Wave breaking in the surf zone and deep water in a non-hydrostatic **RANS** model. Part 2: Turbulence and mean circulation

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Abstract

Field-scale modeling of wave-breaking-induced turbulence and mean circulation is still challenging. Although Boussinesq-type models have been successfully used to study field-scale wave transformation and breaking-driven circulation, they cannot provide turbulence or the vertical structure of velocity field. In addition, the applicability of such models is limited to shallow water. In Part 1 (Derakhti *et al.*, 2016b) of this study, we showed that the non-hydrostatic σ -coordinate RANS model NHWAVE, as described by Derakhti *et al.* (2016a), accurately predicts organized wave motions and total wave-breaking-induced energy dissipation from deep water up to the swash zone using a few vertical σ -layers. In this paper, our goal is to examine what level of detail of wave-breaking-induced turbulence and mean circulation, both in depth- and steepness-limited breaking waves, can be reproduced by NHWAVE. Further, effects of modeled turbulent eddy viscosity on the pre-

dicted time-averaged velocity distribution is discussed. We establish that NHWAVE is capable of predicting the structure of the mean velocity and vorticity fields including large-scale breaking-induced coherent vortices in deep-water breaking events; where the absence of turbulence-induced eddy viscosity results in the overprediction of the velocity and vorticity field in the breaking region.

Keywords:

non-hydrostatic wave model, breaking waves, wave-induced mean flow

1. Introduction

Surface wave breaking is a highly dissipative process, transferring energy and momentum into currents and turbulence, which enhances transport and mixing in the ocean surface layer and surf zone (Peregrine, 1983; Battjes, 1988; Banner & Peregrine, 1993; Melville, 1996; Perlín *et al.*, 2013). It is well-established that wave-breaking-induced turbulent coherent structures, such as obliquely descending eddies, play significant role in the transport of dispersed bubbles and sediment in individual breaking waves in a localized, short-term manner (Nadaoka *et al.*, 1989; Watanabe *et al.*, 2005; Ting & Reimnitz, 2015). However, long-term transport of dispersed particles and passive tracers are mainly dependent on the structure of wave-breaking-driven mean circulation and their evolution. For example, large coherent vortices induced by individual whitecaps in deep and intermediate water (Rapp & Melville, 1990; Pizzo & Melville, 2013; Derakhti & Kirby,

15 2014a) as well as undertow, longshore and rip currents (Longuet-Higgins,
 16 1970; Svendsen, 1984) and large-scale, **vertical-core** eddies (Peregrine, 1998;
 17 Clark *et al.*, 2012) in the surf zone are fairly well-understood wave-breaking-
 18 induced organized motions. Such organized motions need to be reasonably
 19 resolved in any non-hydrostatic model to truly estimate long-term transport
 20 and mixing processes at field scales. The effect of Langmuir circulation cells
 21 should also be taken into account in deep-water mixing (McWilliams *et al.*,
 22 1997).

23 Because of available computational resources, the whole range of the rel-
 24 evant scales can not be resolved in a single high-resolution framework such
 25 as large-eddy simulations (LES) combined with the volume-of-fluid (VOF)
 26 method for free-surface tracking (Watanabe *et al.*, 2005; Lakehal & Liovic,
 27 2011; Derakhti & Kirby, 2014b, 2016; Zhou *et al.*, 2014; Lubin & Glock-
 28 ner, 2015). Recently, several three-dimensional (3-D) wave-resolving non-
 29 hydrostatic models based on Reynolds-averaged Navier-Stokes (RANS) equa-
 30 tions using surface- and terrain-following grid have been developed for coastal
 31 applications (Lin & Li, 2002; Zijlema *et al.*, 2011; Bradford, 2011; Ma *et al.*,
 32 2012; Shirkavand & Badiei, 2014). As opposed to the VOF/LES frame-
 33 work, the free surface is a single-valued function of horizontal location, and
 34 detailed near-surface processes during active breaking, such as turbulence
 35 generation and air entrainment, can not be resolved in this lower-resolution
 36 framework. Due to computational efficiency, however, this framework can
 37 be used to study long-term, $O(\text{days})$, and large-scale, $O(100m \approx 10km)$,

38 wave-breaking-driven circulation as well as transport of dispersed particles
39 and passive tracers.

40 In Part 1 (Derakhti *et al.*, 2016b) of this paper, NHWAVE as described in
41 Derakhti *et al.* (2016a), is shown to predict **organized wave motions** in terms
42 of the evolution of the wave height and phase-averaged free surface eleva-
43 tions and organized wave velocity, spectral evolution as well as the second-
44 and third-order wave statistics in depth-limited regular and irregular break-
45 ing waves as accurate as those predicted by typical VOF/RANS simulations,
46 where vertical grid resolution in all simulated cases is at least an order of
47 magnitude coarser than that of typical VOF-based simulations. It is also
48 shown that the dispersive and nonlinear properties of different wave packet
49 components before and after the break point, the overall wave height de-
50 cay and spectral evolution in steepness-limited unsteady breaking waves are
51 captured by the model.

52 Our goal here is to examine what level of detail of wave-breaking-induced
53 **turbulence and mean circulation** can be reproduced by the non-hydrostatic
54 **RANS model NHWAVE** as described by Derakhti *et al.* (2016a). We carefully
55 **examine NHWAVE predictions of wave-breaking-induced mean circulation in**
56 **terms of wave setup and undertow current in depth-limited breaking waves,**
57 **and in terms of low-frequency forced waves and wave-breaking-driven large-**
58 **scale vortex structures in steepness-limited unsteady breakers. Further, the**
59 **spatio-temporal evolution of wave-breaking-induced turbulence statistics in**
60 **terms of turbulent kinetic energy is compared with corresponding measure-**

ments and the results of the VOF/LES simulation of Derakhti (2016). In addition, the effect of modeled turbulent eddy viscosity, ν_t , on the predicted time-averaged velocity distribution in the surf zone is discussed.

