_p)^2}{1-3.4(\omega/\omega_p)+8.9(\omega/\omega_p)^2}, & 0.56 < \omega/\omega_p \leq 0.95, \\ \frac{0.98-0.19(\omega/\omega_p)+0.0058(\omega/\omega_p)^2}{1-0.26(\omega/\omega_p)+0.12(\omega/\omega_p)^2}, & 0.95 < \omega/\omega_p < 1.6, \\ 0.777, & \text{otherwise,} \end{cases} \quad (12)$$

390 is the Padé-approximated, DHH directional-Stokes-drift-component. Here the
 391 mean wave direction is chosen to be $\hat{e}^w$ (which is constant with depth when
 392 only directional spreading is considered). Substituting (7) and (12) into (11)
 393 (details in Appendices A and B.1),

$$\begin{aligned} \mathbf{u}_{\text{Phil-s}}^S(z) &\approx \hat{e}^w u_{0,\text{Phil-s}}^S \\ &\times \begin{cases} 1, & z = 0, \\ 1.26 [T_1(k_p, z) - B_1(z)T_2(k_p, z)] \\ -0.188 [T_1(k_p^*, z) - B_2(z)T_2(k_p^*, z)], & z < 0 \end{cases} \end{aligned} \quad (13)$$

394 where

$$B_1(z) = 1 + \frac{0.0469}{2k_p|z|}, \quad (14)$$

$$B_2(z) = 1 + \frac{0.503}{2k_p^*|z|}. \quad (15)$$

395 Here $k_p^* = 1.6^2 k_p = 2.56 k_p$ and $u_{0,\text{Phil-s}}^S = |\mathbf{u}_{\text{Phil-s}}^S(0)| \approx 0.811 u_{0,\text{Phil}}^S$ is the
 396 surface Stokes drift including spreading effects. The effect of spreading is to
 397 reduce the surface Stokes drift by nearly 20%.

398 The Stokes drift averaged over a layer from $z = z_0$ ($z_0 < 0$) to $z = 0$ is

399 (details in Appendix B.2),

$$\begin{aligned}
\langle \mathbf{u}_{\text{Phil-s}}^{\text{S}} \rangle_{z_0} \approx & \hat{\mathbf{e}}^{\text{w}} u_{0,\text{Phil-s}}^{\text{S}} \left\{ 0.715 \right. \\
& + \left(\frac{0.151}{k_{\text{p}}|z_0|} - 0.840 \right) [1 - T_1(k_{\text{p}}, z_0)] \\
& - \left(0.840 + \frac{0.0591}{k_{\text{p}}|z_0|} \right) T_2(k_{\text{p}}, z_0) \\
& + \left(\frac{0.0632}{k_{\text{p}}^*|z_0|} + 0.125 \right) [1 - T_1(k_{\text{p}}^*, z_0)] \\
& \left. + \left(0.125 + \frac{0.0946}{k_{\text{p}}^*|z_0|} \right) T_2(k_{\text{p}}^*, z_0) \right\}. \tag{16}
\end{aligned}$$

400 This formula is needed to evaluate the Theory Wave $La_{\text{SL,proj}}$ for an arbitrary
401 boundary layer depth using (4).

402 4.2. Estimating the peak wavenumber

403 In (16) the only unknowns are the peak wavenumber, k_{p} , and the surface
404 magnitude of the Stokes drift, $u_{0,\text{Phil-s}}^{\text{S}}$. The peak wavenumber could be es-
405 timated from the Pierson-Moskowitz spectrum (Pierson and Moskowitz, 1964;
406 Bouws, 1998). But a more direct constraint for our purposes determines the
407 peak wavenumber and other parameters by requiring that the Stokes drift pro-
408 file yields a given Stokes transport and a given surface Stokes drift. Following
409 Breivik et al. (2016), a general peak wavenumber could be estimated from the
410 surface Stokes drift and the total Stokes transport,

$$k_{\text{Phil}} = \frac{u_{0,\text{Phil}}^{\text{S}}}{2V^{\text{S}}} \left(1 - \frac{2\beta}{3} \right) \approx 0.167 \frac{u_{0,\text{Phil}}^{\text{S}}}{V^{\text{S}}} \approx 0.206 \frac{u_{0,\text{Phil-s}}^{\text{S}}}{V^{\text{S}}}, \tag{17}$$

411 where $V^{\text{S}} = |\mathbf{V}^{\text{S}}|$, and $\mathbf{V}^{\text{S}} = \int_{-\infty}^0 \mathbf{u}^{\text{S}}(z) dz$. Breivik et al. (2016) find that $\beta = 1$
412 is an adequate choice, as assumed here.

413 Again, no directional spreading effect is accounted for in (17). An improved
414 formula for the general peak wavenumber with directional spreading effects may
415 be derived by integrating (13), instead of (8), from $z = -\infty$ to $z = 0$ to find
416 (details in Appendix B.3)

$$k_{\text{Phil-s}} \approx 0.176 \frac{u_{0,\text{Phil-s}}^{\text{S}}}{V^{\text{S}}}. \tag{18}$$

417 Including directional spreading here is a modest effect, as seen by comparing
418 (18) to (17).

419 4.3. Stokes transport and surface Stokes drift

420 To close the Theory Wave model, approximations are needed to determine
421 the Stokes transport and the surface Stokes drift from variables available in a
422 climate model. The depth-integrated Stokes drift from the Pierson-Moskowitz
423 spectrum can be used to infer Stokes transport from wind speed, as previously
424 used and assessed for the climate model applications (McWilliams and Restrepo,
425 1999; Webb and Fox-Kemper, 2011). The resulting Stokes transport scales with
426 the first moment of the wave spectrum, or equivalently the mean frequency and
427 significant wave height (e.g. Breivik et al., 2014, 2015),

$$V^S \approx 2\pi c_1 m_1 = \frac{2\pi c_1}{16} \bar{f} H_{m_0}^2. \quad (19)$$

428 Here, c_1 is a magnitude loss coefficient accounting for directional spreading fur-
429 ther discussed below. For the Pierson-Moskowitz spectrum the mean frequency
430 is related to the peak frequency (Webb and Fox-Kemper, 2011) by

$$\bar{f} = \frac{m_1}{m_0} \approx 1.296 f_p. \quad (20)$$

431 The peak frequency and the significant wave height from Pierson-Moskowitz
432 spectrum (Bouws, 1998) are, respectively,

$$f_p \approx 0.877 \frac{g}{2\pi U_{19.5}} \approx 0.816 \frac{g}{2\pi U_{10}}, \quad (21)$$

433 and

$$H_{m_0} = 4\sqrt{m_0} \approx 0.0246 U_{10}^2. \quad (22)$$

434 Here $U_{19.5}$ is the wind speed at 19.5 m above the sea surface in m s^{-1} and H_{m_0}
435 in m. A generally accepted empirical relation, $U_{19.5} \approx 1.075 U_{10}$ (Holthuijsen,
436 2007), is used in (21).

437 While the dominant scaling relationships have been laid out, (19) also in-
438 cludes a magnitude loss coefficient $c_1 < 1$ to account for a number of (presumed

constant) factors that make the unidirectional estimate $2\pi m_1$ an overestimation for the Stokes transport. Some important factors are: (i) ignoring the directional spreading effects overestimates the Stokes transport by about 17% (Ardhuin et al., 2009; Breivik et al., 2014; Webb and Fox-Kemper, 2015); (ii) ignoring the effects from swell by the use of empirical spectrum overestimates the Stokes transport (Webb and Fox-Kemper (2015) report a combined effect of directional spreading and multidirectional waves corresponding to $c_1 = 0.698$); (iii) using the mean frequency and significant wave height from Pierson-Moskowitz spectrum, which represents fully developed wind-waves, overestimates m_1 for real ocean surface waves. By evaluating this estimate as compared to the 3-hourly Stokes transport data over a year from a global wave hindcast simulation with WAVEWATCH III (Rascle and Ardhuin, 2013, denoted as RA13 hereafter), $c_1 = 0.667$ reasonably accounts for the combined effects mentioned and appears to correct the overestimation, as illustrated in Fig. 3a. Furthermore, a crude approximation of swell on the Stokes drift versus the Phillips wind-wave spectrum is accounted for in tuning the magnitude loss coefficient c_1 against RA13. Improvement here is certainly possible, but the goal is to evaluate how well a rough estimate of Langmuir mixing performs.

