Incorporating floating surface objects into a fully dispersive surface wave model

Mark D. Orzech^{1*}, Fengyan Shi², Jayaram Veeramony¹, Samuel Bateman³,
Joseph Calantoni³, and James T. Kirby²

¹*NRL Code 7320, 1009 Balch Blvd, Stennis Space Center, MS 39529 USA*

²*Center for Applied Coastal Research, University of Delaware, Newark DE 19716 USA*

³*NRL Code 7434, 1005 Balch Blvd, Stennis Space Center, MS 39529 USA*

**Corresponding author. Phone: 1-228-688-5974. Fax: 1-228-688-4759.*

Email: mark.orzech@nrlssc.navy.mil

Abstract

The shock-capturing, non-hydrostatic, three-dimensional (3D) finite-volume model NHWAVE was originally developed to simulate wave propagation and landslide-generated tsunamis in finite water depth (Ma et al., 2012). The model is based on the incompressible Navier-Stokes equations, in which the z -axis is transformed to a σ -coordinate that tracks the bed and surface. As part of an ongoing effort to simulate waves in polar marginal ice zones (MIZs), the model has now been adapted to allow objects of arbitrary shape and roughness to float on or near its water surface. The shape of the underside of each floating object is mapped onto an upper σ -level slightly below the surface. In areas without floating objects, this σ -level continues to track the surface and bed as before. Along the sides of each floating object, an immersed boundary method is used to interpolate the effects of the object onto the neighboring fluid volume. Provided with the object's shape, location, and velocity over time, NHWAVE determines the fluid fluxes and pressure variations from the corresponding accelerations at neighboring cell boundaries. The system was validated by comparison with analytical solutions and a VOF model for a 2D floating box and with laboratory measurements of wave generation by a vertically oscillating sphere. A steep wave simulation illustrated the high efficiency of NHWAVE relative to a VOF model. In a more realistic MIZ simulation, the adapted model produced

qualitatively reasonable results for wave attenuation, diffraction, and scattering.

Keywords: wave model, floating surface objects, sigma-coordinate,
Navier-Stokes, finite volume, immersed boundary

1. INTRODUCTION

With the growing focus on global warming and its effects in the polar regions, increasing attention has been paid to modeling the interaction of waves with ice floes in marginal ice zones (MIZs) along the edges of Arctic and Antarctic pack ice. As areas of open water continue to grow in the Arctic during spring and summer months, more powerful waves are playing an increasingly important role in the breakup and recession of the sea ice (Thomson and Rogers, 2014). However, at present their representation in operational models such as WAVEWATCH III (Tolman, 2009) and Arctic Cap (Posey et al., 2010) is either crudely parameterized or completely neglected (Zhao et al., 2015). Development and validation of more accurate, physically based representations of wave-ice interaction is urgently needed.

Although mathematical representations of wave-ice interaction were first developed in the mid-20th century (Keller and Weitz, 1953; Evans and Davies, 1968; Wadhams, 1973; Squire et al., 1995), the sophistication and variety of theoretical and numerical wave-ice models accelerated considerably as polar research became a priority and more sophisticated semi-analytical models were developed (Meylan and Squire, 1994; Shen and Squire, 1998; Peter and Meylan, 2004; Bennetts et al., 2007; Kohout and Meylan, 2007). While some modelers have focused on representing the MIZ as a continuous material with varying viscoelastic properties (Zhao and Shen, 2013; Rogers and Orzech, 2013; Zhao and Shen, 2015), other investigators, beginning with Wadhams (Wadhams, 1986) and Meylan (Meylan, 1993; Meylan and Squire, 1996) have instead modeled individual floes as flexible thin elastic plates (Hermans, 2004; Andrianov and Hermans, 2004; Gayen et al., 2005). A common approach has been to use linear potential flow theory (as described in Squire, 2007) in combination with a thin-

27 plate ice model to simulate wave-floe interaction (Kohout and Meylan, 2008;
 28 Williams et al., 2013; Meylan et al., 1997; Bennetts et al., 2010). While this
 29 method is very useful for estimating attenuation of smaller amplitude waves by
 30 floes, it neglects floe collisions and rafting, as well as the overwash of floes by
 31 steeper waves (Yiew et al., 2016). Results from several recent lab experiments
 32 (Toffoli et al., 2015; Bennetts et al., 2015; Bennetts and Williams, 2015) sug-
 33 gest that floe overwash plays a role in producing increased attenuation rates
 34 measured for steeper waves. Field measurements from Kohout et al. (2014)
 35 and Meylan et al. (2014) also indicate a possible amplitude dependence in wave
 36 transmission rates, which is not captured by linear potential theory or repre-
 37 sented in larger scale ocean models. Bennetts and Williams (2015) note that floe
 38 collisions may reduce wave-energy transmission in more densely packed marginal
 39 ice fields. To address some of the issues associated with ice floe properties and
 40 behavior, other modelers have used the discrete element method (DEM) to di-
 41 rectly represent single or multiple ice floes, simulating their evolution and track-
 42 ing their motion in response to currents and/or waves (Hopkins and Thorndike,
 43 2006; Hopkins and Shen, 2001; Hermans, 2013; Polojärvi and Tuhkuri, 2013; Xu
 44 et al., 2012; Song et al., 2014).

45 The wave model adaptations to be described herein are part of a larger
 46 effort to investigate and address the above issues by implementing a realistic 3D
 47 representation of the interactions between waves and Arctic ice floes in small
 48 domains of $O(1-10 \text{ km}^2)$. In this coupled system, waves are to be represented
 49 using the phase-resolving, fully dispersive surface wave model NHWAVE (Ma
 50 et al., 2012). Ice floes are represented using the open-source DEM package
 51 LIGGGHTS (Kloss et al., 2012), in a manner similar to the latter category of
 52 modelers described above (Orzech, et al., 2016; Bateman, et al., 2014; Orzech,
 53 et al., 2014). The system is limited to first-year ice, for which vertical thickness
 54 ($O(m)$) is small relative to horizontal dimensions ($O(100m)$). This property
 55 makes it convenient to represent ice floe effects on waves as surface boundary
 56 conditions in the wave model.

57 The present article focuses on the wave component of this system, describing

the development of numerical methods for the NHWAVE model to correctly represent the fluid response to rigid objects at its surface. Unlike most of the wave-ice models cited above, in which the motion of the freely floating floes is coupled to the waves, objects considered here are either fixed or moving with a prescribed oscillatory motion. As such, they are generally uncoupled from surrounding fluid oscillations and thus act to alter and/or generate surface waves in their vicinity. Vertically oriented surface object effects are incorporated into the model by directly mapping a near-surface σ -level onto the underside (and, if necessary, the top) of each floating object. Horizontally oriented effects are interpolated to fluid cell walls using extensions of the immersed boundary approach. The model is compared to potential-flow models for linear waves and 2D geometries, and to a lab experiment for a 3D geometry. Additional qualitative evaluations are performed for steep waves and a larger scale field-based case.

The following sections detail the specific changes made to NHWAVE, then describe and provide results from convergence tests, comparative analytical simulations, an experimental validation, and the qualitative steep-wave and large-scale cases. Section 2 presents a comparative overview of the wave model in a historical context, followed by a brief review of the governing equations and numerical methods of NHWAVE. A more detailed description of the theory and methods for incorporating floating objects and mapping σ -levels is provided in Section 3. Validation test parameters and all test results are summarized in Section 4, and discussion and conclusions are presented in Section 5.

2. WAVE MODEL

2.1. Background

The behavior of floating objects and their interaction with water waves and currents have been studied from the earliest days of sailing and shipping. Theoretical and modeling efforts have primarily focused on how waves affect ships or structures, as in studies of nonlinear ship dynamics (Spyrou and Thomp-

son, 2000), conditions leading to capsize (Soliman and Thompson, 1991; McCue and Troesch, 2005), and wave-object or wave-platform interactions (Wall et al., 2007; Clauss, 2002; Isaacson and Nwogu, 1987). Wave-structure interaction is usually modeled by the boundary integral method (BIM; e.g., Skourup et al., 1992; Grilli et al., 1994) or a computational fluid dynamics model (CFD; e.g., Hieu and Tanamoto, 2006; Shen and Chan, 2008). BIM is based on potential flow theory and does not allow for calculation of energy dissipation due to wave breaking and friction between flows and structures. Traditional CFD models use either the Volume-of-Fluid (VOF) method or the Marker-and-Cell (MAC) method to treat the free surface or the wave-structure interface and are thus computationally expensive. The smoothed particle hydrodynamics (SPH) method (Gingold and Monaghan, 1977), which treats the fluid as a collection of discrete elements, has also been used to model wave-object interaction (e.g. Rogers et al., 2010), but it is even more computationally demanding.

Models based on the multi-layer Boussinesq equations have been shown to be robust and efficient at simulation of non-linear wave propagation (Nwogu, 1993; Wei et al., 1995) and extended to represent turbulence in the water column (Kim and Lynett, 2011). However, assumptions inherent to these models (i.e., $kh < 3$) limit them to predicting only weakly dispersive shallow water waves in intermediate water depth. In addition, floating objects and the free surface itself can cut through their computational cells in an arbitrary way, significantly affecting the accuracy of their boundary conditions and ultimately degrading their velocity estimates.

