Limited Fetch Revisited: Comparison of Wind Input Terms, in Surface Wave Modeling

Pushkarev Andrei^{b,c,d,1}, Zakharov Vladimir^{a,b,c,d}

^aUniversity of Arizona, 617 N. Santa Rita Ave., Tucson, AZ 85721, USA

^bLPI, Leninsky Pr. 53, Moscow, 119991 Russia

^cPirogova 2, Novosibirsk State University, Novosibirsk, 630090 Russia

^dWaves and Solitons LLC, 1719 W. Marlette Ave., Phoenix, AZ 85015 USA

Abstract

Results pertaining to numerical solutions of the Hasselmann kinetic equation (*HE*), for wind driven sea spectra, in the fetch limited geometry, are presented. Five versions of source functions, including the recently introduced *ZRP* model [1], have been studied, for the exact expression of S_{nl} and high-frequency implicit dissipation, due to wave-breaking. Four of the five experiments were done in the absence of spectral peak dissipation for various S_{in} terms. They demonstrated the dominance of quadruplet wave-wave interaction, in the energy balance, and the formation of self-similar regimes, of unlimited wave energy growth, along the fetch. Between them was the *ZRP* model, which strongly agreed with dozens of field observations performed in the seas and lakes, since 1947. The fifth, the *WAM3* wind input term experiment, used additional spectral peak dissipation and reproduced the results of a previous, similar, numerical simulation described in [2], but only supported the field experiments for moderate fetches, demonstrating a total energy saturation at half of that of the Pierson-Moscowits limit. The alternative framework for *HE* numerical simulation is proposed, along with a set of tests, allowing one to select physically-justified source terms.

Keywords: Hasselmann equation, wind-wave interaction, wave-breaking dissipation, nonlinear interaction, self-similar solutions, Kolmogorov-Zakharov spectra

¹Corresponding author, *E-mail:* dr.push@gmail.com

1. Introduction

The motivation, for the research presented in the current paper, was to continue the project of finding a firm scientific foundation for the study of wind driven seas.

5 The most important step in this direction was made in 1962, by K.Hasselmann [3, 4, 5], who proposed the kinetic equation for wind waves description

$$\frac{\partial \varepsilon}{\partial t} + \frac{\partial \omega_k}{\partial \vec{k}} \frac{\partial \varepsilon}{\partial \vec{r}} = S_{nl} + S_{in} + S_{diss} \quad (1)$$

similar to equations used in condensed media physics since the 1920's [6], where $\varepsilon = \varepsilon(\omega_k, \theta, \vec{r}, t)$ is the wave energy spectrum, as a function of wave dispersion $\omega_k = \omega(k)$, angle θ , two-dimensional real space coordinate $\vec{r} = (x, y)$ and time
 10 t . S_{nl} , S_{in} and S_{diss} are nonlinear, wind input and wave-breaking dissipation terms, respectively. Hereafter, only the deep water case, $\omega = \sqrt{gk}$ is considered, where g is the gravity acceleration and $k = |\vec{k}|$ is the absolute value of wavenumber $\vec{k} = (k_x, k_y)$.

Eq.(1) is widely accepted in the oceanographic community [7, 2] and has several
 15 names. It is called the Boltzmann equation [2] (while this is not exactly correct), the energy balance equation [7], and the radiation balance equation. We will call it the Hasselmann equation (hereafter *HE*) as a tribute to Hasselmann's pioneering work. At the least, this is consistent with part of the community [8]. The right side of Eq.(1) consists of three terms. The S_{nl} term is completely
 20 known. It was consistently derived from Euler equations and describes quadruplets of waves satisfying resonant conditions

$$\begin{aligned} \vec{k} + \vec{k}_1 &= \vec{k}_2 + \vec{k}_3 \\ \omega_k + \omega_{k_1} &= \omega_{k_2} + \omega_{k_3} \end{aligned} \quad (2)$$

In the papers [9, 10] we introduced the following splitting of the S_{nl}

$$S_{nl}(\omega, \theta) = F(\omega, \theta) - \Gamma(\omega, \theta)\varepsilon(\omega, \theta) \quad (3)$$

The explicit expressions for F and Γ are presented in the Appendix. The motivation for this splitting is very simple. The term $F(k)$, for any spectral distribution $\varepsilon(\omega, \theta)$, is an essentially positive function. We will soon show that this fact is of fundamental importance.

Kinetic equations similar to the Hasselmann equation are routinely used in different areas of theoretical physics. In all cases, the first and central issue is the description of solutions to the stationary equation

$$S_{nl}(\omega, \theta) = 0 \quad (4)$$

Any solution of this equation can be presented in the form

$$\varepsilon(\omega, \theta) = \frac{F(\omega, \theta)}{\Gamma(\omega, \theta)} \quad (5)$$

As far as $\varepsilon(\omega, \theta) > 0$, for all solutions

$$\Gamma(\omega, \theta) > 0 \quad (6)$$

The function Γ also has another physical sense. In the presence of nonlinear wave ensemble, the dispersion law is undergoing the re-normalization

$$\omega(k) \rightarrow \omega(k) + \Delta\omega(k) \quad (7)$$

The re-normalization has real and imaginary parts. The imaginary part is

$$Im\Delta(\omega) = \frac{1}{2}\Gamma(\omega, \theta) \quad (8)$$

Everybody knows that Eq.(4) has solutions with thermodynamic equilibrium. There is Maxwell distribution in the kinetic gas theory, and Plank distribution in quantum statistical dynamics. Physicists believed, for a long time, that the thermodynamic equilibrium spectra are unique solutions of Eq.(4). This is certainly true, if the entropy of a solution is finite. However, Eq.(4) has a broad class of solutions with infinite entropy, governed by fluxes of conservative quantities – energy, momentum and wave action.

These solutions are now called KZ (Kolmogorov-Zakharov) solutions and widely used in different areas of physics (see, for instance [11, 12, 13, 14]). The general theory of KZ solutions is described in the monograph [15].

A more advanced development is contained in the paper [9]. The discovery of KZ spectra was recognized by the physical community, by awarding a Dirac
50 medal in 2003, for this development.

The first KZ solution was found by Zakharov and Filonenko in 1966 (the English version of [16] was published in 1967). It is the isotropic solution of the stationary Hasselmann Eq.(4) (the details are presented in section 4):

$$\varepsilon(\omega) = \frac{\beta_{KZ}}{\omega^4} = C_K \frac{g^{4/3} P^{1/3}}{\omega^4} \quad (9)$$

Here P is the energy flux to the high frequency region. It was soon established,
55 [17], that the solution 9 is only “the tip of the iceberg”. Actually, Eq.(4) has a much bigger class of KZ solutions, outlined in the paper [9, 17]. The most interesting and important solutions, governed by fluxes of energy and momentum, are anisotropic. They are not exactly power-like, seeing their ω - dependence deviates from the ω^{-4} law, but only mildly.

Meanwhile, numerous laboratory and field experiments showed that, in the im-
60 portant band of frequency, right behind the spectral peak (approximately for $1.5\omega_p < 3.5\omega_p$), the observed spectra are very close to the ω^{-4} law. Experimental data obtained before 1985 was summarized in the well known paper of Phillips [18]. Since then, a lot of new data has accumulated (see, for instance,
65 [19], [20, 21, 22], [23, 24]). Some other experimental results were cited in the article [25].

Recall that the exact S_{nl} can be derived, rigorously, from the Euler equation. Opposite to it, the “source function” S_{in} – the energy income from the wind, and the energy dissipation function S_{diss} , due to wave-breaking, are only known ap-
70 proximately. In the oceanographic community, there is no consensus regarding their form. We discuss these questions in sections 2 and 3 of this paper.

The ambiguity of their proper definitions presents the first major issue for wind wave theory, and hinders development of accurate operational models, as well.

75 The other important issue is connected with S_{nl} collision term numerical simu-

lation. It is the complex, non-linear, operator, with deep internal symmetries. Several S_{nl} simulation algorithms are available, at the moment, for example: Webb-Resio-Tracy (*WRT*) [26, 27] (also, see important paper [28]), Lavrenov [29] and Masuda [30]. The Van-Vledder version of the *WRT* algorithm [31] has
80 already been included in the *WavewatchIII* and *SWAN* models, for more than a decade.

All of the above algorithms provide reliable results, but are too slow to provide simultaneous *HE* solutions of the Eq.(1) for tens of thousands of spatial points, faster than real time, as is required by operational wave forecasting. Because
85 of this, existing operational models use much faster substitutes for S_{nl} , in the form of *DIA* and its analogs. This is not fatal, as long as the number of quadruplet configurations used in *DIA* is large enough. However, what is wrong is the commonly practiced “tuning” of the *DIA* algorithm parameters, in the operational models.

90 We must stress, however, that we do not discuss the good and bad sides of different modifications of *DIA* models. The only results discussed are those obtained from the numerical algorithm for solving the exact Hasselmann equation. This code is a modification of the *WRT* algorithm . We hereby call it *XNL*.

We insist that a correct definition of the source function is necessary, and we
95 assert that it is possible to perform these corrections, without new theoretical constructions or new difficult experiments. It is sufficient to use existing experimental data, in a proper way. For 68 years, starting from a well-known work of Sverdrup and Munk [32], oceanographers have accumulated a plethora of experimental facts regarding wave growth rate, with respect to winds. Some
100 of those facts were obtained in water tanks, but the most interesting facts come from ocean measurements.

Nowadays, the results of numerous measurements for “fetch limited” field set-ups, where the off shore wind and the waves are quasi-stationary, have been systematized and published [33].

105 All of those situations are described by the stationary *HE*

$$\frac{\partial \omega}{\partial k} \frac{\partial \varepsilon}{\partial x} = S_{nl} + S_{in} + S_{diss} \quad (10)$$

This equation is solved, in the presented research, for different source functions S_{in} and S_{diss} . Five experiments were carried out, for different wind input functions, and their results were compared to known ocean field experimental data. This comparison actively used the fact that the results of those experiments are well described by *Weak Turbulence Theory* (*WTT*). This theory is explained, in detail, in the monograph [15], and applications of this theory, to ocean experiments, are presented in the publications [34, 35, 36, 33, 23, 24].