Finally, an approximation for surface Stokes drift is needed. For simplicity it is assumed to be proportional to the 10-meter wind speed,

$$u_{0,\text{Wind}}^{\text{S}} \approx c_2 U_{10}. \quad (23)$$

A proportionality constant of $c_2 = 0.016$ is found to agree with the surface Stokes drift from RA13, as illustrated in Fig. 3b. Note that only the slope in Fig. 3b depends on c_2 , and the offset cannot be corrected by the simple approach used here. The systematic overestimation by (23) might be due to contributions from swell, which often reduce the surface Stokes drift (Webb and Fox-Kemper, 2015).

465 *4.4. Predicting the enhancement factor from U_{10} , u^* and H_{BL}*

466 With $z_0 = -0.2H_{BL}$ being the depth of surface layer in (16), and $\alpha = \theta_{ww} =$
 467 0 in (3) and (4), we now have a formula for $\mathcal{E}$ as a function of the 10-meter
 468 wind, U_{10} , the surface friction velocity, u^* , and the boundary layer depth, H_{BL} ,

$$\mathcal{E}_{\text{Theory}} = \mathcal{F}(U_{10}, u^*, H_{BL}). \quad (24)$$

469 Normally, U_{10} , u^* and H_{BL} are readily available from any ocean model. There-
 470 fore, (24) offers a solution for parameterizing the effects of Langmuir mixing
 471 without coupling with a prognostic wave model. This approximation to $\mathcal{E}$ is
 472 termed the *Theory Wave* model hereafter.

473 Note that both the directional spreading of wind-waves and the remotely
 474 generated swell lead to the misalignment of wind and waves, reducing the mean
 475 Stokes drift magnitude by 14~20% and 7~23%, respectively (Webb and Fox-
 476 Kemper, 2015). Both effects are approximated in the Theory Wave model.
 477 The adoption of $\alpha = \theta_{ww} = 0$ here should be distinguished from assuming
 478 aligned wind and waves such as in (26) introduced in the next section. The
 479 systematic overestimation of the enhancement factor by assuming $\alpha = \theta_{ww} = 0$
 480 is compensated by the adoption the magnitude loss coefficient, c_1 , in (19).

481 For quick reference and easy implementation, all the essential equations re-

482 quired in the Theory Wave model, (24), are summarized below.

$$\begin{aligned}
u_0^S &\approx 0.016 U_{10}, \\
V^S &\approx 2.67 \times 10^{-5} g U_{10}^3, \\
k_p &\approx 0.176 \frac{u_0^S}{V^S}, \\
k_p^* &= 2.56 k_p, \\
H_{SL} &= H_{BL}/5, \\
T_1(k, z) &= e^{2kz}, \\
T_2(k, z) &= \sqrt{2\pi k|z|} \operatorname{erfc}\left(\sqrt{2k|z|}\right), \\
u_{SL}^S &\approx u_0^S \left\{ 0.715 \right. \\
&\quad + \left(\frac{0.151}{k_p H_{SL}} - 0.840 \right) [1 - T_1(k_p, H_{SL})] \\
&\quad - \left(0.840 + \frac{0.0591}{k_p H_{SL}} \right) T_2(k_p, H_{SL}) \\
&\quad + \left(\frac{0.0632}{k_p^* H_{SL}} + 0.125 \right) [1 - T_1(k_p^*, H_{SL})] \\
&\quad \left. + \left(0.125 + \frac{0.0946}{k_p^* H_{SL}} \right) T_2(k_p^*, H_{SL}) \right\}, \\
La_{SL} &= \sqrt{\frac{u^*}{u_{SL}^S}}, \\
\mathcal{E} &= \sqrt{1 + (1.5 La_{SL})^{-2} + (5.4 La_{SL})^{-4}}. \tag{25}
\end{aligned}$$

483 All the coefficients are dimensionless except that in the approximation of the
484 Stokes transport, V^S , which has units of $\text{m}^{-2} \text{s}^{-4}$.

485 A comparison between the Theory Wave model and the WW3 simulation is
486 shown by density-shaded scatter plots of $\mathcal{E}$, u_0^S and u_{SL}^S in Fig. 2d, 2e and 2f.
487 Each corresponding variable from the Theory Wave model are calculated from
488 (25) with the 6-hourly WW3 data of U_{10} , u^* and H_{BL} . The $\mathcal{E}$ distribution of
489 the Theory Wave model resembles the WW3 data, with consistent magnitude
490 and variability. A prime discrepancy is that the Theory Wave model does not
491 allow $\mathcal{E} < 1$ whereas WW3 does. This discrepancy is negligible overall (see next

section). Similar to the comparison with RA13 in Fig. 3b, u_0^S follows WW3 data except for an offset leading to systematic overestimation, likely from swell contributions (Fig. 2e). The Theory Wave prediction for u_{SL}^S also agrees with the WW3 data (Fig. 2f), especially for large values. The underestimation of u_{SL}^S at small values has relatively little impact on the resulting enhancement factor.

Fig. 4 shows the comparison of the 10-year averaged enhancement factor map in January and July between the Theory Wave model and the Data Wave model. Generally the Theory Wave estimates match the spatial variability and the seasonality of the enhancement factor in the Data Wave climatology, except for modest overestimation. Beyond comparing just the climatological mean, the enhancement factor from the Theory Wave estimate has a broader distribution than that from the Data Wave climatology (See Fig. 2a,d).

5. Data Wave climatology and Theory Wave estimates in CESM

5.1. Comparing with WAVEWATCH III

Both the Data Wave climatology and the Theory Wave estimates are implemented in the Community Ocean Vertical Mixing Project (CVMix, <https://github.com/CVMix>) and tested in CESM. Appendix C details the CVMix implementation. Two ocean-wave simulations forced by the inter-annually varying CORE-II datasets were performed, denoted as DWAV-FULL and TWAV. These two simulations are identical to WW3 except that WAVEWATCH III is replaced by the Data Wave climatology and the Theory Wave estimates, respectively, as the source of the enhancement factor. All simulations are summarized in Table 2.

Fig. 5 and 6 show the summer and winter mean MLD in both hemispheres in observations, CTRL, WW3, DWAV-FULL and TWAV. The observations are from de Boyer Montégut et al. (2004), updated to include Argo data up through 2012. As in LW16, the definition of mixed layer follows the density threshold method of de Boyer Montégut et al. (2004), locating the depth where the potential density (referenced to surface) changes by 0.03 kg m^{-3} from its surface value. This definition allows direct comparison of the model results

521 with observations. Note that Fig. 5b, 5c and 6b, 6c correspond to Fig. 3c, 3g
 522 and Fig. 4c, 4g in LW16, respectively, except that here the ocean model uses a
 523 finer nominal 1° grid and the color scale is slightly modified. The results here
 524 agree with LW16, suggesting that resolution does not significantly affect the
 525 parameterizations of Langmuir mixing, at least while holding the wave model
 526 resolution fixed.

527 Fig. 5d, 5e and 6d, 6e show the results for the case DWAV-FULL and TWAV.
 528 They are difficult to distinguish from WW3. Thus, the Data Wave climatology
 529 and the Theory Wave estimate retain the Langmuir mixing effects on mean
 530 MLD from coupling with WAVEWATCH III.

531 As a direct comparison with Fig. 6 in LW16, Fig. 7 shows the zonal mean
 532 partial pressure of chlorofluorocarbon-11 (pCFC-11) over the Southern Hemi-
 533 sphere. Chlorofluorocarbons (CFC) are commonly used as passive tracers in
 534 the ocean model as a measure of the integrated effects of the air-sea interaction
 535 and mixing in the ocean (e.g., Fox-Kemper et al., 2011; Danabasoglu et al.,
 536 2012). The CFC concentrations in the atmosphere were set (realistically) near
 537 the end of the third CORE-II cycle in CTRL (at model year 170) and their
 538 concentration history is applied thereafter to the end of the fourth CORE-II
 539 cycle in all the simulations. This forcing allows the simulation results to be
 540 compared directly against observations from the Global Ocean Data Analysis
 541 Project (GLODAP, Key et al., 2004). As the CFC climatology in GLODAP
 542 is based on data collected during the World Ocean Circulation Experiment
 543 (WOCE) program from 1990 to 1998, the simulated pCFC-11 is averaged over
 544 the model year 233, corresponding to the forcing year 1994, providing a central
 545 estimate of the observation window for comparison.