The paper is organized as follows. A brief summary of the model set-up and choice of experimental conditions is given in §2. Results for depth-limited and steepness-limited breaking waves are presented in §3 and §4 respectively. In §5, the relative importance of ν_t in the predicted undertow profiles are discussed. Conclusions are presented in §6.

2. Model set-up and choice of experimental conditions

To avoid unnecessary duplication, the reader is referred to Part 1 (Derakhti *et al.*, 2016*b*) for a brief description of governing equations and dynamic surface boundary conditions. A detailed description of the governing equations and boundary conditions may be found in Derakhti *et al.* (2016*a*). Details of the numerical method may be found in Ma *et al.* (2012) and Derakhti *et al.* (2015). In this section, the choice of experimental conditions is briefly explained.

We consider model performance for the case of regular and irregular depth-limited wave breaking on planar and barred beaches using the data sets of Ting & Kirby (1994) and Scott *et al.* (2004) as well as for the case of steepness-limited unsteady wave breaking using the data set of Rapp & Melville (1990). Table 1 summarizes the input parameters for the simulated cases. Details of numerical set-up and inflow boundary conditions for the

83 depth-limited breaking cases and the steepness-limited breaking case consid-
 84 ered here are presented in §3.1 and §4.1 of Part 1, respectively.

85 Both spilling breaking (hereafter referred as TK1) and plunging breaking
 86 (hereafter referred as TK2) cases of Ting & Kirby (1994) are selected to ex-
 87 amine the model prediction of the temporal evolution of the phase-averaged
 88 velocity and turbulent kinetic energy, k , in depth-limited breaking waves.
 89 This experiment has been widely used by other researchers to validate both
 90 non-hydrostatic models using surface- and terrain-following grid (Ma *et al.*,
 91 2014; Bradford, 2011, 2012; Smit *et al.*, 2013; Shirkavand & Badiei, 2014)
 92 and VOF-based numerical models (Ma *et al.*, 2011; Lin & Liu, 1998*a,b*;
 93 Bradford, 2000; Christensen, 2006; Lakehal & Liovic, 2011). The predicted
 94 time-averaged velocity and k fields for TK1, TK2 and the regular breaking
 95 case of Scott *et al.* (2004) (hereafter referred as S1) are compared with cor-
 96 responding experiments. Besides the measurements of Ting & Kirby (1994),
 97 the model results also compared with the results of the VOF/LES simulation
 98 of Derakhti (2016). The latter follows the experimental set-up of Ting & Nel-
 99 son (2011) which is similar to TK1, as summarized in Table 1. Further, the
 100 predicted time-averaged velocity and k fields for the irregular depth-limited
 101 cases of Mase & Kirby (1992) and Scott *et al.* (2004) are compared with the
 102 simulated regular cases.

103 The model results for one of the unsteady plunging breakers of Rapp &
 104 Melville (1990) (hereafter referred as RM1) in an intermediate-depth regime
 105 with $k_c d \approx 1.9$ are presented, where k_c is the wave number of the center fre-

Table 1: Input parameters for the simulated surf zone regular breaking cases on a planar beach. Here, h_0 is the still water depth in the constant-depth region, H and T are the wave height and period of the cnoidal wave generated by the wavemaker, $(kH)_0$ is the corresponding deep-water wave steepness of the generated wave, $\xi_0 = s/\sqrt{H_0/L_0}$ is the self similarity parameter, and s is the plane slope. For S1 and S2, s is the averaged slope before the bar, assumed as $s \sim 1/12$. For the irregular wave cases, $T = T_p$ and $k = k_p$, where p refers to the peak frequency of the incident waves. For S2, $H = H_{s0}$ is the deep-water characteristic wave height. For the rest of irregular wave cases, $H = H_{rms}$ at the first wave gauge.

Case no.	H (m)	T (s)	$(kH)_0$	$k_0 h_0$	ξ_0	breaking type	Exp.
$TK1$	0.125	2.0	0.126	0.6	0.20	regular, spilling	Ting & Kirby (1994)
$\sim TK1$	0.122	2.0	0.119	0.5	0.22	regular, spilling/weakly plunging	Ting & Nelson (2011)
$TK2$	0.128	5.0	0.015	0.2	0.59	regular, plunging	Ting & Kirby (1994)
$S1$	0.64	4.0	0.148	1.2	0.52	regular, plunging	Scott <i>et al.</i> (2004)
$MK1$	0.04	1.7	0.058	0.9	0.52	irregular, plunging	Mase & Kirby (1992)
$MK2$	0.04	1.0	0.161	2.0	0.31	irregular, spilling	Mase & Kirby (1992)
$S2$	0.59	4.0	0.136	1.2	0.54	irregular, plunging	Scott <i>et al.</i> (2004)

quency wave of the input packet as defined in §4.1 of Part 1. The evolution of the mean velocity field and large mean vortex in case RM1 are compared with the corresponding measurements and the results of the VOF/LES simulation of Derakhti & Kirby (2014a). In this experiment, breaking waves were generated using the dispersive focusing technique, in which an input packet propagates over an constant depth and breaks at a predefined time, t_b , and location, x_b . The input parameters for the simulated case are summarized in Table 2.

3. Depth-limited breaking waves

For regular wave cases, $\langle \rangle$ and $\overline{(\)}$ refer to phase and time averaging over five subsequent waves after the results reach quasi-steady state, respectively. We found that averaging over more than five waves results in approximately

Table 2: Input parameters for the simulated focused wave packets. h is the still water depth, $S = \sum_{i=1}^N a_i k_i$ is the global steepness, N is the number of components in the packet, $a_i k_i$ is the component steepness which is the same for the all components, and the discrete frequencies f_i were uniformly spaced over the band $\Delta f = f_N - f_1$ with a central frequency defined by $f_c = \frac{1}{2}(f_N + f_1)$.

