Evaluation of the arbitrary Lagrangian-Eulerian vertical coordinate method in the MPAS-Ocean Model

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Abstract

The vertical coordinate of the Model for Prediction Across Scales-Ocean (MPAS-Ocean) uses the Arbitrary Lagrangian-Eulerian (ALE) method, which offers a variety of configurations. When fully Eulerian, the vertical coordinate is fixed like a z-level ocean model; when fully Lagrangian there is no vertical transport through the interfaces so that the mesh moves with the fluid; additional options for vertical coordinates exist between these two extremes, including z-star, z-tilde, sigma, and isopycnal coordinates. Here we evaluate spurious diapycnal mixing in MPAS-Ocean in several idealized test cases as well as real-world domains with full bathymetry. Mixing data is compared to several other ocean models, including the Parallel Ocean Program (POP) z-level and z-star formulations. In three-dimensional domains, MPAS-Ocean has lower spurious mixing than other ocean models. A series of simulations show that this is likely due to MPAS-Ocean's hexagon-type horizontal grid cells combined with a flux-corrected transport tracer advection scheme designed for these unstructured meshes.

The frequency-filtered vertical coordinate of Leclair and Madec (2011) (also called z-tilde) has been implemented and analyzed in MPAS-Ocean. This addition allows low-frequency vertical transport to pass through the vertical interface in an Eulerian manner, while high-frequency vertical oscillations, such as internal gravity waves, are treated in a Lagrangian manner.

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Z-tilde leads to a substantial reduction in vertical transport across layer interfaces, and a reduction in spurious diapycnal mixing.

Keywords: ocean modeling, diapycnal ocean mixing, Arbitrary Lagrangian-Eulerian coordinate method, unstructured mesh

1. Introduction

The choice of vertical coordinate is intrinsic to the character of a global ocean model. Historically, ocean models have been classified into groups according to their vertical coordinate. Z-level models have fixed, level layer interfaces (Griffies et al., 2005; Smith et al., 2010). Z-level models are straightforward to implement and have no pressure gradient errors due to tilted layers, but are prone to high spurious diapycnal mixing (Willebrand, 2001; Legg et al., 2006). Isopycnal models substantially reduce diapycnal mixing because adiabatic motion is treated entirely in a Lagrangian manner, thus producing no spurious mixing. Purely isopycnal models have insufficient resolution in the mixed layer and other neutrally-stratified regions (Bleck, 1978). Hybrid coordinate models combine the advantages of z-level in the upper ocean for higher resolution and isopycnal coordinates in the deep ocean to reduce diapycnal mixing (Bleck, 2002). Sigma coordinates are terrain-following so that the full vertical resolution may be used throughout, and are particularly useful for coastal modeling (Shchepetkin and McWilliams, 2005). Special care must be taken to avoid errors in the pressure gradient when layers have large slopes along steep topography in sigma coordinate models (Shchepetkin and McWilliams, 2003, 2011).

Amongst z-level models, several improvements have been introduced that are now standard in this class of models. Partial bottom cells improves the representation of bathymetry, and thus improves advection and mixing processes along the ocean floor (Adcroft et al., 1997; Pacanowski and Gnanadesikan, 1998). Scaling the vertical coordinate in proportion to the sea surface height (SSH), sometimes called z-star, reduces spurious mixing due to surface gravity waves and allows the upper layers to be very thin (Adcroft and Campin, 2004). Without this improvement, z-level models must include all SSH variations in the top layer, so it must be at least several meters thick, or use a rigid lid (Smith et al., 1992).

A new addition to improvements in z-level models is the frequency-filtered vertical coordinate of Leclair and Madec (2011), named z-tilde. This scheme

32 applies a time-frequency filter to the horizontal divergence in each grid cell.
33 High-frequency oscillations that occur on a time scale of less than several
34 days, such as internal gravity waves, are filtered out of the vertical transport,
35 so that the vertical grid moves with these waves in a Lagrangian fashion. The
36 remaining low-frequency motions, such as seasonal oscillations and advection
37 of large water masses, move through the vertical interfaces as they do in a
38 typical z-level model. Because much of the spurious mixing is caused by high-
39 frequency oscillations, this scheme offers the advantage of reduced diapycnal
40 mixing intrinsic to isopycnal models, but avoids the complexity of remapping
41 schemes, choosing target grids, and supporting massless layers associated
42 with hybrid-isopycnal models. The end result is a vertical coordinate that
43 may conveniently use all the machinery of a z-level model, such as topography
44 and initial conditions, but where the vertical grid may follow small, fast
45 oscillations in a Lagrangian manner to reduce spurious mixing.

46 In the Arbitrary Lagrangian-Eulerian (ALE) method, the grid is remapped
47 so that cell interfaces may remain at a fixed location (Eulerian), may move
48 with the fluid (Lagrangian), or anything in between (Hirt et al., 1974). It
49 may be configured for any of the vertical coordinate types discussed above.
50 ALE was developed to take advantage of the properties of Lagrangian algo-
51 rithms, where the model mesh naturally tracks the material’s properties, but
52 to avoid the resulting distortions in the computational domain (Donea et al.,
53 2004). ALE methods were applied to general vertical coordinates in ocean
54 models by Bleck (1978).

55 In a typical ALE algorithm, the mesh may be specified in an arbitrary
56 manner at regular intervals in time, and variables are subsequently remapped
57 to that new mesh. The Model for Prediction Across Scales-Ocean (MPAS-
58 Ocean) implementation determines a desired thickness at each time step
59 based on a number of criteria, and then computes the vertical transport
60 through the layer interface that is required to acquire the desired thickness.
61 Rather than explicitly remapping variables to the new grids, the ocean model
62 simply transports water volume and tracers using this vertical transport in
63 the normal prognostic conservation equations (the mass equation is written as
64 a volume or thickness equation for Boussinesq fluids). This is the Lagrangian
65 treatment of the vertical direction described in Adcroft and Hallberg (2006).
66 The method ensures that water volume and tracers are conserved to machine
67 precision.

68 Given the advantages discussed above, we have chosen an ALE frame-
69 work to model vertical transport within MPAS-Ocean. Vetted configurations

70 of MPAS-Ocean employing a vertical ALE treatment include z-level, z-star
71 (Ringler et al., 2013), idealized isopycnal (Ringler and Gent, 2011), as well
72 as z-tilde, partial bottom cells, and sigma coordinates (all in this paper).
73 All these have been tested in realistic high resolution global domains except
74 sigma and isopycnal coordinates, which have been tested in idealized domains
75 to date.

