Tidally-forced flow in a rotating, stratified, shoaling basin

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Abstract

Baroclinic flow of a rotating, stratified fluid in a parabolic basin is computed in response to barotropic tidal forcing using the nonlinear, non-hydrostatic, Boussinesq equations of motion. The tidal forcing is derived from an imposed, boundary-enhanced free-surface deflection that advances cyclonically around a central amphidrome. The tidal forcing perturbs a shallow pycnocline, sloshing it up and down over the shoaling bottom. Nonlinearities in the near-shore internal tide produce an azimuthally independent ‘set-up’ of the isopycnals that in turn drives an approximately geostrophically balanced, cyclonic, near-shore, sub-surface jet. The sub-surface cyclonic jet is an example of a slowly evolving, nearly balanced flow that is excited and maintained solely by forcing in the fast, super-inertial frequency band. Baroclinic instability of the nearly balanced jet and subsequent interactions between eddies produce a weak transfer of energy back into the inertia-gravity band as swirling motions with super-inertial vorticity stir the stratified fluid and spontaneously emit waves. The sub-surface cyclonic jet is similar in many ways to the poleward flows observed along eastern ocean boundaries, particularly the California Undercurrent. It is conjectured that such currents may be driven by the surface tide rather than by winds and/or along-shore pressure gradients.

Keywords: internal tide, cyclonic jet, baroclinic instability, poleward eastern boundary currents

1. Introduction

Oceanic internal tides are inertia-gravity waves at tidal frequencies, usually the semi-diurnal M_2 , excited by the barotropic tide as it induces vertical

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Preprint submitted to Ocean Modelling

April 17, 2015

displacement of stably-stratified fluid in the vicinity of topographic variability. Because M_2 lies within the inertia-gravity wave band f to N defined by the linear dispersion relation, the interaction between barotropic tidal flows and topography is able to generate freely-propagating internal tides that radiate away from generation sites. Here f and N are the local inertial and buoyancy frequencies respectively. Previous studies have focused on internal tides generated at isolated topographic features within basins such as ridge systems like the Hawaiian Ridge (Merrifield et al., 2001; Merrifield and Holloway, 2002; Rudnick et al., 2003) and the Luzon Strait (Mercier et al., 2013; Li and Farmer, 2011; Zhao, 2014) or at continental slope-shelf breaks where the topographic slope transitions from super- to sub-critical values with respect to the inclination angle of energy propagation for internal tides (Huthnance, 1989; Pingree and New, 1992; Gerkema et al., 2004). Near ridges the interest is mostly in energetic, low-vertical-mode internal tides that evolve as they propagate away to the far field, *e.g.* Hazewinkel and Winters (2011), while at shelf breaks studies tend to focus on the smaller scale components that form tidal beams.

A central issue is the energy balance in the conversion of barotropic to baroclinic motions with particular emphasis on the relative magnitudes of the radiating and trapped baroclinic motions. A better understanding of the environmental parameters and physical mechanisms that produce trapped and radiating motions is necessary in order to better understand and predict the intensity and spatial distribution of tidally driven mixing and dissipation in the ocean, *e.g.* Garrett (2003).

In this study, we consider the effect of an idealized free-surface tide in the absence of isolated topographic features and ask, what is the nature of the baroclinic response in a shoaling, rotating, stratified basin? In this context, the shoaling, near-shore region is the generation site for inertia-gravity waves and the overall baroclinic response consists of both waves that radiate toward the basin interior and trapped, near-shore motions at frequencies less than f . We compute the response of a continuously stratified fluid in a rotating, parabolic basin to idealized tidal forcing. The tide is modeled as an imposed, boundary-enhanced free-surface deflection that rotates cyclonically at frequency $\sigma > f$ about a single amphidrome located at the basin center. The ambient stratification consists of a thin mixed layer and a sharp, near-surface, pycnocline with $N \gg \sigma$ overlying weak stratification with N slightly less than σ at depth. The forcing excites boundary intensified motions that advance cyclonically around the basin, sloshing the pycnocline up and down over the shoaling bottom. The azimuthal wavelength of these motions matches the circumference of the basin.

44 The fast time scales of the problem are associated with the tidal and
 45 inertial frequencies σ and f . We will see that a slow time scale emerges that
 46 is associated with the interaction and pairing of the near-shore eddies. The
 47 fast and slow time scales are dynamically linked in this flow by the eddies
 48 themselves which have a vertical vorticity magnitude comparable to f .

49 Our approach here is to convert the idealized tidal model into a time and
 50 space dependent horizontal body force that is then applied to a rigid-lid nu-
 51 merical model of rotating, stratified flow in a parabolic basin. In taking
 52 this approach, we relax the various assumptions that might make analytical
 53 calculations tractable, namely the assumptions of linearity, hydrostaticity,
 54 uniform basin depth, and simple two-layer stratification, and compute the
 55 baroclinic response to the idealized tidal forcing via non-hydrostatic numer-
 56 ical simulation.

57 The remainder of the paper is organized as follows. The idealized tidal
 58 model is described in section 2 followed by a description of the mathemat-
 59 ical model and a brief sketch of its numerical solution in section 3. The
 60 dimensionless parameters and the asymptotic regimes of the flow are given
 61 in section 4. Simulation results are presented in 5 followed by a discussion
 62 in 6.