The possibility of *WTT* application is based on the fact that, in Eq.(1), S_{nl} is the dominant term. This fact can be explained in the following way. All S_{in} cases considered in the current research are quasi-linear, which means that

$$S_{in} = \gamma(\omega, \theta) \varepsilon(\omega, \theta) \quad (11)$$

$$S_{diss} = -\gamma_{diss}(\omega, \theta) \varepsilon(\omega, \theta) \quad (12)$$

Taking into account S_{nl} splitting, Eqs.(3), (10) take the form

$$\frac{\partial \omega}{\partial k} \frac{\partial \varepsilon}{\partial r} = F(\omega, \theta) - (\Gamma(\omega, \theta) - \gamma(\omega, \theta) + \gamma_{diss}(\omega, \theta)) \varepsilon(\omega, \theta) \quad (13)$$

One should note that γ typically has a fairly small value of $10^{-5} \omega_p$, for waves with the frequencies close to the peak frequency ω_p . The value of γ_{diss} does not exceed γ , or waves are not excited at all. Meanwhile, the value of Γ is rather large, as shown by analytic and numeric calculations. It easily exceeds γ , by orders of magnitude [9, 10] (see Appendix). Therefore, one can substitute in the first approximation Eq.(13) by conservative equation

$$\frac{\partial \omega_k}{\partial k} \frac{\partial \varepsilon}{\partial x} = S_{nl} \quad (14)$$

which is, indeed, the subject of the *WTT* study.

It is customary to use “Kitaigorodsky dimensionalization”, where the fetch variable x , total energy E , and peak frequency ω_p are substituted by dimensionless variables

$$\chi = \frac{xg}{u^2}, \quad \varepsilon = \frac{Eg^2}{u^2}, \quad \hat{\omega} = \frac{\omega_p u}{g} \quad (15)$$

All ocean and wave tank measurements show that $\varepsilon(\chi)$ and $\omega(\chi)$ are the power functions of dimensionless fetch χ :

$$\varepsilon = \varepsilon_0 \chi^p \quad (16)$$

$$\hat{\omega} = \omega_0 \chi^{-q} \quad (17)$$

135 The values of p and q vary in different experiments, but not significantly $0.74 < p < 1$, $0.2 < q < 0.3$. They are connected, with strong accuracy, by the “magic relation”

$$10q - 2p = 1 \quad (18)$$

These facts are explained by *WTT* [35]. Conservative kinetic Eq.(14) has a 4-
140 parameter family of self-similar solutions [35, 34, 36, 33], for which the “magic relation” is fulfilled exactly.

It was shown in [1, 9] that the non-conservative *HE*, with the dissipation, localized in short waves, and forcing chosen as the power function

$$\gamma(\omega, \theta) = f(\theta) \omega^s \quad (19)$$

145 also allows a self-similar solution and preserves the “magic relation” Eq.(18).

All numerical experiments presented in the current paper included short-wave dissipation, but in the “implicit” way: the spectrum at frequencies $f > 1.1 Hz$ was forced to Phillips spectrum $\varepsilon_\omega \simeq \omega^{-5}$. The validity of this approach is discussed in section 3. Similar procedure of matching the spectrum with the
150 powerlike tail at high frequency is routine in the operational models [37].

Four out of five of those experiments assumed absence of long-wave dissipation. It is assumed hereafter that “long waves” denotes the waves with the characteristic wavenumber close to the spectral peak vicinity. Such experimental set-up contradicts existing tradition, but is justified by obtained results. Four
155 existing wind forcing terms have been checked: *ZRP* [1], Chalikov [38, 39], Hsiao-Shemdin [40] and Snyder [41]. The only S_{in} term, in the power form, was the *ZRP* forcing term, and it was only this experiment which showed agreement with the field observation, for which $p = 1$ and $q = 0.3$. The other S_{in} terms

lead to the Eqs.(16), (17), where indices p and q are functions of the dimension-
160 less fetch. It is important to note that the “magic relation” Eq.(18) still holds,
as well, which means that corresponding spectra exhibit “local” self-similarity.
The question is: how valid is the claim that the wave energy and the mean
frequency behave like powers?

The developed *WTT* (Weak Turbulent Theory) does not impose limitations on
165 wave energy growth, with the fetch. The maximum length of the dimensionless
fetch $\chi \simeq 10^5$ was considered in the experiments of Donelan et al. [42]. The
corresponding wave energy maximum was $\varepsilon = 4.07 \cdot 10^{-3}$, without any deviation
from the power law.

We should stress that we now speak about a stationary-in-time wave field, and
170 discuss dependence of its characteristics on the fetch, only. In 1964, Pierson and
Moscowitz formulated the hypothesis that on long enough fetches, the wave
field becomes statistically homogeneous. According to Young [7] this “spatial
saturation” occurs at $\chi \simeq 5 \cdot 10^4$ (see Fig.5.10 in the cited book) and the energy
spectrum is stabilized on the level $\varepsilon \simeq 3.64 \cdot 10^{-3}$. For that time, it was an
175 important achievement, but we must emphasize that this hypothesis was pure
speculative, because Pierson and Moscowitz measured the wave field only at
one spatial point. However, recent analysis of numerous experimental data,
published in [33, 34, 36], does not support the concept of the spatial saturation.
Nevertheless, the idea of a “mature sea” was actively supported in the paper
180 [43] and became a kind of credo for oceanographers. The WAM3 model was
designed specifically to support this idea.

The numerical test of the *WAM3* model, using the *XNL* approach, was per-
formed in the fetch-limited geometry, long ago (see [2], pp.229, 254, Fig.3.7 and
Fig.3.22). It was found that for moderate fetches $10^2 < \chi < 10^3$ this model
185 describes the experimental situation pretty well, however, predicting saturation
at the fetch $\chi \simeq 2 \cdot 10^4$ on the low level $\varepsilon_{Max} \simeq 1.8 \cdot 10^{-3}$. Our numerical experi-
ments confirmed these results. Moreover, we found that in the “practical fetch”
interval $10^2 < \chi < 10^3$, the results of *WAM3* coincide with the results obtained
via the *ZRP* model, without any spectral peak dissipation. For larger fetches,

190 the *ZRP* model demonstrates much better coincidence with field experiments,
than the *WAM3* model does.

Here one can recall William Okham’s principle “It is futile to do with more
things that which can be done with fewer”. Application of this principle leads
to excluding the long wave dissipation from consideration, and to the conclusion
195 that the *WAM3* model is not consistent enough. It is satisfactory in only one
aspect - it passes the ω^{-4} test, explained in section 4.

The obtained results can be seen as a progressive step towards universal, physically-
based, ocean surface wave models, the development of which will require minimal
tuning for different ocean conditions. Other perspectives are discussed in the
200 Conclusion.

2. Current state of wind input source terms

Nowadays, the number of existing models of S_{in} is large, but these models lack
firm, theoretical, justification. Different theoretical approaches argue with each
other. A detailed description of this discussion can be found in the monographs
205 [7, 2], and the papers [44, 45, 46, 38, 47, 40, 48, 42].

The development of wind wave models had begun as far back as the 1920’s, in the
well-known works of Jeffreys [49], [50]. His model is semi-empiric and includes
an unknown “sheltering coefficient”. All other existing theoretical models are
also semi-empiric, with one exclusion – the famous Miles model [47]. This model
210 is rigorous, but is related to an idealized situation – the initial stage of wave
excitation, by laminar wind, with specific wind profile $U(z)$.

Miles theory application is hampered by two circumstances. First is the fact that
the atmospheric boundary layer is the turbulent one, and creating a rigorous,
analytic theory of such turbulence is, as of today, an unsolvable problem.

215 There is the opinion, however, that wind speed turbulent pulsations are small,
with respect to horizontal velocity $U(z)$, [51, 52, 53, 54] and that they should be
neglected in the first approximation [52, 54]. This does not mean that turbulence
is not taken into account, at all. It is suggesting that the role of the turbulence

consists in formation of the averaged horizontal velocity profile.

220 The wide spread opinion is that the horizontal velocity profile is distributed by the logarithmic law

$$U(z) = 2.5u_* \ln \frac{z}{z_0} \quad (20)$$

Here u_* is the friction velocity and z_0 is the roughness parameter

$$z_0 = C_{ch} \frac{u_*^2}{g} \quad (21)$$

where $C_{ch} \simeq 3 \cdot 10^{-2}$ is the experimental and dimensionless Charnock constant. One should note that the appearance of an anomalously small constant, not hav-
 225 ing “formal justification”, is an extremely rare phenomenon in physics. Eqs.(20), (21) mean that the roughness parameter is very small: for typical ocean conditions – wind speed 10 m/sec on the height $z = 10$ m we get $z_0 \simeq 5 \cdot 10^{-4}$ m. Such roughness is only twice the size of the viscid layer, defined from multiple experiments on turbulent wind flow, over smooth metal plates. Notice that
 230 the logarithmic law certainly could not work for a height of the order of few centimeters, where capillary effects are essential.

Usage of Eqs.(20), (21) assumes, therefore, that the ocean behaves as smooth metal surface. This is not correct. Horizontal momentum is transferred to the smooth plate, on the surface itself, while in the ocean this process happens
 235 differently.

Momentum off-take, from the atmospheric boundary layer, is smoothly distributed over the whole width of the boundary layer and begins from the highest “concurrence layer”, i.e. from the height where the phase speed of the fastest wave matches the horizontal velocity.

240 Momentum off-take leads to horizontal velocity distribution $U(z)$ depending on time, the wave’s development level, and energy spectrum. Meanwhile, Miles’s instability increment is extremely sensitive to the horizontal velocity profile (there is no wave excitation for the linear profile $U(z)$, in Miles theory, for example). The velocity profile is especially important for slight elevations, on
 245 the order of several centimeters, over the water surface, which is almost unknown

and difficult for experimental measurements. However, there have been some advances in this direction [55, 51].

The necessity of taking into account the waves feedback, into the horizontal velocity profile, was understood a long time ago, as seen in the works of Fabrikant [52] and Nikolaeva et al. [53]. This approach was later continued by Jannsen [54] and explained, in detail, in the monograph [2] in the form of “quasi-laminar” theory. This theory is lacking.

To consider the theory as self-consistent, even in the approximation of turbulence absence, it is necessary to solve equations describing the horizontal velocity profile $U(z)$, together with the Hasselmann equation, describing the energy spectrum evolution. This is not done yet, either.

That fact aside, many theoreticians do not share the opinion about turbulent pulsations insignificance, and consider them as the leading factor. Corresponding *TBH* theory by Townsend, Belcher and Hunt [44] is an alternative to quasi-laminar theory. Both theories are discussed in [56].