Case no.	h (m)	S	f_c (1/s)	$\Delta f/f_c$	N	breaking type
<i>RM1</i>	0.60	0.352	0.88	0.73	32	plunging

118 similar statistics in the shoaling and surf zone regions. The corresponding
 119 measured averaged variables were calculated by averaging over 102 successive
 120 waves starting at a minimum of 20 minutes after the initial wavemaker move-
 121 ment for TK1 and TK2, and by phase averaging over 150 successive waves
 122 and ensemble averaging over at least 8 realizations for S1. For irregular wave
 123 cases, $\overline{(\quad)}$ refers to long-time averaging over several minutes, or more than
 124 250 waves. The mean water level is defined as $\overline{D} = h + \overline{\eta}$, where h is the still
 125 water depth and $\overline{\eta}$ is the wave-induced set-down/set-up.

126 3.1. *Time-averaged water surface level and velocity field*

127 Comparing with observations, Figure 1 shows that the model accurately
 128 predicts the evolution of the time-averaged (mean) water surface level $\overline{\eta}$ from
 129 outside of the surf zone up to the swash zone. In particular, the maximum
 130 wave set-up is accurately predicted in all cases except in S2, indicating that
 131 NHWAVE accurately predicts the gradient of the wave radiation stress in
 132 both spilling and plunging depth-limited breaking waves. As shown in panel
 133 (a), the jump in the observed set-up in the spilling case TK1 at $x \approx 5\text{m}$ does

not exist in the predicted results; the same discrepancy was also reported in other studies such as Smit *et al.* (2013, Figure 7) and Tonelli & Petti (2010, Figure 9). In S1, the mean water level is accurately resolved from deep water up to the swash zone, as apposed to the VOF/RANS simulation of Jacobsen *et al.* (2014, Figure 4B) which overpredicts wave-induced set-up after the bar.

Figure 2 shows the NHWAVE prediction of the spatial distribution of the time-averaged velocity field using 4, 8 and 16 vertical σ -levels for the spilling case TK1 and that by the 3-D VOF/LES simulation of Derakhti (2016). To obtain the Eulerian mean velocities, the model results in the σ -coordinate system first were interpolated onto a fixed vertical mesh at each cross-shore location using linear interpolation, and then time averaging was performed. The results of the simulations with various vertical resolutions have approximately the same structure in the surf zone. The predicted return current using 4 σ -levels shown in panel (a) has not detached from the bed at $X \sim 0$ in contrast to the simulations with 8 and 16 σ -levels and the 3-D VOF/LES simulation shown in panel (d). Regardless of the various vertical resolutions, the predicted time-averaged velocity in the transition region has a relatively stronger vertical variation compared with the 3-D VOF/LES simulation. A similar pattern of results was found for the plunging case TK2 and is not shown. Figures 3 and 4 show that the slope in the predicted undertow profiles in the transition region is larger than in the measured undertow profiles for both cases. This difference is more noticeable in the

157 plunging case TK2, in which the measured profiles are approximately uniform
 158 with depth. The normalized undertow current for the irregular wave cases
 159 MK1 and MK2 (not shown) have smaller magnitude than that for regular
 160 wave cases TK1 and TK2 with the same vertical structures within the surf
 161 zone. This is consistent with the measurements of Ting (2001) which has the
 162 similar incident wave conditions and experimental set-up compared with the
 163 simulated irregular breaking waves on a planar beach in the present study.

164 Figure 5 shows the spatial distribution of the time-averaged velocity field
 165 using 8 σ -levels for the depth-limited regular (S1) and irregular (S2) breaking
 166 waves on a barred beach. As in TK1, the predicted return current using 4
 167 σ -levels (not shown) has not detached from the bed shoreward of the break-
 168 ing point, as opposed to the simulation with 8 σ -levels shown in Figure 5(a).
 169 In S1, the results of the simulations with 4 (not shown) and 8 σ -levels have
 170 approximately the same structure after the breaking point, where the pre-
 171 dicted undertow current using 8 σ -levels has larger magnitude in the entire
 172 surf zone. In S2, the predicted undertow current using 4 (not shown) and 8
 173 σ -levels (Figure 5b) have approximately the same structure and magnitude
 174 in the entire surf zone. The magnitude of undertow in the irregular case S2 is
 175 smaller than that in the regular case S1; a similar trend was observed in the
 176 simulated cases on a planar beach. The slope of the undertow profile in S1
 177 has a strong spatial variation near the break point as shown in Figure 6(c),
 178 where the slope of the undertow profile at $x = 48.0\text{m}$ (red lines) considerably
 179 decreases compared with that at $x = 51.0\text{m}$ (black lines). This is due to the

180 detachment of the undertow current from the bed, forming negative slopes at
 181 seaward of the break point. Figure 6(c) also shows that the model predicts
 182 breaking seaward of the measured break point as in TK1 and TK2. The ve-
 183 locity measurements were carried out at seven cross-shore locations, shown
 184 in Figure 5(a), using Acoustic Doppler Velocimeters (ADV) sampling at
 185 50 Hz. The measured undertow profiles at two different longshore locations
 186 (shown by open and solid circles) reveal that the time-averaged velocity field
 187 varies in the spanwise direction close to the break point; the 3-D effects are
 188 absent in our two-dimensional simulation.