63 2. Experimental setup

64 We consider a parabolic basin with a stress-free rigid lid in a rotating
 65 reference frame as shown in figure 1. A simple model of a free surface tide
 66 is specified by

$$\eta = \eta_o(r) \cos(\theta - \sigma t) \quad (1)$$

67 where η is the deviation of the surface elevation from its equilibrium value,
 68 r is the radial distance from the domain center (x_o, y_o) , θ is the polar angle
 69 that increases in the counter-clockwise or cyclonic direction, and σ is the
 70 forced tidal frequency. The shape function $\eta_o(r)$ is taken as a polynomial
 71 with $\gamma = 3$,

$$\eta_o(r) = \eta_{\max} \left(\frac{r}{R} \right)^\gamma. \quad (2)$$

72 The free surface deflection η is maximal at the boundary, decays to zero
 73 in the interior and has a phase that propagates in the counter-clockwise
 74 direction with a period of $2\pi/\sigma$ (see figure 2). From this simple model, we
 75 define a body force

$$\mathbf{F}(x, y, z, t) = -g\nabla\tilde{\eta}. \quad (3)$$

76 Here $\mathbf{F}$ is written as an explicit function of z and η with a tilde to indi-
 77 cate that, near the curved boundary, the component of $\nabla\eta$ normal to the

Figure 1: Schematic of a parabolic basin with surface radius R and depth H . An initially quiescent fluid is set into motion by a body force $\mathbf{F}$, which models forcing by a barotropic tide advancing cyclonically along the coastline with an amphidrome at the basin center. The stratification is characterized by a surface mixed layer, a strong pycnocline and a weak stratification at depth.

78 immersed boundary is smoothly tapered to zero over a scale that is com-
 79 parable to the grid spacing. We regard the resulting $\mathbf{F}$ as an externally
 80 imposed body force that models the influence of a barotropic tide for an
 81 irregularly shaped basin with a rigid lid. Our objective is to determine the
 82 internal response to this forcing.

83 3. Mathematical model

84 3.1. Basin shape

85 The basin shape is defined within a cubic computational domain of di-
 86 mensions L_x, L_y and L_z by the height of the bottom boundary $h(x, y)$ where

$$h(x, y) = h_o + \frac{r^2}{\alpha^2} \quad (4)$$

87 Here α^2 is equal to $\frac{1}{4}(0.95L_x)^2/L_z$. For small but finite $h_o > 0$, the deepest
 88 point in the basin is slightly above $z = 0$ and the basin outcrops the upper

89 surface $z = L_z$ slightly inside the vertical computational boundaries. The
 90 effective dimensions of the basin are thus R and H , as shown in figure
 91 1, which are slightly smaller than $L_x/2 = L_y/2$ and L_z respectively. In
 92 presenting results, we will shift the coordinate origin to the point $(x_o, y_o, z -$
 93 $h_o)$ and normalize horizontal and vertical positions by R and H respectively.

94 3.2. Equations of motion

95 The flow is modeled using the Boussinesq equations of motion for a
 96 rotating stratified fluid on an f -plane augmented with the tidal body force
 97 $\mathbf{F}$

$$\frac{\partial}{\partial t}u + \vec{u} \cdot \nabla u - fv = -\frac{1}{\rho_o} \frac{\partial p'}{\partial x} + \mathbf{F} \cdot \hat{x} + \mathcal{D}_u \quad (5)$$

$$\frac{\partial}{\partial t}v + \vec{u} \cdot \nabla v + fu = -\frac{1}{\rho_o} \frac{\partial p'}{\partial y} + \mathbf{F} \cdot \hat{y} + \mathcal{D}_v \quad (6)$$

$$\frac{\partial}{\partial t}w + \vec{u} \cdot \nabla w + \frac{g}{\rho_o} \rho' = -\frac{1}{\rho_o} \frac{\partial p'}{\partial z} + \mathcal{D}_w \quad (7)$$

$$\frac{\partial}{\partial t}\rho + \vec{u} \cdot \nabla \rho = \mathcal{D}_\rho \quad (8)$$

$$\nabla \cdot \vec{u} = 0. \quad (9)$$

102 Here $\hat{x}$ and $\hat{y}$ are horizontally oriented unit vectors, f is the Coriolis param-
 103 eter and g is the gravitational acceleration. The $\mathcal{D}$ terms are dissipation
 104 operators discussed further below.

105 The perturbation density ρ' is defined with respect to a time independent
 106 ambient vertical profile $\bar{\rho}$ where

$$\rho = \bar{\rho}(z) + \rho'(x, y, z, t) \quad (10)$$

107 and $\bar{\rho}(z)$ is the initial density profile. Pressure is decomposed into a depth
 108 dependent part that is in hydrostatic balance with $\bar{\rho}$ and a perturbation
 109 component p' defined via

$$p = -g \int^z \bar{\rho}(\tilde{z}) d\tilde{z} + p'(x, y, z, t). \quad (11)$$

110 The hydrostatic balance drops out of Eq. (7) in the usual way and so only
 111 ρ' appears in that equation while p' appears throughout.

112 The $\mathcal{D}$ terms are a combination of Laplacian operators with molecular
 113 viscosity and “hyper-diffusivity” operators. For example

$$\mathcal{D}_u = \nu \nabla^2 u + \nu_h \left(\frac{\partial^6}{\partial x^6} + \frac{\partial^6}{\partial y^6} \right) u + \nu_v \left(\frac{\partial^6}{\partial z^6} \right) u. \quad (12)$$

114 The operators for the other fields are similar. The coefficients of the “hyper-
 115 diffusivity” operators have dimensions m^6/s and are specified so that the
 116 relatively weak downscale variance cascades are efficiently removed at the
 117 smallest resolvable scales while permitting very small molecular diffusivities
 118 to be prescribed. This is important here as we wish to conduct numerical
 119 experiments over hundreds of forcing periods without having the ambient
 120 stratification diffuse so much that the wave environment changes consider-
 121 ably. Specifying only Laplacian diffusion is problematic: diffusivity values
 122 large enough to ensure numerical stability diffuse the ambient stratification
 123 excessively while diffusivities small enough to nearly maintain the initial
 124 stratification are not sufficient to arrest the downscale cascade and main-
 125 tain numerical stability. Because the underlying numerical method is based
 126 on Fourier expansions, these dissipation operators are easily and accurately
 127 computed.