546 Consistent with LW16, including the Langmuir mixing parameterization ap-
 547 pears to reduce the low concentration biases of pCFC-11 in the Southern Ocean
 548 to a modest extent. Most important here is the agreement in Langmuir mixing
 549 effects on pCFC-11 among the WW3, DWAV-FULL and TWAV.

550 Therefore, it appears that the Data Wave climatology and the Theory Wave
 551 estimate perform comparably in terms of Langmuir mixing effects to the prog-

552 nistic wave model WAVEWATCH III. In the following section, it will be demon-
553 strated that the consistency among these results is not random, and indeed it
554 is easy to generate other estimates that disagree more.

555 5.2. Significance assessment versus contrasting simulations

556 Four additional ocean-wave simulations were performed to assess the sen-
557 sitivity of the ocean model to variations in enhancement factor, denoted as
558 DWAV-CON1, DWAV-CON2, DWAV-AL and DWAV-RA13. To avoid the pos-
559 sible unphysical effects near the sea ice margin, the same sea ice mask from
560 DWAV-FULL was applied to these four simulations. Simulation details are
561 described below; Table 2 summarizes them.

562 Two simulations with constant enhancement factor at all locations at times
563 ($\mathcal{E} = 1.52$ and $\mathcal{E} = 1.33$) were performed, denoted as DWAV-CON1 and DWAV-
564 CON2, respectively. The former value results from assuming fully-developed
565 wind wave conditions with $La_t = 0.3$ and ignoring the effects of Stokes depth
566 and misaligned wind and waves. The latter value is the global and time mean
567 of the Data Wave climatology. Thus, comparing this simulation to the Data
568 Wave simulation quantifies the importance of spatial variability and seasonality
569 in the Data Wave climatology. The $\mathcal{E} = 1.33$ value also agrees with an en-
570 hancement factor of $\sqrt{1.75} \approx 1.32$ to $\sqrt{2} \approx 1.41$ as implied from the buoyant
571 float measurements of mean square vertical velocity by D’Asaro (2001); Tseng
572 and D’Asaro (2004), whereas the former ($\mathcal{E} = 1.52$) value is larger than these
573 observational estimates. The simulation DWAV-CON1 is contrasted against
574 TWAV to emphasize the effect of Stokes depth, and perhaps also misaligned
575 wind and waves to a lesser extent (as TWAV includes some spreading effects).
576 And DWAV-CON2 compared to DWAV-FULL reveals the importance of spatial
577 and seasonal variations. In addition, DWAV-CON1 and DWAV-CON2 together
578 offer an assessment of the ocean model sensitivity to the magnitude of the en-
579 hancement factor.

580 In contrast to all of the other simulations here, the monthly $\mathcal{E}$ climatology in
581 DWAV-RA13 is derived from the dataset of a higher resolution WAVEWATCH

III global wave hindcast with the latest source term parameterization (Raschle and Ardhuin, 2013). However, as this is not an ocean-wave coupled simulation and there is no boundary layer depth information in this dataset, the enhancement factor is estimated from surface values only, using a different formula optimized to neglect misalignment and Stokes depth effects (Van Roekel et al., 2012), which is

$$\mathcal{E}_a = \sqrt{1 + (3.1La_t)^{-2} + (5.4La_t)^{-4}}, \quad (26)$$

To evaluate the importance of high-resolution, another simulation denoted DWAV-AL uses this same formula but with the coarse resolution wave model used in WW3 and DWAV-FULL. This simulation corresponds to the VR12-AL parameterization scheme in LW16–neglecting wave-wind misalignment and Stokes depth effects. The greatest difference between DWAV-AL and DWAV-FULL simulations is that the former mixes too strongly in the winter hemisphere extratropical regions. Comparing the DWAV-RA13 and DWAV-AL simulations assesses the possibility for potential improvements in estimating the $\mathcal{E}$ climatology by using output data from wave simulations with higher resolution and probably better wave physics. Note that DWAV-RA13 makes use of better wave data, but of a poor Langmuir mixing parameterization scheme.

To be consistent with the Data Wave climatology the 10-year hindcast RA13 data from 2000 to 2009 was used. The calculation is performed on the original grid (0.5° spatial resolution) and averaged in time to get the monthly climatology. Then the monthly climatology is interpolated to the coarse resolution wave grid ($3.2^\circ \times 4^\circ$ spatial resolution) as used in our coupled system. As the averaged field is spatially smooth, the information loss by interpolating to the coarse resolution is limited.

The left panels of Fig. 8 illustrate the seasonal cycle of mean MLD biases as compared with observations in the Northern Hemisphere extratropical regions ($30^\circ\text{N} - 90^\circ\text{N}$), the equatorial regions ($30^\circ\text{S} - 30^\circ\text{N}$) and the Southern Hemisphere extratropical regions ($90^\circ\text{S} - 30^\circ\text{S}$), respectively. The right panels of Fig. 8 are the corresponding reductions of root mean square error (RMSE)

611 as compared with CTRL. The black curves in the left panels represent the
 612 mean biases of the CTRL in which no Langmuir mixing effect is included. Note
 613 the persistent shallow biases in the Southern Hemisphere extratropical regions
 614 (Fig. 8e). The red, blue and purple curves represent results for WW3, DWAV-
 615 FULL and TWAV, and the solid, dashed, dotted and dashed-dotted curves
 616 in gray are for the simulations DWAV-CON1, DWAV-CON2, DWAV-AL and
 617 DWAV-RA13, respectively. Note that adding the Langmuir mixing effects is a
 618 major change to modeled behavior, but distinctions among different Langmuir
 619 treatments are comparatively small. However, two groups of curves distinguish
 620 themselves, color versus gray in Fig. 8.

621 The consistency among WW3, DWAV-FULL and TWAV (colored curves)
 622 is remarkable when compared with other cases (gray curves). In particular,
 623 although the mean MLD biases in the Northern Hemisphere winter are increased
 624 in all cases as compared with CTRL, the three colored cases are clearly less
 625 biased (Fig. 8a). And more importantly, the reductions in RMSE are only found
 626 in the three colored cases (Fig. 8b). In the equatorial regions, the three colored
 627 cases introduce the least degradation, though DWAV-CON2 and DWAV-AL
 628 seems to be also indistinguishable from the colored cases (Fig. 8c, 8d). In the
 629 Southern Hemisphere extratropical regions, where improvements are found in
 630 all cases, the distinctions between the colored cases and the gray cases are clear
 631 in both the mean biases (Fig. 8e) and the RMSE reductions (Fig. 8f).

632 In all the regions throughout the year, DWAV-CON1 performs the worst.
 633 This is expected as an enhancement factor of $\mathcal{E} = 1.52$ overestimates the Lang-
 634 muir mixing effects in most cases, especially in winter. However, this does not
 635 imply that $La_t = 0.3$ everywhere is a bad approximation. In fact, $La_t = 0.3$
 636 is a reasonable estimate for most regions (See, e.g., Belcher et al., 2012). Note
 637 that in the Theory Wave, $c_2 = 0.016$ in (23) gives $La_t \approx 0.3$. It is ignoring the
 638 Stokes depth effects versus the boundary layer depth by scaling the VKE only
 639 with La_t , as well as misalignment, that contributes to the overestimation of the
 640 Langmuir mixing effects.

641 Simply tuning the constant enhancement factor to a smaller and more rea-

sonable value, e.g., $\mathcal{E} = 1.33$ in DWAV-CON2, appears to give a fair approximation to the Langmuir mixing effects, except during winter (summer) in the Northern (Southern) Hemisphere extratropical regions, when $\mathcal{E} = 1.33$ appears to be an overestimation (underestimation). Comparing DWAV-CON2 with DWAV-CON1, an increase of the enhancement factor from $\mathcal{E} = 1.33$ to $\mathcal{E} = 1.52$ roughly deepens the winter mean MLD in the extratropical regions by 15 m (Fig. 8a and 8e).