189 Comparing the results with the measured undertow profiles, it is seen that
 190 the undertow current on top of and after the bar as well as in the inner surf
 191 zone are fairly reasonably captured using as few as 4 σ -levels. Hereafter, we
 192 use the statistical method developed by Willmott (1981) to asses the model
 193 predictive skill for various breaking-related properties. The model predictive
 194 skill is given by

$$\text{Skill} = 1 - \frac{\sum_{i=1}^N |\psi_{i,\text{sim}} - \psi_{i,\text{obs}}|^2}{\sum_{i=1}^N \left[|\psi_{i,\text{sim}} - \bar{\psi}_{\text{obs}}| + |\psi_{i,\text{obs}} - \bar{\psi}_{\text{obs}}| \right]^2}, \quad (1)$$

195 where ψ is the variable being compared, $\bar{\psi}$ is its sample mean value, and
 196 subscripts $()_{\text{sim}}$ and $()_{\text{obs}}$ stand for the simulated results and observations,
 197 respectively. This skill parameter measures the degree to which the observed
 198 deviations about the observed mean correspond with the predicted deviations
 199 about the observed mean. Perfect agreement between the simulated results

200 and observations yields a skill of 1.0 whereas complete disagreement yields
 201 a skill of 0 (Willmott, 1981). In the spilling case TK1, the model predictive
 202 skills using 8 σ -levels for the undertow profiles varies from 0.7 near the initial
 203 break point up to 0.95 in the inner surf zone. In the plunging case TK2, the
 204 predictive skills are between 0.8 and 0.9. As apposed to TK1, the model skill
 205 for $\bar{u}$ decreases at shallower depths. In S1, the model shows predictive skills
 206 higher than 0.8 on the top of and shoreward face of the bar (Figure 6 *d, e, f*),
 207 while in other regions the predicted undertow profiles have an opposite slope
 208 compared with observations resulting in the predictive skills lower than 0.2.
 209 In all cases, the increase of vertical resolution from 4 to 8 σ -levels improves
 210 the corresponding model skill especially in the outer surf zone, while the
 211 change in the predictive skills due to further increase of vertical resolution
 212 from 8 to 16 σ -levels is negligibly small. A relatively larger slope in the
 213 predicted undertow profiles compared with observations is discussed in §5.

214 3.2. *Turbulence Statistics*

215 Figure 7 shows snapshots of the predicted instantaneous turbulent kinetic
 216 energy k distribution using 4, 8 and 16 σ -levels for TK1 and that by the 3-D
 217 VOF/LES simulation of Derakhti (2016). In general, the overall distribution
 218 of k using various vertical resolutions is the same. However, increasing the
 219 vertical resolution decreases the predicted k levels near the wave front region.
 220 The same trend is also observed for TK2 (not shown). Further, the predicted
 221 k levels above the trough levels are smaller than the 3-D VOF/LES simula-

tion, mainly due to the absence of near-surface turbulence generation in the present study.

Figure 8 shows a comparison of measured and simulated phase-averaged turbulent kinetic energy $\langle k \rangle$ time series close to the bed and trough level at different cross-shore locations using 4, 8 and 16 σ -levels for the spilling case TK1. $\langle k \rangle$ is overestimated higher in the water column during the entire wave period especially close to the break point. This overestimation has been also reported in previous VOF-based $k - \epsilon$ studies (Lin & Liu, 1998a; Ma *et al.*, 2011). Lin & Liu (1998a) argued that this is because the RANS simulation can not accurately predict the initiation of turbulence in a rapidly distorted shear flow such as breaking waves. Alternately, Ma *et al.* (2011) incorporated bubble effects into the conventional single phase $k - \epsilon$ model, and concluded that the exclusion of bubble-induced turbulence suppression is the main reason for the overestimation of turbulence intensity by single phase $k - \epsilon$. Comparing Figure 8 with the corresponding results from the VOF-based model Ma *et al.* (2011, Figure 7), we conclude that predicted $\langle k \rangle$ values below trough levels in spilling breaking waves by NHWAVE using as few as 4 σ -levels are at least as accurate as the VOF-based simulation without bubbles.

In the plunging case TK2, a different behavior is observed in the predicted $\langle k \rangle$ values shown in Figure 9 compared with the corresponding results for the spilling case TK1, regardless of the various vertical resolutions. After the initial break point, $\langle k \rangle$ is underpredicted especially for lower elevations.

245 The model could not resolve the sudden injection of k into the deeper depths
 246 at the initial stage of active breaking and thus, there is a considerable un-
 247 derprediction of $\langle k \rangle$ at the beginning of active breaking below trough level.

248 The NHWAVE predictions of near-bed and above-trough time-averaged
 249 turbulent kinetic energy $\bar{k}$ are smaller than the corresponding $\bar{k}$ predictions
 250 by the 3-D VOF/LES simulation (not shown). The increase of the vertical
 251 resolution leads to a more concentrated patch of $\bar{k}$ in TK1; a similar trend
 252 is also observed for TK2 (not shown). Figures 10, 11 and 12 show the com-
 253 parison of modeled and measured $\bar{k}$ profiles at different cross-shore locations
 254 before and after the initial break point for TK1, TK2 and S1 respectively. In
 255 the plunging case TK2, the noticeable underprediction of $\langle k \rangle$ at the initial
 256 stage of active breaking shown in Figure 9 compensates relatively smaller
 257 overprediction of $\langle k \rangle$ at the other phases, resulting to apparent smaller $\bar{k}$
 258 values than those in the measurement in the shoreward end of the transi-
 259 tion region and inner surf zone, as shown in Figure 11($B - D$). In S1, the
 260 model accurately predicts the cross-shore variation of $\bar{k}$, with the large $\bar{k}$ lev-
 261 els across the breaker bar, where the waves are breaking, and the subsequent
 262 decay of $\bar{k}$ level on the seaward face as well as after the bar. In the spilling
 263 case TK1, the model predictive skills for $\bar{k}$ varies from 0.2 close to the initial
 264 break point up to 0.9 in the inner surf zone using as few as 4 σ -levels, where
 265 the skill values continuously increase across the transition region and inner
 266 surf zone. In the plunging case TK2, however, the corresponding model skills
 267 are lower than 0.6 in the surf zone regardless of various vertical resolutions;

268 with the minimum value of 0.2 in the inner surf zone. In S1, the model skills
 269 are between 0.7 and 0.9 across and shoreward of the breaker bar.