76 MPAS-Ocean is an open-source, global ocean model (see <http://mpas-dev.github.io>). MPAS-Ocean is one component of a family of climate models
77 within the MPAS framework, including atmosphere (Skamarock et al., 2012),
78 sea-ice, and land-ice models. MPAS components use horizontally unstruc-
79 tured meshes based on Voronoi tessellations, which allow high-resolution re-
80 gions within a low-resolution global mesh, with a smoothly varying mesh
81 transition between them. The motivation for this design is to provide a sin-
82 gle ocean modeling system that can be used with traditional quasi-uniform
83 meshes as well as regional configurations to investigate regional effects of cli-
84 mate change. All simulations presented here use MPAS-Ocean version 2.0,
85 released in November 2013.
86

87 The purpose of this paper is two-fold: (1) to verify the behavior of MPAS-
88 Ocean against published results for numerous idealized test cases and real-
89 world domains; and (2) to demonstrate the advantages of z-tilde in reduc-
90 ing spurious vertical mixing. Ilicak et al. (2012) proposed four idealized
91 test cases that are repeated here, and provided comparison data for Modu-
92 lar Ocean Model (MOM), MIT General Circulation Model (MITgcm), Re-
93 gional Ocean Modeling System (ROMS), and the Generalized Ocean Layered
94 Model (GOLD). To that we have added data from MPAS-Ocean run in z-
95 level, z-star, isopycnal, sigma, and z-tilde coordinates, as well as POP, a
96 quadrilateral-grid ocean model developed at Los Alamos National Labora-
97 tory, using z-level and z-star coordinates. The test cases include the (1) lock
98 exchange, (2) overflow, (3) internal wave, (4) baroclinic eddies, and (5) real
99 world. These cases progress from simple domains and initial conditions to
100 more complex dynamics and topography, so that spurious mixing may be
101 measured under a variety of conditions.

102 The design of the MPAS unstructured horizontal mesh has already been
103 described in several publications (Ringler et al., 2008; Thuburn et al., 2009;
104 Ringler et al., 2010), while high-resolution and variable-resolution global sim-
105 ulations were verified against observations in Ringler et al. (2013). The cur-
106 rent work may be considered a follow-on to Ringler et al. (2013), where the
107 prior is a detailed vetting of the horizontal infrastructure of MPAS-Ocean,

108 while the current does the same for the vertical.

109 This paper is organized as follows. Section 2 details the algorithms of the
 110 vertical grid, with subsections on the overall ALE formulation and z-tilde.
 111 Section 3 presents the results from four idealized test cases, while section
 112 4 proceeds to realistic simulations of global oceans with full bathymetry.
 113 Finally, the work is summarized in the concluding Section 5.

114 2. Numerical Algorithm

115 This section describes the equations and discretization of the MPAS-
 116 Ocean vertical coordinate. All horizontal variables and operations are con-
 117 tinuous to simplify the presentation. The horizontal discretization is dis-
 118 cussed in Sections 2, 3.1, and A.4 of Ringler et al. (2013), and further details
 119 may be found in the MPAS-Ocean User’s Guide (Jacobsen et al., 2013). All
 120 functionality described here is available in MPAS-Ocean version 2.0.

121 The MPAS-Ocean governing equations are for a hydrostatic, incompress-
 122 ible fluid using the Boussinesq approximation. Conservation of momentum,
 123 volume, and tracers are written for layer k as

$$\begin{aligned} \frac{\partial u_k}{\partial t} + q_k h_k u_k^\perp + \overline{[w^t \delta z^t(u)]}_k^m \\ = -\frac{1}{\rho_0} \nabla p_k - \frac{\rho_k g}{\rho_0} \nabla z_k^{mid} - \nabla K_k + [\mathbf{D}_h^u]_k + [\mathbf{D}_v^u]_k + \mathcal{F}_k^u \end{aligned} \quad (1)$$

$$\frac{\partial h_k}{\partial t} + \nabla \cdot (h_k \mathbf{u}_k) + w_k^t - w_{k+1}^t = 0, \quad (2)$$

$$\frac{\partial (h_k \varphi_k)}{\partial t} + \nabla \cdot (h_k \mathbf{u}_k \varphi_k) + \overline{\varphi}_k^t w_k^t - \overline{\varphi}_{k+1}^t w_{k+1}^t = [\mathbf{D}_h^\phi]_k + [\mathbf{D}_v^\phi]_k + \mathcal{F}_k^\phi \quad (3)$$

124 where all symbols are defined in Tables 1 and 2. Additional equations diag-
 125 nose the pressure from hydrostatic balance and the density from an equation
 126 of state, as described in section A.1 of Ringler et al. (2013). Overbars in-
 127 dicate that variables are vertically averaged to the top (t) or middle (m) of
 128 the layer, and δz^t is a first-order derivative in the vertical. The term ∇z_k^{mid}
 129 does not appear in z-level models, but is required to compensate for pressure
 130 gradients due to the tilting of layers (Adcroft and Campin, 2004, Appendix
 131 A).

132 2.1. ALE Algorithm

133 The ALE algorithm in the MPAS-Ocean code provides the vertical trans-
 134 port of fluid through the top cell interface, w^t , which is then used in the

Table 1: Latin variables used in prognostic equation set. All variables are located at the center of the layer in the vertical unless noted.

symbol	name	notes
$\mathbf{D}_h^u, \mathbf{D}_v^u$	mom. diffusion terms	h horizontal, v vertical
D_h^φ, D_v^φ	tracer diffusion terms	
f	Coriolis parameter	
$\mathcal{F}^u$	momentum forcing	
$\mathcal{F}^\varphi$	tracer forcing	
g	grav. acceleration	constant
h	layer thickness	
$\mathbf{k}$	vertical unit vector	
K	kinetic energy	$K = \mathbf{u} ^2 / 2$
p	pressure	
q	potential vorticity	$q = \eta / h$
S	salinity	a tracer φ
t	time	-
u	horizontal velocity	normal component to edge
$u^\perp$	horizontal velocity	tangential component to edge
$\mathbf{u}$	horizontal velocity	-
w^t	vertical transport	top of layer in vertical
z	vertical coordinate	positive upward
z^{mid}	z-location of middle of layer	

time-integration of the thickness, momentum and tracer equations. The ALE vertical grid moves in time; the cell interface locations may be determined by summing layer thicknesses h_k from the bottom bathymetry up.

A summary of the ALE algorithm is as follows, with variables defined in Table 4. This is executed within each time step, where all variables at the previous time step, n , are known.

1. Compute desired thickness for the new time,

$$h_k^{ALE} = h_k^{rest} + h_k^{SSH} + h_k^{hf} + h_k^{limit} \quad (4)$$

2. Solve for vertical transport w^t from (2),

$$w_k^t = w_{k+1}^t - D_k - \frac{h_k^{ALE} - h_k^n}{\Delta t} \quad (5)$$

Table 2: Greek variables used in prognostic equation set. All variables are located at the center of the layer in the vertical.

symbol	name	notes
ω	relative vorticity	$\omega = \mathbf{k} \cdot (\nabla \times \mathbf{u})$
η	absolute vorticity	$\eta = \omega + f$
Θ	potential temperature	a tracer φ
κ_h, κ_v	tracer diffusion	horizontal and vertical
ν_h, ν_v	viscosity	horizontal and vertical
ρ	density	
ρ_0	reference density	constant
φ	generic tracer	e.g. Θ, S

143 3. Solve for new thickness, h_k^{n+1} , using the continuity equation (2) within
144 the time integration routine.

145 Step 1 is isolated from 2 and 3 so that the vertical transport calculation
146 (step 2) and its application (step 3) is independent of the “prescription” for
147 determining the target layer thickness (step 1). Thus, new algorithms for de-
148 termining layer position can be readily implemented and tested. In addition,
149 this partitioning insures that w_k^t is used consistently in the time-stepping
150 algorithm for both volume and tracer evolution (3). This is the Lagrangian
151 treatment of the vertical direction described in Adcroft and Hallberg (2006),
152 where layer thickness is prognostic and sea surface height is diagnostic. For
153 fully Lagrangian layers, the vertical transport is set to zero in step 2. This
154 setting is used for isopycnal simulations.