The relatively poor performance in DWAV-AL confirms the importance of the Stokes depth and misaligned wind and waves effects. The results are slightly better in DWAV-RA13 (presumably due to better simulated surface Stokes drift) suggesting potential improvements from utilizing a higher resolution prognostic wave model with better wave physics. However, DWAV-RA13 still performs significantly worse than DWAV-FULL, especially in the extratropical regions in winter, which suggests that the improvements from a better wave model are limited by the Langmuir mixing parameterization. Consistently, TWAV includes only the barest essentials of wave prediction physics, but includes the effects of Stokes depth and limited wind-wave misalignment, and delivers a closer approximation to DWAV-FULL and WW3 than DWAV-RA13.

The vertical profiles of the pCFC-11 RMSE reductions as compared with CTRL in different regions are shown in Fig. 9, in which a negative value corresponds to an improvement as compared with CTRL, whereas a positive value corresponds to a degradation. The color coding is the same as Fig. 8. The black dashed curve (CTRL*) in each subfigure represents the case in which the root mean square difference from the GLODAP observation (essentially representing the year 1994) is calculated with the simulated mean pCFC-11 from one year later (corresponding to the year 1995). Since the simulated ventilation is insufficient, as suggested by the low pCFC-11 concentration biases (Fig. 7), allowing the ocean to mix one year longer apparently reduces the root mean square difference from observations. This offers an estimate of the significance of the pCFC-11 RMSE reductions induced by parameterizing Langmuir mixing.

Generally, a greater reduction in the low pCFC-11 concentration biases at

intermediate depth is associated with a stronger degradation near the surface, all brought by a stronger winter mixing (e.g., comparing DWAV-CON1 with DWAV-CON2, or comparing DWAV-AL with DWAV-FULL). Clearly enhancing the vertical mixing within the ocean surface boundary layer alone cannot fix this problem, but fixes are left to future studies. The important results here are, again, that cases WW3, DWAV-FULL and TWAV (colored curves) agree with each other, and differentiate themselves from the other cases.

5.3. *Climate impacts in fully coupled simulations*

In this section, results from a set of the Atmosphere-Ocean-Land-Ice-Wave fully coupled simulations will be discussed, aiming at assessing the performance of the Data Wave climatology and the Theory Wave estimates in the face of the climate feedbacks. Possible feedbacks are as follows. Langmuir mixing deepens the MLD. As a result, the sea surface temperature (SST) and the subsurface temperature adjust accordingly. In the low- to mid-latitudes, SST drops but the subsurface temperature increases, associated with an increase in the surface heat flux into the ocean. Presumably, the surface wind will adjust to the changed surface conditions such as the atmospheric stability and feeds back to the waves, and thereby the Langmuir mixing.

Note that the impact of changing wind on the Langmuir mixing is twofold. On one hand, a change in wind modifies the enhancement factor by changing the mixing regimes with a different turbulent Langmuir number (e.g., [Belcher et al., 2012](#)) or changing the state of wind-wave equilibrium with a different wave age (e.g., [Harcourt and D’Asaro, 2008](#)). On the other hand, a change in wind also affects the net effects of Langmuir mixing even with a fixed enhancement factor by changing u^* in the turbulent velocity scale in KPP. A fixed enhancement factor corresponds to a fixed wind-wave equilibrium state, e.g., fully developed wind-waves described by [Pierson and Moskowitz \(1964\)](#). Both effects exist in the fully coupled simulation with a prognostic wave model, while only the latter one exists in the Data Wave climatological representation. In the case with Theory Wave estimates the former effect is partially represented, but not as in

703 a prognostic wave model, where the wind can modify the enhancement factor
704 through (24). However, since the enhancement factor is robust under change
705 in wind climate, as shown in Section 3.1, it is not expected that the former
706 effect is significant. In fact, as will be shown in this section, the coupled model
707 results with both the Data Wave climatology and the Theory Wave estimate
708 resemble those with WAVEWATCH III—that is, coupled model feedbacks onto
709 enhancement factor are not important.

710 Both the Data Wave climatology and the Theory Wave estimates were tested
711 under the pre-industrial forcing scenario. To begin this suite of simulations,
712 a control simulation without any Langmuir mixing parameterization was inte-
713 grated for 100 years (PI-CTRL). Starting at year 21, three simulations including
714 Langmuir mixing effects were branched from PI-CTRL and run for 80 years,
715 one with a prognostic wave model, WAVEWATCH III, denoted as PI-WW3,
716 the second one with the Data Wave climatology, denoted as PI-DWAV, and the
717 third one with the Theory Wave estimate, denoted as PI-TWAV. In addition,
718 the control simulation without any Langmuir mixing parameterization (20C-
719 CTRL) and a simulation with WAVEWATCH III (20C-WW3) were performed
720 under the 20th century transient scenario for direct comparison to observations.
721 See Table 2 for a brief summary.

722 Consistent with the ocean-wave only simulations, the deepening in simulated
723 MLD and enhancement in the vertical mixing of the pCFC11 are found in fully
724 coupled simulations. The results of both PI-DWAV and PI-TWAV resemble that
725 of PI-WW3, thus detailed comparison of these results is not needed. Instead, a
726 focus on climate variables that are indirectly affected by the Langmuir mixing
727 and related feedbacks follows.

728 Fig. 10 illustrates the vertical profiles of the annual mean ocean temper-
729 ature error averaged over different regions for both PI (solid lines) and 20C
730 (dashed lines) simulations. The errors are computed as model minus observa-
731 tion. The observations are from the Polar Science Center Hydrographic Clima-
732 tology (PHC3.0) dataset, which is a combination of the National Oceanographic
733 Data Center’s (NODC) World Ocean Atlas (WOA) 1998 (Levitus et al., 1998)

734 and an updated version by [Steele et al. \(2001\)](#) for the Arctic Ocean.

735 Corresponding to the deepening in simulated MLD, enhanced vertical mix-
736 ing at the sea surface efficiently transports heat downward and mixes up the
737 relatively cooler water from below. As a result, the SST decreases and the sub-
738 surface temperature increases. As shown in Fig. 10, the global averaged annual
739 mean SST is cooled by 0.12°C in PI-DWAV and 0.13°C in PI-TWAV, versus
740 0.15°C in PI-WW3. Whereas the subsurface temperature is warmed by as much
741 as about 0.5°C between about 100 m and 200 m. The cooling in the SST is
742 mostly found in the Southern Ocean, where the deepening of the MLD is most
743 significant. The warming in the subsurface layer is mainly in the equatorial
744 regions. As most of the subsurface layer water in the equatorial regions and the
745 southern hemisphere extratropical regions are biased warm near the surface and
746 biased cool in the subsurface layer, the redistribution of the heat over the upper
747 ocean by the Langmuir mixing partly alleviates the biases in the temperature
748 distribution. The bias reduction is similar among DWAV, TWAV, and WW3
749 coupled simulations.

750 Since the boundary layer is deepened by Langmuir mixing effects, the ampli-
751 tude of the SST seasonal cycle is reduced due to increased heat capacity. This is
752 clearly seen in Fig. 11, which shows the seasonal cycle of the SST averaged over
753 the extratropical regions. Note that in all cases the annual mean is subtracted
754 from the monthly mean to show the phase and amplitude of the seasonal cycle.
755 In both hemispheres the mean SST seasonal cycle amplitude is reduced by about
756 0.4°C . This reduces the too strong bias of SST seasonal cycle in the Southern
757 Hemisphere extratropical region by about 35%. However, it does not improve
758 the simulated SST seasonal cycle in the Northern Hemisphere extratropical re-
759 gion significantly, where the dominating bias is the too early warming occurring
760 in the summer.

761 Consistent with LW16, the salinity does not appear to be significantly af-
762 fected by the parameterized Langmuir mixing. The sea surface water is only
763 slightly fresher in the Southern Ocean and tropics, and saltier in the subtropical
764 regions.

765 All these changes are robust between PI and 20C simulations and are reason-
766 ably retained when switching from coupling with a prognostic wave model to the
767 Data Wave climatology or the Theory Wave estimation. Therefore, both non-
768 prognostic-model approaches appear to work reasonably well in representing the
769 Langmuir mixing effects in fully coupled simulations.