128 3.3. Boundary conditions

129 The upper boundary is a rigid lid with w , $\frac{\partial u}{\partial z}$, $\frac{\partial v}{\partial z}$ and $\frac{\partial \rho}{\partial z}$ all prescribed
 130 to be zero. To avoid having to resolve small viscous boundary layers, the
 131 curved bottom boundary is taken as a free-slip wall with $\vec{u} \cdot \hat{n} = 0$, where
 132 $\hat{n}$ is the local unit vector normal to the boundary. No boundary condition
 133 is imposed on ρ at this boundary. These are the exact boundary conditions
 134 desired in the limiting case of an inviscid, nondiffusive fluid. We recall
 135 for example that, for an ideal fluid, the exact solution for a linear internal
 136 Kelvin wave at a vertical sidewall (Gill, 1982; Csanady, 1967) has non-
 137 vanishing normal gradients of both the along-wall speed and density. In
 138 deriving these solutions, the only boundary condition at the wall is the no
 139 penetration condition. Stress and scalar diffusion through the wall vanish
 140 by virtue of vanishing molecular diffusivities, not vanishing normal gradients
 141 of the solutions. We adopt this approach here under the idealization that
 142 unless a no-slip condition is imposed and the tangential flow is required to
 143 vanish at the curved boundary, the action of the viscous terms is negligible
 144 and thus vanishing normal flow at the wall is the only allowable boundary
 145 condition to prescribe. This approach then permits small but physically
 146 negligible stresses and scalar fluxes at the boundary when the molecular
 147 diffusivities are taken sufficiently small as discussed in section 4.

148 *3.4. Initial conditions*

149 The experiments start from a state of rest at $t = 0$:

$$\vec{u}(\vec{x}, 0) = 0. \quad (13)$$

150 The initial density profile consists of a surface mixed layer, a thin, strongly
 151 stratified pycnocline and a weaker stratification at depth. To specify the
 152 initial profile, we first define several parameters. Let z_i and δ be the mid-
 153 height and thickness of the pycnocline respectively and $z_1 = z_i + \delta/2$ and
 154 $z_2 = z_i - \delta/2$. Let $\rho_{\max}$, ρ_1 and ρ_2 be the density values at $z = 0$, at z_1
 155 and at z_2 respectively. We then define the characteristic magnitude of the
 156 density gradients in the deep basin and in the pycnocline as

$$c_1 = \frac{\rho_{\max} - \rho_2}{z_1} \quad \text{and} \quad c_2 = \frac{\rho_2 - \rho_1}{\delta} \quad (14)$$

157 as shown in figure 2. Introducing a small transition scale $\beta = \delta/10$, and a
 158 superposition of approximate step functions using hyperbolic tangents, we
 159 integrate in z to obtain

$$\begin{aligned} \bar{\rho}(z) = & \frac{c_1}{2} (\beta \log(\cosh((z - z_1)/\beta)) - z) \\ & + \frac{c_2}{2} (\beta \log(\cosh((z - z_1)/\beta))) \\ & - \frac{c_2}{2} (\beta \log(\cosh((z - z_2)/\beta))) - \bar{\rho}_s \end{aligned} \quad (15)$$

160 where $\bar{\rho}_s$ is a constant chosen such that the value of $\bar{\rho}$ at the surface is equal
 161 to zero. We set $\rho(\vec{x}, 0) = \bar{\rho}(z)$, which gives the initial profiles shown in figure
 162 2. The characteristic squared buoyancy frequencies in the pycnocline and in
 163 the deep basin are given by

$$N_{\text{deep}}^2 = \frac{g}{\rho_o} c_1 \quad \text{and} \quad N_{\text{pyc}}^2 = \frac{g}{\rho_o} c_2. \quad (16)$$

164 As indicated in figure 2, the forcing frequency σ is less than N within the
 165 main pycnocline and greater than N below.

166 *3.5. Numerical solution*

167 The equations of motion are solved using trigonometric expansions for
 168 the dependent variables in a cubic domain of size L_x, L_y and L_z . All variables
 169 are expanded in cosine series in both horizontal directions. In the vertical, u ,
 170 v and ρ are expanded in cosine series while w is expanded in a sine series and
 171 so the prescribed boundary conditions at the rigid lid are satisfied implicitly.

Figure 2: Left: Initial density profile $\bar{\rho}(z)$ and corresponding profile of the squared buoyancy frequency N^2 . The depth range $z_2 < z < z_1$ is shaded. The dashed vertical line shows the square of the tidal frequency σ . Right: Magnitude of η (color) at $t = 0$, the convention for the azimuthal angle θ and the cyclonically rotating tidal phase lines.

172 The boundary conditions on the curved bottom boundary are implemented
 173 using an immersed boundary approach (Winters and de la Fuente, 2012).

174 The equations are integrated for $O(250)$ forcing periods using a 2-D
 175 domain decomposition on distributed memory machines with 256 or 512
 176 processors. The primary output consists of data, u, v, w, ρ' and p' , from
 177 horizontal planes just below the base of the mixed layer, within, and below
 178 the pycnocline, and on the two vertical planes $y = 0$ and $x = 0$. Data from
 179 these two-dimensional planes are saved ≈ 100 times per forcing period. Full
 180 three-dimensional snapshots are saved less frequently and can be used to
 181 restart the simulations to generate different or additional data as desired.

182 4. Ordering of scales

183 The dimensionless parameters defining the simulation are listed in Table
 184 1. Here we summarize briefly the asymptotic sizes of a few of the key di-
 185 mensionless parameters. First, $N_{\text{pyc}}/f = O(10)$ and $\sigma/f > 2$. Thus, forcing
 186 at frequency σ can potentially excite free internal waves at both higher and
 187 lower frequencies and the overall frequency bandwidth is comparable to, but
 188 smaller than, that in the mid-latitude ocean thermocline where $N/f \approx 50$.

189 The external Rossby radius of deformation a_{ext} is imposed implicitly via
 190 $\eta_o(r)$ and is smaller than, but of the same order, as the surface basin radius
 191 R . The internal Rossby radius $a_{\text{int}} \approx c/f \approx 0.1R$, i.e. is asymptotically
 192 small compared to R but larger than typical ocean values. Here $c = \sqrt{\frac{g'h_1h_2}{H}}$,
 193 h_1 is the depth of the pycnocline center, $h_2 = H - h_1$ and $g' = g\Delta\rho/\rho_o$ with
 194 $\Delta\rho = \rho_2 - \rho_1$, the density difference across the pycnocline. Increasing both
 195 N_{pyc}/f and R would yield a wider band of internal wave frequencies and a
 196 larger scale separation between the internal and external deformation radii,
 197 but at considerably greater computational expense.