The recently developed model NHWAVE (Ma et al., 2012) uses a different approach, solving the Navier-Stokes equations in a transformed domain with a surface/bed-following σ -coordinate instead of a Cartesian z -coordinate. The free surface is treated as a single-valued function of horizontal location, which resolves the issues with arbitrary intersection of cells discussed above. The model defines dynamic pressure at vertically-facing cell sides (i.e., along the σ -levels), which allows for accurate application of the pressure boundary condition at the free surface. With this model configuration, a very small number (i.e., fewer

than 10) of vertical levels is required to accurately describe wave dispersion. The Harten Lax and van Leer (HLL) Riemann approximation (Harten et al., 1983) is used to determine fluxes at cell faces. A shock-capturing Godunov-type approach is employed, allowing the model to deal with discontinuous flow conditions resulting from breaking waves or sudden surface impacts. These simplifications also considerably reduce the computational requirements of the model.

The Pressure Decimation and Interpolation (PDI) Method was added to NHWAVE by Shi et al. (2015), who confirmed that the dynamic pressure can be modeled accurately with only a small number of vertical layers. This significantly increased model efficiency for simulating non-hydrostatic, baroclinic processes. Most recently, Derakhti et al. (2016b,c) carried out extensive model validations of NHWAVE against laboratory data. The focus of their study was to examine the model's capability of predicting breaking waves in both the surf zone and deep water. Results showed that NHWAVE accurately predicted depth-limited breaking waves using only 4 σ -layers and steepness-limited breaking waves using as few as 8 σ -layers.

2.2. Governing Equations

The complete NHWAVE equations can be found in Ma et al. (2012, 2013a,b). Here, we only list the equations related to the present implementation. Following the notation of Ma et al. (2012), the incompressible Navier-Stokes equations in Cartesian coordinates are given by

$$\frac{\partial u_i}{\partial x_i} = 0 \quad (1)$$

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial P}{\partial x_i} + g_i + \frac{\partial \tau_{ij}}{\partial x_j}, \quad (2)$$

where i and j equal 1, 2, and 3, with $(x_1, x_2, x_3) = (x, y, z)$, P equal to total pressure, ρ the water density, g the gravitational acceleration, τ_{ij} the turbulent shear stress, and u_i the velocity component in the x_i direction. The z coordinate points up with an origin at the still water surface. With the transformation

$$\sigma = (z + h)/(\eta + h), \quad (3)$$

145 in which $h(x, y)$ is water depth and $\eta(x, y)$ is free surface elevation relative to
 146 still water level, (1) and (2) can be written in a compact, conservative form in
 147 the σ -coordinate (Shi et al., 2015):

$$\frac{\partial \Psi}{\partial t} + \nabla \cdot \Theta(\Psi) = \mathbf{S}, \quad (4)$$

148 where $\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial \sigma} \right)$. Once it is discretized, (3) defines the σ -levels, which
 149 run from $\sigma = 1$ at the free surface (“first” σ -level) to $\sigma = 0$ at the seabed
 150 (“last” σ -level). Ψ and $\Theta(\Psi)$ are the vector of conserved variables and the
 151 flux vector, respectively.

$$\Psi = \begin{pmatrix} D \\ Du \\ Dv \\ Dw \end{pmatrix} \quad (5)$$

$$\Theta = \begin{pmatrix} Dui + Dvj + \omega \mathbf{k} \\ (Duu + (\frac{1}{2}g\eta^2 + gh\eta))\mathbf{i} + Duv\mathbf{j} + u\omega\mathbf{k} \\ Duv\mathbf{i} + (Dvv + (\frac{1}{2}g\eta^2 + gh\eta))\mathbf{j} + v\omega\mathbf{k} \\ Duw\mathbf{i} + Dvw\mathbf{j} + w\omega\mathbf{k} \end{pmatrix}. \quad (6)$$

152 where $D = h + \eta$, (u, v, w) represent velocity components in the Cartesian coor-
 153 dinate system (x^*, y^*, z^*) , and ω is defined as the vertical velocity in the σ image
 154 domain (the relationship between ω and u_3 can be found in (Ma et al., 2012)).
 155 The source term on the right hand side includes several source components,

$$\mathbf{S} = \mathbf{S}_h + \mathbf{S}_p + \mathbf{S}_\tau, \quad (7)$$

156 where $\mathbf{S}_h$, $\mathbf{S}_p$, and $\mathbf{S}_\tau$ represent the bottom slope term, dynamic pressure gra-
 157 dient, and turbulent mixing, respectively. These terms are expressed as

$$\mathbf{S}_h = \begin{pmatrix} g\eta \frac{\partial h}{\partial x} \\ g\eta \frac{\partial h}{\partial y} \\ 0 \end{pmatrix} \quad \mathbf{S}_p = \begin{pmatrix} -\frac{D}{\rho} \left(\frac{\partial p}{\partial x} + \frac{\partial p}{\partial \sigma} \frac{\partial \sigma}{\partial x^*} \right) \\ -\frac{D}{\rho} \left(\frac{\partial p}{\partial y} + \frac{\partial p}{\partial \sigma} \frac{\partial \sigma}{\partial y^*} \right) \\ -\frac{1}{\rho} \frac{\partial p}{\partial \sigma} \end{pmatrix} \quad \mathbf{S}_\tau = \begin{pmatrix} DS_{\tau x} \\ DS_{\tau y} \\ DS_{\tau z} \end{pmatrix}, \quad (8)$$

158 where p represents the dynamic pressure. The $k-\epsilon$ closure model and the stan-
 159 dard Smagorinsky (1963) Large Eddy Simulation (LES) model are implemented
 160 in NHWAVE (Derakhti et al. (2016a); note that these features are not used in
 161 the tests described below, for which objects are treated as frictionless). $\partial\sigma/\partial x^*$
 162 and $\partial\sigma/\partial y^*$ are calculated by

$$\begin{aligned} \frac{\partial \sigma}{\partial x^*} &= \frac{1}{D} \frac{\partial h}{\partial x} - \frac{\sigma}{D} \frac{\partial D}{\partial x} \\ \frac{\partial \sigma}{\partial y^*} &= \frac{1}{D} \frac{\partial h}{\partial y} - \frac{\sigma}{D} \frac{\partial D}{\partial y}. \end{aligned} \quad (9)$$

163 The governing equation for the free surface can be obtained by integrating the
 164 mass conservation equation over the full range of σ (Ma et al., 2015) and is
 165 written as

$$\frac{\partial D}{\partial t} + \frac{\partial}{\partial x} \left(D \int_0^1 u d\sigma \right) + \frac{\partial}{\partial y} \left(D \int_0^1 v d\sigma \right) = 0. \quad (10)$$

166 2.3. Numerical Implementation

167 Numerically, NHWAVE uses a combined finite-volume and finite-difference
 168 scheme with a Godunov-type method for the spatial discretization and a two-
 169 stage (second-order) SSP Runge-Kutta (R-K) scheme (Gottlieb et al., 2001) for
 170 time stepping. The two-step projection method, which splits the time integra-
 171 tion into a hydrostatic predictor phase and the non-hydrostatic corrector phase,
 172 is used within each R-K stage. The computational grid is Cartesian in the hor-
 173 izontal direction and follows the σ -coordinate in the vertical direction. Here,
 174 we illustrate just the first stage of R-K for brevity.

175 In the hydrostatic phase, an intermediate quantity, $\mathbf{U} = (Du, Dv, Dw)^T$,
 176 is evaluated using the momentum equations with the dynamic pressure term
 177 neglected,

$$\frac{\mathbf{U}^* - \mathbf{U}^n}{\Delta t} = -\nabla \cdot \Theta(\mathbf{U})^n + \mathbf{S}_h^n + \mathbf{S}_\rho^n + \mathbf{S}_\tau^n, \quad (11)$$

178 where $\mathbf{U}^n$ represents $\mathbf{U}$ value at time level n and $\mathbf{U}^*$ is the intermediate value
 179 from the hydrostatic phase. The numerical fluxes $\Theta(\mathbf{U})$ in (11) are discretized
 180 using a second-order Godunov-type finite volume method.