There is another approach, not connected with experimental analysis - numerical simulation of the boundary atmospheric layer, in the frame of empiric theories of turbulence. It was developed in the works [45, 46, 38, 39]. Since those theories are insufficiently substantiated, the same relates to the correspondingly derived wind input terms.

Across the wide variety of theoretical approaches to defining S_{in} , almost all of them are “quasi-linear” [10] where the standard relation [7, 2]

$$\gamma(\omega, \phi) = \frac{\rho_a}{\rho_w} \omega \beta\left(\frac{\omega}{\omega_0}, \phi\right) \quad (22)$$

is being used. Here $\omega_0 = \frac{g}{u}$, where u is the wind speed, defined differently in individual models. The function β is dimensionless and is growing with the growth of $\frac{\omega}{\omega_0}$.

However, even for the models exhibiting the strongest wind input, the value of β belongs to the interval $0 < \beta < 1$, for ξ from the interval $0 < \xi < 3$. In some models (see, for example [38]) β becomes negative for the waves propagating

faster than the wind, or under large angle, with respect to the wind.

275 Looking at multiple experimental attempts to define S_{in} , one should notice that they need to be critically analyzed. The criticism is not about the integrity of measurements itself, but about the methodology used, the validity of data interpretation, and the possibility of transferring conclusions made in artificial environments to real ocean conditions.

280 A significant amount of the experiments, belonging to the so-called “fractional growth method” category, have been performed, through energy spectrum measurement in time, and calculation of the corresponding γ through

$$\gamma(\omega, \phi) = \frac{1}{\varepsilon(\omega, \phi)} \frac{\partial \varepsilon(\omega, \phi)}{\partial t} \quad (23)$$

Eq.(23) is, in fact, the linear part, or just two terms of the HE Eq.(1). This method is intrinsically wrong, since it assumes that either advection $\frac{\partial \omega_k}{\partial k} \frac{\partial \varepsilon}{\partial r}$ and
285 nonlinear S_{nl} terms of Eq.(1) are absent altogether, or relation

$$\frac{\partial \omega}{\partial \vec{k}} \frac{\partial \varepsilon}{\partial \vec{r}} = S_{nl} \quad (24)$$

is fulfilled.

The first assumption is simply not correct, since neglected terms are defining ocean conditions.

The second assumption is almost fulfilled, indeed, since the sea is described by
290 the WTT . But the terms in Eq.(24) are large with respect to the terms in Eq.(23), and therefore there is no reason to neglect the terms of Eq.(24).

Regarding the “fractional growth method”, we are just citing the single publication by Plant [57] where, it seems, the author understood its scarcity.

As a matter of fact, it is natural to ask about the spectral correlation function
295 $Q(\omega)$ between the surface elevation η and the wind-induced pressure on the surface P :

$$Q(\omega) = \langle \eta(\omega) P^*(\omega) \rangle \quad (25)$$

where brackets denote ensemble averaging, in Fourier space, and asterisk refers to the complex conjugate.

Unfortunately, the number of such experiments is limited, and not all of them
300 have significant value for describing ocean phenomena. Also, one should not consider the experiments performed in laboratory conditions.

Consider, for example, the set of experiments described in [58]. These experiments were performed in the wave tank of 40 *m* length and 1 *m* depth. The wind was blowing at speeds up to 16 *m/sec*, but they only studied short waves,
305 no longer than 3 *m*, moving no faster than 3.3 *m/sec*. Therefore, they studied the very short, wave-tail of the function β , in conditions far from those of the ocean. Another problem with flume data is upper physical confinement of the vertical velocity profile. The value of these measurements is not significant.

The same arguments apply to multiple precisely-performed measurements on
310 the Lake George, Australia [59]. The depth of this lake is, on average, about 1 *m*. That is why the waves slower than 3.3 *m/sec* can propagate on its surface. The typical wind speed, corresponding to these measurements was 8–12 *m/sec*. Therefore, while the results of these measurements are quite interesting, the obtained expression for S_{in} is questionable, because it runs completely against
315 quasi-linear theory. The quasi-linear theory predicts smoothing of the velocity profile $U(z)$, with the wave’s development. The wind input growth rate, however, was increasing with the wave’s energy level, in the experiments [60].

After critically analyzing experiments on S_{in} measurements, only three of them deserve attention. Those are the experiments by Snyder et al. [41], Hsiao et al.
320 al. [40] and Hasselmann et al. [61]. These experiments were performed in the open ocean and measured direct correlations of surface speed change and the pressure. The accuracy was not ideal and the data scatter was significant, presumably due to contemporary technologies. Therefore, their interpretation is quite ambiguous. The fact of this uncertainty was highlighted in the paper
325 [38]. Either way, these experiments produced two well-known formulas for β . Next, we present β ’s expressions, for the cases analyzed in the current paper.

For Snyder et al. [41], Hasselmann-Bosenberg [61] case

$$\beta_{SHB} = \begin{cases} 0.24(\xi - 1) & \text{for } 1 < \xi < 4 \\ 0, & \text{for } \xi < 1 \end{cases} \quad (26)$$

For Hsiao-Shemdin case [40]

$$\beta_{HS} = \begin{cases} 0.12(0.85\xi - 1)^2 & \text{for } \xi \geq 1 \\ 0, & \text{for } \xi < 1 \end{cases} \quad (27)$$

330 Let us notice that for *ZRP* case [1]

$$\beta_{ZRP} = 0.05\xi^{4/3} \quad (28)$$

The differences between various S_{in} , corresponding to Eqs.(26)-(28), are significant. For many practical purposes, the spectral peak is located in the interval $1.5 < \xi < 2.5$, where the difference between Snyder and Hsiao-Shemdin functions is huge. Indeed

$$\beta_S(1.5) = 0.12 \quad \beta_{HS}(1.5) = 0.009 \quad (29)$$

$$\beta_S(2.5) = 0.36 \quad \beta_{HS}(2.5) = 0.15 \quad (30)$$

This serious difference is explained by lack of accuracy in both experiments (see [38]). Fig.6 of paper [41] and Fig.4 of the paper [40] show that the experimental data scatter has the same order as the mean values. Thus, the offered forms of the source functions Eqs.(26), (27) are not seriously justified. However, Hsiao-Shemdin data appears to be more trustworthy. It seems quite obvious that the Snyder function overestimates the wind input, by several times. Presented numerical experiments justify this conjecture.

For the *ZRP* function

$$\beta_{ZRP}(1.5) = 0.086 \quad \beta_{ZRP}(2.5) = 0.17 \quad (31)$$

345 and in the interval $1 < \xi < 3$

$$\beta_{SHB}(\xi) < \beta_{ZRP}(\xi) < \beta_S(\xi) \quad (32)$$

Fig.1 presents one-dimensional plots of four functions, $\beta(\xi)$, studied in numerical experiments presented below. We intentionally did not include the description

Figure 1: Four cases of function $\beta(\xi)$ along the wind ($\theta = 0$) used in the numerical experiments. Solid line: Snyder-Hasselmann-Bosenberg case Eq.(26) , dashed line: Hsiao-Shemdin case Eq.(27), dashed-dotted line: Chalikov case [39] and dotted line: ZRP case Eq.(28)

of the sophisticated Chalikov algorithm [39] for corresponding $\beta(\xi)$, for the sake
 350 of space.

We can conclude that, at the moment, there is no reliable parameterization of S_{in} , accepted by the entire oceanographic community. Keeping that fact in a mind, we decided to go our own way; this way is not focused on building new theoretical models, nor will it reconsider measurements of S_{in} .

355 For 68 years, since the works of Sverdrup and Munk [32], physical oceanography assimilated tremendous amount of wind-wave experimental data - wave energy and spectral peak frequency- as functions of limited fetch. Such experiments are analyzed in works [36, 33, 23, 24]. On the other hand, numerical methods for solving the HE Eq.(1), with exact S_{nl} terms, have been improved significantly,
 360 for duration-limited and fetch-limited domains, as well.

Therefore, a new, purely pragmatic, approach for defining S_{in} was proposed. The function S_{in} has been chosen in such a way, so that numerical solutions of the Hasselmann equation explain the most known field experiments. The result was the S_{in} function, described in detail in [1], and named thereafter: the *ZRP*
 365 function.

It is important to emphasize that work [1] assumed localization of energy dissipation, in short waves. This assumption contradicts widely accepted concepts, but we explain the differences in the following section.

3. Two scenarios of wave-breaking dissipation term: spectral peak or high-frequency domination?

In this section, we explain why there is no need to use dissipation, in the spectral peak area.

Spectral peak frequency damping is a widely accepted practice, and is included as an option in the operational models *WAM*, *SWAN* and *WaveWatchIII*.

Notice that different operational models use completely different long-wave dissipation functions.

The form of *WAM3* spectral peak dissipation used in this paper is given by the following definition [37] (the original notations are preserved):

$$S_{ds}(k, \theta) = C_{ds} \hat{\sigma} \frac{k}{\hat{k}} \left(\frac{\hat{\alpha}}{\hat{\alpha}_{PM}} \right)^2 N(k, \theta) \quad (33)$$

$$\hat{\sigma} = \left(\overline{\sigma^{-1}} \right)^{-1}$$

$$\hat{\alpha} = E \hat{\sigma}^4 g^{-2}$$

where $N(k, \theta)$ is the wave action spectrum, σ is the frequency, k is the wavenumber, θ is the angle, E is the total energy, $C_{ds} = -2.36 \cdot 10^{-5}$, $\hat{\alpha}_{PM} = 3.02 \cdot 10^{-3}$ is the value of $\hat{\alpha}$ for *PM* spectrum. Overline notation in $\left(\overline{\sigma^{-1}} \right)^{-1}$ means averaging over the spectrum.

This formula implicitly assumes that dissipation is concentrated in the long-wave region, and numerical experiments, below, show that it is indeed realized that way, see Fig.23.

It is important to emphasize that Eq.(33) is not supported by laboratory or field experiments, nor by analytical theory, nor numerical simulations, in the framework of phase-resolving numerical models. This is a heuristic construction, and it is important to trace its origin. Eq.(33) appeared in the paper of Komen

390 et al. [43], and exerted strong influences on future developments of physical oceanography.