198 The strength of the tidal forcing is quantified by comparing the char-
 199 acteristic magnitude of the imposed vertical acceleration of the pycnocline
 200 to the restoring force per unit mass induced when this occurs, g' . The
 201 characteristic horizontal acceleration is $|g\nabla\eta| \approx g\eta_{\text{max}}/R$. Horizontally ac-
 202 celerated flow is deflected vertically owing to the parabolic basin shape and
 203 so the characteristic vertical acceleration induced by the tidal forcing is given
 204 by $g(\eta_{\text{max}}/R)(H/R)$, where H/R is the overall basin slope. The ratio of the
 205 forced vertical acceleration to the restoring force per unit mass g' is then
 206 $(\rho_o/\Delta\rho)(\eta_{\text{max}}/R)(H/R)$, a measure of how strongly the pycnocline is being
 207 vertically displaced by the forcing. Here we take

$$\frac{\rho_o}{\Delta\rho} \frac{\eta_{\text{max}}}{R} \frac{H}{R} \ll 1 \quad (17)$$

Table 1: Dimensionless parameters defining the flow regime and the model resolution. Symbols are defined in section 4.

parameter	value	comment
N_{pyc}/f	11.6	strongly stratified pycnocline
$N_{\text{deep}}/N_{\text{pyc}}$	0.19	weak stratification at depth
σ/f	2.7	$2f < N_{\text{deep}} < \sigma < N_{\text{pyc}}$
δ/H	0.13	thin pycnocline
h_1/h_2	0.06	near-surface pycnocline
H/R	0.12	shallow basin
$(\rho_o/\Delta\rho)(\eta_{\text{max}}/R)(H/R)$	0.046	weak tidal forcing
$[\eta/\partial\eta/\partial r]/R$	≈ 1	external Rossby deformation radius
$a_{\text{int}}/R = (c/f)/R$	0.12	internal Rossby deformation radius
$\delta^2 f/\nu$	4.6×10^3	Laplacian dissipation is negligible
$(\nu_h k_*^6 \Delta t)^{-1}$	5	k_* is horizontal Nyquist wavenumber
$(\nu_v m_*^6 \Delta t)^{-1}$	5	m_* is vertical Nyquist wavenumber
$\Delta x/R = \Delta y/R$	0.004	horizontal grid spacing
$\Delta z/\Delta x$	0.22	grid aspect ratio
$\sigma \Delta t$	0.025, 0.0125	time step

and in this sense, internal waves in the basin are weakly forced by the tide.

The importance of dissipation is quantified by two dimensionless parameters reflective of the two operators in Eq. (12). $\delta^2 f/\nu$ is the ratio of the time required to diffuse momentum or mass across the pycnocline thickness δ to the inertial time scale $1/f$. This quantity is $O(10^3)$ and so the Laplacian terms are negligible over the simulation time of the experiments. $1/(\nu_h k_*^6 \Delta t)$ is the decay time associated with the sixth order operator at the spatial scale corresponding to the maximum resolved horizontal wavenumber k_* divided by the time step Δt . Similar quantities define the effective strength of the vertical diffusion of momentum and the horizontal and vertical diffusion of mass. These quantities prescribed to be $O(1)$ so that variance that is transferred from larger scales to the smallest resolvable scales is dissipated quickly, *i.e.* within a few time steps. The influence of these higher order operators, however, is strongly restricted to motions near the grid scale. Motions at scales of $O(10\Delta x)$ for example are dissipated on a time scale 10^6 times longer. Thus, except for a weak filtering effect near the grid scale, the computed flow is governed by equations that are essentially inviscid and non-diffusive.

Figure 3: Time series of along shore speed v at $y = 0$, $x/R = \pm 0.93$ and $z/H = 0.925$. At $\theta = \pi$, v is the negative of the azimuthal speed v_θ , which is positive for cyclonic flow.

5. Results

Results from a single experiment with parameters listed in Table 1 are now presented in some detail. We defer a systematic parameter study to future work and focus on presenting and explaining the relatively rich flow field that develops.

5.1. Overview: near-shore time series

Figure 3 shows near-shore time series of the y component of velocity v taken from within the pycnocline at $\theta = 0$ and π . The time series are normalized by the wave speed c used to define the internal Rossby deformation radius a_{int} in section 4. Both time series show high frequency fluctuations on the ‘fast’ time scale of the tidal forcing. The magnitude of these fluctuations increases over about the first 20 tidal periods and then remains approximately constant for about 70 periods. Very quickly after startup however, the flow develops a residual component, becoming always positive at $\theta = 0$ and always negative at $\theta = \pi$. The time averaged flow at both locations is in the counterclockwise or cyclonic sense. After about the first 20 tidal periods, the along-shore currents at these locations oscillate regularly about a slowly increasing mean speed. After about 90 tidal periods however, both time series become much more irregular, though the time average currents change very little.

Except for temporal phase, the flow prior to $t\sigma/2\pi \approx 90$ is essentially independent of θ once it is resolved into azimuthal and radial components.

Figure 4: Black: Mean isopycnal positions at $\theta = \pi$, averaged over the tidal period $40 < \sigma t / 2\pi < 41$. Colored isopycnals show the instantaneous positions corresponding to those shown with heavy black lines throughout the period. Here $\Delta\rho$ is the initial density jump across the pycnocline and contour values are uniformly spaced within the range $1/4 \leq \rho/\Delta\rho \leq 3/4$.

248 In displaying the cross-shore structure of the flow, we show results taken
 249 from the region surrounding the location of the $\theta = \pi$ time series shown in
 250 figure 3. For cross reference with these time series, the time values for the
 251 remaining figures are marked in the figure.