181 In the non-hydrostatic phase, the velocity field is corrected using the dynamic
 182 pressure term:

$$\frac{\mathbf{U}^{(1)} - \mathbf{U}^*}{\Delta t} = \mathbf{S}_p^{(1)}, \quad (12)$$

183 where the superscript $()^{(1)}$ represents the first stage for the R-K scheme. Sub-
 184 stituting (12) into the continuity equation,

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial \sigma} \frac{\partial \sigma}{\partial x^*} + \frac{\partial v}{\partial y} + \frac{\partial v}{\partial \sigma} \frac{\partial \sigma}{\partial y^*} + \frac{1}{D} \frac{\partial w}{\partial \sigma} = 0, \quad (13)$$

185 yields the Poisson equation of the dynamic pressure,

$$\begin{aligned} & \frac{\partial}{\partial x} \left[\frac{\partial p}{\partial x} + \frac{\partial p}{\partial \sigma} \frac{\partial \sigma}{\partial x^*} \right] + \frac{\partial}{\partial y} \left[\frac{\partial p}{\partial y} + \frac{\partial p}{\partial \sigma} \frac{\partial \sigma}{\partial y^*} \right] + \frac{\partial}{\partial \sigma} \left(\frac{\partial p}{\partial x} \right) \frac{\partial \sigma}{\partial x^*} + \\ & \frac{\partial}{\partial \sigma} \left(\frac{\partial p}{\partial y} \right) \frac{\partial \sigma}{\partial y^*} + \left[\left(\frac{\partial \sigma}{\partial x^*} \right)^2 + \left(\frac{\partial \sigma}{\partial y^*} \right)^2 + \frac{1}{D^2} \right] \frac{\partial}{\partial \sigma} \left(\frac{\partial p}{\partial \sigma} \right) = \\ & \frac{\rho}{\Delta t} \left(\frac{\partial u^*}{\partial x} + \frac{\partial u^*}{\partial \sigma} \frac{\partial \sigma}{\partial x^*} + \frac{\partial v^*}{\partial y} + \frac{\partial v^*}{\partial \sigma} \frac{\partial \sigma}{\partial y^*} + \frac{1}{D} \frac{\partial w^*}{\partial \sigma} \right). \end{aligned} \quad (14)$$

186 The same procedure is used for the second stage of the R-K scheme.

187 The velocity and tracer variables are defined at the cell centers. The dynamic
 188 pressure for the Keller-box scheme is defined at the cell faces (Stelling and
 189 Zijlema, 2003). The velocity values at the cell faces, (u^*, v^*, w^*) , are obtained by
 190 linear interpolation using the adjacent cell-centered values for the discretization
 191 of the Poisson equation (14).

192 To model moving boundaries such as shorelines, NHWAVE includes a 2D
 193 moving mask scheme for alternately wet and dry cells. At each time step, the
 194 water depth is determined for each cell, and the cell is assigned a mask value
 195 of one (wet) if the depth is greater than a specified D_{min} and zero (dry) when

depth is less than D_{min} . Dry cells have zero normal flux at cell faces, and wave speeds in neighboring wet cells are modified (see Ma et al., 2012).

3. INCLUSION OF FLOATING OBJECTS

To accomodate floating objects at the water surface, NHWAVE is extended to allow for modification of cell dimensions based on the characteristics of the object boundaries as well as those of the surface and bed (George, 2010). For a partially submerged floating object, the free surface (i.e., first σ -level) is broken at the object's lateral (or top) boundaries. (3) cannot be applied to surface regions occupied by an object. In these regions, NHWAVE is adapted to track the object's vertical motion by mapping a subsurface σ -level directly onto the underside of the object. The model's existing 2D moving-mask capability is upgraded to 3D to represent the interruption of the free surface by fully three-dimensional objects. Kinematic and dynamic boundary conditions at lateral walls of cells touched by an object are determined using the immersed boundary (IB) method (Mittal and Iaccarino, 2005). Critical flow velocities are generally not an issue, as relevant applications involve slow-moving objects floating at (or near) the water surface. Wave breaking is included when warranted by more energetic conditions.

3.1. Coordinate transformation

In the vertical direction, a flexible σ -coordinate transformation is made for the given z coordinates of the top and bottom of the object, z_t and z_b , respectively (Figure 1). The σ -layers are divided into three regions, with constant sublayer thicknesses, Δz , for any horizontal location in each region specified as

$$\Delta z_b = \frac{h + z_b}{n_b} \quad \sigma = 0 \sim k_b \quad (15)$$

$$\Delta z_o = \frac{z_t - z_b}{n_o} \quad \sigma = k_b \sim k_t \quad (16)$$

$$\Delta z_t = \frac{\eta - z_t}{n_t} \quad \sigma = k_t \sim 1, \quad (17)$$

where subscripts b , o , and t designate layers below, inclusive of, and above the object, respectively. k_b and k_t are σ -values of the bottom and top surfaces of the object, and n_b, n_o and n_t represent the number of layers in each region. For ice-floe applications, objects were restricted to the second layer from the top, with $n_t = 1$, $n_o = 1$.

In contrast to the free-surface form expressed by (9), $\frac{\partial \sigma}{\partial x}$ and $\frac{\partial \sigma}{\partial y}$ are calculated using a more general form as

$$\frac{\partial \sigma}{\partial x} = - \frac{\frac{\partial z}{\partial x} \big|_{\sigma}}{\frac{\partial z}{\partial \sigma} \big|_x} \quad (18)$$

$$\frac{\partial \sigma}{\partial y} = - \frac{\frac{\partial z}{\partial y} \big|_{\sigma}}{\frac{\partial z}{\partial \sigma} \big|_y}. \quad (19)$$

3.2. 3D moving masks

The existing code of NHWAVE uses the wetting-drying algorithm to simulate moving boundary processes. The wetting-drying method is implemented using a two-dimensional (2D) horizontal mask (e.g. Shi et al., 2012). The 2D mask is not applicable to modeling wave interactions with ice floes located at the water surface where waves and ice floes interact in three dimensions and waves diffract as they pass under floes. Therefore, the extension of the 2D mask to three dimensions (3D) was necessary.

The implementation was straightforward, following the existing 2D mask approach but taking into account the 3D geometry of individual objects. The mask array was initialized in three dimensions as $\text{MASK}(i, j, k)$ with values of 0 or 1 assigned to cells containing object or fluid, respectively. A 3D object with geometry described in Cartesian coordinates (x, y, z) was first projected onto the NHWAVE horizontal grid, (x, y) as shown in Figure 2, after which the top and bottom surfaces of the object were fitted by the σ -coordinate as described

242 in (15) - (17). Specifically, the mask values are assigned by

$$\begin{aligned} \text{MASK} &= 0 && \text{if object projection includes } u,v \text{ point } (i,j) \text{ and } k = k_b \sim k_t \\ \text{MASK} &= 1 && \text{otherwise.} \end{aligned} \quad (20)$$

243 The influence of objects moving in partially filled cells was taken into account
 244 with an immersed boundary method. The fluxes and accelerations (pressure ef-
 245 fects) of an object were mapped to “boundary points” defined in fluid-filled cells
 246 immediately neighboring the partially filled cells marked by circled crosses in
 247 Figure 2. The boundary points were then used to apply the immersed boundary
 248 conditions. The implementation of this method in the kinematic and dynamic
 249 lateral boundary conditions is described in the following two sections, along
 250 with the simplified vertical boundary conditions resulting from the mapping of
 251 σ -levels.

252 3.3. Dynamic boundary conditions

253 In accordance with the pressure Poisson equation (14), the Neumann bound-
 254 ary condition is used for the dynamic pressure along object boundaries. For an
 255 object at the top or bottom of a cell,

$$\frac{\partial p}{\partial \sigma} = -\rho D \frac{dw_o}{dt} \quad (21)$$

256 where $\frac{dw_o}{dt}$ is the acceleration of the object in the vertical direction. Pressure
 257 components associated with the cell boundary conditions along mapped σ -layer
 258 boundaries are obtained using a central difference in the σ direction. As illus-
 259 trated in Figure 3, the central difference can be expressed by

$$\frac{p_{i,j,k+1} - p_{i,j,k-1}}{2\Delta\sigma} = -\rho_0 D \frac{dw_o}{dt} \quad (22)$$

260 In the horizontal direction, an object may move within a partially filled fluid
 261 cell. Under such conditions, we use the the direct-forcing immersed boundary
 262 method following Ma et al. (2015). An additional virtual forcing term, S_{IB} , was
 263 added to the non-hydrostatic phase,

$$\frac{\mathbf{U}^{(1)} - U^*}{\Delta t} = \mathbf{S}_p^{(1)} + S_{IB}^{(1)}, \quad (23)$$

where S_{IB} is defined as a Dirac delta function, which has non-zero values only at the boundary points defined in Figure 2. $()^{(1)}$ is the first stage of the R-K scheme. The second stage should be performed in the same way. The virtual forcing at the boundary points was given by

$$S_{IB}^{(1)} = \frac{\mathbf{u}_b - U^*}{\Delta t} - \mathbf{S}_p^{(1)} \quad (24)$$

where $\mathbf{u}_b = (u_b, v_b)$, the velocity at a boundary point. The boundary velocity, $\mathbf{u}_b$, can be evaluated by linear interpolation or extrapolation using the fluid velocity at neighboring fluid points and the velocity of the object at the so-called marker points, as shown in Figure 3. For example, in the case demonstrated in Figure 3, u_b is obtained using the fluid velocity $u_{i-2,j,k}$ and the object velocity u_o (marked by red arrow). Detailed model implementation and validation for the immersed boundary method can be found in Ma et al. (2015).

3.4. Kinematic boundary conditions

In the vertical direction, because the top and the bottom of the object are fitted by the σ -coordinate at every time step, the vertical velocity component w (or u_3) can be imposed directly through the kinematic boundary condition

$$w|_{z_o} = -\frac{\partial z_o}{\partial t} - u|_{z_o} \frac{\partial z_o}{\partial x} - v|_{z_o} \frac{\partial z_o}{\partial y} \quad (25)$$

where z_o is the z coordinate of the object surface. (25) is of the same form as the kinematic boundary conditions used at the free surface and the bottom for bottom deformation problems (Ma et al., 2012). In addition, the vertical velocity ω in the image domain satisfies

$$\omega = 0. \quad (26)$$

Note that (26) is derived from the σ -coordinate transformation at the top and bottom surfaces of the object.