The authors of article [43] analyzed the energy balance, in the surface of the wind-driven sea, and concluded that the introduction of artificial dissipation term Eqs.(33) is necessary for explanation of experimental facts. This analysis
395 was unsatisfactory for two reasons.

The authors of [43] considered that existence of the "fully developed" sea, which is not only stationary in time, but homogeneous in space, as an obvious fact. For this reason they neglected the advection term $C_g \frac{\partial \varepsilon}{\partial x}$ in their analysis. In fact, all known stationary spectra vary with the fetch. It makes the concept of
400 the "fully developed sea" doubtful.

Another weak point of the paper [43] is its uncritical use of the Snyder source function. As was shown before, this function has shaky foundations. Our numerical calculations show that it overestimates the energy growth rate by a factor of $5 \div 6$. The authors of Eqs.(33) were using the Hsiao-Shemdin source
405 function, which would hardly support Eqs.(33) dissipation function, seeing in this case the balance had to be shifted to the dissipation side.

Anyways, for more than thirty years the dissipation term Eq.(33), together with Snyder input term, dominated in the operating models. These choices became a sort of credo, in physical oceanography. The purpose of the presented paper
410 is its revision.

The dissipation of water surface waves, due to white-capping, is an extremely important physical phenomenon, not yet properly studied. M.Longuet-Higgins spent a lot of effort to develop an analytical theory of the wave-breaking [62, 63, 64, 65]. He found the set of interesting exact and approximate solutions of the
415 Euler equations, describing potential flow, of ideal fluid with the free surface, but didn't solve the problem completely.

The difficulty of development in wave-breaking analytical theory is explained by sophisticated mathematical reasoning. Most probably, the system of Euler equations, for incompressible ideal fluid potential flow, with free surface on deep
420 water in $1 + 1$ geometry (i.e. depth coordinate and one horizontal coordinate) is

the completely integrable system. It has too many peculiar features: cancellation of non-trivial four-wave interactions [66], presence of an indefinite number of extra motion constants [67], partial solutions describing propagating capillary waves, expressed in elementary functions (the “Crapper solution” [68]). So far, 425 the exact integrability, for the general time-dependent problem, is established in the exotic case of “asymptotically upwelling flow”, in the absence of gravity [69]. An infinite number of exact solutions were found in the paper. Some of those solutions were published long ago by Longuet-Higgins [65].

Integrability makes the theory of white-capping complicated for the following 430 reason. Integrability means absence of a universal scenario, of this effect. From the view-point of general nonlinear wave dynamics, wave-breaking is an example of “weak-collapse” [70]. Such collapses are described, as a rule, by self-similar solutions.

Breakers, described by self-similar solutions

$$\eta(x, t) = g(t_0 - t)^2 F\left(\frac{x - x_0}{g(t_0 - t)^2}\right) \quad (34)$$

435 were studied analytically and numerically, in the framework of the simplified (and non-integrable!) *MMT* (Maida-McLaughlin-Tabak) model of Euler equations [71]. Here $\eta(x, t)$ is the water surface elevation, x and t are spatial coordinate and time, respectively, and F is self-similar function.

Solution Eq.(34) describes formation of the wedge, with the top at $x = x_0$, 440 at the moment of time $t = t_0$. Exact Euler equations have similar solutions, describing formation of locally stationary “corner flow solution”, with angle 120° (see Longuet-Higgins [63]). But in the *MMT* model, the non-integrable case, the self-similar breaker is a “global attractor”. In other words, all breakers are self-similar. The exact Euler equation has the same self-similar solution 445 describing formation of the “Stokes corner flow” [63], but now it is not a generic scenario. In the general case, formation of the wedge is only the first stage of the breaker evolution. Later it ejects an inclined “Dirichlet jet” [62], which plunges back into the water and transforms mechanical energy to heat. This scenario

was qualitatively described in an article of Longuet-Higgins, and is supported
 450 by many numerical simulations [72]-[73], laboratory experiments [74], and field
 observations [73]-[75]. The literature on this subject is huge, and only a small
 portion of it is cited.

In spite of the complexity of this scenario, in terms of Fourier transform, the
 physical picture of the phenomenon is, more or less, universal. On the stage
 455 of wedge formation, the spatial Fourier spectrum, of energy, forms a “fat tail”.
 Up to a certain moment of time, this spectrum is reversible in time. Plunging
 of the jets causes formation of the drops and bubbles, leading to dissipation
 of the energy and irreversibility. This is the mechanism of “high-frequency
 dissipation”. The presence of high-frequency dissipation “chops off” the end of
 460 the tail, and violates the tail invertability. Low and high harmonics, however, are
 strongly coupled in this event, due to strong, nonlinear, non-local, interaction,
 and deformed high wave-numbers, so the tail, almost immediately, returns to the
 spectral peak area. As soon as the fat spectral tail returns to the spectral peak
 area, total energy in the spectrum diminishes, causing settling of the spectral
 465 peak at a lower level of energy. This process of “shooting” of the spectral tail
 toward high wave-numbers, and its returning back, due to wave breaking, is
 the real reason of “sagging down” of the energy profile in the spectral peak
 area. This was erroneously associated with the presence of the damping in the
 spectral peak area. This explanation suggests that individual wave-breaking
 470 studies [60, 76] do not prove the presence of spectral peak damping.

There is another question of fundamental importance. What is the speed, C_b ,
 of breaker propagation? This C_b is connected with the characteristic length of a
 breaker by $P_b \simeq \frac{C_b^2}{g}$. The breaker propagation speed is the subject of direct
 measurements. Breakers produce strips of foam, and propagation of these strips
 475 can be traced relatively easily. The results of numerous experiments performed
 by Huang et al. [77, 78, 79, 80], Gemmrich et al. [81] gave, approximately,
 the same result: most of the breakers are “slow”. Their propagation speed
 is $C_b \simeq 0.2C_p$. Slow breakers are quasi-one-dimensional, but they propagate
 in a broad sector of angles with respect to the wind. Some breakers are fast

480 ($C_b \simeq C_p$). Fast breakers propagate in the same direction as the leading wave. What is important is that “slow” and “fast” breakers are formed, but for completely different reasons. Let us look at the KZ spectrum

$$\varepsilon_\omega \simeq \omega^{-4}, \quad I_k \simeq k^{-5/2} \quad (35)$$

This spectrum is concentrated on fractals, non-smooth functions, and cannot be extended very far in the high frequency. At $\omega \simeq 3\omega_p$ the KZ spectrum turns into
 485 the Phillips spectrum $\varepsilon \simeq \omega^{-5}$, $I_k \simeq k^{-3}$, which is concentrated on wedge-like functions. Thus, formation of “slow breakers” is an unavoidable consequence of the energy flux to high frequency region, provided by four-wave, nonlinear, wave interaction. Thus, it is reasonable to suppose that wave-breaking dissipation is localized in short scales.

490 The population of waves having frequencies $3 \div 4$ times bigger than spectral maximum frequency is called “Phillips sea” [82, 18, 83], which we call for the sake of brevity the “short waves”. The “Phillips sea” contains no more than 2% of the total wave energy, but the whole energy dissipation, fueled by energy flux from long waves, is happening right there. It is proved, experimentally,
 495 that “Phillips sea” is described by a universal Phillips spectrum $\varepsilon \simeq \frac{\alpha g^2}{\omega^5}$, where $\alpha \simeq 0.01$ is a dimensionless constant, while for the Pierson-Moscowitz spectrum $\alpha = 0.0081$ [82, 18, 83].

“Phillips sea” is quite an interesting physical object. It contains breakers of different, statistically uniformly distributed, sizes [83], down to characteristic
 500 wave length $\lambda \simeq 3 \div 5$ cm, where capillary effects become essential.

The exact form of the “Phillips sea” energy dissipation function is unknown. Recently, a quite plausible model of such function has been presented in [84], which is hoped to become the subject of oceanographic community discussion. What’s about the “fast breakers”? They rarely appear in $1+1$ geometry, where
 505 nontrivial four-wave interactions are canceled out, and there is no energy flux to high wavenumbers.

The steepness, in the conditions of typically developed wave turbulence, is not

big: $\mu = \langle \nabla \eta^2 \rangle^{1/2} \sim 0.1$, or even smaller. Because this value is very far from limiting steepness of Stokes wave $\mu \simeq 0.3$, these waves are, essentially,
510 weakly-nonlinear. Besides those waves, shorter waves inevitably develop, having steepness approaching the critical one, and those waves break.

However, there is also another process- the modulational instability or “wave grouping” - which leads to spatial inhomogeneity and formation of “rough waves” propagating, with the spectral peak velocity. Theory of rogue wave formation
515 is a separate, and interesting phenomenon, which is discussed in many articles (see, for instance [85, 86, 87]), but is not the subject of the current paper.

It is important that direct numerical solutions of both exact [88] and approximate [89] primordial Euler equations show that dissipation of the rogue waves does not make any significant contribution into energy balance of wind-driven
520 seas. Thus, the main conclusion about the dissipation taking place in short scales remains unchanged.

4. Numerical experiments set-up

The subject of numerical simulation was the stationary *HE* Eq.(10), for different wind input functions. A total of 5 different wind inputs have been tested in the
525 frame of stationary *HE*:

$$\frac{\partial \omega_k}{\partial \vec{k}} \frac{\partial \varepsilon}{\partial \vec{r}} = S_{nl} + S_{in} + S_{diss} \quad (36)$$

The first 4 tests were done assuming the absence of low-frequency dissipation and the presence of “implicit” high-frequency dissipation.

All simulations were performed with the help of the *WRT* method [26, 90, 28],
530 previously used in [91, 20, 21, 22, 84, 27, 92], on the grid of 71 points in frequency and 36 points in angle domains. A constant wind of speed 10 *m/sec* is assumed to be blowing away from the shore line, along the fetch. The assumption of the constant wind speed is a necessary simplification, due to the fact that the numerical simulation is being compared to various data from field
535 experiments, and the considered set-up is the simplest physical situation, which can be modeled.

4.1. The details of “implicit” damping implementation

One should specifically stop and note details of the “implicit” high-frequency damping, used in all five numerical simulations. Including the “implicit” damp-
 540 ing consists in continuation of the spectral tail by Phillips law [82] $A(\omega_0) \cdot \omega^{-5}$, where $A(\omega_0)$ is the parameter dynamically changing in time.