155 The desired ALE thickness in step 1 includes contributions from four
156 terms (4):

- 157 1. **Resting thickness**, h^{rest} , is the layer thickness when the ocean is at
158 rest, i.e. without SSH or internal perturbations. For z-type coordinates,
159 the resting thickness is constant in each horizontal layer, and cells below
160 bathymetry are unused. For sigma coordinates, all layers are used, and
161 h^{rest} varies horizontally in proportion to the column’s total depth.
- 162 2. **SSH alteration**, h^{SSH} , alters layer thicknesses so that changes are in
163 proportion to the sea surface height (SSH),

$$h_k^{SSH} = \zeta \frac{W_k h_k^{rest}}{\sum_{k'=1}^{kmax} W_{k'} h_{k'}^{rest}}. \quad (6)$$

- 164 The weights W_k determine how SSH oscillations are distributed amongst
 165 the layers, as described in Table 3.
- 166 3. **High-frequency thickness**, h^{hf} , allows high-frequency thickness os-
 167 cillations, such as internal gravity waves, to be treated in a Lagrangian
 168 manner. This is the “z-tilde” scheme of Leclair and Madec (2011) de-
 169 scribed in the next section.
- 170 4. **Minimum/maximum thickness alteration**, h^{limit} , is the change in
 171 thickness required to enforce the minimum and maximum thickness
 172 in each cell. For example, when a cell is too thin, h^{limit} is positive,
 173 while nearby cells in the vertical incur a corresponding negative h^{limit}
 174 to conserve volume in the column.

175 Of the four terms, resting thickness is always positive, while the others are
 176 small alterations about zero. Summing (4) over a column,

$$\begin{aligned}
 \sum_{k=1}^{kmax} h_k^{ALE} &= \sum_{k=1}^{kmax} h_k^{rest} + \sum_{k=1}^{kmax} h_k^{SSH} + \sum_{k=1}^{kmax} h_k^{hf} + \sum_{k=1}^{kmax} h_k^{limit} \\
 &= H + \zeta + 0 + 0.
 \end{aligned} \tag{7}$$

177 Thus the first two terms are always included so that the column thickness
 178 sums to $H + \zeta$, while the second two terms are optional.

179 The MPAS-Ocean ALE algorithm broadens the choices in vertical co-
 180 ordinate settings from traditional vertical coordinate names. For example,
 181 the choice of weights in (6) may place all SSH oscillations in the top layer
 182 only ($W_1 = 1, W_k = 0$ otherwise), or distribute SSH oscillations amongst
 183 all layers, in proportion to the resting thickness ($W_k = 1$ for all k). If the
 184 initial layer thicknesses are constant in the horizontal, these settings corre-
 185 spond to z-level and z-star vertical coordinates. A third option is weighted
 186 z-star, with user-specified weights for W_k (Table 3), where one could spec-
 187 ify $W_k = 1$ in the upper layers and taper W_k off to zero at depth, so that
 188 SSH oscillations primarily affect the mixed layer. The advantage of z-star
 189 and weighted z-star is that layers may be much thinner than the maximum
 190 SSH, which is typically 1–2 m. The high-frequency thickness (z-tilde) and
 191 minimum/maximum thickness alteration may be applied *in addition* to the
 192 other settings. Simulations presented as z-tilde in this paper use SSH alter-
 193 ation weights of $W_k = 1$ for all layers, so it is effectively z-tilde applied to
 194 a z-star vertical coordinate. Partial bottom cells (PBCs) may also be used
 195 independent of the other vertical coordinate settings. The overflow test case
 196 was conducted with and without PBCs, and all global simulations use PBCs.

Table 3: Vertical coordinate settings for traditional names. The flag column is the MPAS-Ocean setting for the configuration flag `config_vert_coord_movement`

name	weights	flag
z-level	$W_k = \begin{cases} 1 & k = 1 \\ 0 & k > 1 \end{cases}$	'fixed'
z-star	$W_k = 1 \quad \forall \quad k$	'uniform_stretching'
weighted z-star	any W_k	'user_specified'

Table 4: Variables used in ALE equation sets. A subscript k indicates the layer index. The ∇ indicates a horizontal gradient within a single layer.

symbol	name	notes
D	thickness-weighted divergence	see (8)
$\bar{D}$	barotropic divergence	see (9)
D'	baroclinic divergence	$D'_k = D_k - \bar{D}$
D^{lf}	low-frequency divergence	see (12)
D^{hf}	high-frequency divergence	$D_k^{hf} = D'_k - D_k^{lf}$
h^{ALE}	desired ALE thickness	see (4)
h^{rest}	resting thickness	
h^{SSH}	SSH thickness alteration	see (6)
h^{hf}	high-freq. thickness alteration	see (13)
h^{limit}	min/max thickness alteration	
H	total resting thickness	$H = \sum_{k=1}^{kmax} h_k^{rest}$
$kmax$	index of deepest water cell	
W	SSH thickness weights	
τ_{Dlf}	frequency filter time scale	constant
τ_{hhf}	restoring time scale for h^{hf}	constant
κ_{hhf}	h^{hf} diffusion	constant
ζ	sea surface height	$\zeta = \sum_{k=1}^{kmax} h_k - H$

197 2.2. High-frequency thickness algorithm (z -tilde)

198 The high-frequency thickness alteration, h^{hf} , in (4) allows layer thickness
199 to oscillate so that high-frequency motions, such as internal gravity waves, are
200 treated in a Lagrangian manner. Low-frequency motions, such as seasonal

201 changes or slow motions of water masses, are treated in an Eulerian manner.
 202 This is the “z-tilde” scheme of Leclair and Madec (2011), which generally
 203 reduces spurious vertical mixing and better preserves water mass properties.
 204 To begin, consider the horizontal divergence in each cell,

$$D_k = \nabla \cdot (h_k \mathbf{u}_k). \quad (8)$$

205 In each column the barotropic divergence is the vertical sum,

$$\overline{D} = \frac{\sum_{k=1}^{kmax} h_k D_k}{\sum_{k=1}^{kmax} h_k}, \quad (9)$$

206 so that the divergence in a cell may be written as the sum of the barotropic
 207 part and the remaining baroclinic part,

$$D_k = \overline{D} + D'_k. \quad (10)$$

208 Furthermore, the baroclinic divergence is split between a low-frequency bar-
 209 clinic divergence and a high-frequency baroclinic divergence,

$$D'_k = D_k^{lf} + D_k^{hf}. \quad (11)$$

210 Two additional prognostic equations are solved in this configuration,

$$\frac{\partial D_k^{lf}}{\partial t} = -\frac{2\pi}{\tau_{Dlf}} (D_k^{lf} - D'_k), \quad (12)$$

$$\frac{\partial h_k^{hf}}{\partial t} = -D_k^{hf} - \frac{2\pi}{\tau_{hhf}} h_k^{hf} + \nabla_h \cdot (\kappa_{hhf} \nabla_h h_k^{hf}), \quad (13)$$

211 where the first is a simple low-pass filter for the baroclinic divergence and
 212 the second is a prognostic equation for the high-frequency thickness, and
 213 variables are defined in Table 4. The filter time scale τ_{Dlf} is typically set
 214 to five days, so that internal gravity waves and other short time scale adia-
 215 batic phenomena are included in the high-frequency divergence, D^{hf} . This
 216 is computed from the low-frequency divergence D^{lf} with the simple diag-
 217 nostic equations (8–11). The high-frequency thickness equation is forced by
 218 the high-frequency divergence, and includes a restoring term and horizontal
 219 diffusion term to avoid long-term drift of h^{hf} from zero. The restoring time

scale, τ_{hhf} , is typically set to 30 days, and the horizontal diffusion is zero for the simulations in this paper. However, these two terms are ad-hoc methods to ensure reasonable vertical grids, and may need to be adjusted for other domains.