In the horizontal direction, as an object shifts within a partially filled fluid cell, its surface creates fluxes into or out of the cell. The numerical fluxes at

the interface of a boundary cell, $\mathbf{u}_s$, can again be obtained by a linear interpolation/extrapolation method using the fluid velocity and the object velocity $\mathbf{u}_o$, as shown in Figure 3. Using (5) and (6), the kinematic boundary conditions can be expressed by

$$Du = Du_s \quad (27)$$

$$\Theta(Du) = Du_s u_s + \frac{1}{2} g \eta^2 + g h \eta \quad (28)$$

in the x direction and

$$Dv = Dv_s \quad (29)$$

$$\Theta(Dv) = Dv_s v_s + \frac{1}{2} g \eta^2 + g h \eta \quad (30)$$

in the y direction.

4. NUMERICAL TESTS AND VALIDATION

4.1. Convergence tests

The test of model convergence with the number of vertical layers in the σ coordinate has been carried out in Ma et al. (2012). The test not only gave a good indication of convergence associated with the vertical discretization, but also verified that the Keller-box scheme (Stelling and Zijlema, 2003) can model surface waves accurately with relative few vertical layers.

In this study, we conduct general tests of model convergence with both horizontal space and time discretization. Following a general approach to a convergence test for a finite-difference or finite volume model, we set up the model in a rectangular domain of 800 m $\times$ 800 m, with a constant water depth of 10 m. The initial condition is given by a motionless Gaussian hump of water with its center located at the center of the domain (x_c, y_c) ,

$$\eta(x, y, t = 0) = a_0 \exp -\gamma[(x - x_c)^2 + (y - y_c)^2]/L^2, \quad (31)$$

where a_0 is the initial height of the hump, $L = 10^4$ m is a scale length, and $\gamma = 0.5$ is the shape coefficient.

310 To test the convergence with respect to the horizontal spacing, we adopt a
 311 sequence of different grid spacing, $dx \times n$ and $dy \times n$, where $dx = dy = 0.25$
 312 m, and $n = 1, 2, \dots, 8$. The time step is kept as a constant, i.e., $dt = 0.0125$ s
 313 (The Courant Numbers are less than 0.5 in all the cases). Figure 4 shows the
 314 convergence rate with the horizontal grid refinement that is demonstrated by
 315 the RMS differences of simulated surface displacements between cases n and
 316 $n + 1$ at $t = 20$ s. It shows that the logarithmic RMS differences decrease
 317 linearly as grid spacing decreases. The averaged Cauchy convergence rate is
 318 1.11, which is consistent with the Godunov-type HLL scheme used in the model
 319 (HLL-Contact implemented in NHWAVE is not used for this study owing to its
 320 numerical instability).

321 The convergence with respect to time discretization is examined in a similar
 322 manner by using a sequence of time steps from 0.0125 to 0.2 s and keeping a
 323 constant horizontal grid spacing $dx = 2$ m. The convergence rate with time
 324 step refinement is shown in Figure 5. It can be seen that the convergence
 325 rate decreases with a decrease of time step. The averaged convergence rate
 326 is 1.25, which is a little lower than expected in terms of the second-order SSP
 327 Runge-Kutta scheme used in the time discretization. The reason for the reduced
 328 convergence rate at a smaller time step is unknown.

329 *4.2. Wave reflection and transmission by fixed floating objects*

330 Reflection and transmission play an important role in reshaping waves that
 331 interact with floating objects. The model accuracy can be evaluated using
 332 wave reflection and transmission coefficients defined by $K_r = H_r/H_i$ and $K_t =$
 333 H_t/H_i , where H_i , H_r and H_t are the incident, reflected and transmitted wave
 334 height, respectively. An analytical solution for surface wave scattering by a
 335 rectangular floating object is provided by Mei and Black (1969), who use linear
 336 potential flow theory based on the formulation and numerical method of Miles
 337 (1967) for the summation of an infinite series.

338 Here we considered a two-dimensional floating object configured as a rect-
 339 angular shape with a length of $2a$ and a draft of d , in water of depth h (Figure

6). To compare with the solution of Mei and Black (1969), NHWAVE was set up in a vertical 2D computational domain 1600 m long and 10 m deep, with the horizontal coordinate specified by $x = 0 - 1600$ m. The rectangular object was centered at $x = 800$ m with a draft of $d = 5$ m. Monochromatic waves were generated by a wavemaker (Lin and Liu, 1998) located at $x = 400$ m. We adopted a small incident wave amplitude $A_0 = 0.05$ m to allow for comparison of results to the linear wave solution from Mei and Black (1969).

To avoid wave reflection from domain boundaries, sponge layers with a thickness of 300 m were used on both sides of the domain. Two configurations of the rectangular object were tested, with $a/d = 1$ and $a/d = 3$. The numerical experiments were performed using a series of wave periods, $T = 5, 6, 8, 10, 12$ and 15 s, to examine the transmission rates in response to different kh values. These resulted in wavelengths of 36 – 144 m and wave steepness values $ka = 0.002 - 0.009$. The horizontal grid size was 1 m except for the case of $T = 5$ s where a grid size of 0.5 m was used in order to resolve the short waves. Five vertical layers were applied to all cases with the object specified in the two top layers.

Figure 7 shows, as an example, the surface elevation from the model with the configuration $a/d = 3$ and $T = 10$ s. The model predicted the transmission of progressive waves of constant amplitude past the object as shown on the right side of the figure ($x = 800 - 1400$ m). On the left side of the object ($x = 400 - 800$ m), partial standing waves were generated by the superposition of the incident waves and partially reflected waves. Model output was compared to results from Mei and Black (1969) in Figure 8. The model predicted transmission rates decreasing with increasing kh values, which agrees very well with the analytical solutions. The model accuracy in predicting wave transmission was evaluated using the normalized root-mean-square error (NRMSE) with respect to the theoretical solutions. The NRMSE formula is defined as

$$NRMSE = \sqrt{\frac{\sum_{i=1}^N (X_i - X'_i)^2}{\sum_{i=1}^N X_i^2}}, \quad (32)$$

where N is the number of data values. X' and X represent the model result and theoretical solution, respectively. NRMSEs were 1.2% and 3.2% for $a/d = 1$ and 3, respectively.

4.3. Wave radiation from a moving object

Another important test for wave-object interaction is the radiation and scattering of water waves by a moving object. Theoretical solutions of wave radiation from a moving object, such as heaving, surging and pitching, have been derived by several authors. The solutions are based on linear wave theory and may be used for the present model tests given approximately linear wave conditions.

Analytical solutions of wave radiation from a heaving object can be found in Black et al. (1971), where Schwinger's variational formulation is applied to the radiation of surface waves due to small oscillations of bodies. In the numerical model tests, we set up the model in the same manner as in the wave refraction tests shown in Figure 6. To compare with Black et al. (1971), the rectangular object was specified with $d = 5$ m and $a = 5$ m. The object oscillated with vertically sinusoidal motion and a small amplitude ($A = 0.1$ m), with several different periods. Figure 9 shows the modeled wave amplitude Amp normalized by the oscillation amplitude A in comparison to the analytical solutions. Model output was close to theory, with a slight underprediction of normalized amplitudes at some frequencies. The NRMSE defined by (32) was 8.3 % for this case.

Wave radiation and scattering from a surging and pitching object were tested against analytical solutions from Zheng et al. (2004). The model setups were similar to the heaving test case except that the motions of the object were prescribed to be surging and pitching. For the surging case, a sinusoidal motion was specified in the x direction with amplitude A again equal to 0.1 m and a range of oscillation periods. Figure 10 shows the model results with comparisons to the analytical solutions, in the same manner as in Figure 9. The model predicted the wave amplitudes reasonably well with some underprediction for lower frequency motions. NRMSE was 5.9 %. Further tests with higher model

398 resolution (0.5 m) reduced this error slightly to 5.6 %.

399 In the tests with a pitching object, the object motion was specified by the
400 amount of pitch with respect to the centroid of the object, i.e., $x = 0$ m and
401 $z = 0$ m. In this case, a maximum pitch amplitude of $A = 0.1$ m was prescribed
402 at the outer edge of the object (i.e., at $x = a$). Figure 11 shows the comparison
403 between the model results and analytical solutions. The model predicted the
404 peak of the wave height at the resonant frequency well and exhibited good
405 agreement with the analytical solutions for all kh values. NRMSE was 2.2 %
406 for this case.

407 The configuration of the heaving, surging, and pitching cases above is atyp-
408 ical for the adapted NHWAVE model, which is configured to represent surface
409 objects that are thin relative to overall water depth, restricting those objects to
410 the first two σ -layers near the surface. A block with a draft of 5 m in a depth
411 of 10 m causes considerable distortion of the fitted upper σ -levels in the wave
412 model. Despite these conditions, the output of the model in all three cases is
413 reasonably close to theory, with an exceptionally small error in the pitching test.