252 5.2. Internal wave ‘set-up’

253 The cross-shore structure of the near-shore isopycnals is shown in figure
 254 4. The black contours are isopycnals computed after time averaging the
 255 density field over the 41st tidal forcing period indicated in figure 3. During
 256 this time interval the tidal fluctuations are regular and the magnitude of
 257 the near- and along-shore flow v is accelerating only slowly. The density
 258 values are normalized by the initial density contrast across the pycnocline
 259 $\Delta\rho$ and the time averaged positions of $\rho/\Delta\rho = 1/4$ and $3/4$ are shown
 260 with heavy lines. Also shown are instantaneous positions of these same
 261 isopycnals at equally spaced intervals throughout the tidal cycle in red and
 262 blue respectively.

263 The time averaged isopycnal response is asymmetric about the mid-
 264 pycnocline value $\rho/\Delta\rho = 1/2$. Near the sloping boundary the upper portion
 265 of the pycnocline is lifted significantly upward on average, while the lower
 266 portion is deflected only mildly downward. At $x/R = -.95$ for example, the
 267 time averaged upward deflection of the $\rho/\Delta\rho = 1/4$ is approximately one

268 quarter of the local depth. The upper portion of the pycnocline sloshes up
 269 and down over the sloping boundary at the tidal frequency but with resid-
 270 ual positions that are lifted upward compared to the basin interior by an
 271 amount that is significant compared to the local depth.

272 The internal wave ‘set-up’ and ‘set-down’ shown in figure 4 are a con-
 273 sequence of the re-adjustment of mass and momentum required to cancel
 274 the Stokes drift for waves at a sloping boundary in the absence of mixing
 275 (Wunsch, 1971; Longuet-Higgins and Stewart, 1964). Based on an analytical
 276 calculation for weakly nonlinear, two-dimensional internal waves in a shoal-
 277 ing region with constant slope, Wunsch (1971) gives a scaling argument for
 278 the vertical setup of the isopycnals Z_{setup} that can be written in terms of
 279 the magnitude $\mathcal{Z}$ of the tidal frequency isopycnal excursions and the local
 280 depth D

$$Z_{\text{setup}} \approx \frac{\mathcal{Z}^2}{D} \quad [m]. \quad (18)$$

281 Wunsch (1971) further argues that this can be a measurable effect in the
 282 ocean for small amplitude waves for which his theoretical calculations are
 283 formally valid and a potentially very large effect for finite amplitude internal
 284 waves.

285 This quantity can be estimated at $x/R = -0.95$. Choosing $\mathcal{Z} \approx D/2$
 286 to be the approximate vertical excursion scale of the $\rho/\Delta\rho = 1/4$ isopycnal
 287 implies that

$$Z_{\text{setup}}/D \approx 1/4, \quad (19)$$

288 which matches the value inferred from figure 4.

289 *5.3. Time averaged near-shore cyclonic jet*

290 The consequences of the internal wave ‘set-up’ induced by tidal forcing
 291 in a shoaling basin are significant owing to the effects of rotation. The time
 292 averaged upward deflection of the near-shore isopycnals shown in figure 4
 293 occurs at all azimuthal positions. This induces an azimuthally-independent,
 294 residual cross-shore pressure gradient that drives a corresponding along-
 295 shore flow in the cyclonic direction. The residual of this cyclonic response
 296 gives rise to the offsets of the time series seen in figure 3.

297 The stability of jets over sloping boundaries has been previously investi-
 298 gated for hydrostatic, two-layer flow (Poulin and Flierl, 2005; Poulin et al.,
 299 2014). Wind-driven, rectified flow in a rotating lake has also been studied
 300 in a two-layer framework (Ou and Bennett, 1979). The flow here, however,
 301 is continually energized by the tidal forcing and is continuously stratified.
 302 Continuous stratification is likely to be important as the setup shown in

Figure 5: Azimuthal speed v_θ at $\theta = \pi$, averaged over the same tidal period shown in figure 4. Also shown are the time averaged isopycnals.

figures 4 and 5 has a distinct varicose character that is not captured in a two-layer formulation. In addition, in the depth range of the main pycnocline, the boundary slope $\frac{\partial h}{\partial r}$ is within about 5% of the critical angle for internal waves in a fluid characterized by f and N_{pyc} . These waves have aspect ratios of horizontal to vertical scales of $O(1)$ and so the nearly resonant waves excited close to the boundary in this flow are non-hydrostatic.

The along-shore flow at $\theta = \pi$, averaged over the same tidal period as in figure 4, is shown in figure 5 along with the corresponding isopycnals. The along-shore flow takes the form of a near-shore, bottom trapped cyclonic jet. The jet has a maximum speed at about $x/R = -0.95$ and weakens rapidly toward the basin interior. The horizontal scale of the jet is about $0.1R$ which is approximately equal to the internal deformation radius a_{int} .

If the near-shore cyclonic jet were to be in geostrophic balance with the sloping isopycnals, then, assuming hydrostatic pressure

$$\frac{\partial v}{\partial z} \approx -\frac{g}{\rho_o f} \frac{\partial \rho}{\partial x}. \quad (20)$$

The degree to which this thermal wind balance holds can be estimated by

318 computing the ratio of the characteristic magnitudes of each term. Taking

$$\left| \frac{\partial v}{\partial z} \right| \approx \frac{c}{0.1H} \quad \text{and} \quad \left| \frac{\partial \rho}{\partial x} \right| \approx \frac{.25\Delta\rho}{0.05R}, \quad (21)$$

319 the ratio of the size of the pressure gradient term to the vertical shear is
 320 about 1.1, suggesting the interpretation of this jet as a rotationally induced
 321 response to the tidally forced set-up of the near-shore pycnocline.

322 The simple view that emerges is that wave stresses induce a pycnocline
 323 setup near the sloping boundary and that this setup then produces an along-
 324 shore geostrophic jet. In detail however, a three term balance between the
 325 setup, rotation and wave stresses would be expected. Wunsch (1971) derives
 326 such a balance but notes that it is degenerate in the sense that arbitrary
 327 along-shore currents can be added that induce correspondingly arbitrary
 328 changes to the isopycnal setup.