The enforcement of a minimum and maximum layer thickness in (4) is an addition to the algorithm presented in Leclair and Madec (2011) that was needed to run test cases with large initial perturbations. In those situations, a high frequency divergence would include the evacuation of all mass from some cells. This can be avoided with the simple minimum thickness requirement. In realistic global simulations starting from climatological initial conditions, the minimum and maximum were often not required, but are still a useful safeguard against very thin layers when the dynamics occur faster than the restoring time scale. An optional maximum layer thickness is included to prevent severely distorted layers, and was required to run z-tilde in domains with strongly transient flows.

2.3. Resting Potential Energy

The primary metric used in this study to diagnose spurious mixing is the resting potential energy (RPE), where the potential density $\rho(x, y, z)$ is sorted and redistributed in the model domain (Ilicak et al., 2012). The sorted density, $\rho^*(z)$, is constant horizontally and monotonically increasing with depth. The RPE is

$$RPE = g \int_{\Omega} z \rho^*(z) dV, \quad (14)$$

where Ω is the domain. An equally valid interpretation of RPE is a measure of the fluid’s vertical center of mass. Vertical mixing results in a monotonically-increasing center of mass with time. Thus, when surface buoyancy forcing is absent, as is the case in the simulations presented below, the time-rate-of change of RPE is a measure of vertical mixing. A visual example is that RPE increases as more cells are in the mid-range of the color bars in Figures 1 and 4.

An important limitation of this method is that it is only valid for cases with zero buoyancy fluxes. Thus all cases presented here are spin-down simulations from an initial condition or, for real-world cases, from an equilibrated condition. The RPE method cannot be used to diagnose mixing in typical realistic simulations that include surface forcing.

253 The algorithm for computing RPE for discrete fields is as follows. For each
 254 cell in the domain, there is an associated potential density $\rho(i, k)$ and volume
 255 $V(i, k)$ where i and k are horizontal and vertical indices. After sorting, the
 256 new density $\rho^*(j)$ is monotonically increasing, and the volume $V^*(j)$ must
 257 be rearranged in the same way, so that each ρ^* corresponds to the same cell
 258 volume as in the original variables (note V^* is not sorted to be monotonically
 259 increasing). Each cell is now given a new mid-depth $z^*(j)$, horizontal area
 260 $A^*(j)$, and thickness $h^*(j) = A^*(j)/V^*(j)$ by piling thin cells in a stack
 261 from the top to the bottom of the domain. The horizontal area is the cross-
 262 sectional area of the domain at a depth of z^* . The RPE is then computed
 263 as

$$RPE = g \sum_j z^*(j) \rho^*(j) V^*(j), \quad (15)$$

264 which has units of Joules. The initial value RPE of a particular simulation
 265 depends on the domain and initial conditions, and so is not relevant to the
 266 mixing. Thus plots of RPE use values normalized to time zero,

$$\frac{RPE(t) - RPE(0)}{RPE(0)}. \quad (16)$$

267 This same argument holds for the time derivative of RPE, but the non-
 268 normalized value is used to compare to data published in Ilicak et al. (2012).
 269 Plots showing $dRPE/dt$ use units of W m^{-2} , and are normalized only by the
 270 surface area of the domain.

271 Real-world cases use the nonlinear equation of state by Jackett and Mc-
 272 Dougall (1995), where density is a function pressure as well as temperature
 273 and salinity. Thus potential density was used for ρ^* in (15), following the
 274 ρ_{2000} method in Ilicak et al. (2012) [Appendix A]. (We used potential density
 275 referenced to the surface, but this is a minor difference.) This method of
 276 RPE calculation does not account for changes in potential density due to
 277 pressure after the potential density is resorted to a new depth. That would
 278 require an expensive iterative algorithm, which has been shown to change
 279 RPE by less than 1% (Ilicak et al., 2012, Appendix A). See Saenz et al.
 280 (2014) for an alternative efficient method that accounts for pressure changes
 281 when resorting potential density. An additional issue with real-world cases is
 282 that our RPE computation allows water parcels to be sorted throughout the
 283 world's oceans without constraints due to the bathymetry of ocean basins.
 284 Sorting density separately within distinct basins would change the value of

285 RPE, but would not alter relative mixing rates when comparing simulations
 286 using the same domain.

287 For real-world simulations the mean density and depth must be sub-
 288 tracted before computing the RPE (Ilicak et al., 2012, Appendix A). Define
 289 $V = \int_{\Omega} dV$, $\bar{\rho} = \frac{1}{V} \int_{\Omega} \rho dV$, and $\bar{z} = \frac{1}{V} \int_{\Omega} z dV$, so that the perturbation
 290 variables are $\rho' = \rho - \bar{\rho}$ and $z' = z - \bar{z}$. Then (15) is revised to be

$$RPE = g \sum_j (z^*)'(j) (\rho^*)'(j) V^*(j). \quad (17)$$

291 Note that the order of operations of sorting and subtracting the mean are
 292 interchangeable.

293 Idealized test cases use a harmonic operator for horizontal momentum
 294 diffusion of the form $\nu_h \nabla^2$. In this case, RPE results are shown as a function
 295 of the grid Reynolds number,

$$Re_{\Delta} = \frac{U \Delta x}{\nu_h}, \quad (18)$$

296 where U is the characteristic velocity scale, Δx is the horizontal mesh spacing,
 297 and ν_h is the specified horizontal viscosity with units of m²/s (Bryan et al.,
 298 1975; Griffies, 2004; Ilicak et al., 2012). For the lock exchange and overflow
 299 test cases, the velocity scale is the theoretical wave speed in a rectangular
 300 channel; for the internal wave and baroclinic eddies test cases, it is computed
 301 from the root-mean-squared velocity computed over the domain.

302 For real-world simulations, a horizontal biharmonic operator is used, with
 303 coefficient ν_{4h} and units m⁴/s. Then the grid Reynolds number is

$$Re_{\Delta} = \frac{U(\Delta x)^3}{8\nu_{4h}}, \quad (19)$$

304 where the velocity scale U is computed from the globally averaged kinetic
 305 energy (Griffies and Hallberg, 2000).

306 3. Idealized test cases

307 In this section we evaluate vertical mixing in MPAS-Ocean using four
 308 idealized test cases introduced in Ilicak et al. (2012), and make direct com-
 309 parisons to other ocean models under identical conditions whenever the data
 310 is available. Parameters that vary amongst the four idealized test cases are

311 listed in Table 5. The remainder of this section describes settings that are
 312 the same in all four idealized test cases.