770 6. Discussion

771 The Data Wave climatology for the enhancement factor was aimed at mim-
772 icking the mean Langmuir mixing effects due to coupling with a prognostic wave
773 model and is based on the relation between the wind forcing and the resulting
774 vertical mixing in the ocean surface boundary layer. Note that a monthly mean
775 enhancement factor does not imply that the high frequency forcing from the
776 wind is smoothed out. It is only a particular realization of the rather stochas-
777 tic wave state (the enhancement factor from a prognostic wave model) that is
778 replaced with the most likely realization under similar wind forcing (the Data
779 Wave). The high frequency wind forcing still affects the ocean surface vertical
780 mixing through its proportionality with u^* . The monthly climatology is used
781 here simply to retain the seasonality of the Langmuir mixing effects, which is
782 associated with the seasonality of the forcing regimes as will be discussed below.
783 In fact, the Stokes drift has a relatively persistent character, with the autocor-
784 relation zero crossing time varying from 9 to 15 days for the surface values and
785 longer for the subsurface values (Haney et al., 2015). Therefore, further refin-
786 ing the enhancement factor climatology with intermediate values (e.g., daily)
787 does not seem to be necessary. We have tried a 6-hourly enhancement factor
788 climatology averaged over the same 10 years, and the results are identical to
789 that presented here with a monthly climatology. Note that the Data Wave cli-
790 matology is derived from simulations with present-day forcing, and may not be
791 robust under climate change with significant shift of the large scale wave pat-
792 terns (Hemer et al., 2013; Fan et al., 2014). However, as shown in Table 3, its
793 application under moderate change of wind climate is justified .

794 The Theory Wave estimate is based on similar ideas. But instead of trying
 795 to find a statistical description for the enhancement factor, we started from
 796 a statistical description for the surface waves, the empirical wave spectra, and
 797 approximated the enhancement factor from that spectrum. The Stokes drift pro-
 798 file is first approximated based on the work of Breivik et al. (2014, 2016), with
 799 added directional spreading effects from Webb and Fox-Kemper (2015). The
 800 enhancement factor is then calculated from the surface layer averaged Stokes
 801 drift, where the boundary layer depth diagnosed from the ocean model is in-
 802 volved. So unlike the Data Wave climatology, the enhancement factor in the
 803 Theory Wave estimate is still diagnosed online. But unlike a prognostic wave
 804 model, the wave physics is replaced by wave statistics. Note that, following
 805 the procedures in Breivik et al. (2016), the Phillips spectrum is used only to
 806 determine the shape of the Stokes drift profile. The final profile is constrained
 807 by the total Stokes transport, which is estimated from the Pierson-Moskowitz
 808 spectrum with a magnitude loss coefficient c_1 in (19), and the surface Stokes
 809 drift, which is assumed to be proportional to the wind with a proportionality
 810 constant c_2 in (23). Both coefficients are determined by matching the Ras-
 811 cle and Ardhuin (2013) global wave hindcast data. No effects from swell are ex-
 812 plicitly accounted for, though the approximated mean effect is included by the
 813 magnitude loss coefficient. The magnitude loss coefficient used here is consis-
 814 tent with the one reported in Webb and Fox-Kemper (2015) and the resulting
 815 enhancement factor agrees reasonably well with that estimated from the full
 816 spectrum wave model (see, e.g., Fig. 2d). This insensitivity of the enhancement
 817 factor, and thereby the Langmuir mixing effects, to the detailed structure of the
 818 Stokes drift profile is partially expected from the LES studies (Harcourt and
 819 D’Asaro, 2008; Van Roekel et al., 2012), in which the vertical turbulent kinetic
 820 energy collapse into a single curve when the surface layer averaged Stokes drift
 821 is used. The Theory Wave approach is also expected to be more robust than
 822 the Data Wave one under climate change, as it does not depend on regionally
 823 specific empirical parameters, although changes to the swell climate will not be
 824 picked up without information from a wave model.

Overall, the CESM ocean model component appears to be relatively insensitive to the variations of the enhancement factor, showing some tolerance to errors. However, two factors are identified to be important to make both the Data Wave climatology and the Theory Wave estimate a success in parameterizing the Langmuir mixing effects. The first one is the seasonality of the enhancement factor, brought by the effect of Stokes depth versus the boundary layer depth. It accounts for the dependence of the Langmuir mixing effects on the forcing regimes. The smaller enhancement factor in winter seems counter-intuitive at first, as ocean surface waves are more vigorous in winter. However, note that the enhancement factor is somehow a correction to the wind effects (to account for the wave effects) on the ocean surface boundary layer vertical mixing. In winter when the boundary layer is deep and dominated by convection, the relative wave effects should not be as large as that in summer, when the wave effects are competing with wind effects and stratification opposes turbulence mixing. The idea is somewhat similar to the modifications proposed by Smyth et al. (2002) on the McWilliams and Sullivan (2000) formulation of the enhancement factor. But here this effect is accounted for by using the surface layer averaged Stokes drift.

The second factor is the effect of misaligned wind and waves, or directional spreading and crossing swell. Swell misaligned with the local wind will reduce the magnitude and change the direction of the Stokes drift (Webb and Fox-Kemper, 2015) and suppress the Langmuir turbulence (Van Roekel et al., 2012). Therefore, the Langmuir mixing effects will generally be overestimated without considering swells. The effect of swells is accounted for in Van Roekel et al. (2012) and LW16 by projecting both the wind and Stokes drift onto the direction of Langmuir cells, assuming that only the component of wind stress parallel to the direction of the Langmuir cells energizes the Langmuir turbulence. The resulting enhancement factor depends on the angle between wind and waves, θ_{ww} , and the angle between wind and Langmuir cells, α , as in (3). As a result, the mean effect of swells is incorporated into Data Wave when averaging the enhancement factor as a whole. In comparison, this effect is only

856 partially accounted for in the Theory Wave estimate by allowing a magnitude
 857 loss coefficient $c_1 < 1$ in (19) to match the prediction of the total Stokes trans-
 858 port from the Pierson-Moskowitz spectrum with the RA13 wave hindcast data.
 859 In addition, setting $c_1 = 1$ will result in an overestimation of the enhancement
 860 factor (not shown). Potential improvements are possible by incorporating a
 861 higher resolution prognostic wave model with better wave physics. However,
 862 the improvements will be limited without considering these two factors.

863 Note that the Data Wave and the Theory Wave are derived particularly for
 864 the Langmuir mixing parameterization. There are other surface wave effects
 865 that are not considered here that require better representation, such as the
 866 modification of the momentum flux through both a roughened ocean surface and
 867 the redistribution of wind-driven momentum across the mixed layer (Sullivan
 868 et al., 2004, 2014; Garfinkel et al., 2011; Breivik et al., 2015), the breaking waves
 869 (Phillips, 1985; Sullivan et al., 2004, 2007) and interaction between the surface
 870 waves and the submesoscale processes (Hamlington et al., 2014; Haney et al.,
 871 2015; Suzuki et al., 2016). In cases that these surface wave effects should be
 872 considered and parameterized, a prognostic wave model coupled with the climate
 873 model is likely required. However, it would be interesting to see whether the
 874 effects of these processes could be parameterized statistically.

875 The statistical modeling approaches explored in this paper were aimed at
 876 reproducing the Langmuir mixing effects in LW16 with a much lower com-
 877 putational cost. Of course, computers are continually getting more powerful.
 878 Algorithmic developments (e.g., Webb, 2013) may also provide prognostic spec-
 879 tral wave models at a lower computational cost. However, the major cost that
 880 is avoided by the theory wave approach developed here is the human cost of
 881 implementing a wave model into a coupled model framework. Aside from the
 882 modest computational cost (3-10% of the overall climate model system used
 883 here), getting WAVEWATCH III included and vetted in CESM took nearly a
 884 decade of work by our team. There are advantages to having a full wave model
 885 in a climate model beyond Langmuir mixing (Cavaleri et al., 2012), but the
 886 advantage of the Theory Wave is that these improvements can be assessed with

low *computational and human* costs *before* deciding to implement a full wave modeling system.

Finally, it is worth noting that the Langmuir mixing parameterization developed here (Van Roekel et al., 2012), as well as similar ones (McWilliams and Sullivan, 2000; Smyth et al., 2002; Harcourt and D’Asaro, 2008; Reichl et al., 2016; Kantha and Clayson, 2004; Harcourt, 2013; Pearson et al., 2015, etc.), represent incremental improvements over previous non-Langmuir schemes. It may be that a wholly new approach to boundary layer mixing is needed to produce a leap in climate model accuracy. Surely however, errors in other components of the modeling system (atmosphere, mesoscale parameterization, etc.) contribute errors in the ocean boundary layer that are irreducible by direct effort alone.