414 *4.4. Steep waves passing a series of floating objects*

415 The above model tests used a small wave amplitude and floating objects
416 with small aspect ratios in order to compare with theoretical solutions based on
417 linear wave theory. In contrast, the main applications of the planned wave-ice
418 system will be to scenarios where waves may be steep and the aspect ratio of
419 typical first-year ice floes may be much larger than those in the above tests.
420 We conducted an additional test in which steeper waves pass through a series
421 of floating objects with a larger aspect ratio. Lacking experimental data and
422 theoretical solutions for such a problem, the model results from NHWAVE are
423 compared with a validated VOF-type Navier-Stokes solver (Shi et al., 2004). As
424 mentioned in the introduction, a VOF-type model uses a function of a volume
425 of fluid to capture the wave surface and the solid-fluid interface. A high grid
426 resolution is needed for this application.

427 NHWAVE and the VOF model were set up in a 800 m long and 10 m deep

428 computational domain. Four fixed floating objects with a width of 50 m and
 429 draft of 1 m were added at locations from 250 m to 750 m as shown in Figure 12.
 430 To make the wave generation exactly the same in the two models, waves were
 431 generated by specifying the theoretical solution at the left boundary instead of
 432 using the internal wavemaker (the internal wavemaker theories are different for
 433 the two models). Sponge layers were not applied. In NHWAVE, the horizontal
 434 grid size was 1 m, and five σ layers were used. In the VOF model, the horizontal
 435 grid size was also 1 m, and a stretched grid was adopted in the vertical direction
 436 with the minimum grid size of 0.01 m around the still water level in order to
 437 capture the wave surface and the water-object interface. The total number of
 438 vertical layers was 120.

439 Waves were generated based on second-order Stokes wave theory. The inci-
 440 dent wave height was 1 m and the wave period was 8 s, resulting in an incident
 441 steepness $ka \approx 0.09$. The waves did not overwash the floes. Figure 12 shows
 442 comparisons of surface elevation between the NHWAVE results (blue solid lines)
 443 and the VOF-model results (red dashed lines) at $t = 20s, 40s, 60s$ and $80s$. Wave
 444 transmission and reflection processes are demonstrated in the snapshots of sur-
 445 face elevation from the two models, where the wave amplitudes and phases
 446 were very close to each other. Qualitatively, these results display the reflection
 447 and transmission characteristics typically seen for steep waves in ice floes; i.e.,
 448 transmitted waves become progressively less steep as they pass successive ice
 449 floes. Note that the VOF-model uses a much higher vertical resolution than
 450 NHWAVE. While results are similar, the computational cost of the VOF model
 451 in this application is an order of magnitude higher than that of NHWAVE.

452 4.5. *Waves generated by an oscillating sphere*

453 Model output is further validated by comparison with the results of a 3D lab-
 454 oratory experiment. While there are many physical tests measuring the effects
 455 of waves on floating objects, relatively few examine the waves and other distur-
 456 bances generated in a fluid by the motions of surface objects. The most common
 457 type of study that includes object-generated wave effects is the analysis of ship

wakes (e.g., Soomere, 2007; Miyata et al., 1992; Torsvik et al., 2009; Soomere, 2005), where waves tend to be large in size and fluid velocities are relatively high. Most laboratory experiments examine the effects of bodies impacting the water at high speed, as seen in landslides (e.g., Panizzo et al., 2002; Heller et al., 2008; Di Risio et al., 2009). We were unable to find any three-dimensional studies in which waves were recorded in a reproducible manner from an object oscillating vertically at more moderate speeds. To obtain such a dataset, we found it necessary to design and implement our own lab experiment.

The experiment was conducted in a cylindrical water tank of 3 m diameter and 2 m depth (Figure 13), in which a 22 cm diameter solid sphere (bowling ball) was attached to a thin cable and manually oscillated up and down at the water surface for several seconds. A metal rod was bolted to the top of the sphere and passed through a vertical pipe on the mounting bracket to ensure consistent vertical motion. The positions of the sphere and the resulting waves were tracked along a horizontal axis with a Riegl VZ1000 scanning LIDAR system (Riegl, 2014). Small pieces of confetti paper were added to the water to enhance the LIDAR reception of surface shape. The LIDAR was operated in line scan mode with a scanning rate of 10 Hz.

After allowing the water surface to become completely calm, the sphere was oscillated over a vertical range of about ± 5 cm at a frequency of roughly 2 Hz for approximately 10 seconds. The positions of the sphere and the water surface (± 2 -3 mm) were recorded by the LIDAR. To avoid contamination by waves reflected from the tank wall, only data from the first 2.5 s were utilized. A higher resolution time series ($dt = 0.01s$) of sphere position was required for the NHWAVE computation and was extracted from the 10 Hz LIDAR data by running a cubic spline interpolation. The vertical scale of the test was very small, with measured wave heights not exceeding 4 cm.

To compare with the experimental results, NHWAVE was set up in a 3m $\times$ 3 m rectangular domain with a configuration otherwise matching that in the tank experiment and set to record water level time series at several locations increasingly farther from the sphere. The grid size was 0.5 m. The NHWAVE

489 surface elevation time series are plotted against LIDAR elevation data at four
 490 locations in Figure 14.

491 Model performance was reasonably good, with wave amplitudes generally
 492 predicted well but with slight phase shifts in some wave peaks. The relative
 493 errors of the maximum wave height, calculated by $|H_{\text{data}} - H_{\text{model}}|/H_{\text{data}}$,
 494 where H_{data} and H_{model} represent the maximum envelope of the surface eleva-
 495 tion, were 1.1%, 2.3%, 1.71% and 0.70%, respectively, at the four measurement
 496 locations ($r = 0.45 \text{ m} \sim 1.05 \text{ m}$). Absolute errors in maximum wave height
 497 ranged from $0.2 - 0.5 \text{ mm}$. Factors that may contribute to these errors include
 498 neglected surface and form drag, nonlinear behavior of the fluid in the immedi-
 499 ate vicinity of the sphere, and errors in interpretation of the LIDAR elevation
 500 data for the sphere and the water surface.

501 Surface and form drag were both small for the smooth bowling ball, which
 502 oscillated over a less-than 10 cm range at under 2 Hz. These conditions also
 503 limited the role of nonlinear effects resulting from flow separation and associated
 504 turbulence. The waves in the 2 m tank were deep water waves, with $kh \approx 32$. It
 505 is likely that measurements based on LIDAR – including surface elevation, hor-
 506 izontal locations, and possibly also time steps – are the greatest contributors to
 507 overall error in the experiment. Analysis of the LIDAR dataset revealed several
 508 millimeters of uncertainty in surface elevations, as described above. We were
 509 unable to determine the relative quality of individual elevation measurements,
 510 so the final elevation results were determined by a simple averaging of elevation
 511 values recorded within $\pm 5 \text{ mm}$ (horizontally) of each selected location at each
 512 time step. This averaging of LIDAR horizontal and vertical data very likely
 513 contributed to both the amplitude and the phase differences between measured
 514 and modeled surface time series.

515 4.6. A large domain simulation

516 To examine the model’s performance in simulating wave-ice floe interaction
 517 for a larger computational domain with more realistic floe size distribution, we
 518 set up a test in a $1.2 \text{ km} \times 1 \text{ km}$ domain with a constant water depth of 10 m.

519 The horizontal coordinates were configured as $x = 0 \sim 1200$ m and $y = 0 \sim 1000$
 520 m. Ice floes with different sizes and a uniform draft of 2 m were distributed at
 521 fixed locations throughout the domain (motion of the floes was not considered).
 522 The ice floe size distribution followed Toyota et al. (2006)'s field observation,
 523 i.e., the ice floe distribution N was estimated empirically as a function of floe
 524 diameter d as

$$N(d) = N_0 d^{-\alpha} \quad (33)$$

525 where N_0 is a parameter representing the general density of ice floes and α is an
 526 empirical coefficient obtained by the least squares method using field data. The
 527 parameter α has a large range and depends on the size range of the ice floes.
 528 For floes smaller than 40 m, a reasonable estimate is $\alpha = 1.15$. In this case,
 529 we adopted $N_0 = 1500.0$ and $\alpha = 1.15$. The floes were represented as square
 530 blocks, with side lengths discretized into 20, 25, 30, 35, and 40 m. The total
 531 number of ice floes was 160 based on (33). The floes were distributed in the
 532 region $x = 300 \sim 1100$ m and $y = 100 \sim 900$ m. A random function was used
 533 to assign each floe location.

534 The model grid size was 2 m in both x and y directions. Five vertical
 535 layers were used. The internal wavemaker was located at $x = 100$ m. Normally
 536 incident waves were generated using the JONSWAP spectral distribution with
 537 a significant wave height of 1 m and peak period of 8 s. Waves did not overwash
 538 the floes. Sponge layers of 50 m were used at the four lateral boundaries.

539 Figure 15 shows a snapshot of wave surface elevation and ice floes in the do-
 540 main (wavemaker and sponge layers are not shown). Results are qualitatively
 541 reasonable. As waves pass through the irregularly distributed floes, stems are
 542 formed due to the combination of reflected and transmitted waves. Wave diffrac-
 543 tion can be seen behind groups of ice floes and larger individual floes. Wave
 544 magnitude attenuates with increasing distance through the field of ice floes.