313 The tracer advection scheme, which is evaluated in detail in Section 3.6, is
 314 monotonic flux-corrected transport (Skamarock and Gassmann, 2011), with
 315 third-order reconstruction of the tracer values at the cell edges. The time-
 316 stepping algorithm is split-explicit, with the long baroclinic time step (Δt)
 317 given in Table 5. The two-dimensional barotropic stage uses 40 subcycles to
 318 step through $2\Delta t$, as described in Appendix A5 of Ringler et al. (2013). The
 319 horizontal turbulence closure for these cases is

$$\mathbf{D}_h^u = \nu_h \nabla^2 \mathbf{u} \quad (20)$$

320 for momentum, with a constant viscosity ν_h , and $\mathbf{D}_h^\phi = 0$ for tracers. There is
 321 no surface forcing by winds, tracer fluxes, or tracer restoring. Further details
 322 of how these settings are applied in MPAS-Ocean can be found in Jacobsen
 323 et al. (2013).

324 The idealized domains are in (x, y, z) Cartesian space, and individual
 325 cells are regularly spaced hexagons. A handful of tests were conducted using
 326 quadrilateral cells as well, with nearly identical results. The lock exchange,
 327 overflow, and internal wave test cases (1–3) are effectively two-dimensional in
 328 (x, z) , but simulations are three-dimensional and use four cells with periodic
 329 boundaries in the y -direction. For this study, a single resolution was sufficient
 330 for each of cases 1–3, and three horizontal resolutions were used for the
 331 baroclinic eddies, case 4 (see grid cell size in Table 5). This is a reduction
 332 from the number of horizontal and vertical resolutions presented in Ilıcak
 333 et al. (2012). The domains are partitioned into blocks to run on multiple
 334 processors, ranging in number from eight for the lock exchange to several
 335 hundred for the 1 km baroclinic eddies case.

336 All four idealized test cases use a linear equation of state of the form

$$\rho(\Theta, S) = \rho_{ref} - \alpha(\Theta - \Theta_{ref}) + \beta(S - S_{ref}) \quad (21)$$

337 where $\rho_{ref} = 1000 \text{ kg m}^{-3}$, $\alpha = 0.2 \text{ kg m}^{-3} \text{ C}^{-1}$, and $\Theta_{ref} = 5 \text{ C}$. The
 338 initial salinity is set uniformly as $S = S_{ref}$ so that density depends only
 339 on temperature. For a linear equation of state there is no explicit depth or
 340 pressure dependence, so potential density is identical to density. All cases
 341 begin with zero velocity.

Table 5: Parameter settings for idealized test cases. Isopycnal and z-type initial conditions are specified by ρ -layer and z-layer, respectively. RPE avg is the time over which $dRPE/dt$ was averaged.

	case 1	case 2	case 3	case 4
	lock exchange	overflow	internal wave	baroclinic eddies
domain size x	64 km	200 km	250 km	160 km
domain size y	-	-	-	500 km
domain size z	20 m	2000 m	500 m	1000 m
grid cell size	0.5 km	1 km	5 km	1, 4, 10 km
number ρ -layers	2	2	20	-
number z-layers	20	100	20	20
z-layer thickness	1 m	20 m	25 m	50 m
time step Δt , s	30–120	5–20	300	30, 120, 300
ν_h , m^2/s	0.01, 0.1, 1, 10, 100, 200	0.01, 0.1, 1, 10, 100, 1000	0.01, 1 15, 150	1, 5, 10, 20, 200
ν_v , m^2/s	0.0001	0.0001	0.0001	0.0001
κ_h , m^2/s	0	0	0	0
κ_v , m^2/s	0	0	0	0
bottom drag	-	-	-	0.01
Coriolis, s^{-1}	-	-	-	-1.2×10^{-4}
run duration	17 hrs	40 hrs	200 days	320 days
RPE avg	17 hrs	13–37 hrs	10–100 days	1–320 days
Re_Δ velocity	u_f	u_f	U_c	U_c

3.1. Lock exchange test case

The lock exchange case provides a way to measure mixing in the simplest possible configuration. The initial condition,

$$\Theta(x, z) = \begin{cases} 5 & x < 32 \text{ km} \\ 30 & x \geq 32 \text{ km} \end{cases} \quad (22)$$

may be thought of as two basins of water that are allowed to interact at time zero. Warm water flows over the cold water from right to left, and vice versa. Simulations were performed for a range of horizontal viscosities, while the vertical viscosity is kept constant and explicit tracer diffusion is zero (Table 5). Higher viscosities generate a smooth, tilted interface between the warm and cold fluid, and produce less mixing, as quantified by a lower

351 time-rate-of-change in RPE. Simulations with lower viscosities are more en-
 352 energetic, have a higher grid Reynolds number, and produce more oscillations
 353 in the temperature field (Figures 1 and 2). The stability condition to pre-
 354 vent spurious numerical modes and unphysical extrema is $Re_{\Delta} < 2$ (Griffies,
 355 2004). Comparing grid Reynolds numbers of 2.5 and 250 in Figure 1 shows
 356 that the later contains grid-scale noise, and is not well-resolved. The time
 357 step of the high viscosity runs were limited by the viscous CFL condition;
 358 a time step of 120 seconds was used for lower viscosities, but was decreased
 359 to 60 and 30 seconds at $\nu_h = 100$ and $200 \text{ m}^2\text{s}^{-1}$, respectively. All models
 360 considered here are hydrostatic, so the vertical momentum equation lacks an
 361 acceleration term. Non-hydrostatic simulations would include the generation
 362 of Kelvin-Helmholtz billows (Fringer et al., 2006).

363 Because all diffusion has been turned off, the correct solution is where no
 364 mixing occurs, and the front propagates with no intermediate temperatures
 365 between 5C and 30C. This is demonstrated by MPAS-Ocean in isopycnal
 366 mode and the GOLD simulations in Ilicak et al. (2012), where the RPE
 367 remains constant. The MPAS-Ocean isopycnal simulations used only two
 368 layers, with temperatures of 5C and 30C. MPAS-Ocean does not support
 369 massless layers, so the initial thickness is 1 cm for the warmer top layer in the
 370 left half of the domain, and 19.99 m in the right half, while the cold bottom
 371 layer is opposite. As the isopycnal simulations proceed, the thin layers fill at
 372 the wave front (Figure 1). There is zero vertical advection between the two
 373 layers, so the temperature of each layer remains constant in time. The shape
 374 of the layer interface varies with grid Reynold’s number, where the interface
 375 changes from tilted to flat as the grid Reynold’s number increases.

376 The amount of mixing by MPAS-Ocean in z-star mode is less than MIT-
 377 gcm and MOM, but more than ROMS (Figure 2a). The ROMS low value of
 378 *RPE* is likely due to its third-order, upstream-biased, dissipative advection
 379 scheme for momentum (Shchepetkin and McWilliams, 2005), which adds lo-
 380 cal dissipation to enforce stability. The time derivative of RPE at 17 hours
 381 is lower for MPAS-Ocean than for MITgcm over the range of viscosities
 382 tested (Figure 2c). Results for z-level and z-star vertical coordinate settings
 383 in MPAS-Ocean are nearly identical, as one would expect for an internally
 384 propagating wave.