7. Conclusion

In this paper two approximations to the enhancement factor, the Data Wave climatology and the Theory Wave estimation, are explored and tested in CESM as substitutes for the prognostic wave model utilized in the Langmuir mixing parameterization in LW16. Both appear to retain the Langmuir mixing effects brought by coupling with WAVEWATCH III, but with significantly less computational cost (Table 1). The data and source code for both approximations are easily accessible through the CVMix project (Appendix C). The essential equations required in the Theory Wave model are summarized in (25) for quick reference.

Two factors are shown to be important in parameterizing the global Langmuir mixing effects: the seasonality of the enhancement factor, brought by the effect of Stokes depth versus the boundary layer depth, and the effect of misaligned wind and waves, or directional spreading and crossing swell. Both factors help avoiding too much mixing in winter. Without considering these two factors, the potential improvements by incorporating results from higher resolution prognostic wave models with better wave physics will be limited.

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929 Appendix A The depth-averaged Stokes drift under a 1D Phillips- 930 type spectrum

931 It is useful to know the average Stokes drift over a part of the water column,
 932 for example to compute the average Stokes drift experienced by a submerged
 933 object. By assuming the 1D Phillips spectrum (7) to be a reasonable approxi-
 934 mation to the full spectrum, it is possible to provide a closed-form expression
 935 for the integral of Eq (8), i.e., the Stokes drift transport between the vertical
 936 level z_0 and the surface. The Stokes transport under Eq (8) can be written

$$V(y_0) = \frac{u_0^S}{2k_p} \int_{y_0}^0 e^{-y} dy - \beta \sqrt{\pi} \underbrace{\int_{y_0}^0 \sqrt{y} \operatorname{erfc}(\sqrt{y}) dy}_{I_1}. \quad (\text{A.1})$$

937 Here we have introduced the variable substitution $y = -2k_p z$ (also applies to
 938 variables with subscript 0) and u_0^S is the surface Stokes drift speed. The integral
 939 I_1 can be solved by first substituting $w = \sqrt{y}$,

$$I_1 = 2 \int_0^{w_0} w^2 \operatorname{erfc} w dw. \quad (\text{A.2})$$

940 Then, integrating by parts gives

$$I_1 = \frac{2}{3}w_0^3 \operatorname{erfc} w_0 + \underbrace{\frac{4}{3\sqrt{\pi}} \int_0^{w_0} w^3 e^{-w^2} dw}_{I_2}. \quad (\text{A.3})$$

941 The integral I_2 can be found analytically (see Eq (3.321.6) of [Gradshteyn and](#)
942 [Ryzhik 2007](#)),

$$I_2 = 1 - (1 + w_0^2)e^{-w_0^2}. \quad (\text{A.4})$$

943 Eq (A.1) can now be solved,

$$V(y_0) = \frac{u_{\text{Phil}}^0}{2k_p} \left\{ 1 - e^{-y_0} - \frac{2\beta}{3} \left[1 + \sqrt{\pi} y_0^{3/2} \operatorname{erfc} \sqrt{y_0} - (1 + y_0)e^{-y_0} \right] \right\}. \quad (\text{A.5})$$

944 Reverting back to dimensional variables we get

$$V(z_0) = \frac{u_0^S}{2k_p} \left\{ 1 - e^{-2k_p|z_0|} - \frac{2\beta}{3} \left[1 + \sqrt{\pi} (2k_p|z_0|)^{3/2} \operatorname{erfc} \left(\sqrt{2k_p|z_0|} \right) - (1 + 2k_p|z_0|) e^{-2k_p|z_0|} \right] \right\}. \quad (\text{A.6})$$

945 In order to find the average Stokes drift between a lower level z_0 and an
946 upper level z_1 all that is needed is to use Eq (A.6) twice to find

$$\bar{u}_0^S = \frac{V(z_0) - V(z_1)}{z_1 - z_0}. \quad (\text{A.7})$$

947 **Appendix B Wave directional spreading refinement for a 1D Phillips-** 948 **type spectrum**

949 *B.1 Depth-dependent Stokes drift*

950 The depth-dependent Stokes drift for a 1D Phillips-type spectrum ([Breivik](#)
951 [et al., 2016](#)) can be modified to include wave directional spreading effects by
952 using the Padé-approximated, DHH directional-Stokes-drift-component from

953 Webb and Fox-Kemper (2015). The resulting Stokes drift profile is,

$$\begin{aligned}
\mathbf{u}_{\text{Phil-s}}^{\text{S}}(z) &= \hat{e}^{\text{w}} \frac{2}{g} \int_0^\infty H_{\text{DHH}}(\omega/\omega_{\text{p}}) \omega^3 S_{\text{Phil}}(\omega) e^{2kz} d\omega \\
&= \hat{e}^{\text{w}} 2\alpha g \int_{\omega_{\text{p}}}^\infty H_{\text{DHH}}(\omega/\omega_{\text{p}}) \omega^{-2} e^{2z\omega^2/g} d\omega \\
&= \hat{e}^{\text{w}} 2\alpha g \left\{ \int_{\omega_{\text{p}}}^{1.6\omega_{\text{p}}} \left(\frac{0.98 - 0.19(\omega/\omega_{\text{p}}) + 0.0058(\omega/\omega_{\text{p}})^2}{1 - 0.26(\omega/\omega_{\text{p}}) + 0.12(\omega/\omega_{\text{p}})^2} \right) \omega^{-2} e^{2z\omega^2/g} d\omega \right. \\
&\quad \left. + \int_{1.6\omega_{\text{p}}}^\infty 0.777 \omega^{-2} e^{2z\omega^2/g} d\omega \right\}. \tag{B.1}
\end{aligned}$$

954 Instead of evaluating $\mathbf{u}_{\text{Phil-s}}^{\text{S}}$ directly for the first interval, it is much easier
955 to work with an approximation of the Padé-approximant. For $r = \omega/\omega_{\text{p}}$ and
956 $a_0, a_2 > 0$ (a_1 arbitrary), let

$$f(r) = a_0 + a_1 r - a_2 r^2, \tag{B.2}$$

957 such that following conditions hold:

$$\begin{aligned}
f(1) &= H_{\text{DHH}}(1), \\
f(1.6) &= H_{\text{DHH}}(1.6), \\
\int_1^{1.6} f(r) dr &\approx \int_1^{1.6} H_{\text{DHH}}(r) dr. \tag{B.3}
\end{aligned}$$

958 One possible solution is to let $a_2 = 0.0958$, $a_1 = 0$ and $a_0 = 1.021$. Then
959 (B.2) holds to within a relative error of 1.1×10^{-3} (in the interval) of the Padé-
960 approximant.

961 Then (B.1) can be written as

$$\begin{aligned}
\mathbf{u}_{\text{Phil-s}}^{\text{S}}(z) &= \hat{e}^{\text{w}} 2\alpha g \left\{ \int_{\omega_{\text{p}}}^{1.6\omega_{\text{p}}} \left[(a_0 - 0.777) \omega^{-2} - \frac{a_2}{\omega_{\text{p}}^2} \right] e^{2z\omega^2/g} d\omega \right. \\
&\quad \left. + 0.777 \int_{\omega_{\text{p}}}^\infty \omega^{-2} e^{2z\omega^2/g} d\omega \right\}. \tag{B.4}
\end{aligned}$$

962 At the surface ($z = 0$), this simplifies to

$$\begin{aligned}
u_{0,\text{Phil-s}}^{\text{S}} &= |\mathbf{u}_{\text{Phil-s}}^{\text{S}}(0)| \\
&= u_{0,\text{Phil}}^{\text{S}} \left\{ \int_{\omega_{\text{p}}}^{1.6\omega_{\text{p}}} \left[(a_0 - 0.777)\omega_{\text{p}}\omega^{-2} - \frac{a_2}{\omega_{\text{p}}} \right] d\omega + 0.777 \right\} \\
&\approx 0.811 u_{0,\text{Phil}}^{\text{S}},
\end{aligned} \tag{B.5}$$

963 where $u_{0,\text{Phil}}^{\text{S}} = 2\alpha g / \omega_{\text{p}}$.