270 Ting (2001) observed that the mean of the highest one-third wave-averaged
 271 k values in his irregular waves in the middle surf zone was about the same as
 272 $\bar{k}$ in a regular wave case TK1, where deep-water wave height to wavelength
 273 ratio of those two cases was on the same order. Here, results show that the
 274 values of $\sqrt{\bar{k}/g\bar{D}}$ for the irregular wave case MK2 (not shown) are at the
 275 same order or even larger than those in regular breaking cases in the middle
 276 and inner surf zone. In the outer surf zone, however, the values of $\sqrt{\bar{k}/g\bar{D}}$ are
 277 smaller than those in regular breaking cases. Although the $\bar{k}$ values decrease
 278 near the bottom in the outer surf zone similar to regular breaking cases, they
 279 have small vertical and cross-shore variations in the inner surf zone. Further,
 280 the predicted $\bar{k}$ using 4 and 8 σ -levels have approximately the same structure
 281 and magnitude in the entire surf zone in all cases.

282 4. Steepness-limited unsteady breaking waves

283 In this section, the normalized time and locations are defined as

$$x^* = \frac{x - x_{ob}}{L_c}, \quad z^* = \frac{z}{L_c}, \quad t^* = \frac{t - t_{ob}}{T_c}, \quad (2)$$

284 where T_c and L_c are the period and wavelength of the center frequency wave
 285 of the input packet, respectively. Here, t_{ob} and x_{ob} are the time and location
 286 at which the forward jet hits the free surface, obtained from the corresponding

287 VOF simulations.

288 4.1. Wave-breaking-induced mean circulation

289 Comprehensive experimental work by Rapp & Melville (1990) and Drazen
290 & Melville (2009) has revealed the main characteristics of the ensemble-
291 averaged flow field in unsteady breaking waves, especially after active break-
292 ing. The ensemble-averaged velocity field may be decomposed into

$$\langle \mathbf{u} \rangle = \mathbf{u}_w + \mathbf{u}_{fw} + \mathbf{u}_c, \quad (3)$$

293 where $\mathbf{u}_w$ is the orbital velocity of the surface waves, $\mathbf{u}_{fw}$ is the velocity of
294 the forced long-waves induced by breaking, and $\mathbf{u}_c$ is the Eulerian-mean cur-
295 rent stemming from the momentum loss during the breaking and/or Stokes
296 drift. Rapp & Melville (1990) measured the velocity field using LDV at seven
297 elevations and seven x locations in the breaking region. The measured ve-
298 locity signals are low-pass filtered using the threshold frequency of 0.3 Hz, to
299 remove the surface waves as in Rapp & Melville (1990), where the frequency
300 range of the input surface waves is $0.56 < f_i < 1.20$ Hz. Figure 13 shows
301 the low-pass filtered results and the corresponding measurements for RM1
302 at $x^* = 0.15$ and $x^* = 0.60$, from very close to the free surface to $z^* = -0.15$
303 ($\approx z = -d/2$). The smaller low-passed filtered velocity field is due to the
304 smaller wave dissipation and smaller wave forcing, predicted by the model.

305 The Eulerian-mean current can be calculated by time averaging of the

ensemble-averaged velocity signal,

$$\mathbf{u}_c = \overline{\mathbf{u}} = \frac{1}{t_2^* - t_1^*} \int_{t_1^*}^{t_2^*} \langle \mathbf{u} \rangle dt^*, \quad (4)$$

where t_1^* and t_2^* cover the entire wave packet. During time integration for each grid point, when the point is above the free surface the velocity signal is zero. Figure 14 shows the spatial distribution of the normalized mean current and its horizontal-averaged between $x^* = 0$ and 1.5, as well as the normalized horizontal-averaged mass flux below the depth z^* , $\widehat{M}^*(z^*) = \int_{z_1^*}^{z^*} \widehat{u}_c^* dz^*$ where $z_1^* = -0.31$ is the bottom elevation, for RM1 using 8 σ -levels (top panels) together with the VOF/LES results by Derakhti & Kirby (2014a) (bottom panels). The positive current near the surface, the return negative current at lower depths and the two distinct circulation cells are captured by the model as in the VOF/LES results. Comparing with the measurements of (Rapp & Melville, 1990, Figure 43) and the VOF/LES simulation, we can see that the model generated a large mean vortex with relatively stronger velocity field. We believe this is due to the absence of an enhanced eddy viscosity that would be present as a result of the turbulence, which was not captured by NHWAVE in unsteady breaking cases. In addition, the model predicts relatively larger cells than those predicted by the VOF/LES simulation, especially in the x direction. The predicted patch of persistent vorticity (not shown) is consistent with Drazen & Melville (2009, Figure 4) and the VOF/LES simulation of Derakhti & Kirby (2014a, Figure 4.16),

326 having larger vorticity values due to underestimation of effective viscosity in
327 the absence of turbulence.

328 5. Discussion

329 5.1. Effects of eddy viscosity on undertow profiles

330 In this section, we examine why the model predictions of undertow pro-
331 files, shown in §2, had a relatively larger slope compared with observations
332 in the transition region of the depth-limited breaking waves, where the dis-
333 crepancy was more pronounced in the plunging breakers.

334 Considering available undertow models using an eddy viscosity closure
335 scheme (see Garcez Faria *et al.*, 2000, among others), it is known that the
336 three factors determine the vertical profile of undertow currents; including
337 (i) bottom boundary layer (BBL) processes, leading to a landward stream-
338 ing velocity (Longuet-Higgins, 1953; Phillips, 1977) or a seaward streaming
339 velocity due to a time-varying eddy viscosity within the wave turbulent BBL
340 (Trowbridge & Madsen, 1984), close to the bed; (ii) vertical variations of
341 the eddy viscosity ν_t , affected mainly by breaking-generated turbulence; and
342 (iii) wave forcing due to the cross-shore gradients of radiation stress, set-
343 up, and convective acceleration of the depth-averaged flow. As explained by
344 Garcez Faria *et al.* (2000), the slope of the undertow profile is a function of
345 both wave forcing and ν_t . Large values of wave forcing generates more ver-
346 tical shear, resulting in a parabolic profile, whereas large values of ν_t reduce
347 vertical shear, leading to a more uniform velocity profile with depth.