329 At this point, we can now see that the $\theta = \pi$ time series in figure 3 is
 330 taken from within the near-shore cyclonic jet shown in figure 5. The speed
 331 of the jet increases steadily over the first 20 tidal periods as the isopycnals
 332 ‘set up’ in response to the forcing. After the initial spinup, the speed of
 333 the jet increases only slowly, with relatively little change over the next 70
 334 periods, consistent with the approximately geostrophic balance estimated
 335 for $40 < t\sigma/2\pi < 41$.

336 5.4. Cross-shore flow

337 While figure 5 shows the position and structure of the cyclonic jet when
 338 averaged over a tidal period, directly forced cross-shore motions cause the
 339 jet to move up and down on the slope at the tidal frequency. Figure 6
 340 shows vertical profiles of the cross-shore speed v_r at $\theta = \pi$, $x/R = -0.93$.
 341 The radial flow sloshes on- and off-shore with both a barotropic component
 342 of approximate magnitude $c/4$ as well as a baroclinic, bottom intensified
 343 component. This cross-shore flow is directly forced by the tide and thus has
 344 a frequency equal to σ . Averaging over a forcing period essentially removes
 345 the cross-shore flow.

346 The bottom intensification of the flow is reminiscent of the vertical struc-
 347 ture of topographic Rossby waves over a sloping bottom see *e.g.* (Rhines,
 348 1970; Pedlosky, 1987). Topographic Rossby waves are, however, freely prop-
 349 agating modes with frequencies much smaller than f . For frequencies ap-
 350 proaching f from below, freely propagating bottom-trapped waves (Rhines,
 351 1970) also decay away from the boundary, but in this limit the vertical decay
 352 scale matches the cross-shore wavelength, whereas it is much smaller than

Figure 6: Profiles of cross-shore flow speed v_r at $\theta = \pi$ and $x/R = -0.93$ for the time period $40 < t\sigma/2\pi < 41$. Left: Instantaneous profiles sampled at equal time intervals. Right: The same profiles with the instantaneous vertical mean speed $\bar{v}_r(t)$ removed. All speeds normalized by c .

353 this scale here. The observed cross-shore flow is the forced, rather than
 354 freely propagating, response to tidal forcing of a rotating, stratified fluid
 355 over a sloping bottom. Though the frequency of these motions σ is much
 356 higher than the frequency of free topographic Rossby or bottom-trapped
 357 waves, the forced response retains the bottom trapped character of these
 358 free modes.

359 5.5. Baroclinic instability of the near-shore jet

360 Figure 3 shows that the near-shore cyclonic jet transitions from a slowly
 361 accelerating feature with regular oscillations on the tidal time scale to a much
 362 more irregular flow that is no longer accelerating. The transition occurs at
 363 about $t\sigma/2\pi = 90$ and the irregular nature of the jet persists throughout
 364 the remainder of the experiment.

365 Figure 7 is a series of snapshots from this second regime for $159 <$
 366 $t\sigma/2\pi < 160$. Shown is the density field on a near-surface horizontal plane
 367 that slices through near-shore cyclonic jet and the corresponding time av-
 368 eraged, uplifted positions of the upper portion of the pycnocline shown in
 369 figure 4. Also shown are the positive and negative pressure contours. Over
 370 the fast time scale of a tidal period, the pressure contours rotate cyclonically
 371 making one full rotation while the density field changes relatively little. The
 372 azimuthal modal structure of the density field is approximately constant over

373 a tidal cycle. The structure of the density field is consistent with baroclinic
 374 instability of the approximately geostrophically balanced cyclonic jet.

375 Orlanski and Cox (1972) give a scaling estimate for the azimuthal wave-
 376 length λ_{bc} of baroclinically unstable waves on a geostrophically balanced jet
 377 as

$$\lambda_{bc} \approx \frac{2}{L} \frac{N^2 D^2}{f^2} \quad (22)$$

378 where L is the cross-shore scale over which the jet varies and D is the total
 379 depth. Taking $N = N_{pyc}$ and $L = 0.1R$ gives an estimate for the azimuthal
 380 mode number $C_{0.95}/\lambda_{bc}$ of approximately 12.3 where $C_{0.95}$ is the circum-
 381 ference at $r = 0.95R$. This estimate is slightly higher than the observed
 382 value of approximately 10 obtained by counting distinct dark spots in figure
 383 7 (note the two ongoing mergers at about 10 and 4 o'clock in each figure) ,
 384 but is a reasonable match for a simple scaling argument.

385 While it is not obvious from the figure, the baroclinically unstable waves
 386 are evolving on a much slower time scale than the tidal period and adjacent
 387 cores, or dark spots in the density images, interact with one another and
 388 occasionally merge. During the period shown, this is occurring near $\theta =$
 389 $3\pi/8$ and $-\pi/8$. The slow evolution and modification of these features gives
 390 rise to the irregularity of the later portion of the time series in figure 3.

391 5.6. Near-shore eddies

392 Baroclinic instability of the near-shore cyclonic jet produces slowly evolv-
 393 ing vortical structures or near-shore eddies that provide a mechanism for
 394 cross-shore advection and transport. Figure 8 shows snapshots of the ver-
 395 tical vorticity ζ normalized by f , a measure of the local Rossby number
 396 characterizing the relative importance of the nonlinear terms to the Coriolis
 397 terms in the horizontal momentum equations. Throughout the interior of
 398 the domain, the magnitude of the vorticity is small, indicating that rotation
 399 dominates nonlinearity there and that the time scale of vortical motions is
 400 much longer than $1/f$.

401 At the edges however, where distinct eddies are present, $|\zeta/f|$ exceeds 1,
 402 indicating that nonlinearity is at least as important as rotation in the near-
 403 shore flow and that vortical and inertial time scales are comparable. The
 404 coherent vortices are cyclonic with high density cores and thus are examples
 405 of ‘cold-core’ eddies. They appear to slowly migrate cyclonically around
 406 the edge of the domain as they interact with one another. Blue bands with
 407 $\zeta/f < -1$ are regions where the total vertical vorticity $f + \zeta$ is anticyclonic.