964 To simplify the integration for subsurface values, let

$$\begin{aligned}
T_1(k, z) &= e^{2kz}, \\
T_2(k, z) &= \sqrt{2\pi k|z|} \operatorname{erfc}\left(\sqrt{2k|z|}\right).
\end{aligned} \tag{B.6}$$

965 and $k_{\text{p}}^* = (1.6\omega_{\text{p}})^2/g = 2.56k_{\text{p}}$. After evaluating the integrals and rearranging
966 terms using (B.6), (B.4) can be rewritten (for $z < 0$) as

$$\begin{aligned}
\mathbf{u}_{\text{Phil-s}}^{\text{S}}(z) &= \hat{\mathbf{e}}^{\text{w}} \frac{2\alpha g}{\omega_{\text{p}}} \left\{ a_0 \left[T_1(k_{\text{p}}, z) - \left(1 + \frac{a_2}{a_0} \frac{1}{4k_{\text{p}}|z|} \right) T_2(k_{\text{p}}, z) \right] \right. \\
&\quad \left. - \frac{a_0 - 0.777}{1.6} \left[T_1(k_{\text{p}}^*, z) - \left(1 + \frac{1}{a_0 - 0.777} \frac{1.6^2}{4k_{\text{p}}^*|z|} \right) T_2(k_{\text{p}}^*, z) \right] \right\}.
\end{aligned} \tag{B.7}$$

967 To further simplify, let

$$B_1(z) = 1 + \frac{0.0469}{2k_{\text{p}}|z|} \approx 1 + \frac{a_2}{a_0} \frac{1}{4k_{\text{p}}|z|}, \tag{B.8}$$

$$B_2(z) = 1 + \frac{0.503}{2k_{\text{p}}^*|z|} \approx 1 + \frac{1}{a_0 - 0.777} \frac{1.6^2}{4k_{\text{p}}^*|z|}. \tag{B.9}$$

968 Then (B.7) can be rewritten as

$$\begin{aligned}
\mathbf{u}_{\text{Phil-s}}^{\text{S}}(z) &= \hat{\mathbf{e}}^{\text{w}} u_{0,\text{Phil}}^{\text{S}} \left\{ a_0 [T_1(k_{\text{p}}, z) - B_1(z)T_2(k_{\text{p}}, z)] \right. \\
&\quad \left. - \frac{a_0 - 0.777}{1.6} [T_1(k_{\text{p}}^*, z) - B_2(z)T_2(k_{\text{p}}^*, z)] \right\} \\
&\approx \hat{\mathbf{e}}^{\text{w}} u_{0,\text{Phil-s}}^{\text{S}} \left\{ 1.26 [T_1(k_{\text{p}}, z) - B_1(z)T_2(k_{\text{p}}, z)] \right. \\
&\quad \left. - 0.188 [T_1(k_{\text{p}}^*, z) - B_2(z)T_2(k_{\text{p}}^*, z)] \right\}.
\end{aligned} \tag{B.10}$$

969 *B.2 Depth-averaged Stokes drift*

970 From (B.10), the depth-averaged Stokes drift over a layer from $z = z_0$ ($z_0 <$
971 0) to the surface can be written as,

$$\begin{aligned}\langle \mathbf{u}_{\text{Phil-s}}^S \rangle_{z_0} &= \frac{1}{|z_0|} \int_{z_0}^0 \mathbf{u}_{\text{Phil-s}}^S(z) dz \\ &= \hat{\mathbf{e}}^w \frac{u_{0,\text{Phil-s}}^S}{|z_0|} \left\{ 1.26 [J_1(k_p) - J_2(k_p) - 0.0469 J_3(k_p)] \right. \\ &\quad \left. - 0.188 [J_1(k_p^*) - J_2(k_p^*) - 0.503 J_3(k_p^*)] \right\},\end{aligned}\quad (\text{B.11})$$

972 where

$$J_1(k) = \int_{z_0}^0 T_1(k, z) dz = \frac{1}{2k} (1 - T_1(k, z_0)), \quad (\text{B.12})$$

$$\begin{aligned}J_2(k) &= \int_{z_0}^0 T_2(k, z) dz \\ &= \frac{2|z_0|}{3} T_2(k, z_0) + \frac{1}{3k} - \left(\frac{1}{3k} + \frac{2|z_0|}{3} \right) T_1(k, z_0),\end{aligned}\quad (\text{B.13})$$

$$J_3(k) = \int_{z_0}^0 \frac{1}{2k|z|} T_2(k, z) dz = \frac{1}{k} [1 + T_2(k, z_0) - T_1(k, z_0)]. \quad (\text{B.14})$$

973 See also Appendix A for the 1D profile without spreading. Substituting (B.12)

974 - (B.14) back into (B.11), we get,

$$\begin{aligned}\langle \mathbf{u}_{\text{Phil-s}}^S \rangle_{z_0} &\approx \hat{\mathbf{e}}^w u_{0,\text{Phil-s}}^S \left\{ 0.715 \right. \\ &\quad + \left(\frac{0.151}{k_p |z_0|} - 0.840 \right) [1 - T_1(k_p, z_0)] \\ &\quad - \left(0.840 + \frac{0.0591}{k_p |z_0|} \right) T_2(k_p, z_0) \\ &\quad + \left(\frac{0.0632}{k_p^* |z_0|} + 0.125 \right) [1 - T_1(k_p^*, z_0)] \\ &\quad \left. + \left(0.125 + \frac{0.0946}{k_p^* |z_0|} \right) T_2(k_p^*, z_0) \right\}.\end{aligned}\quad (\text{B.15})$$

975 *B.3 Total Stokes transport*

976 The total Stokes transport is derived by integrating (B.10) from $z = -\infty$ to
 977 $z = 0$. Notice that

$$\int_{-\infty}^0 T_1(k, z) dz = \frac{1}{2k}, \quad (\text{B.16})$$

$$\int_{-\infty}^0 T_2(k, z) dz = \frac{1}{3k}, \quad (\text{B.17})$$

$$\int_{-\infty}^0 \frac{1}{2k|z|} T_2(k, z) dz = \frac{1}{k}. \quad (\text{B.18})$$

978 The total Stokes transport is then

$$\begin{aligned} V_{\text{Phil-s}}^S &= \int_{-\infty}^0 \mathbf{u}_{\text{Phil-s}}^S(z) dz \\ &= \hat{e}^w u_{0,\text{Phil-s}}^S \left[1.26 \left(\frac{1}{2k_p} - \frac{1}{3k_p} - \frac{0.0469}{k_p} \right) \right. \\ &\quad \left. - 0.188 \left(\frac{1}{k_p^*} - \frac{1}{3k_p^*} - \frac{0.503}{k_p^*} \right) \right] \\ &\approx \hat{e}^w \frac{0.176 u_{0,\text{Phil-s}}^S}{k_p}. \end{aligned} \quad (\text{B.19})$$

979 **Appendix C The Data Wave and Theory Wave in CVMix**

980 The Theory Wave is implemented in the Community Ocean Vertical Mixing
 981 Project (CVMix, <https://github.com/CVMix>) as a public function,

982 `cvmix_kpp_efactor_model(u10, ustar, hbl, Cvmix_params_in)`

983 with the four arguments being the 10-meter wind, the water side surface fric-
 984 tion velocity, the boundary layer depth, and a global parameter of the type
 985 `cvmix_global_params_type`, respectively, all with SI units. This function re-
 986 turns the enhancement factor and should be called before passing it to the
 987 subroutines `cvmix_kpp_compute_turbulent_scales` and `cvmix_coeffs_kpp`.

988 The Data Wave requires file I/O and therefore is suggested to be imple-
 989 mented in the ocean model outside the CVMix framework. However, for
 990 testing purpose, another public function is available in a fork of CVMix
 991 (<https://github.com/qingli411/CVMix-src>),

992 `cvmix_kpp_efactor_read(infile, lon, lat, time)`

993 with the four arguments being the filename of the input enhancement factor
994 data, longitude, latitude and day of a year to be interpolated. The monthly
995 enhancement factor climatology data is saved in Fortran binary format for
996 the application considered here as well as in netCDF format for more gen-
997 eral applications, all of which are available in the Brown Digital Repository
998 (<https://doi.org/10.7301/Z0348H86>).