348 Figure 15 shows the predicted undertow profiles using 8 σ -levels for the
 349 regular depth-limited spilling breaking TK1 and plunging breaking TK2 cases
 350 together with the corresponding measurements, where the different model
 351 predictions are obtained by using constant ($0, 100\nu, 500\nu$, and 1000ν , $\nu =$
 352 $10^{-6}\text{m}^2/\text{s}$) and spatio-temporal varying ν_t calculated based on the $k - \epsilon$
 353 turbulence model. As the magnitude of ν_t increases the slope of the undertow
 354 profiles significantly decreases in both cases. Figure 16 shows the snapshots of
 355 the distribution of modeled ν_t for TK1 using 4, 8 and 16 σ -levels. Far behind
 356 each bore-front region, for example $1 < X < 3$, the spatial structure of the
 357 predicted ν_t is similar to that reported in the VOF/RANS simulation in Lin
 358 & Liu (1998*a*, Figures 12c and 13c), where the magnitude of the former is up
 359 to 30% smaller than the latter. Close to the bore-front regions, the NHWAVE
 360 predicted values of ν_t are significantly smaller than those reported in Lin &
 361 Liu (1998*a*, Figures 12c and 13c), where increasing vertical resolution results
 362 in a greater deviation compared with the VOF/RANS simulation.

363 Compared with observations, we conclude that the underprediction of
 364 modeled ν_t results in greater vertical shear in the predicted undertow profiles,
 365 where the larger discrepancy in TK2 is due to a more noticeable underpre-
 366 diction of ν_t in TK2 compared with that in TK1. In addition, the lack of
 367 second-order BBL effects, and thus the absence of the associated streaming
 368 velocity in the present simulations may explain some part of the difference
 369 between the predicted and measured return current close to the bed.

370 5.2. *Computational efficiency and CPU timing*

371 In this section, we compare the computational cost of the non-hydrostatic
372 σ -coordinate RANS model NHWAVE with our high-resolution VOF/LES
373 model Derakhti & Kirby (2014b) for the regular depth-limited spilling break-
374 ing waves case TK1. The same horizontal grid resolution and Courant num-
375 ber are used in all simulations. Various vertical resolutions of 4, 8 and 16
376 σ -layers are considered for the NHWAVE simulations. Both NHWAVE and
377 VOF/LES model use the HYPRE parallel solver and preconditioner library
378 to solve the linear systems. All simulations are run for a 100s simulation
379 time on the same HPC cluster by using 20 processors.

380 As summarized in Table 3, the NHWAVE simulation with 4 σ -layers are
381 about two order of magnitude faster than the corresponding single-phase
382 VOF/LES simulation. If we assume that the required horizontal resolution
383 for an NHWAVE simulation can be coarser than that for a VOF/LES sim-
384 ulation by a factor of r , the speed ratio of the required CPU-time between
385 two simulations increases by an additional factor of r^3 . Such a considerable
386 decrease in the required computational resources for a typical NHWAVE
387 simulation compared with its comparable VOF/LES simulation makes the
388 NHWAVE model suitable for field-modeling of wave-breaking-induced turbu-
389 lence and mean circulation, providing vertical structure of 3-D velocity field
390 and turbulence statistics over the inner shelf and nearshore regions.

Table 3: Normalized CPU-time of the simulations by the σ -coordinate RANS model NHWAVE using various vertical resolutions and the VOF/LES model Derakhti & Kirby (2014b) for the regular depth-limited spilling breaking waves case TK1. The reference value is the CPU time of the NHWAVE simulation with 4 σ -layers for a 100s simulation time, which is equal to 13 procs.hour.

Cases	R_{CPU}
NHWAVE, 4 σ -layers	1
NHWAVE, 8 σ -layers	1.7
NHWAVE, 16 σ -layers	3
VOF/LES, multi-layer	91

391 6. Conclusions

392 In this paper, we examined wave-breaking predictions ranging from shallow-
393 to deep-water conditions using a 3-D non-hydrostatic σ -coordinate RANS
394 model, NHWAVE (Ma *et al.*, 2012), comparing results both with correspond-
395 ing experiments and with the results of a VOF/Navier-Stokes solver (Ma
396 *et al.*, 2011; Derakhti & Kirby, 2014a; Derakhti, 2016). The new version
397 of NHWAVE has been described in Derakhti *et al.* (2016a), including the
398 new governing equations and consistent surface and bottom boundary con-
399 ditions. We considered regular and irregular depth-limited breaking waves
400 on planar and barred beaches as well as steepness-limited unsteady breaking
401 focused wave packets in intermediate and deep depths. The same equations
402 and numerical methods were used for the various depth regimes and involved
403 no ad-hoc treatment, such as imposing hydrostatic conditions, in triggering
404 breaking. In Part 1 (Derakhti *et al.*, 2016b) of this paper, we showed that
405 NHWAVE, as described in Derakhti *et al.* (2016a), accurately captured or-

ganized wave motions in terms of free-surface evolution, spectral evolution and wave statistics as accurate as those predicted by typical VOF/RANS simulations, where vertical grid resolution in all simulated cases was at least an order of magnitude coarser than that of typical VOF-based simulations. In Part 2, we examined the model reproduction of wave-breaking-induced turbulence and mean circulation. The main conclusions of the present study can be categorized as follows:

(a) Depth-limited breaking waves: using as few as 4 σ -levels, the model was shown to fairly reasonably predict the evolution of the wave-induced set-up as well as the evolution of the ensemble-averaged turbulent kinetic energy, k , distribution under trough levels for depth-limited breaking waves both on a planar and barred beaches as accurate as those predicted by typical VOF/RANS simulations without bubble effects. Under strong plunging breakers, the rapid advection of high k to lower depths can not captured by the model due to the unresolved jet impact and subsequent splash processes. Further, the turbulent eddy viscosity ν_t was underestimated in both spilling and plunging breakers, especially in the breaking crest regions. The underestimation of ν_t was shown to result in the overprediction of the slope of undertow profiles, especially in the transition region. We believe that both the underprediction of k in plunging breakers and the underprediction of ν_t are mainly due to the ignorance of turbulent stress at the free surface in bore-front regions, which the model assumes to be smooth, rather than a selected number of vertical σ -levels. The k prediction above trough levels and the

429 ν_t estimation near crest regions may be further improved by replacing the
430 zero-tangential-stress boundary condition or the zero-gradient boundary con-
431 dition for k with physics-based models such as those proposed by Brocchini
432 & Peregrine (2001) and Brocchini (2002).