408 Figure 9 shows a plan view of w at $z/H = 0.95$ at $t\sigma/2\pi = 160$. The
 409 radial pattern in the basin interior is a signature of the forced waves at

Figure 7: Density at $z/H = 0.95$ during last simulated tidal period. Also shown are positive (solid, +) and negative (dashed, -) pressure contours. Time, scaled by the tidal period, increases moving to the right and down. The phase of the pressure field propagates cyclonically while the eddies remain approximately stationary on the fast, tidal time scale.

410 frequency σ propagating inward from the edges. The radial wavelength
 411 corresponds roughly to c/σ . In addition to these signals, smaller scale waves
 412 are emitted from each of the near-shore vortices as indicated by the arrows.
 413 We have already noted that these motions stir the fluid at frequencies greater
 414 than f and so it is not surprising radiating gravity waves would be excited,
 415 e.g. Danioux et al. (2012), Vanneste (2013) and references therein. As in
 416 Danioux et al. (2012), these waves are relatively weak: the frequency spectra
 417 shown in section 5.8 are dominated by motions at the forcing frequency and
 418 its harmonics.

419 5.7. *Along-shore structure near the coast*

420 The along-shore structure of the near-shore jet is shown in figure 10. The
 421 upper panel shows a snapshot of the azimuthal speed v_θ at $t\sigma/2\pi = 160$, well
 422 after the jet has become baroclinically unstable. Distinct, nearly barotropic
 423 columns of relatively fast cyclonic flow correspond to the individual eddies
 424 apparent in figure 8. Between these eddies however, for example at $\theta = 0$,
 425 fast azimuthal flow is confined to the lower half of the water column as was
 426 seen prior to the formation of the eddies and shown in figure 5. The lower
 427 panel shows a snapshot of the azimuthal structure of the cross-shore flow
 428 v_r . The cross-shore flow has an azimuthal mode one structure and closely
 429 follows the temporal phase of the tidally forced pressure field which are
 430 shown in figure 8. The cross shore flow is on-shore, toward low near-shore
 431 pressure, around $\theta = 0$ and off-shore, away from high near-shore pressure
 432 near $\theta = \pm\pi$. The presence of the eddies only slightly distorts this overall
 433 structure. The radial flow is bottom intensified with significant shear at
 434 about $z/H = 0.9$. This near-bottom flow leads the overlying flow in phase
 435 by about $\pi/4$ which appears to be largely responsible for the observed shear.
 436 The overall structure of v_r shown here matches the cross-shore structure
 437 observed prior to the excitation of the eddies shown in figure 6.

438 5.8. *Frequency spectra*

439 Time series of the azimuthal, radial and vertical components of veloc-
 440 ity within the near-shore jet were extracted over time intervals preceding
 441 and following the instability of the jet and the formation of near-shore ed-
 442 dies. The dimensionless spectra $\mathcal{S}$ is shown as a function of dimensionless
 443 frequency ω/σ in figure 11. The spectra are normalized such that

$$\frac{1}{\sigma} \int \mathcal{S}(\omega) d\omega = \frac{1}{T} \int \phi^2(t) dt \quad (23)$$

Figure 8: Vertical vorticity $\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ normalized by f at $z/H = 0.95$ and at the $t\sigma/2\pi$ values of 160, 170, 180 and 190.

Figure 9: Vertical speed w normalized by c at $t\sigma/2\pi = 160$ and $z/H = 0.95$, corresponding to the last snapshot in figure 7 and the first in figure 8. Arrows indicate regions of spontaneous emission of fast gravity waves.

Figure 10: Top: Azimuthal speed $v_\theta(\theta, z/H)/c$ at $r/R = 0.93$ at $t\sigma/2\pi = 160$. Positive azimuthal speeds indicate cyclonic flow along the boundary. Bottom: Corresponding radial flow v_r/c .

Figure 11: Frequency spectra for the azimuthal, radial and vertical currents at $\theta = \pi$, $x/R = -0.93$ and $z/H = 0.925$. The spectra for the azimuthal speed corresponds to the lower time series shown in figure 3. Spectra are shown for two time intervals spanning 20-80 and 80-240 forcing periods. The inertial and tidal frequencies are indicated.

where T is the time span of the interval and ϕ is the de-meaned, de-trended and windowed time series of speed values normalized by c .

Spectra corresponding to the time span prior to the excitation of eddies at about $\sigma t/2\pi = 90$ are shown in thin lines. These spectra have distinct spectral lines at the tidal frequency $\omega/\sigma = 1$ and at each tidal harmonic over a frequency range extending over more than a decade. Between the harmonics, there is very little variance. At the forced tidal frequency, the cross-shore component of the flow dominates and, across all frequencies, the horizontal flow speeds exceed vertical speeds.

Spectra computed after the onset of eddy motions are shown in dark lines. Compared to the earlier spectra, these spectra have significantly increased variance between the harmonic peaks and substantially more variance at low, sub-inertial frequencies. This is particularly true of the azimuthal spectra which has a broad maxima at frequencies about $f/10$.

At frequencies higher than tidal but within the internal wave band $1 < \omega/\sigma < 4.4$, the vertical and radial spectra for the later time period decay with frequency at a slope of approximately minus two. The corresponding azimuthal spectra falls off more rapidly with frequency.

Spectra of w taken from near-shore and interior locations at the same

Figure 12: Comparison of near-shore ($x/R = -0.93$) and interior ($x/R = -0.59$) vertical velocity spectra at $\theta = \pi$ and $z/H = 0.925$. Spectra are shown for the same time intervals as in figure 11.

height are shown in figure 12. Because the tidal forcing directly exerts only purely horizontal forcing, vertical motions in the interior pycnocline are a signature of radiation of internal waves from the near-shore boundaries. In comparison to the near-shore spectra, the interior spectra are reduced in strength by about two orders of magnitude in the internal wave band and about four orders of magnitude at sub-inertial frequencies.