545 This test was performed on the Linux cluster Farber located at the University
 546 of Delaware. Using 48 cores, it took about 8 hours for a simulation of 1000 s.
 547 The 10 m water depth was selected to reduce the computational time step (via

the CFL condition). With a depth of 100 m, this simulation would require
roughly 24 hours. A more detailed analysis of wave scattering, transmission
rates and wave breaking, including validation with available laboratory and
field data, will be conducted when the fully coupled NHWAVE-DEM system
becomes operational.

553 5. DISCUSSION AND CONCLUSIONS

554 As part of a larger effort to model wave interaction with Arctic ice floes, the
555 non-hydrostatic, finite volume model NHWAVE has been modified to accomo-
556 date floating surface objects. A flexible coordinate transformation was added
557 to the σ -coordinate (i.e., vertical) dimension of the model, allowing σ -layers in
558 the vicinity of a floating object to be mapped directly onto the top and bottom
559 surfaces of the object. The existing 2D horizontal mask for wetting and drying
560 was extended to 3D to keep track of fluid cells that are partially or completely
561 occupied by a floating object. Using an immersed boundary method, kinematic
562 boundary conditions were modified to account for the fluid fluxes generated
563 in neighboring cells by the motion of the fluid/object interface, and dynamic
564 boundary conditions were added to calculate changes in fluid pressure due to
565 vertical and horizontal accelerations of the floating object.

566 Model convergence rates for space and time discretization were verified to
567 be within normal ranges. The adapted model was then validated by compari-
568 son to analytical solutions for wave transmission, reflection, and generation by
569 a floating rectangular block. Model-estimated results compared well to theory
570 for all tests, although generated wave heights were somewhat underestimated at
571 lower oscillation frequencies for a heaving or surging block. The modest under-
572 estimations resulted from the relatively large draft of the floating object used,
573 the oversimplified representation of the object motion in the theoretical deriva-
574 tion, and the smaller amplitude motions not fully resolved in virtual forcing
575 computed by the immersed boundary method.

576 A qualitative comparison was provided for a case of steep waves passing
577 through multiple ice floes. Results from NHWAVE closely matched those from
578 a VOF solver, with both models predicting transmission of lower frequency
579 wave components and reflection of higher frequency wave components, reducing
580 the steepness of transmitted waves. The adapted model was also much more
581 efficient, producing the output roughly ten times faster than the VOF model.

582 NHWAVE was next validated by comparison to measurements from a labora-

583 tory experiment in which waves were created by a vertically oscillating sphere.
584 Surface elevation time series of small amplitude ($O(10\text{ cm})$) waves were rea-
585 sonably well matched by the model at four locations, with a slight difference in
586 phase apparent at some locations. The phase difference was likely due primarily
587 to LIDAR-related errors in measurement of surface elevations. A smaller contri-
588 bution to model error came from surface and form drag effects that were not yet
589 included in this version of NHWAVE. The implementation of these additional
590 features represents future work.

591 For a final qualitative evaluation, the adapted model was used in a larger-
592 scale domain to simulate wave transmission, reflection, and diffraction by a field
593 of ice floes with a realistic floe size distribution. Simulation results indicated
594 significant reflection and scattering of incident waves by the floes, with evidence
595 of diffraction behind larger ice pieces. Wave attenuation resulting from overwash
596 of floes was not included in this case or the earlier steep wave case, but tests will
597 be performed in the near future with the coupled wave-ice system to compare
598 model output to recent lab results from Toffoli et al. (2015) and Bennetts et al.
599 (2015).

600 Overall wave model performance was satisfactory in all validations of NHWAVE
601 with floating objects present in the simulation domain. The modifications to
602 NHWAVE presented here represent a key component to the larger wave-ice
603 modeling project. Future work will be focused on coupling NHWAVE with the
604 discrete element method to represent ice floes as collections of small bonded
605 particles. When fully coupled, the two models will simulate the exchange of
606 momentum and energy between waves and ice floes, including refraction, reflec-
607 tion, and diffraction of waves as well as losses due to shear, fracturing, overwash,
608 and collision of floes. While this research is primarily oriented toward ice floes
609 in waves, the adapted model described here could potentially be applied to
610 other floating objects in deep water such as barges, ships, platforms, flotsam or
611 garbage patches.

612 6. ACKNOWLEDGMENTS

613 This work was supported under base funding to the Naval Research Labo-
614 ratory from the Office of Naval Research. The work was also supported in part
615 by a grant of computer time from the DoD High Performance Computing Mod-
616 ernization Program at the ERDC DSRC and Navy DSRC. FS received support
617 from Office of Naval Research Award #N0014-14-0569. We thank the technical
618 support team from code 7434 for their invaluable assistance in completing the
619 infamous "bowling ball experiment". We also thank the three anonymous re-
620 viewers for their detailed and constructive comments, which have significantly
621 improved this manuscript.

622 REFERENCES

- 623 Andrianov, A.I., Hermans, A.J., 2004. Hydroelasticity of elastic circular plate.
624 In: Landrini, M., Campana, E.F., Iafrati, A. (Eds.), Proc. 19th Intl. Work-
625 shop on Water Waves and Floating Bodies. Istituto Nazionale Studi ed Espe-
626 rienze di Architettura Navale, Cortona, Italy.
- 627 Bateman, S., Shi, F., Orzech, M., Veeramony, J., Calantoni, J., 2014. Discrete
628 element method simulations of ice floe dynamics. AGU 2014 Fall Meeting,
629 Abstract EP43C-3580, San Francisco, CA.
- 630 Bennetts, L.G., Briggs, N.R.T., Porter, D., 2007. A multi-mode approximation
631 to wave scattering by ice sheets of varying thickness. J. Fluid Mech. 579,
632 413-443.
- 633 Bennetts, L.G., Peter, M.A., Squire, V.A., Meylan, M.H., 2010. A three-
634 dimensional model of wave attenuation in the marginal ice zone. J. Geophys.
635 Res. 115, C12043.
- 636 Bennetts, L.G., Alberello, A., Meylan, M.H., Cavaliere, C., Babanin, A.V.,
637 Toffoli, A., 2015. An idealised experimental model of ocean surface wave
638 transmission by an ice floe. Oc. Mod. 96, 85-92.

639 Bennetts, L.G., Williams, T.D., 2015. Water wave transmission by an array of
640 floating discs. *Proc. R. Soc. A* 471, 20140698.

641 Black, J.L., Mei, C.C., Bray, M.C.G., 1971. Radiation and scattering of water
642 waves by rigid bodies. *J. Fluid Mech.* 46, 151-164.

643 Derakhti, M., Kirby, J. T., Shi, F., and Ma, G., 2016a. NHWAVE: Govern-
644 ing equations, consistent boundary conditions and turbulence modeling. *Oc.*
645 *Mod.*, under review.

646 Derakhti, M., Kirby, J. T., Shi, F., and Ma, G., 2016b. Wave breaking in the
647 surf zone and deep water in a non-hydrostatic model. Part 1: Cross-shore
648 wave transformation. *Oc. Mod.*, in revision.

649 Derakhti, M., Kirby, J. T., Shi, F., and Ma, G., 2016c. Wave breaking in the surf
650 zone and deep water in a non-hydrostatic model. Part 2: Breaking-induced
651 velocity and turbulence. *Ocean Modelling*, in revision.

652 Clauss, G. F., 2002. Dramas of the sea: episodic waves and their impact on
653 offshore structures. *Appl. Oc. Res* 24, 147-161.

654 Di Risio, M., Bellotti, G., Panizzo, A., 2009. Three-dimensional experiments on
655 landslide generated waves at a sloping coast. *Coast. Engrg.* 56(5-6), 659-671.

656 Doctors, L.J., 2003. The influence of viscosity on the wavemaking of a model
657 catamaran. *Proc. 18th Int'l Wkshp on Water Waves and Floating Bodies*, Le
658 Croisie, France 12, 1-4.

659 Eldhuset, K., 1996. An automatic ship and ship wake detection system for
660 spaceborne SAR images in coastal regions. *IEEE Trans. on Geosci. and Rem.*
661 *Sens.* 34(4), 1010-1019.

662 Evans, D.V., Davies, T.V., 1968. Wave-ice interaction. Rep. 1313, Davidson
663 Lab., Stevens Inst. Technol., Hoboken, NJ.

664 Gayen, R., Mandal, B.N., Chakrabarti, A., 2005. Water-wave scattering by an
665 ice-strip. *J. Eng. Math.* 53(1), 21-37.

666 George, D.L., 2010. Adaptive finite volume methods with well-balanced Rie-
 667 mann solvers for modeling floods in rugged terrain: application to the Mal-
 668 passet dam-break flood (France, 1959). *Int. J. Numer. Meth. Fluids* 66, 1000-
 669 1018.

670 Gingold, R.A., Monaghan, J.J., 1977. Smoothed particle hydrodynamics: theory
 671 and application to non-spherical stars. *Mon. Not. R. Astron. Soc.* 181, 375-
 672 389.