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Table 1: A summary of computational cost^a for simulations without Langmuir mixing parameterization (CTRL), coupled with WAVEWATCH III (WW3), with the Data Wave (DWAV) and the Theory Wave (TWAV).

Case name	CTRL	WW3	DWAV	TWAV	NP ^b	NS ^c	Description
CORE-II	100±0.7	103.2±1.4	100.7±1.2	99.7±0.4	64	7	CORE-II forced ocean-wave
PI	100±0.6	110.1±0.9	100.3±0.4	100.6±0.5	128	16	Preindustrial fully coupled; CAM4
20C	100±1.1	105.4±1.2	-	-	128	38	20th century fully coupled; CAM5

^a Measured in processor element-hours per simulated-year as mean $\pm$ standard deviation, normalized by the mean cost of CTRL for each case and then multiplied by 100 to show the percentage. All simulations were performed on the NCAR supercomputer Yellowstone, with a nominal 1° resolution for the ocean model and 1.9°×2.5° for the atmosphere model when applicable.

^b Number of processors. Fully sequential component layout is used.

^c Number of timing samples represented in the statistics.

Table 2: A summary of simulations. Components included in each simulation are indicated by the tag *Type*, with O: CORE-II forced ocean only; OW: CORE-II forced ocean-wave coupled; F: Atmosphere-Ocean-Land-Ice-Wave fully coupled. The tag *LM* represent Langmuir mixing parameterization.

Case name	Type	LM	Years	Yr Analyzed	Enhancement factor
CTRL	O	off	1-248	199-248	1.0
WW3	OW	on	195-248	199-248	Diagnosed online from WAVEWATCH III with (3)
DWAV-FULL	OW	on	195-248	199-248	The Data Wave, monthly $\mathcal{E}$ climatology with (3)
TWAV	OW	on	195-248	199-248	The Theory Wave, calculated online from (25)
DWAV-CON1	OW	on	195-248	199-248	1.52
DWAV-CON2	OW	on	195-248	199-248	1.33
DWAV-AL	OW	on	195-248	199-248	Monthly $\mathcal{E}_a$ climatology with (26)
DWAV-RA13	OW	on	195-248	199-248	Same as DWAV-AL, but with data from RA13
PI-CTRL	F	off	1-100	71-100	1.0
PI-WW3	F	on	21-100	71-100	Same as WW3
PI-DWAV	F	on	21-100	71-100	Same as DWAV-FULL
PI-TWAV	F	on	21-100	71-100	Same as TWAV
20C-CTRL	F	off	1850-2001	1970-2001	1.0
20C-WW3	F	on	1850-2001	1970-2001	Same as WW3

Table 3: Comparison of the 10-year averaged wave statistics over the last 10 years (denoted as T_2 ; corresponding to forcing years 2000-2009) versus the first 10 years (T_1 ; corresponding to forcing years 1960-1969) of the 50-year data. See text for more details.

Case	Measure	$\Delta U_{10}(\%)$	$\Delta u^*(\%)$	$\Delta H_{m_0}(\%)$	$\Delta u_0^S(\%)$	$\Delta \mathcal{E}(\%)$
Global	NMD ^a	2.5	3.4	3.9	3.8	0.0
	NRMSD ^b	10.0	12.3	9.1	15.7	2.8
90°S-30°S	NMD	5.3	7.0	7.9	7.8	0.3
	NRMSD	8.5	10.9	10.6	12.2	2.4
30°S-30°N	NMD	0.5	0.4	0.9	0.3	-0.1
	NRMSD	11.4	13.9	8.6	18.4	3.0
30°N-90°N	NMD	1.2	2.0	1.1	1.9	-0.2
	NRMSD	6.0	7.7	7.3	9.3	2.4

^a Normalized Mean Difference: $\frac{\overline{x(T_2)} - \overline{x(T_1)}}{\overline{x(T_1)}} \times 100\%$

^b Normalized Root Mean Square Difference: $\sqrt{\left(\frac{\overline{x(T_2)} - \overline{x(T_1)}}{\overline{x(T_1)}}\right)^2} \times 100\%$

Figure 1: The maps of the monthly mean enhancement factor from the Data Wave.

Figure 2: The density-shaded scatter plots of (a,d) the enhancement factor, $\mathcal{E}$, (b,e) surface Stokes drift magnitude (m s^{-1}), u_0^S and (c,f) surface layer averaged Stokes drift magnitude (m s^{-1}), u_{SL}^S . Comparisons are made between (a,b,c) Data Wave and WW3, and (d,e,f) Theory Wave and WW3. The scatter plots are generated from the 6-hourly output data over the last 10 years of the simulation WW3. The values for Data Wave is obtained from the 10-year monthly average, followed by a linear interpolation to expand the data to 6-hourly frequency. The values for Theory Wave is calculated from (25). For all the figures, yellow, green and blue represent the highest 0-30%, 31-60% and 61-90% centered distribution, respectively.

Figure 3: The density-shaded scatter plots of (a) total Stokes transport, V^S and (b) surface Stokes drift, u_0^S , generated from one year of the 3-hourly output data from RA13. Comparison is made between the Theory Wave prediction and the model diagnosed data in RA13. The total Stokes transport and surface Stokes drift in Theory Wave are calculated from (19) and (23), respectively, with $c_1 = 0.667$ and $c_2 = 0.016$. See Fig. 2 for the explanation for the colors.

Figure 4: Comparison of the January (a,b) and July (c,d) mean enhancement factor map between Theory Wave (a,c) and Data Wave (b,d).

Figure 5: Impact of Langmuir mixing on the summer mean mixed layer depth (MLD; m) for both hemispheres. Subfigure (a) shows the observation from [de Boyer Montégut et al. \(2004\)](#), updated to include the ARGO data to 2012. (b) shows the case CTRL, (c) the case WW3, (d) the case DWAV-FULL and (e) the case TWAV. MLDs are averaged over Jul., Aug. and Sep. for the Northern Hemisphere and Jan., Feb. and Mar. for the Southern Hemisphere.

Figure 6: Same as Fig. 5, but for winter mean MLD. Note the different color scalings here.

Figure 7: Impact of Langmuir mixing on the zonal mean pCFC-11 (patm) in the Southern Hemisphere. (a) shows the observation from [Key et al. \(2004\)](#). (b)-(e) show the anomaly from the observation for the cases CTRL, WW3, DWAV-FULL and TWAV, respectively.

Figure 8: The seasonal cycle of the mean MLD biases versus observation (a, c, e) and the corresponding reductions in root mean square errors (RMSE) after including Langmuir mixing (b, d, f), averaged over (a, b) $30^{\circ}\text{N} - 90^{\circ}\text{N}$, (c, d) $30^{\circ}\text{S} - 30^{\circ}\text{N}$ and (e, f) $90^{\circ}\text{S} - 30^{\circ}\text{S}$. The control simulation (CTRL) without Langmuir mixing effect is colored in black, the simulation with WAVEWATCH III (WW3) in red, the Data Wave (DWAV-FULL) in blue and the Theory Wave (TWAV) in purple. Overlaid solid, dashed, dotted and dash-dotted curves in gray are for DWAV-CON1, DWAV-CON2, DWAV-AL and DWAV-RA13, respectively. Note that (b), (d) and (f) show the differences in RMSE as compared with CTRL.

Figure 9: The vertical profiles of the RMSE reductions of pCFC-11 (patm) as compared with CTRL over regions: (a) Global; (b) 30°N - 90°N; (c) 30°S - 30°N; (d) 90°S - 30°S. The color coding is the same with Fig. 8, except an additional black dashed line here, which illustrates the difference of pCFC-11 RMSE in CTRL between the year 1995 and the year 1994 (see text for more details). All RMSEs are calculated against the observation from [Key et al. \(2004\)](#), which basically represents the year 1994.

Figure 10: The vertical profiles of the mean error of temperature ($^{\circ}\text{C}$) as compared with the observation from [Steele et al. \(2001\)](#) (PHC3.0) over regions: (a) Global; (b) $30^{\circ}\text{N} - 90^{\circ}\text{N}$; (c) $30^{\circ}\text{S} - 30^{\circ}\text{N}$; (d) $90^{\circ}\text{S} - 30^{\circ}\text{S}$.

Figure 11: The seasonal cycle (annual mean subtracted) of the sea surface temperature (SST; °C) averaged over (a) 90°S - 30°S and (b) 30°N - 90°N.