6. Discussion

The interaction between nearly geostrophically ‘balanced’ motions, with slow time scales much longer than $1/f$, and ‘unbalanced’ inertia-gravity waves with fast time scales of order $1/f$ or smaller is a fundamental dynamical issue in both the atmosphere and the oceans. Often the interaction is cast in terms of the adjustment of a nearly balanced flow to or toward a balanced state via the radiation or spontaneous emission of fast inertia-gravity waves. Theories describing these interactions (quasi-geostrophic, semi-geostrophic etc) typically exploit a separation of time scales via the assumption of a small Rossby number U/fL , where U and L are the characteristic velocity

479 and length scales of the nearly balanced motions and thus L/U is the corre-
 480 sponding slow time scale. Often, but not always, these theories rely on the
 481 additional assumption of a separation in energies, with fast scale motions
 482 taken to be comparatively weak. Such low Rossby number flows have been
 483 shown to emit fast scale motions only weakly (Zeitlin, 2008).

484 Recent work *e.g.* (Zeitlin, 2008; Barkan et al., 2014) has examined the
 485 mechanisms by which rotating, stratified flows subject to steady large scale
 486 forcing can spontaneously develop localized features with sufficiently large
 487 Rossby numbers to bypass the theoretical constraints operative at small
 488 Rossby number and thereby generate fast, unbalanced motions more effi-
 489 ciently.

490 The problem discussed here is different in that the nearly balanced state
 491 is not the energy source for the fast, unbalanced wave motions. Rather,
 492 energy transferred from directly forced motions at the super-inertial tidal
 493 frequency drives the nearly balanced flow itself. The dynamics responsible
 494 for exciting and maintaining the slow, nearly balanced flow are fundamen-
 495 tally nonlinear and stem from the set-up of the pycnocline (figure 4) in the
 496 near-shore region of a shoaling basin. The slow flow takes the form of a
 497 cyclonic, near-shore, bottom-intensified jet (figure 5) that is prone to baro-
 498 clinic instability (figure 7). The resulting vortices have Rossby numbers of
 499 order one (figure 8) and are therefore nonlinear motions with time scales
 500 L/U of order $1/f$. Interactions among these eddies transfer energy to larger
 501 scales and lower frequencies as the eddies merge.

502 The presence of these eddies alters the internal wave field by sponta-
 503 neously emitting waves (figure 9) at frequencies comparable to the vorticity
 504 ζ and perhaps also by altering the background conditions within which non-
 505 linear wave-wave interactions take place. The spectra in the gravity wave
 506 band, while spiky with peaks at the tidal frequency and its harmonics, are
 507 nevertheless continuous in frequency once the eddies have appeared (figures
 508 11 and 12).

509 This problem is an example in which purely monochromatic forcing
 510 within the fast, internal wave frequency band produces a relatively rich
 511 flow with a nearly balanced along-shore jet, near-shore eddies and a broad-
 512 banded internal wave field. Compared to the trapped, non-propagating
 513 vortical motions near shore, the radiating inertia-gravity wave field in the
 514 interior is weak. Thus, in the absence of isolated topographic features,
 515 barotropic tidal forcing in a shoaling basin generates coastally amplified,
 516 energetic motions at tidal, sub-, and super-inertial frequencies, in addition
 517 to a much weaker internal wave field that radiates to the basin interior.

518 The bottom trapped cyclonic jet that develops in this simple experiment

519 is in many ways similar to the subsurface, poleward flows observed along
 520 eastern ocean boundaries (Warren, 1990; Wooster and Reid, 1963). Such
 521 flows are ubiquitous at mid-latitudes in the major ocean basins (Smith,
 522 1989), with perhaps the most comprehensive observations having been made
 523 off the west coast of North America (see *e.g.* Pierce et al 2000), where the
 524 undercurrent is located at approximately 200-275 *m* depth, 20-25 *km* off-
 525 shore of the shelf break and has a mean speed of about 10 *cm/s*. While
 526 a convincing dynamical explanation for these poleward undercurrents has
 527 not yet emerged, most explanations invoke an along-shore pressure gradient
 528 of unknown origin, perhaps linked to regional wind gradients and/or local
 529 topographic features such as capes (Mooers, 1989). Given the seasonal and
 530 geographical persistence of these currents, and the expected complexity and
 531 variability of along-shore pressure gradients set up by wind and topography,
 532 such explanations should probably be taken tentatively.

533 As an alternative hypothesis, the dynamics illustrated here are intriguing,
 534 in part because they are generic. They rely neither on the wind nor
 535 on local topographic features yet provide a simple mechanism for driving
 536 cyclonic subsurface currents at continental slopes that is dynamically consistent
 537 with observations. While the tendency of the cyclonic jet modeled
 538 here to undergo baroclinic instability is reminiscent of the squirts, jets and
 539 eddies that are ubiquitous in eastern oceans, observed eddies originating
 540 in the California Undercurrent are about twice as likely to be anticyclonic
 541 than cyclonic (Collins et al., 2013), with the bias perhaps explained by topographic
 542 triggering near specific Points and Capes or nearshore bottom stress
 543 (Molemaker et al., 2015). Nevertheless, the idealized experiment discussed
 544 here shows that forced, tidal motions of a stratified fluid at sloping boundaries
 545 has the intrinsic tendency to ‘set-up’ a subsurface thermocline, drive
 546 or enhance a near-bottom cyclonic jet as a consequence of geostrophy, and
 547 ultimately, to produce along-shore variability and cross-shore transport via
 548 baroclinic instability. Such flows are the result of energy transfer from the
 549 surface tide to low-frequency currents and eddies.

550 Acknowledgements:

551 This work was supported by the National Science Foundation, grant number
 552 OCE-1155121. XSEDE computing resources were made available under
 553 grant TG-OCE120004. Discussions with Falk Feddersen, Clint Winant, Jim
 554 Lerczak, Jennifer Thomas and Roy Barkan are gratefully acknowledged and
 555 appreciated.

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