The coefficient $A(\omega_0)$, in front of ω^{-5} , is not exactly known, but is not required to be defined in explicit form - it is dynamically determined from the continuity condition of the spectrum, at frequency ω_0 , on every time step. In other words,
 545 the starting point of the Phillips spectrum coincides with the last point of the dynamically changing spectrum, at the frequency point $\omega_{crit} = 2\pi f_{crit}$, where $f_{crit} \simeq 1.1 \text{ Hz}$, as per Resio and Long experimental observations [22]. This is the way the high frequency “implicit” damping is incorporated into the alternative computational framework of *HE*.

One should note recently developed analytical models, which automatically de-
 550 scribe the transition from the *KZ* spectrum ω^{-4} to Phillips tail ω^{-5} [84]. Such modification of the “implicit” damping is in future plans, but the question of the finer details of the high-frequency “implicit” damping structure is of secondary importance, at the current “proof of the concept” stage, of the alternative frame-
 555 work development.

4.2. WTT facts used in numerical simulation

As a rule, confirmed by field and numerical observations, the wave energy spec-
 trum has sufficiently sharp peak at $\omega \simeq \omega_p$. However, almost immediately after
 the spectral peak at $\omega \simeq 1.5\omega_p$ the advection term $\frac{\partial \omega}{\partial k} \frac{\partial \varepsilon}{\partial r}$ becomes insignificant,
 560 and the original stationary *HE* Eq.(10) is transformed into

$$S_{nl} + S_{in} + S_{diss} = 0 \quad (37)$$

Comparison with Eq.(13) shows that Eq.(37) can be rewritten in the form

$$\varepsilon(\omega, \theta) = \frac{F(\omega, \theta)}{\Gamma(\omega, \theta) - \gamma(\omega, \theta) + \gamma_{diss}(\omega, \theta)} \quad (38)$$

As it was shown in [9, 10], nonlinear dissipation $\Gamma(\omega, \theta)$ in the “universal area” $\omega > 1.5\omega_p$ is several times greater than wind forcing term. Therefore, Eq.(37)

565 can be rewritten, as a first approximation, by

$$S_{nl}(\omega, \theta) = 0 \quad (39)$$

or

$$\varepsilon(\omega, \theta) = \frac{F}{\Gamma} \quad (40)$$

As was mentioned before, this equation has a rich family of solutions. The simplest and best known solution is the isotropic Zakharov-Filonenko solution [16]

$$\varepsilon(\omega, \theta) = \frac{C_p g^{4/3}}{\omega^4} P^{1/3} = \frac{\beta_{KZ}}{\omega^4} \quad (41)$$

570 Here P is the energy flux into the high wavenumbers region.

The energy density flux per square unit, in the atmosphere, is $P_{sq} = \rho_a U^3$, where ρ_a is the atmosphere density and U is the wind speed. For $U = 10 \text{ m/sec}$, $P_{sq} \simeq 1.2 \text{ kW}$. A relatively small part of this flux is transferred to the water. According to Hwang and co-authors [79, 80], the estimated amount, transferred
575 to the ocean, is $P_0 \simeq 0.1 \text{ Wt}$. Approximately one third of this amount is spent into energy flux formation, toward high wave numbers. In “oceanographic” normalization this flux has to be divided by $\rho_w g$, where $\rho_w \simeq 10^3 \text{ kg/m}^3$. Finally, the energy flux toward small scales is

$$P \sim 2 \div 3 \cdot 10^{-6} \text{ m}^2/\text{sec} \quad (42)$$

580 This expression agrees with the presented numerical experiments. The Kolmogorov constant in the Eq.(41) can be found numerically [9]. Recently, its value has been found more rigorously [93]:

$$C_p = 4\pi \cdot 0.194 = 2.43 \quad (43)$$

585 One can estimate the characteristic value of β_{KZ} :

$$\beta_{KZ} = C_p g^{4/3} P^{1/3} \simeq 0.6 \text{ m}^2/\text{sec}^3 \quad (44)$$

According to *WTT*, the value of β_{KZ} should be constant, somewhere in the region $1.5\omega_p < \omega < \omega_{crit}$. Here, $\omega_{crit} = 2\pi \cdot 1.1 = 6.91$ is the critical frequency at which the “implicit” damping is turning on. The energy flux, for this area,

is diminishing proportionally to ω^{-3} , and β_{KZ} is not constant anymore - one
 590 has to substitute β_{KZ} by $\beta_{KZ} \cdot \frac{\omega_{crit}}{\omega}$.

The general, anisotropic KZ solution, with zero wave-action flux from $\omega \rightarrow \infty$,
 can be presented in the form [9]

$$\varepsilon(\omega, \theta) = \frac{\beta_{KZ}}{\omega^4} F\left(\frac{\omega_s}{\omega}, \theta\right) \quad (45)$$

where $\omega_s = \frac{M}{P}$, M - the momentum flux to the small scale region. The frequency
 ω_s depends on the shape of S_{in} , in a typical case $\omega_s \simeq \omega_p$. $F\left(\frac{\omega_s}{\omega}, \theta\right)$ is a
 595 “structural function”. It is established that in the limit $\frac{\omega_s}{\omega} \rightarrow 0$ (see Katz et al.
 [17], Zakharov [9]):

$$F\left(\frac{\omega_s}{\omega}, \theta\right) \rightarrow 1 + c_2 \frac{\omega_s}{\omega} \cos \theta \quad (46)$$

where c_2 is the “second Kolmogorov constant”.

If one uses, for S_{nl} , the “diffusion approximation” [94], the structural function
 is known

$$F\left(\frac{\omega_s}{\omega}, \theta\right) = \left(1 + \frac{\omega_s}{\omega} \cos \theta\right)^{1/3} \quad (47)$$

600 As far as $F \rightarrow 1$ at $\omega \rightarrow \infty$, the KZ solution Eq.(45) describes well-known
 “angular spreading”. This solution becomes isotropic at $\omega \rightarrow \infty$.

One can introduce

$$f\left(\frac{\omega_s}{\omega}\right) = \frac{1}{2\pi} \int_0^{2\pi} F\left(\frac{\omega_s}{\omega}, \theta\right) d\theta \quad (48)$$

From Eq.(46), one can see that $f\left(\frac{\omega_s}{\omega}\right) \rightarrow +\lambda \left(\frac{\omega_s}{\omega}\right)^2 + \dots$

A more detailed study, of the functions $F\left(\frac{\omega_s}{\omega}, \theta\right)$ and $f\left(\frac{\omega_s}{\omega}\right)$, is an urgent the-
 605 oretical problem, but is out of the scope of the current paper. One can expect,
 however, that $f\left(\frac{\omega_s}{\omega}\right)$, for $\omega > 2\omega_p$, is close to 1. Presented calculations con-
 firm this conjecture: the compensated angle-averaged spectrum $\langle \epsilon \rangle \omega^4$ is
 constant, up to 20% accuracy inside, the spectral band $0.4Hz < f < 1Hz$.

Notice that the average energy spectrum, $\langle \varepsilon \rangle$, decays in this spectral band,
 610 by a factor of 40, and the difference of the KZ spectrum and Phillips spectrum
 is essential.

Let's discuss self-similar solutions of the conservative HE Eq.(24). This equa-
 tion has a family of self-similar solutions, which can be conveniently rewritten
 615 in the form

$$\varepsilon(\omega, \theta, \chi) = \chi^{p+q} F(\xi, \theta) \quad (49)$$

$$\xi = \omega \chi^q \quad (50)$$

where $F(\chi, \theta)$ is the function, satisfying the relation

$$\frac{\cos \theta}{2\xi} \left[(p+q)F + q\xi \frac{\partial F}{\partial \xi} \right] = S_{nl} \quad (51)$$

and q and p are the constants, connected by the “magic relation” Eq.(18). If
 the self-similar solution is realized, then dimensionless energy and frequency are
 620 the power functions, of the dimensionless fetch; see Eq.(16), (17).

It was shown in [1] that self-similar solutions also exist for the case of wind
 input, if $\beta(\xi)$ in Eq.(22) is the power function of the frequency:

$$\beta = \omega^s f(\theta) \quad (52)$$

625 The constants p and q are defined now, unambiguously as

$$q = \frac{1}{2+s} \quad (53)$$

$$p = \frac{8-s}{2(2+s)} \quad (54)$$

As was already mentioned, practically all ocean field measurements demonstrate
 power dependencies Eq.(16), (17). However, there is scattering in the definition
 of the exponent p , in the range $0.74 < p < 1.1$. It's quite possible that this
 scattering is due to absence of the universal expression for S_{in} , suitable for any
 630 atmospheric boundary layer state.

The experiments of Kahma (see [25] for references), performed for “stable” and
 “unstable” atmosphere, gave different values of p . However, the “magic relation”

Eq.(18) still holds true for those different cases. This fact holds the promise that *WTT* always works.

635 More than the half of the numerical experiments have the values $p = 1$, $q = 0.3$. Such self-similarity occurs if $s = 4/3$. This fact, together with field experimental data [22], leads to the appearance of the *ZRP* wind input term [1].

5. Numerical study of different wind input models

The current section presents the results of numerical simulation, of different
640 wind input models, in the alternative framework of *HE*, for limited fetch state-
ment, based on the following:

1. Exact S_{nl} term
2. Absence of spectral maximum dissipation (excluding special *WAM3* case)
3. High-frequency “implicit” dissipation

645 The first numerically studied wind input model is described in the previous section’s *ZRP* model:

$$S_{in}(\omega, \phi) = \gamma(\omega, \phi) \cdot \varepsilon(\omega, \phi) \quad (55)$$

$$\gamma = 0.05 \frac{\rho_{air}}{\rho_{water}} \omega \left(\frac{\omega}{\omega_0} \right)^{4/3} f(\theta) \quad (56)$$

$$f(\theta) = \begin{cases} \cos^2 \theta & \text{for } -\pi/2 \leq \theta \leq \pi/2 \\ 0 & \text{otherwise} \end{cases} \quad (57)$$

$$\omega_0 = \frac{g}{u_{10}}, \quad \frac{\rho_{air}}{\rho_{water}} = 1.3 \cdot 10^{-3} \quad (58)$$

Fig.2a shows total energy growing along the fetch, by the power law

$$\hat{\varepsilon} = \varepsilon_0 \chi$$

650 in accordance with Eq.(16) for $p = 1.0$, see Fig.2b.

Dependence of mean frequency, on the fetch, shown in Fig.3a, demonstrates the law

$$\hat{\omega} = \omega_0 \chi^{-0.3}$$

Figure 2

in good correspondence with self-similar dependence Eq.(17), for $q = 0.3$, see Fig.3b.