673 Gottlieb S., Shu C.-W. and Tadmor E., 2001. Strong stability-preserving high-
 674 order time discretization methods. *SIAM Review* 43, 89-112.

675 Grilli, S.T., Losada, M.A. and Martin, F., 1994. Characteristics of solitary wave
 676 breaking induced by breakwaters. *J. Water. Port Coastal & Oc. Engrg.* 120,
 677 1, 74-92.

678 Harten, A., Lax, P., van Leer, B., 1983. On upstream differencing and Godunov
 679 type methods for hyperbolic conservation laws. *SIAM Review* 25(1), 35-61.

680 Heller, V., Hager, W. H., Minor, H.E., 2008. Scale effects in subaerial landslide
 681 generated impulse waves. *Exp. in Fluids* 44(5), 691-703.

682 Hermans, A., 2013. Interaction of free-surface waves with floating flexible strips.
 683 *J. Eng. Math.* 49(2), 133-147.

684 Hermans, A., 2013. Numerical modeling of force and contact networks in frag-
 685 mented sea ice. *Annals Glaciol.* 54(62), 114-120.

686 Hieu, P.D. and Tanimoto, K., 2001. Verification of a VOF-based two-phase
 687 flow model for wave breaking and wave-structure interactions. *Oc. Engrg.* 33,
 688 11-12 1565-1588.

689 Hopkins, M. A., Shen, H. H., 2001. Simulation of pancake ice in a wave field.
 690 *Annals Glaciol.* 33, 355-360.

691 Hopkins, M. A., Thorndike, A. S., 2006. Floe formation in Arctic sea ice. *J.*
 692 *Geophys. Res.* 111, C11S23.

693 Isaacson, M., Nwogu, O. U., 1987. Wave loads and motions of long structures
694 in directional seas. *Trans. ASME* 109, 126-132.

695 Keller, J.B., Weitz, M., 1953. Reflection and transmission coefficients for waves
696 entering or leaving an icefield. *Commun. Pure Appl. Math.* 6(3), 415-417.

697 Kim, D. H., Lynett, P. J., 2011. Turbulent mixing and passive scalar transport
698 in shallow flows. *Phys. Fluids* 23(1), 016603.

699 Kloss, C., Goniva, C., Hager, G., Amberger, S., Pirker, S., 2012. Models,
700 algorithms and validation for opensource DEM and CFD-DEM. *Progress in*
701 *Comp. Fluid Dyn., An Int. J.* 12(2/3), 140-152.

702 Kohout, A.L., Meylan, M.H., 2007. A model for wave scattering in the marginal
703 ice zone based on a two-dimensional floating elastic plate solution. *Ann.*
704 *Glaciol.* 44, 101-107.

705 Kohout, A.L., Meylan, M.H., 2008. An elastic plate model for wave attenuation
706 and ice floe breaking in the marginal ice zone. *J. Geophys. Res.* 113(C9),
707 C09016.

708 Kohout, A.L., Williams, M.J.M., Dean, S.M., Meylan, M.H., 2014. Storm-
709 induced sea-ice breakup and the implications for ice extent. *Nature* 509,
710 13262.

711 Lee, C.M., Cole, S., Doble, M., Freitag, L., Hwang, P., Jayne, S., Jeffries, M.,
712 Krishfield, R., Maksym, T., Maslowski, W., Owens, B., Posey, P., Rainville,
713 L., Roberts, A., Shaw, W., Stanton, T., Thomson, J., Timmermans, M.,
714 Toole, J., Wadhams, P., Wilkinson, J.,
715 bibinfoauthorZhang, J., bibinfoyear2012. Marginal Ice Zone (MIZ) Program:
716 Science and Experiment Plan. APL-UW TR1201, Technical Report, Applied
717 Physics Laboratory, University of Washington, Seattle, 48pp..

718 Liang, Q., Marche, F., 2009. Numerical resolution of well-balanced shallow
719 water equations with complex source terms. *Adv. Water Res.* 32, 873-884.

720 Lin, P., Liu, P.L.F., 1998. Turbulence transport, vorticity dynamics, and solute
721 mixing under plunging breaking waves in surf zone. *J. Geophys. Res.* 103(C8),
722 15,677-15,694.

723 Ma, G., Shi, F., Kirby, J. T., 2012. Shock-capturing non-hydrostatic model for
724 fully dispersive surface wave processes. *Ocean Modelling* 43-44, 22-35.

725 Ma, G., F. Shi, and J. T. Kirby, 2013a. Numerical simulation of tsunami waves
726 generated by deformable submarine landslides, *Oc. Mod.*, 69, 146-165.

727 Ma, G., J. T. Kirby, S. F. Su, J. Figlus and F. Shi, 2013b, Numerical study
728 of turbulence and wave damping induced by vegetation canopies, *Coastal*
729 *Engineering*, 80, 68-78.

730 Ma, G., A. A. Farahani and F. Shi, 2015, Modeling wave-structure interactions
731 by an immersed boundary method in a σ -coordinate model, in preparation.

732 McCue, L., Troesch, A., 2005. Probabilistic determination of critical wave height
733 for a multi-degree of freedom capsizing model. *Oc. Engrg.* 32, 1608-1622.

734 Mei, C.C., Black, J.L., 1969. Scattering of surface waves by rectangular obstacles
735 in waters of finite depth. *J. Fluid Mech.* 38, 499-511.

736 Meylan, M.H., 1993. The behaviour of sea ice in ocean waves. PhD Thesis,
737 University of Otago, Dunedin, New Zealand.

738 Meylan, M.H., Squire, V.A., Fox, C., 1997. Towards realism in modeling ocean
739 wave behavior in marginal ice zones. *J. Geophys. Res.* 102, 22981-22991.

740 Meylan, M.H., Bennetts, L.G., Kohout, A.L., 2014. In situ measurements and
741 analysis of ocean waves in the Antarctic marginal ice zone. *Geophys. Res.*
742 *Lett.* 41, 5046-5051.

743 Meylan, M.H., Squire, V.A., 1994. The response of ice floes to ocean waves. *J.*
744 *Geophys. Res.* 99(C1), 899-900.

745 Meylan, M.H., Squire, V.A., 1996. Response of a circular ice floe to ocean waves.
746 J. Geophys. Res. 101(C4), 8869-8884.

747 Miles, J.W., 1967. Electrostatics of slender bodies. J. Appl. Phys. 38(1), 192-
748 196.

749 Mittal, R., Iaccarino, G., 2005. Immersed boundary methods. Annual Review
750 of Fluid Mechanics 37, 239–261.

751 Miyata, H., Ming, Z., Watanabe, O., 1992. Numerical study on a viscous-flow
752 with free-surface waves about a ship in steady straight course by a finite
753 volume method. J. of Ship Res. 36(4), 332-345.

754 Nwogu, O., 1993. Alternative form of Boussinesq equations for nearshore wave
755 propagation. J. Waterway, Port, Coastal and Ocean Engrg 119, 618-638.

756 Nwogu, O., 2007. Numerical modeling of waves generated by high-speed vessels
757 in shallow water with a coupled Boussinesq-panel method. 9th Int'l Conf. on
758 Num. Ship Hydrodyn., Ann Arbor, MI, Aug 5-8, 2007.

759 Orzech, M., Veeramony, J., Shi, F., Bateman, S., Calantoni, J., 2016. Modeling
760 small-scale physics of waves and ice in the MIZ. AGU 2016 Ocean Sciences
761 Meeting, Session 9483, Abstract 89970, New Orleans, LA.

762 Orzech, M., Shi, F., Calantoni, J., Bateman, S., Veeramony, J., 2014. Small-
763 scale modeling of waves and floes in the marginal ice zone. AGU 2014 Fall
764 Meeting, Abstract C11A-0347, San Francisco, CA.

765 Panizzo, A., Belotti, G., De Girolamo, P., 2002. Application of wavelet trans-
766 form analysis to landslide generated waves. Coast. Engrg. 44(4), 321-338.

767 Peter, M.A., Meylan, M.H., 2004. Infinite-depth interaction theory for arbitrary
768 floating bodies applied to wave forcing of ice floes. J. Fluid Mech. 500, 145-
769 167.

770 Polojärvi, A., Tuhkuri, J., 2013. On modeling cohesive ridge keel punch through
771 tests with a combined finite-discrete element method. *Cold Reg. Sci. Tech.*
772 85, 191-205.

773 Posey, P.G., Metzger, E.J., Wallcraft, A.J., Smedstad, O.M., Phelps, M.W.,
774 2010. Validation of the 1/12 Arctic Cap Nowcast/Forecast System (ACNFS).
775 NRL Report NRL/MR/7320-10-9287.

776 Riegl Laser Measurement Systems, 2014. Produktdetail, Riegl VZ-1000.
777 <http://www.riegl.com/nc/products/terrestrial-scanning/>, visited Dec. 9, 2014.

778 Rogers, B., Dalrymple, R., Stansby, P., 2010. Simulation of caisson breakwater
779 movement using 2-D SPH. *J. Hydraul. Res.* 48, 135-141.

780 Rogers, W.E., Orzech, M.D., 2013. Implementation and testing of ice and mud
781 source functions in WAVEWATCH III. Naval Research Laboratory Memo-
782 randum Report 7320-13-9462, April 2013.