433 (b) Steepness-limited breaking waves: Although the absence of turbulence-
434 induced eddy viscosity in unsteady breaking waves predictions led to the
435 overprediction of the velocity and vorticity field in the breaking region, the
436 model was shown to predict the structures of the mean velocity field including
437 large breaking-induced coherent vortices. The near-surface turbulence model
438 for whitecap events, e.g., the model proposed by Brocchini (2002) to set a
439 boundary condition for k , is needed to provide sufficient k levels during active
440 breaking, with which the model will produce the turbulence field, leading to
441 an enhance eddy viscosity and an appropriate amount of breaking-induced
442 dissipation in the breaking region.

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Figure 1: Cross-shore variation of wave-induced set-down/set-up for the regular depth-limited breaking cases (a) TK1, (A) TK2, and (b) S1; and irregular depth-limited breaking cases (B) S2, (c) MK1, and (C) MK2. Comparison between NHWAVE results with 4 σ -levels (dashed lines) and 8 σ -levels (solid lines), and the corresponding measurements (circle markers). Regular waves in TK1, TK2 and S1 were observed to break at $x = 6.4\text{m}$, 7.8m and 53m respectively. Irregular waves in S2 were observed to break as far offshore as $x = 42\text{m}$. In MK1 and MK2 irregular waves start to break approximately at $h_b = 12.5\text{cm}$. In S1 and S2, the top of the bar is located between $x = 52.8\text{m}$ and $x = 56.5\text{m}$. In each case, H_0 is the farthest offshore measured wave height.

Figure 2: Time-averaged velocity field, $\bar{\mathbf{u}}$, for the regular depth-limited spilling breaking case TK1. Comparison between NHWAVE results with (a) 4 σ -levels, (b) 8 σ -levels, and (c) 16 σ -levels as well as (d) the 3-D VOF/LES simulation of Derakhti (2016). Dash lines show the crest $\langle \eta \rangle_{max}$ and trough $\langle \eta \rangle_{min}$ elevations. Colors show $\bar{u}/\sqrt{gD}$. $X = x - x_b$ is the horizontal distance from the initial break point.

Figure 3: Time-averaged normalized horizontal velocity (undertow) profiles for the regular depth-limited spilling breaking case TK1 at (a – d) different cross-shore locations, and (A – D) different cross-shore locations in which the x location of the predicted and observed results are shifted with respect to the simulated and observed break point, where the former is 0.7m far more offshore than the latter in both cases. Here, $X = x - x_b$ represents the horizontal distance from the break point. Comparison between NHWAVE results with 4 σ -levels (dashed lines), 8 σ -levels (solid lines), 16 σ -levels (dotted-dashed lines) and the measurements (circle markers). Here, $z^* = (z - \bar{\eta})/\bar{D}$ where $\bar{D} = h + \bar{\eta}$ is the mean water depth.

Figure 4: Time-averaged normalized horizontal velocity (undertow) profiles for the regular depth-limited plunging breaking case TK2 at (a – d) different cross-shore locations, and (A – D) different cross-shore locations in which the x location of the predicted and observed results are shifted with respect to the simulated and observed break point, where the former is 0.7m far more offshore than the latter in both cases. Here, $X = x - x_b$ represents the horizontal distance from the break point. Comparison between NHWAVE results with 4 σ -levels (dashed lines), 8 σ -levels (solid lines), 16 σ -levels (dotted-dashed lines) and the measurements (circle markers). Here, $z^* = (z - \bar{\eta})/\bar{D}$ where $\bar{D} = h + \bar{\eta}$ is the mean water depth.

Figure 5: Time-averaged velocity field, $\bar{\mathbf{u}}$, for the (a) regular S1 and (b) irregular S2 depth-limited breaking waves on a barred beach using 8 σ -levels. In (a), dash lines show the crest $\langle \eta \rangle_{max}$ and trough $\langle \eta \rangle_{min}$ elevations. In (b), the dash line shows $H_{rms} + \bar{\eta}$. Colors show $\bar{u}/\sqrt{g\bar{D}}$ where $\bar{D} = h + \bar{\eta}$ is the mean water depth. Vertical dashed lines show the cross-shore locations of the results shown in Figure 6. The regular waves were observed to plunge at $x = 53\text{m}$. The random waves were observed to be both plunging and spilling as far offshore as $x = 42\text{m}$.

Figure 6: Time-averaged normalized horizontal velocity (undertow) profiles for the regular depth-limited plunging breaking waves on a barred beach case S1 at (a–g) different cross-shore locations before and after the bar. Comparison between NHWAVE results with 4 σ -levels (dashed lines), 8 σ -levels (solid lines), and the measurements at two different longshore locations (open and solid circle markers). Red lines at (c) show the results 3m seaward of the corresponding measurement location. The regular waves were observed to plunge at $x = 53\text{m}$. Here, $\bar{D} = h + \bar{\eta}$ is the mean water depth.