Fig.3a presents, not only mean frequency, but also the maximum spectral frequency. Their difference, however, is so small, that we will not distinguish between them, hereafter.

The result of the “magic relation” check is presented in Fig.4. It presents the relation as a function of the fetch. It strongly agrees with the self-similar prediction of Eq.(18).

Table 1 presents the results [33] of calculating the exponents p and q (see Eqs.(16), (17)), for 12 different experimental observations, with the last row corresponding to a limited fetch growth numerical experiment, within the alternative *ZRP* framework. The value of $C = 10q - 2p$, averaged over the experiments, is $\langle C \rangle = 0.95$. One can see correspondence with the predicted, theoretical, value $C_t = 1$, as well as the numerical result. One should note interpretations of the *JONSWAP* experiment, by different experts, provided different values of p and q , and, correspondingly, C .

Let’s proceed with the analysis of numerical spectra. Typical, angle averaged,

(a) Dimensionless mean frequency, as a function of dimensionless fetch (solid line), calculated as $\langle f \rangle = \frac{1}{2\pi} \frac{\int \omega n d\omega d\theta}{\int n d\omega d\theta}$, where $n(\omega, \theta) = \frac{\varepsilon(\omega, \theta)}{\omega}$ is the wave action spectrum. The dotted line is the peak frequency $f_p = \frac{\omega_p}{2\pi}$, and the dashed is the theoretical fit $3.4 \cdot \left(\frac{xg}{U^2}\right)^{-0.3}$.

(b) Local exponent $-q$ of the mean frequency as the function of dimensionless fetch.

Figure 3

Figure 4: “Magic relation” $(10q - 2p)$ as the function of dimensionless fetch for *ZRP* wind input term.

Experiment	p	q	$C = 10q - 2p$
Babanin, Soloviev 1998 Black Sea	0.89	0.28	1.02
Walsh et al. (1989) US coast	1.0	0.29	0.90
Kahma, Calkoen (1992) unstable	0.94	0.28	0.92
Kahma, Pettersson (1994)	0.93	0.28	0.94
JONSWAP by Davidan (1980)	1.0	0.28	0.80
JONSWAP by Phillips (1977)	1.0	0.25	0.75
Kahma, Calkoen (1992) composite	0.9	0.27	0.90
Kahma (1981, 1986) rapid growth	1.0	0.33	1.03
Kahma (1986) average growth	1.0	0.33	1.03
Donelan <i>et al.</i> (1992) St Claire	1.0	0.33	1.03
JONSWAP by Hasselmann et al. (1973)	1.0	0.33	1.03
Mitsuyasu et al. (1971)	1.0	0.33	1.03
ZRP numerics	1.0	0.3	1.00

Table 1: Exponents p and q (see Eqs.(16),(17)) for 12 different experimental observations [33] with the last row corresponding to limited fetch growth numerical experiment within alternative *ZRP* framework.

Figure 5: Typical, angle averaged, wind input function density $\langle S_{in} \rangle = \frac{1}{2\pi} \int \gamma(\omega, \theta) \varepsilon(\omega, \theta) d\theta$ and angle averaged spectrum $\langle \varepsilon \rangle = \frac{1}{2\pi} \int \varepsilon(\omega, \theta) d\theta$ (solid line) as the functions of the frequency $f = \frac{\omega}{2\pi}$.

wind input function density $\langle S_{in} \rangle$ and angle averaged spectrum, in linear coordinates, are presented on Fig.5. It is seen that a major portion of the wind forcing is concentrated in the spectral peak vicinity.

For the sake of brevity, the calculation of density flux to high wavenumbers is omitted, and only the final result is presented: $P \simeq 2 \cdot 10^{-6} \text{ m}^2/\text{sec}$, which gives the value $\beta_{KZ} \simeq 0.5 \text{ m}^2/\text{sec}^3$. An approximation of β_{KZ} is given by angle averaged compensated spectrum $\varepsilon\omega^{-4}$, shown in Fig.6.

Finally, Fig.7 presents angle averaged energy spectrum, as the function of frequency, in logarithmic coordinates. One can see that it consists of segments of:

1. Spectral maximum area
2. Kolmogorov-Zakharov spectrum ω^{-4}
3. Phillips high frequency tail ω^{-5}

It follows from Fig.6, 7 that, in the interval $0.4 \text{ Hz} < f < 1 \text{ Hz}$, the energy spectrum is close to the Zakharov-Filonenko spectrum Eq.(41), with the accuracy 20%. One cannot expect higher accuracy, due to the anisotropy of the realized spectrum and the influence of the high frequency dissipation, as well. One can state, nevertheless, that in the domain of frequencies exceeding spectral

Figure 6: Angle averaged, energy compensated, spectrum $\langle \varepsilon \omega^4 \rangle = \frac{1}{2\pi} \int \varepsilon(\omega, \theta) \omega^4 d\theta$ as the function of decimal logarithm of frequency $f = \frac{\omega}{2\pi}$ shows the behavior close to the theoretically predicted value $\beta \simeq C_K (g^4 P)^{1/3} \simeq 0.5$ in the area of the “plateau”.

Figure 7: Decimal logarithm of angle averaged spectral energy density $\langle \varepsilon \rangle = \frac{1}{2\pi} \int_0^{2\pi} \varepsilon d\theta$, as a function of the decimal logarithm of the frequency $f = \frac{\omega}{2\pi}$ - solid line. Dashed line - fit $\sim \omega^{-4}$, dash-dotted line - fit $\sim \omega^{-5}$.

maximum frequencies, the energy spectrum is fairly close to the one described
 690 by equation $S_{nl} = 0$, which confirms the view that the energy balance of wind
 excited surface waves, as presented in current research.

The analysis carried out in the previous section shows that the quality of the
 different versions of wind input terms S_{in} should be estimated by the following
 criteria:

- 695 1. Checking powers of the observed energy and mean frequency dependen-
 cies Eq.(16), (17) along the fetch, versus what is predicted by self-similar
 solutions.
2. Checking the “magic relations” Eq.(18) between exponents p and q , for
 observed energy and frequency dependencies, along the fetch.
- 700 3. Checking the exponents of directional (angle averaged) spectral energy
 dependencies versus the Kolmogorov-Zakharov exponent -4 .

We applied such tests to the results of HE simulations, which used the following
 popular wind input terms, within alternative framework:

1. Chalikov S_{in} term [39, 38]
- 705 2. Snyder S_{in} term [41]
3. Hsiao-Shemdin S_{in} term [40]
4. $WAM3$ S_{in} term [37]

6. Test of Chalikov wind input term

The sophisticated Chalikov wind input term algorithm is not presented in the
 710 current paper, due to space. Curious readers can find it in [38, 39].

Fig.8a shows that total energy growth, along the fetch, significantly exceeds
 what is observed in ZRP simulation. This dependence is not the power function
 of the fetch, see Fig.8b, but can be approximated by relatively slowly changing
 values of the exponent p , ranging from 0.8 to 0.5, along the fetch. While $p \simeq 0.8$
 715 is still observed in some experiments, the value $p \simeq 0.5$ is completely unrealistic.

Figure 8: Same as Fig.2, but for Chalikov S_{in}

Figure 9: Same as Fig.3, but for Chalikov S_{in}

The same relates to the mean frequency dependence, against the fetch, shown in Fig.9a, with the values of the exponent q shown in Fig.9b. The value of q is also not constant, but slowly diminishes with the fetch. One should note that the value $q \simeq 0.25$ has been detected in the experiments, while $q < 0.2$ has never, apparently, occurred.

Fig.10 presents the combination, $(10q - 2p)$, as a function of the fetch. It is surprising that it is in good accordance with the relation Eq.(18). It means that despite incorrect values p and q along the fetch, their combination $(10q - 2p)$ still holds in complete accordance with theoretical prediction, and the spectra

Figure 10: “Magic relation” ($10q - 2p$) as a function of the dimensionless fetch for Chalikov wind input term.

are “locally self-similar”, in accordance with *WTT*.

Fig.11 presents directional (angle averaged) spectrum, as a function of frequency, in logarithmic coordinates. One can see Kolmogorov-Zakharov $\sim \omega^{-4}$ and Phillips high frequency tail $\sim \omega^{-5}$, as well.

Fig.12 confirms the presence of ω^{-4} spectrum, through existence of the “plateau” section, to the right of the spectral peak area, in the frequency range $0.45Hz < f < 1Hz$.

7. Test of Snyder wind input term

Snyder wind input term Eq.(26) is especially important, since it is included, as an option, in operational models.

The main disadvantage of the Snyder wind input term is rapid energy growth, with the fetch, presented in Fig.13a. For dimensionless fetches $\chi \simeq 5 \cdot 10^3$ it shows values approximately three times bigger than those experimentally observed and those obtained in the more realistic *ZRP* model. Apparently, that fact, together with a non-critical belief in the Snyder wind input function, caused the myth about long-wave dissipation, due to breaking of the long waves. Despite unrealistic energy growth along the fetch, Fig.13b shows that energy growth is close to the power function Eq.(16), with the index p slowly changing

Figure 11: Same as Fig.7, but for Chalikov S_{in} .

Figure 12: Angle averaged energy compensated spectrum $\langle \varepsilon \omega^4 \rangle = \frac{1}{2\pi} \int \varepsilon(\omega, \theta) \omega^4 d\theta$ as the function of decimal logarithm of frequency $f = \frac{\omega}{2\pi}$ for Chalikov S_{in} .

Figure 13: Same as Fig.2, but for Snyder S_{in}

from 1 to 0.6, which is, generally, significantly lower than observed in field experiments.

745 The same relates to the mean frequency dependence, against the fetch, shown in Fig.14a, with the values of the exponent q shown in Fig.14b. The value of q is not constant, but slowly diminishes with the fetch. One should note that the value $q \simeq 0.25$ has been detected in experiments, while $q < 0.2$ has never, apparently, occurred.

750 Dependence of the mean frequency, against the fetch, shown in Fig.14a, is lower than ZRP numerical results, but can be also approximated by a power function of the fetch Eq.(17), with the values of q slowly diminishing along the fetch, from 0.3 to 0.25, see Fig.14b.