783 Shen, L. and Chan, E.S., 2008. Numerical simulation of fluid-structure interac-
784 tion using a combined volume of fluid and immersed boundary method. *Oc.*
785 *Engrg.* 35(8-9), 939-952.

786 Shen, H.H., Squire, V.A., 1998. Wave damping in compact pancake ice fields
787 due to interactions between ice cakes. In: Jeffries, M. (Ed.), *Antarctic Sci.*
788 *Res. Series*, Amer. Geophys. Union 74, 325-341.

789 Shi, F., Zhao, Q., Kirby, J. T., Lee, D.S., Seo, S.N., 2004. Modeling of wave
790 interaction with complex coastal structures using an enhanced VOF model.
791 *Proc. 29th Conf. Coastal Engrg.*, Lisbon, Portugal, 13pp.

792 Shi, F., Kirby, J. T., Harris, J. C., Geiman, J. D., Grilli, S. T., 2012. A high-
793 order adaptive time-stepping TVD solver for Boussinesq modeling of breaking
794 waves and coastal inundation. *Oc. Mod.* 43-44, 36-51.

795 Shi, J., Shi, F., Kirby, J. T., Ma, G., Wu, G., Tong, C., Zheng, J., 2015. Pressure
796 decimation and interpolation (PDI) method for a baroclinic non-hydrostatic
797 model. *Oc. Mod.* 96, 265-279.

798 Skourup, J., Sterndorff, M.J., Hansen, E.A., 1992. Numerical modelling of
799 wave-structure interaction by a three-dimensional non-linear boundary ele-
800 ment method: A step towards the numerical wave tank. *Oc. Engrg.* 5,
801 437-460.

802 Smagorinsky, J., 1963. General circulation experiments with the primitive equa-
803 tions, i. the basic experiment. *Monthly Weather Rev.* 91, 99-164.

804 Soliman, M. S., Thompson, J. M. T., 1991. Transient and steady state analysis
805 of capsize phenomena. *Appl. Oc. Res.* 13(2), 82-92.

806 Song, A., Morriss, B.F., Deeb, E.J., Richter-Menge, J., Perovich, D.K., Hopkins,
807 M.A., 2014. High resolution sea ice dynamics modeling using the discrete
808 element method. *Amer. Geophys. Union Fall Mtg. 2014*, Abstract C54A-08.

809 Soomere, Tarmo, 2005. Fast ferry traffic as a qualitatively new forcing factor of
810 environmental processes in non-tidal sea areas: a case study in Tallinn Bay,
811 Baltic Sea. *Envir. Fl. Mech.* 5, 293-323.

812 Soomere, T., 2007. Nonlinear components of ship wake waves. *Appl. Mech.*
813 *Rev.* 60(1-6), 120-138.

814 Spyrou, K. J., Thompson, J. M. T., 2000. The nonlinear dynamics of ship
815 motions: a field overview and some recent developments. *Phil. Trans. R. Soc.*
816 *Lond. A* 358, 1735-1760.

817 Squire, V.A., Dugan, J.P., Wadhams, P., Rottier, P.J., Liu, A.K., 1995. Of
818 ocean waves and sea ice. *Annu. Rev. Fluid Mech.* 27, 115-168.

819 Squire, V.A., 2007. Of ocean waves and sea-ice revisited. *Cold Reg. Sci. and*
820 *Tech.* 49, 110-133.

821 Stelling, G., Zijlema, M., 2003. An accurate and efficient finite-difference algo-
822 rithm for non-hydrostatic free surface flow with application to wave propaga-
823 tion. *Intl. J. Numer. Methods Fluids* 43, 1-23.

Thomson, J., Squire, V., Rogers, E., Babanin, A., Guest, P., Maksym, T.,
 Wadhams, P., Stammerjohn, S., Fairall, C., Persson, O., Doble, M., Graber,
 H., Shen, H., Gemmrich, J., Lehner, S., Holt, B., Williams, T., Meylan,
 M., Bidlot, J., 2013. Sea state and boundary layer physics of the emerging
 Arctic Ocean. Technical Report, APL-UW TR1306, Appl. Phys. Lab., U. of
 Washington, Seattle, 59pp.

Thomson, J., Rogers, W.E., 2014. Swell and sea in the emerging Arctic Ocean.
 Geophys. Res. Letters 41, 3136-3140.

Toffoli, A., Bennetts, L.G., Meylan, M.H., Cavaliere, C., Alberello, A., Elsnab,
 J., Monty, J.P., 2015. Sea ice floes dissipate the energy of steep ocean waves.
 Geophys. Res. Lett., 42, 8547-8554.

Tolman, H.L., 2009. User Manual and System Documentation of WAVEWATCH
 III(TM) Version 3.14. Tech. Note, NOAA/NWS/NCEP/MMAB, 220 pp..

Torsvik, T., Didenkulova, I., Soomere, T., 2009. Variability in spatial patterns
 of long nonlinear waves from fast ferries in Tallinn Bay. Nonlinear Proc. in
 Geophys. 16(2), 351-363.

Toyota, T., Takatsuji, S., Nakayama, M., 2006. Characteristics of sea ice floe
 size distribution in the seasonal ice zone. Geophys. Res. Ltrs. 33, L02616.

Wadhams, P., 1973. The effect of a sea ice cover on ocean surface waves. PhD
 thesis, Univ. Cambridge, UK, 223 pp.

Wadhams, P., 1986. The seasonal ice zone/The Geophysics of Sea Ice. Springer,
 USA, 825-991.

Wadhams, P., Squire, V.A., Goodman, D.J., Cowan, A.M., Moore, S.C., 1988.
 The attenuation rates of ocean waves in the marginal ice zone. J. Geophys.
 Res. 93(C6), 6799-6818.

Wall, W. A., Genkinger, S., Ramm, E., 2007. A strong coupling partitioned ap-
 proach for fluid-structure interaction with free surfaces. Computers & Fluids
 36, 169-183.

852 Wei, G., Kirby, J. T., Grilli, S. T., Subramanya, R., 1995. A fully nonlinear
853 Boussinesq model for surface waves. Part 1. Highly nonlinear unsteady waves.
854 J. Fluid Mech. 294, 71-92.

855 Williams, T.D., Bennetts, L.G., Dumont, D., Squire, V.A., Bertino, L., 2013.
856 Wave-ice interactions in the marginal ice zone, Part 1: Theoretical founda-
857 tions. Ocean Mod. 71, 81-91.

858 Xu, Z., Tartakovsky, A. M., Pan, W., 2012. Discrete-element model for the
859 interaction between ocean waves and sea ice. Phys. Rev. E 85, 016703.

860 Yiew, L.J., Bennetts, L.G., Meylan, M.H., French, B.J., Thomas, G.A., 2016.
861 Hydrodynamic responses of a thin floating disk to regular waves. Ocean Mod.
862 97, 52-64.

863 Zheng, Y. H., You, Y. G., Shen, Y. M., 2004. On the radiation and diffraction
864 of water waves by a rectangular buoy. Ocean Engrg. 31, 1063-1082.

865 Zhao, X., Shen, H.H., 2015. Ocean wave transmission and reflection by vis-
866 coelastic ice covers. Ocean Mod. 92, 1-10.

867 Zhao, X., Shen, H.H., Cheng, S., 2015. Modeling ocean wave propagation under
868 sea ice covers. Acta Mech. Sinica 31(1), 1-15.

869 Zhao, X., Shen, H.H., 2013. Ocean wave transmission and reflection between
870 two connecting viscoelastic ice covers: An approximate solution. Ocean Mod.
871 71, 102-113.

Figure 1: σ -coordinate transformation.

Figure 2: 3D masks (gray color) in the x-y plane. Object is plotted in blue. Crosses: u and v points. Circled crosses: boundary points for the lateral immersed boundary conditions.

Figure 3: Sketch for the immersed boundary condition in the x - z plane with pressure (circles), velocity (crosses), boundary points for the lateral immersed boundary condition (circled crosses), cell boundary fluxes (squares), and marker points for local object velocity (red circles/arrows).

Figure 4: Convergence rates with horizontal grid refinement.

Figure 5: Convergence rates with time step refinement.

Figure 6: Sketch of 2D floating rectangular object.

Figure 7: Snapshot of wave reflection and transmission by a floating object.

Figure 8: Comparisons between model results and analytical solution of wave transmission past a floating object.

Figure 9: Comparisons between model results and analytical solution of waves generated by object heaving.

Figure 10: Comparisons between model results and analytical solution of waves generated by object surging.

Figure 11: Comparisons between model results and analytical solution of waves generated by object pitching.

Figure 12: Comparisons between NHWAVE (blue solid lines) and VOF model (red dashed lines) showing transmission and reflection of steep wave components by floating objects (light blue rectangles).

Figure 13: Experimental setup for wave generation by oscillating sphere, side view.

Figure 14: Elevation time series from oscillating sphere experiment, showing vertical position of sphere center (top panel) and water elevation at increasing distance (r) from sphere center (remaining panels). Solid line is model, x is LIDAR data, and dashed line is fitted to data.

Figure 15: Snapshot of surface elevation in a simulation of irregular waves passing random distributed ice floes. Colorbar represents surface elevation.