385 Using Bernoulli’s equation for an ideal fluid, the speed of a gravity current
 386 in a rectangular channel is

$$u_f = \frac{1}{2} \sqrt{gH(\delta\rho/\rho_0)} \quad (23)$$

387 (Benjamin, 1968), which is 0.496 m/s for this test problem. This predicts
388 that the right front location is at 62.4 m at 17 hours. The front in MPAS-
389 Ocean simulations is at 60 m and 62 m for the $\nu_h = 1 \text{ m}^2/\text{s}$ and $\nu_h = 100$
390 m^2/s , respectively (Figure 1), which is very close to the front location in
391 MITgcm and MOM simulations (Ilicak et al., 2012, Figure 5). The wave front
392 propagates slightly faster in isopycnal models. The theoretical wave speed
393 u_f is used to compute the grid Reynolds number in (18). The root-mean-
394 squared velocity, computed over the domain, begins at zero and increases to
395 0.5 m/s at 16 hours, so is in good agreement with u_f .

396 Overall, the lock exchange allows modelers to test mixing and advection
397 without the complications of topography, Coriolis forces, bottom drag, and
398 any surface forcing. The remaining test cases add these in one by one, so
399 that the model developer may gain confidence through step-wise validation
400 exercises. Mixing rates are highly sensitive to the choice of advection scheme,
401 as will be shown in Section 3.6.

402 3.2. Appropriate use of z -tilde

403 The z -tilde scheme was tested in the lock exchange with both the z -
404 level and isopycnal initial conditions described in the previous section. The
405 results depend on the frequency filter time scale, τ_{Dlf} (Figure 3). The results
406 for z -level initial conditions (20 one-meter-thick layers) are as follows. When
407 $\tau_{Dlf} < 1$ hour, the wave dynamic, which is moving at the time scale of several
408 hours, is filtered out of the high-frequency thickness, and the grid remains
409 at its initial Eulerian position. In this case, $dRPE/dt$ values converge to
410 the z -level values as τ_{Dlf} goes to zero. At the other extreme, when $\tau_{Dlf} >$
411 12 hours, the wave is included in the high frequency thickness, and the grid
412 behaves in a Lagrangian manner: as the wave proceeds from the center of
413 the domain, layers behind the wave fill to the maximum layer thickness,
414 and those in the remainder of the column evacuate to the minimum layer
415 thickness (typically set to 10m and 0.1m, respectively). In this case, the
416 grid becomes very distorted with oscillations, and produces higher $dRPE/dt$
417 values than the original z -level simulations. When τ_{Dlf} is between 1 and 12
418 hours, the grid moves in a weakly Lagrangian manner, and never reaches the
419 minimum or maximum thickness. In this case, the grid is less distorted, and
420 there is some reduction in mixing.

421 When the initial conditions are isopycnal (i.e. two layers, each with con-
422 stant temperature) $dRPE/dt$ values can be made arbitrarily small by increas-
423 ing τ_{Dlf} , until it converges to a purely Lagrangian grid where $dRPE/dt = 0$.

Figure 1: Lock exchange test case: Temperature, degrees C, at 17 hours for MPAS-Ocean using isopycnal (a, b) and z-star vertical coordinates (c, d). The horizontal viscosity is $\nu_h = 1.0$ (a, c) and $\nu_h = 100$ (b, d), corresponding to grid Reynolds numbers of $Re_\Delta = 250$ and $Re_\Delta = 2.5$. In (b), the wave front has just reached the edge of the domain at 17 hours.

424 When $\tau_{Dlf} < 100$ days, the grid moves move slowly, and the thick (originally
 425 isopycnal) layers on the left and right experience a great deal of mixing.

426 The application of z-tilde to the lock exchange shows that z-tilde is not
 427 appropriate for test cases with strongly transient flows. When the grid follows
 428 fast dynamics in a Lagrangian manner, those dynamics must be small per-
 429 turbations or the grid will be strongly distorted, resulting in similar or higher
 430 mixing rates compared to an Eulerian grid. Adding high-frequency thickness
 431 diffusion (κ_{hhf}) improves the smoothness of the grid, but does not substan-
 432 tially reduce the mixing. The only way to have the z-tilde scheme converge
 433 to zero mixing in the Lagrangian limit of high τ_{Dlf} is to use isopycnal-layer
 434 initial conditions. Then z-tilde reduces to an isopycnal-layer model, but this
 435 defeats the purpose of z-tilde, which is meant to reduce mixing for small, fast
 436 perturbations within z-level-type vertical coordinates.

437 3.3. Overflow test case

438 The overflow test case is similar to the lock exchange, but includes a
 439 drop in topography that induces vertical advection and mixing. The bottom

440 depth, shown in Figure 4, is

$$d(x) = d_1 + \frac{1}{2} (d_2 - d_1) \left(1 + \tanh \left(\frac{x - x_0}{\sigma} \right) \right) \quad (24)$$

441 with $d_1 = 500$ m, $d_2 = 2000$ m, $x_0 = 40$ km, and $\sigma = 7$ km. The initial
442 temperature is

$$\Theta(x, z) = \begin{cases} 10 & x < 20 \text{ km} \\ 20 & x \geq 20 \text{ km} \end{cases} \quad (25)$$

443 As the wave descends the steep topography, mixing depends on the hori-
444 zontal viscosity, the advection scheme, the vertical coordinate, and the treat-
445 ment of the bottom topography. Like the lock exchange, low horizontal
446 viscosity results in higher kinetic energy and higher Reynolds number. At
447 low viscosity the interface between warm and cold water has more oscilla-
448 tions (Figure 4). The higher kinetic energy induces more mixing, resulting in
449 higher RPE at higher Reynolds numbers (Figure 5). Z-level and z-star verti-
450 cal coordinates produce nearly identical results. The overflow was also tested
451 in isopycnal and sigma coordinates, but not z-tilde, for the same reasons as
452 the lock exchange (Section 3.2).

453 Simulations with MPAS-Ocean isopycnal coordinates use two layers, each
454 with a uniform temperature of 10C and 20C. The inactive layer is initialized
455 with a layer thickness of 1 cm for high viscosities, and 1 m for low viscosities.
456 These layers are allowed to activate when the average thickness of a cell and
457 its neighbor is greater than 2 cm and 2 m, respectively. This check prevents
458 the inactive layer from descending the slope prematurely. In addition, a
459 minimum layer thickness of 1 cm to 1 m is enforced once the layer is activated.
460 Smaller timesteps, in some cases down to one second, were required to prevent
461 negative layer thickness just behind the head. A bottom drag coefficient of
462 0.01 was required for the isopycnal simulations to run stably.

463 In order to show the versatility of the MPAS-Ocean ALE vertical coordi-
464 nate, we include terrain-following sigma-coordinate simulations of the over-
465 flow test case (Figure 4i-l). The other test cases are flat-bottom, so sigma and
466 z-star would be identical. When the overflow test case is configured with a
467 sigma coordinate, the 100 layers are uniformly distributed between the ocean
468 surface and ocean bottom. This produces layer thicknesses that are 5 m over
469 the shelf and 20 m in the full-depth region. These sigma coordinate tests
470 include the pressure gradient correction due to tilted coordinate surfaces—
471 this is the ∇z_k^{mid} term in (1), computed as in Shchepetkin and McWilliams

(2003, Section 2, method 4). Weighted averaging of the density and vertical coordinate (Song, 1998; Shchepetkin and McWilliams, 2003, Section 3) is not included. Errors in sigma coordinate models due to the compressibility of sea water (Shchepetkin and McWilliams, 2003, Section 7) are not relevant for this test case, which uses a linear equation of state.