The Kolmogorov-Zakharov spectrum $\sim \omega^{-4}$ and Phillips high frequency tail
755 $\sim \omega^{-5}$ can be seen in Fig.15, presenting directional spectrum as a function of frequency, in logarithmic coordinates. The span of the Kolmogorov-Zakharov $\sim \omega^{-4}$ segment can be estimated from Fig.16.

Fig.17 presents the combination $(10q - 2p)$ as a function of the fetch. Again, it strongly agrees with the theoretical relation Eq.(18). As in the Chalikov case, it
760 means that, despite imperfect values of p and q and rapid energy growth along the fetch, their combination $(10q - 2p)$ still holds in complete accordance with

Figure 14: Same as Fig.3, but for Snyder S_{in}

Figure 15: Same as Fig.7, but for Snyder S_{in} .

Figure 16: Angle averaged, energy compensated, spectrum $\langle \varepsilon \omega^4 \rangle = \frac{1}{2\pi} \int \varepsilon(\omega, \theta) \omega^4 d\theta$ as the function of decimal logarithm of the frequency $f = \frac{\omega}{2\pi}$, for Snyder S_{in} .

theoretical prediction, i.e. self-similarity is also fulfilled, locally, in the Snyder case.

8. Test of *Hsiao-Shemdin* wind input term

765 Fig.18a shows that total energy growth, along the fetch, underestimates *ZRP* simulation. It obeys the power law Eq.(16) with the exponent $p \approx 0.5$, see Fig.18b.

Fig.19a demonstrates mean frequency dependence, on the fetch, by power law Eq.(17), with asymptotic value of the index $q \approx 0.21$, see Fig.19b.

770 Kolmogorov-Zakharov segment $\sim \omega^{-4}$ and Phillips high frequency tail $\sim \omega^{-5}$ can be seen in Fig.20, presenting directional spectrum as a function of frequency, in logarithmic coordinates. The span of the Kolmogorov-Zakharov spectrum can be estimated using Fig.21.

Fig.22 presents combination $(10q - 2p)$ as the function of the fetch. It is in 775 total agreement with the theoretical predictions Eq.(18), which means that self-similarity is also fulfilled locally in *Hsiao-Shemdin* case.

Figure 17: Relation $(10q - 2p)$ as a function of the fetch x for Snyder wind input term.

Figure 18: Same as Fig.2, but for Hsiao-Shemdin S_{in}

Figure 19: Same as Fig.3, but for Hsiao-Shemdin S_{in}

Figure 20: Same as Fig.7, but for Hsiao-Shemdin S_{in} .

Figure 21: Angle averaged, energy compensated, spectrum $\langle \varepsilon \omega^4 \rangle = \frac{1}{2\pi} \int \varepsilon(\omega, \theta) \omega^4 d\theta$ as the function of decimal logarithm of the frequency $f = \frac{\omega}{2\pi}$ for Hsiao-Shemdin S_{in} .

Figure 22: Relation $(10q - 2p)$ as the function of the fetch for Hsiao-Shemdin wind input term.

9. Test of *WAM3* input terms

The current section presents what is, arguably, the most important part of the paper. It concerns the numerical simulation performed in the frame of *WAM3* model [43, 48, 37], using exact expression for S_{nl} term. Similar experiments were performed by different authors, more than twenty years ago, see the monograph [2]. The results presented in the current paper do not contradict them (see Fig.24) and reveal some new features.

The source term for WAM cycles 1 through 3 contain, not only wind input term, but also long-wave dissipation [43, 48, 37].

The input source term was used in Snyder form, as per [37]:

$$S_{in}(k, \theta) = C_{in} \frac{\rho_a}{\rho_w} \max \left[0, \left(\frac{28u_*}{c} \cos(\theta - \theta_w) - 1 \right) \right] \omega \varepsilon(k, \theta) \quad (59)$$

$$u_* = u_{10} \sqrt{(0.8 + 0.065u_{10})10^{-3}} \quad (60)$$

where $C_{in} = 0.25$, ρ_a and ρ_w are the densities of air and water, u_* is the wind friction velocity, and c is the wave phase velocity.

White capping dissipation was defined by Eq.(33) [37]. Turning on such dissipation radically changes the whole physical picture of the dissipation-free Snyder case, both quantitatively and qualitatively as well.

First, the dissipation maximum coincides with the spectral maximum. Fig.23 demonstrating that *WAM3* dissipation can be called the “spectral peak dissipation”, indeed due to unambiguous, spectral peak frequency area localization. Fig.25a shows that total energy growth, along the fetch, becomes constant, at the dimensionless fetch value $\chi = 5 \cdot 10^4$, which for wind speed $U = 10 \text{ m/sec}$ means 500 km dimensional fetch.

The *WAM3* model predicts saturation and formation of the “mature sea”. The limiting level of energy $\varepsilon \cdot 10^{-3}$ is half that predicted by Pierson-Moscowitz $\varepsilon_{max} \simeq 3.64 \cdot 10^{-3}$. One should notice that Donelan has predicted $\varepsilon_{max} \simeq 4.07 \cdot 10^{-3}$, Young has predicted $\varepsilon_{max} \simeq (3.6 \pm 0.9)10^{-3}$. Hence, the *WAM3* model essentially underestimates the energy growth, for large values of the fetch.

Figure 23: Angle averaged energy dissipation function $\langle S_{diss} \rangle = \frac{1}{2\pi} \int_0^{2\pi} \gamma_{diss} \varepsilon(\omega, \theta) d\theta$ (dotted line) and angle averaged scaled energy spectrum $\langle \varepsilon \rangle = \frac{1}{2\pi} \int_0^{2\pi} \varepsilon(\omega, \theta) d\theta$ (solid line) as the functions of the frequency $f = \frac{1}{2\pi}$.

The exact same results are described in the monograph [2] (see the corresponding Fig.3.7).

805 The novelty is the following – for moderate fetches, $10 < \chi < 10^3$, the *WAM3* model gives the same results as *ZRP* model, *without any spectral dissipation*. This statement is illustrated by Fig.24, Fig25a.

The value of the exponent p versus the fetch, asymptotically, goes to 0, see Fig.25b. This demonstrates strong discrepancies, with *ZRP* results, for large
810 fetches.

The stationary level of energy corresponds to $0.2 m^2$, which is approximately 1.5 times less than $0.36 m^2$, the corresponding Pierson-Moscovitz spectrum.

Similarly to energy, the dependence of the mean frequency, against the fetch, shown in Fig.26a, becomes constant at the dimensionless fetch value $\chi = 5 \cdot$
815 10^4 . The value of corresponding index q goes asymptotically to 0, see Fig.26b. Indicating discrepancies amongst *ZRP* results.

Nevertheless, Fig.27 demonstrates the Kolmogorov-Zakharov segment of the spectrum $\sim \omega^{-4}$, for small fetch value $\sim 20 km$.

The solid line on Fig.28 presents, angle averaged, compensated wave energy
820 spectrum, for stationary state, corresponding to dimensional fetch value $\sim$

Figure 24: Decimal logarithm of total dimensionless energy, as a function of decimal logarithm, of the dimensional fetch. Solid line – *WAM3* case, dashed line – *ZRP* case approximated by the fit $2.9 \cdot 10^{-7} \frac{xg}{U^2}$, stars – data from Fig.3.7 in monograph [2] recalculated in the assumption $u_* = \frac{1}{27} U_{10}$.

Figure 25: Same as Fig.2, but for *WAM3* S_{in}

Figure 26: Same as Fig.3, but for $WAM3 S_{in}$

Figure 27: Same as Fig.7, but for $WAM3 S_{in}$.

Figure 28: Angle averaged energy compensated spectrum $\langle \varepsilon \omega^4 \rangle = \frac{1}{2\pi} \int \varepsilon(\omega, \theta) \omega^4 d\theta$ as the function of frequency $f = \frac{1}{2\pi}$ for WAM3 S_{in} .

500 km (solid line). The dashed line demonstrates compensated Pierson-Moscowitz spectrum. Both spectra have coinciding high-frequency behavior, but are completely different at lower f . Almost perfect correspondence at higher frequencies could mean that WAM3 input terms were tuned to match the experimental results only in spectral tail area.

Fig.29 presents combination $(10q - 2p)$ as a function of the fetch. It is in total disagreement with the theoretical predictions. There is no indication of “magic relation” Eq.(18) fulfillment.

Comparison of WAM3 model with JONSWAP experiments shows that it describes them fairly well, for small fetches of the order of 10 km, i.e., in the region far from saturation and realization of the stationary state. Applicability of the model, for longer fetches, is questionable, at least for the reason that “magic relation” ceases to be realized.

10. Conclusion

Series of numerical experiments have been performed, for four different variants of wind input terms S_{in} , in the frame of the alternative numerical framework, which assumed:

1. exact nonlinear term S_{nl}

Figure 29: “Magic relation” ($10q - 2p$) as a function of the dimensionless fetch for *WAM3* wind input term.

Experiment	p -test	q -test	KZ -spectrum	Magic relation	Energy growth
<i>ZRP</i>	YES	YES	YES	YES	YES
<i>Chalikov</i>	NO	NO	YES	YES	NO
<i>Snyder</i>	NO	NO	YES	YES	NO
<i>Hsiao – Shemdin</i>	NO	NO	YES	YES	NO
<i>WAM3</i>	NO	NO	YES	NO	NO

Table 2: Summary of the tests performed on five models of wind input S_{in}

2. absence of spectral peak dissipation

840 3. “implicit” high-wavenumbers dissipation in the form of Phillips tail ω^{-5}

The fifth numerical experiment contained *WAM3* spectral peak dissipation, “by definition”, but all other aspects of the numerical simulation correspond to the above alternative framework.

845 The results of all five numerical experiments were subjected to the five tests described below, with the summary presented in Table 2.

The p - and q - tests are the checks for Eq.(16) and Eq.(17) respectively; they check if the energy and mean frequency are power functions of the fetch, with proper self-similar exponents $p = 1$ and $q = 0.3$.

The KZ - spectrum test is a direct check of WTT 's validity, according to which,
 850 the directional (angle averaged) energy spectrum $\langle \varepsilon \omega^4 \rangle$ has to be, with up to
 20% accuracy, constant in the inertial interval $1.5\omega_p < \omega < 3.5\omega_p$. Fulfilling this
 test directly points to the fact that HE , in the universal domain, is described
 by stationary Eq.(4), which is caused by mutual cancellation of the “in” and
 “out” terms, as the dominating process in S_{nl} .