Figure 7: Snapshots of the turbulent kinetic energy, $k(\text{m}^2/\text{s}^2)$, distribution at $(a - d)$ $(t - t_0)/T = 0.25$ and $(A - D)$ $(t - t_0)/T = 0.75$ for the regular depth-limited spilling breaking case TK1. Comparison between NHWAVE results with (a, A) 4 σ -levels, (b, B) 8 σ -levels, and (c, C) 16 σ -levels as well as (d, D) the 3-D VOF/LES simulation of Derakhti (2016). Here, $X = x - x_b$ is the horizontal distance from the initial break point.

Figure 8: Phase-averaged k time series for the regular depth-limited spilling breaking case TK1 close to the (a – d) bed and (A – D) trough level, at different cross-shore locations before and after the initial break point. Comparison between NHWAVE results with 4 σ -levels (dashed lines), 8 σ -levels (solid lines), 16 σ -levels (dotted-dashed lines) and measurements (circle markers). Here, $z^* = (z - \bar{\eta})/\bar{D}$ where $\bar{D} = h + \bar{\eta}$ is the mean water depth. The observed and simulated initial break points are at $x_b = 6.4\text{ m}$ and 5.7 m respectively.

Figure 9: Phase-averaged k time series for the regular depth-limited plunging breaking case TK2 close to the (a – d) bed and (A – D) trough level, at different cross-shore locations before and after the initial break point. Comparison between NHWAVE results with 4 σ -levels (dashed lines), 8 σ -levels (solid lines), 16 σ -levels (dotted-dashed lines) and measurements (circle markers). Here, $z^* = (z - \bar{\eta})/\bar{D}$ where $\bar{D} = h + \bar{\eta}$ is the mean water depth, and $X = x - x_b$ is the horizontal distance from the initial break point, where the observed and simulated initial break points are at $x_b = 7.8$ m and 7.1 m respectively.

Figure 10: Time-averaged normalized k profiles for the regular depth-limited spilling breaking case TK1 at at $(a-d)$ different cross-shore locations, and $(A-D)$ different cross-shore locations in which the x location of the predicted and observed results are shifted with respect to the simulated and observed break point, where the former is 0.7m far more offshore than the latter in both cases. Here, $X = x - x_b$ represents the horizontal distance from the break point, and $z^* = (z - \bar{\eta})/\bar{D}$ where $\bar{D} = h + \bar{\eta}$ is the mean water depth. Comparison between NHWAVE results with 4 σ -levels (dashed lines), 8 σ -levels (solid lines), 16 σ -levels (dotted-dashed lines) and the measurements (circle markers).

Figure 11: Time-averaged normalized k profiles for the regular depth-limited plunging breaking case TK2 at (a – d) different cross-shore locations, and (A – D) different cross-shore locations in which the x location of the predicted and observed results are shifted with respect to the simulated and observed break point, where the former is 0.7m far more offshore than the latter in both cases. Here, $X = x - x_b$ represents the horizontal distance from the break point, and $z^* = (z - \bar{\eta})/\bar{D}$ where $\bar{D} = h + \bar{\eta}$ is the mean water depth. Comparison between NHWAVE results with 4 σ -levels (dashed lines), 8 σ -levels (solid lines), 16 σ -levels (dotted-dashed lines) and the measurements (circle markers).

Figure 12: Time-averaged normalized k profiles for the surf zone regular breaking waves on a barred beach case S1 at different cross-shore locations before and after the bar. Comparison between NHWAVE results with 4 σ -levels (dashed lines), 8 σ -levels (solid lines), and the measurements (circle markers). Red lines at (c) show the results 3m seaward of the corresponding measurement location. Here, $z^* = (z - \bar{\eta})/\bar{D}$ where $\bar{D} = h + \bar{\eta}$ is the mean water depth.

Figure 13: Normalized low-pass filtered horizontal and vertical velocities for RM1 using 8 σ -levels (dashed lines) and 16 σ -levels (solid lines), at $(a - h)$ $x^* = 0.15$ and $(A - H)$ $x^* = 0.60$ at different elevations (a, A, e, E) $z^* = -0.016$, (b, B, f, F) $z^* = -0.031$, (c, C, g, G) $z^* = -0.078$, (d, D, h, H) $z^* = -0.155$. The circles are the measurements of the corresponding case adopted from Rapp & Melville (1990), Figure 42. The reference value is the wave phase speed of the center frequency wave component of the input packet.

Figure 14: (a, d) Spatial distribution of the normalized mean current, $\mathbf{u}_c^*$; (b, e) normalized horizontal-averaged mean current in the streamwise direction, $\widehat{u}_c^*$ and (c, f) normalized accumulative horizontal-averaged mass flux, $\widehat{M}^*$, in the breaking region for RM1. (a – c) NHWAVE results with 8 σ -levels and (d – f) VOF/LES results by Derakhti & Kirby (2014a).

Figure 15: Time-averaged normalized horizontal velocity (undertow) profiles for the regular depth-limited ($a - d$) spilling TK1 and ($A - D$) plunging TK2 breaking cases at different cross-shore locations before and after the initial break point. Comparison between the NHWAVE results in which ν_t is calculated based on the $k - \epsilon$ turbulence model (black thick solid lines), $\nu_t = 0$ (blue dotted lines), $\nu_t/\nu = 100$ (red dashed lines), $\nu_t/\nu = 500$ (red dotted-dashed lines), and $\nu_t/\nu = 1000$ (red thin solid lines) as well as the measurements of Ting & Kirby (1994) (markers). The observed break point for TK1 and TK2 are at $x_b = 6.4$ m and 7.8 m, respectively.

Figure 16: Snapshots of the normalized turbulent eddy viscosity, ν_t/ν , distribution at $(a - c)$ $(t - t_0)/T = 0.25$ and $(A - C)$ $(t - t_0)/T = 0.75$ for the regular depth-limited spilling breaking case TK1 using (a, A) 4 σ -levels, (b, B) 8 σ -levels and (c, C) 16 σ -levels. Here, $X = x - x_b$ is the horizontal distance from the initial break point.