The flow begins as a wave front in a rectangular channel, with a theoretical velocity of $u_f = 1.57$ m/s from (23) using $H = 500$ m for the upper shelf. This places the wave front at 37 km at 3 hours, which is near the simulated wave front in all the models, as shown in Figure 4 and Ilicak et al. (2012, Figure 9). By six hours, the wave front has reached the ridge bottom, but locations vary among models: 47 km for MPAS-Ocean, 55 km for MOM, 60 km for GOLD, 70 km for MITgcm, and 78 km for MPAS-Ocean with sigma coordinates (all using $\nu_h = 1000$ m²/s). Unlike the lock exchange test case, there is no theoretical solution to compare to for the head location of the overflow, so it is difficult to say which models are more accurate. At high viscosity, the overflow plug of water keeps a coherent shape as it descends, with significant mixing at the head. At low viscosity, the cold water is no longer a continuous plug, but is completely dispersed with waves behind the head for many of the models. Z-level and z-star vertical coordinates induce more mixing, which slows the descent down the slope. MPAS-Ocean isopycnal and sigma coordinate simulations descend further in the same amount of time.

All overflow simulations were conducted with zero horizontal and vertical tracer diffusion and a small value of vertical viscosity ($\nu_v = 10^{-4}$). In MPAS-Ocean z-level and z-star coordinates, the cold wave head remained above several cells of warm water as it initially descended down the slope (Figure 4a,c). This behavior is more apparent in MPAS-Ocean at three hours than in MITgcm and MOM, and may be responsible for the head location of MPAS-Ocean being further left at nine hours than the other models. Typically ocean models use a convective adjustment scheme or a Richardson number-based vertical mixing to avoid unstable density profiles, but these are not used here in order to measure only spurious mixing. Some simulations using z-star and partial bottom cells did not run to completion due to this instability (Figure 5, $Re_\Delta > 100$), but proceed normally when Richardson number-based vertical mixing is added (not shown).

Mixing rates, as measured by $dRPE/dt$, are highest for z-star using full cells, lower for z-star with partial bottom cells, and lowest for sigma coordinate simulations (Figure 5). This progression is expected, because full cells

510 have the largest vertical steps to cause mixing. In sigma coordinates, the cold
 511 head falls down the slope by advecting horizontally in grid space, so much
 512 less mixing is induced. One exception is the higher value at $Re_\Delta = 1$, which
 513 appears to be caused by interaction with the right wall. MITgcm and MOM
 514 mixing rates lie between the MPAS-Ocean z-star and sigma values. In all the
 515 high viscosity cases, the head hits the right wall at 200 km before 37 hours,
 516 so some mixing in Figure 5 is due to wall interactions. In low viscosity cases,
 517 the head is sufficiently dispersed that it never reaches the right wall. The
 518 time window of 13–37 hours was used in order to make comparisons with the
 519 other ocean models.

520 3.4. *Internal waves test case*

521 Internal waves are ubiquitous in the ocean. They are driven by winds,
 522 tides, and surface pressure oscillations, and propagate through the stratified
 523 ocean. Breaking internal gravity waves are a significant source of mixing in
 524 ocean observations (Alford, 2003), but linear internal waves tend to produce
 525 vertical mixing in ocean models, particularly in those with fixed vertical
 526 levels such as z-level and z-star (Gouillon, 2010). Models that can follow
 527 isopycnal surfaces in a Lagrangian manner, such as those with isopycnal
 528 and hybrid-isopycnal coordinates, can significantly reduce excessive mixing
 529 because layers move with the internal waves and restrict fluid transport be-
 530 tween layers. The z-tilde scheme restricts mixing in a similar way, but may
 531 be applied to level-type coordinates.

532 The lock exchange and overflow test cases begin with only two values of
 533 temperature, and so are extreme circumstances to measure a model’s behav-
 534 ior. The internal wave test case is vertically stratified, begins with a small
 535 perturbation, and induces internal wave propagation similar to that found in
 536 realistic global simulations. This test has the advantage that the simulation
 537 may proceed for hundreds of days, rather than 20-40 hours as is the case for
 538 the lock exchange and overflow, so that mixing may be measured over a long
 539 period.

540 The initial temperature distribution is $\Theta_0(z) + \Theta'(x, z)$, which has a back-
 541 ground stratification of

$$\Theta_0(z) = \Theta_{bot} + (\Theta_{top} - \Theta_{bot}) \frac{z_{bot} - z}{z_{bot}} \quad (26)$$

542 where $\Theta_{bot} = 10.1$ C, $\Theta_{top} = 20.1$ C, and $z_{bot} = -487.5$ m (note sign conven-

tion on z). The initial wave is

$$\Theta'(x, z) = -A \cos\left(\frac{\pi}{2L}(x - x_0)\right) \sin\left(\pi \frac{z + \Delta z/2}{z_{bot} + \Delta z/2}\right) \quad (27)$$

for $x_0 - L < x < x_0 + L$, where $L = 50$ km, $x_0 = 125$ km, and $\Delta z = 25$ m. We present a single case with $A = 2.0$ C, which is the high amplitude case shown in Ilicak et al. (2012, Figure 11b). The perturbation is a single hump of displaced isotherms that initiate symmetric waves propagating out from the center. The waves meet the solid (non-periodic) boundary after a day, and then return to the center, forming higher wavenumber modes as the waves interact and numerical dispersion takes place. After 200 days the domain is still active with internal waves, but the models differ substantially in their shapes and positions, as shown in Figure 6, and Ilicak et al. (2012, Figure 11).

As in the first two test cases, mixing is a strong function of horizontal viscosity (Figure 7b); RPE increases in time and with lower viscosity. This is summarized in Figure 7c, where each simulation is represented by a symbol. The grid Reynolds number is computed using (18) with a characteristic velocity of $U = \sqrt{2KE}$, where KE is the average kinetic energy from 10–100 days. Isopycnal models such as GOLD have zero mixing for the duration of the simulation. MPAS-Ocean, MITgcm, and MOM have similar mixing for the first 50 days, and diverge somewhat thereafter (Figure 7a). MPAS-Ocean results are identical for z-level and z-star vertical coordinate settings because internal waves do not influence the sea surface height. This case was also tested in isopycnal coordinates, using 20 layers of constant temperature.

The internal wave case is the perfect test for the z-tilde scheme, introduced in Section 2.2. In the z-tilde scheme, layers move vertically with the internal waves, as they do in an isopycnal model. Layers are initially level and equally spaced for all simulations. Animations of z-tilde simulations show the grid moving with the internal waves, so that by 200 days the grid nearly follows the temperature profile (Figure 6). In z-tilde mode, the grid may be initialized as level layers, or the initial layers may follow density contours.

The main parameter in the z-tilde scheme is the frequency filter time scale τ_{Dlf} . When it is smaller than the physical time scales in the model, everything is included in the low-frequency component, so the high-frequency thickness alteration (h^{hf}) approaches zero. In this case the grid does not move and the model behaves as it would with z-star. As the filter time scale is increased,

577 more of the physical processes are included in the high frequency divergence,
 578 the grid moves more, and mixing is reduced. The internal wave test provides
 579 an illustrative example of how spurious mixing may be reduced by increasing
 580 the filter time scale (Figure 7). All z -tilde tests used a time step of 300 s, a
 581 high-frequency thickness restoring timescale (τ_{hhf}) of 30 days, and no high
 582 frequency thickness diffusion ($\kappa_{hhf} = 0$).