855 The “magic relation” test is a check for the “magic relation” Eq.(18) and is more
 liberal than the p – and q – tests, since it assumes that power dependencies of the
 energy and mean frequency, along the fetch, are local, i.e. exponents $p = p(\chi)$
 and $q = q(\chi)$ are slow functions of the fetch, but the “magic relation” can still
 be fulfilled, for any value of the fetch coordinate.

860 The “energy growth” test is a check that the energy growth rate, versus the
 fetch, compares with corresponding ZRP dependence and the results of the 12
 field experiments.

The following is a discussion of the above tests, applied to five simulations:

1. It is no surprise that the ZRP wind input function passed p – and q – tests,
 865 since it was especially designed with the purpose to satisfy them at $p = 1$.
 and $q = 0.3$. It also passes KZ , “magic relation”, and energy growth tests,
 since it reproduces more than a dozen of the field experiments. Therefore,
 it can serve as the benchmark.
2. All other wind input terms also pass the KZ – test. Validity of that result,
 870 for all five versions of the wind input, suggests its universality. One can
 say that, if not for all, then for a very wide choice of the wind input
 functions, the spectrum $\varepsilon \sim \omega^{-4}$ will be realized, due to domination of
 the conservative terms in the HE .
3. All the cases, except $WAM3$, passed the “magic relation” test. This
 875 means, practically, that for any form of quasi-linear S_{in} , for large fetches,
 there is formation of a local, self-similar, regime, with indices p and q
 slowly changing, with the fetch. It also confirms WTT .
4. Chalikov and Hsiao-Shemdin cases fail the p – and q – tests, but are in

qualitative agreement with the field experiments.

880 5. All the cases, except *ZRP*, fail the energy growth test.

The Table 1, of section 5, presented the results of 12 experiments, confirming the law of $p = 1$ and $q = 0.3$ indices. Publication [33] presents the data of 24 field experiments - practically everything found in the literature. As it was already mentioned, half of those experiments satisfied p - and q -tests. Simple
885 calculation shows that “magic relation” test is satisfied, with the accuracy of 30%, for 2/3 of the described experiments. The reason for poor performance on the “magic test”, by other experiments, is discussed, in detail, in publication [33] and is explained, first of all, by data processing imperfections. More recent experiments, confirming the “magic relation”, are presented in publica-
890 tion [24]. One can conclude that the “magic relation” is confirmed, nowadays, experimentally.

As far as the *WAM3* model is concerned, while it does not pass any of the tests, except for *KZ* spectrum test, it realizes speculative phenomenon of the “mature sea”, not confirmed by serious experiments, but described in publication [43].
895 In any case, none of the field experiments analyzed in [33], [24] resemble, even remotely, formation of the “mature sea”. It would seem rational to refrain from this hypothesis.

The main obstacle for self-consistent wind-wave theory creation is the ambiguity of the analytical expression for S_{diss} . Although, it can be resolved, through a
900 numerical solution of, not *HE*, but primordial Euler equations. Such experiments are already carried out and their result are partially published [88], [95]. They unequivocally show that long wave dissipation occurs due to wave breaking of the short waves. Long wave dissipation, due to this process, is realized in rogue waves, but they are rare phenomena, and their contribution to energy
905 balance is, at least, orders of magnitude lower than assumed in *WAM3* and *WAM4* models. This fact also justifies the author’s lack of desire to supply the numerical model, by the long-wave dissipation.

Finally, let’s discuss research perspectives. So far, the authors solved the *HE*,

either as time evolution at a single spatial point [1], [34], [36] (“duration limited
 910 setup”), or as a spatially stationary solution [1] (“fetch limited setup”). Recent
 progress in the algorithms, computer software, and hardware development allow
 one to numerically solve the HE , using an exact S_{nl} expression, for a fairly large
 amount of points in the temporal-spatial domain. Preliminary results on the
 HE solution, for 40 equidistant points, along the fetch, were already presented
 915 at the *WISE* meeting [96]. Obtained results, for many more points (orders of
 magnitude larger) will be published soon.

It is expected that the plans to numerically simulate the wind wave development,
 using exact S_{nl} , are realistic, for fairly small domains like the Black Sea, Lake
 Michigan, and the Gulf of Bothnia.

920 The authors believe that the presented research is a step toward radical im-
 provement of the existing operating models. The source functions S_{in} and S_{diss}
 must be carefully revised and optimized, by numerical experiments using the
 XNL ’s nonlinear term. As soon as this is done, one should choose a proper ap-
 proximation, for XNL modeling – some version of DIA , with optimally chosen
 925 quadruplets. Ideally, during forecasting, the model will not require parameter
 tuning.

The oceanographic society must start using solid, justified, physical models and
 abandon the “black box with tuning knobs” approach. A new model will require
 minimal, if any, tuning for different ocean conditions.

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12. Appendix

It is convenient to present the Hasselmann equation, not for the energy spectrum $\varepsilon(\omega, \theta)$, but for the wave action spectrum $n(k)$. The spectra are connected by the relation

$$\omega_k n_k dk = \varepsilon(\omega, \theta) \omega d\theta \quad (61)$$

The Hasselmann equation reads

$$\begin{aligned} \frac{\partial n_k}{\partial t} &= 2\pi g^2 \int_{k_2, k_3, k_4} (T_{k k_2 k_3 k_4})^2 \times \\ &\times (n_k n_{k_3} n_{k_4} + n_{k_2} n_{k_3} n_{k_4} - n_k n_{k_2} n_{k_3} - n_k n_{k_2} n_{k_4}) \times \\ &\times \delta(\omega + \omega_2 - \omega_3 - \omega_4) \delta(k_1 + k_2 - k_3 - k_4) dk_1 dk_2 dk_3 dk_4 \end{aligned} \quad (62)$$

Here $\omega_k = \sqrt{gk}$ and $T_{k_1 k_2 k_3 k_4} = \frac{1}{2} (\tilde{T}_{k_1 k_2 k_3 k_4} + \tilde{T}_{k_2 k_1 k_3 k_4})$

$$\begin{aligned} T_{k_1 k_2 k_3 k_4} &= \frac{1}{4} \frac{1}{(k_1 k_2 k_3 k_4)^{1/4}} \left\{ \right. \\ &+ \frac{1}{2} (k_{1+2}^2 - (\omega_1 + \omega_2)^4) \left[(\vec{k}_1 \vec{k}_2 - k_1 k_2) + (\vec{k}_3 \vec{k}_4 - k_3 k_4) \right] \\ &- \frac{1}{2} (k_{1-3}^2 - (\omega_1 - \omega_3)^4) \left[(\vec{k}_1 \vec{k}_3 - k_1 k_3) + (\vec{k}_2 \vec{k}_4 + k_2 k_4) \right] \\ &- \frac{1}{2} (k_{1-4}^2 - (\omega_1 - \omega_4)^4) \left[(\vec{k}_1 \vec{k}_4 + k_1 k_4) + (\vec{k}_2 \vec{k}_3 + k_2 k_3) \right] \\ &+ \left(\frac{4(\omega_1 + \omega_2)^2}{k_{1+2} - (\omega_1 + \omega_2)^2} - 1 \right) (\vec{k}_1 \vec{k}_2 - k_1 k_2) (\vec{k}_3 \vec{k}_4 + k_3 k_4) \\ &+ \left(\frac{4(\omega_1 - \omega_3)^2}{k_{1-3} - (\omega_1 - \omega_3)^2} - 1 \right) (\vec{k}_1 \vec{k}_3 + k_1 k_3) (\vec{k}_2 \vec{k}_4 + k_2 k_4) \\ &+ \left. \left(\frac{4(\omega_1 - \omega_4)^2}{k_{1-4} - (\omega_1 - \omega_4)^2} - 1 \right) (\vec{k}_1 \vec{k}_4 + k_1 k_4) (\vec{k}_2 \vec{k}_3 + k_2 k_3) \right\} \end{aligned} \quad (63)$$

where notation $k_{1+2} = |\vec{k}_1 + \vec{k}_2|$.

Eq.(63) can be rewritten as follows:

$$\frac{\partial n_k}{\partial t} = S_{nl} = F_k - \Gamma_k n_k \quad (64)$$

where

$$F_k = 2\pi g^2 \int |T_{kk_1k_2k_3}|^2 n_{k_1} n_{k_2} n_{k_3} \times \delta(\vec{k} + \vec{k}_1 - \vec{k}_2 - \vec{k}_3) \delta(\omega_k + \omega_{k_1} - \omega_{k_2} - \omega_{k_3}) dk_1 dk_2 dk_3 \quad (65)$$

$$\Gamma_k = 2\pi g^2 \int |T_{kk_1k_2k_3}|^2 (n_{k_1} n_{k_2} + n_{k_1} n_{k_3} - n_{k_2} n_{k_3}) \times \delta(\vec{k} + \vec{k}_1 - \vec{k}_2 - \vec{k}_3) \delta(\omega_k + \omega_{k_1} - \omega_{k_2} - \omega_{k_3}) dk_1 dk_2 dk_3 \quad (66)$$

KZ stationary spectra are given by the expression

$$n_k = \frac{F_k}{\Gamma_k} \quad (67)$$

As far as $n_k > 0$, $\Gamma_k > 0$. The stationary kinetic equation, in the presence of wind input and damping, reads

$$S_{nl} + \gamma_k n_k = 0 \quad (68)$$

950 Here $\gamma_k = \gamma_{in} - \gamma_{diss}$. The solution to this equation is

$$n_k = \frac{F_k}{\Gamma_k - \gamma_k} \quad (69)$$

As far as $n_k > 0$, then $\Gamma_k > \gamma_k$. In fact, in the real situation $\Gamma_k \gg \gamma_k$. This is clear from Fig.30, adopted from paper [10].

It should be stressed out that our modernized code made possible separate
955 calculation of F_k and Γ_k .

Here the dashed line is the theoretical calculation of Γ_k , made narrow in the angle spectrum, and the solid line is the numerical experiment. More details can be found in paper [9].

Excess of Γ_k over γ_k , by orders of magnitude, explains the dominant role of S_{nl} .

Figure 30: Coefficient of nonlinear dissipation Γ_k : theoretical estimate(dashed line) [9, 10] and numerical calculation (solid line). Known parameterizations, of the wave energy growth rate, are shown on the legend. Essential exceeding of Γ_k over γ_k explains applicability of the equation $S_{nl} = 0$

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