583 The vertical cross-coordinate transport is correlated with RPE, and pro-
 584 vides an indirect measure of mixing; in a stratified fluid, vertical cross-
 585 coordinate transport induces mixing. Vertical cross-coordinate transport is
 586 reduced as the filter time scale increases (Figure 8). Note that this measure
 587 is different from vertical velocity; in a fully Lagrangian vertical coordinate,
 588 vertical cross-coordinate transport is zero, and the fluid’s vertical velocity is
 589 identical to the vertical motion of the grid. The z -tilde coordinate approaches
 590 a fully Lagrangian coordinate as $\tau_{Dlf} \rightarrow \infty$.

591 3.5. Baroclinic eddies test case

592 The baroclinic eddies test case is the only one of the four idealized cases to
 593 use three dimensions and include the Coriolis force in the momentum equa-
 594 tion. Horizontal mesh resolutions vary from 10 km, which is eddy permitting,
 595 to 1 km, where the large-scale structures of the eddies are well resolved, so
 596 this test case measures mixing under conditions closer to a realistic high-
 597 resolution global ocean models than previous cases.

598 The domain is an idealization of the Antarctic Circumpolar Current. It
 599 is configured with periodic boundaries in x and solid boundaries in y . Unlike
 600 similar idealized domains in the literature (Karsten et al., 2002; Hecht et al.,
 601 2008), this is a spin-down experiment with no surface wind or buoyancy forc-
 602 ing, so that the domain is a closed system. This ensures that RPE measures
 603 mixing without contamination by external influences.

604 The initial condition is stratified in the vertical, and shifts from warmer
 605 waters in the north to colder waters in the south within a narrow band,
 606 centered about a sine oscillation in the x - y plane (Ilicak et al., 2012, Figure
 607 15). The temperature distribution in the northern section is

$$\Theta_0(z) = \Theta_{bot} + (\Theta_{top} - \Theta_{bot}) \frac{z_{bot} - z}{z_{bot}}, \quad (28)$$

608 where $\Theta_{bot} = 10.1$ C, $\Theta_{top} = 13.1$ C, and $z_{bot} = -975$ m (note sign convention
 609 on z). The southern portion of the domain is uniformly $\Delta\Theta = 1.2$ C cooler.

610 The boundary between the northern and southern portions is defined by

$$y_w(x) = y_0 - y_A \sin\left(2\pi k \frac{x}{L_x}\right) \quad (29)$$

611 where $y_0 = 250$ km, $y_A = 40$ km, $k = 3$, and $L_x = 160$ km, so that three
612 wavelengths fill a domain of width L_x . The temperature distribution is then

$$\Theta(x, y, z) = \begin{cases} \Theta_0(z) & y \geq y_w(x) + \Delta y \\ \Theta_0(z) - \Delta\Theta \left(1 - \frac{y - y_w(x)}{\Delta y}\right) & y_w < y < y_w + \Delta y \\ \Theta_0(z) - \Delta\Theta & y \leq y_w(x) \end{cases} \quad (30)$$

613 where the temperature is interpolated from north to south over a distance of
614 $\Delta y = 40$ km. An additional perturbation is added to one of the wave crests
615 to incite the instability,

$$y'_w(x) = y_0 - \frac{y_A}{2} \sin\left(\pi \frac{x - x_2}{x_3 - x_2}\right) \quad (31)$$

$$\Theta'(x, y) = \Delta\Theta' \left(1 - \frac{y - y'_w(x)}{\Delta y/2}\right) \quad (32)$$

616 so that the temperature is $\Theta + \Theta'$ within the patch $x_2 \leq x \leq x_3$, $y'_w - \Delta y/2 \leq$
617 $y \leq y'_w + \Delta y/2$, where $x_2 = 110$ km, $x_3 = 130$ km, and $\Delta\Theta' = 0.3$ C.

618 A quadratic bottom drag is applied with a dimensionless coefficient of
619 0.01. The simulation is conducted on a southern hemisphere f-plane with a
620 Coriolis parameter of $-1.2 \times 10^{-4} \text{ s}^{-1}$.

621 The tilted isopycnals are the source of potential energy for the baroclinic
622 instability. Simulations are performed at resolutions with grid cell sizes of 1,
623 4, and 10 km and horizontal viscosities ranging from 1 to 200 $\text{m}^2 \text{ s}^{-1}$. At high
624 viscosities (low Reynolds numbers) kinetic energy is low, eddies are large and
625 smooth, and mixing is slower. Low viscosity simulations (higher Reynolds
626 numbers) are more energetic, the eddies mix the background temperature
627 gradient, and RPE increases more quickly. Higher horizontal resolution re-
628 duces the mixing rate because the grid can resolve larger gradients between
629 cold and warm water, particularly around eddies. As time proceeds the ini-
630 tial distribution of tilted isopycnals flatten out, and the eddies spread to
631 the boundaries (Figures 9 and 10). The rate of mixing slows significantly
632 after 100 days when much of the initial temperature distribution has been
633 smoothed (Figure 11). The simulations shown in Figure 12 were run with

634 five values of viscosity ($\nu_h = 1, 5, 10, 20, 200$) for the 10 km and 4 km resolu-
 635 tions, and the lower four values for the 1 km resolution. For each viscosity,
 636 $dRPE/dt$ from POP z-level, POP z-star, MOM, and MITgcm are tightly
 637 clustered. MPAS-Ocean has lower mixing rates than the others; this is in-
 638 vestigated further in the next section. Models vary in the southern-most
 639 y-extent of warm water at day 200, with MITgcm at 60 km, ROMS between
 640 0 and 50 km (Ilicak et al., 2012, Figure 16a,d), and MPAS-Ocean and POP
 641 at 0 km (Figures 9h and 10b), where all of these images are for $\nu_h = 20$ and
 642 $\Delta x = 1$ km. Differences in y-extent of the warm water appear to be unre-
 643 lated to differences in $dRPE/dt$, as POP and MITgcm have similar values
 644 in Figure 12 compared to MPAS-Ocean.

645 The baroclinic eddies test case is a good setting to compare z-level ver-
 646 sus z-star in the same model, because it is three-dimensional, energetic, and
 647 runs for several hundred days. In z-star, the layer thickness is the same as
 648 in fixed z-levels, but with an additional small contribution to distribute SSH
 649 perturbations throughout the water column. These small changes in layer
 650 thickness do not reduce mixing in z-star. Indeed, $dRPE/dt$ results were
 651 nearly identical to z-level simulations for MPAS-Ocean in all cases (data not
 652 shown). Z-star is useful because the model may use extremely thin layers
 653 in the upper ocean. MPAS-Ocean has been successfully tested using z-star
 654 in realistic ocean domains with centimeter-thick layers. In addition, z-star
 655 can accommodate strong surface fluxes due to precipitation and evaporation
 656 without evacuating the top layer. The test cases and statistics presented here
 657 allow us to verify that z-star is working correctly. Figure 12 includes data
 658 from the POP ocean model with the traditional z-level configuration (Mal-
 659 trud and McClean, 2005), and the newly implemented z-star formulation,
 660 which is detailed in Appendix A.

661 MPAS-Ocean tests using z-tilde on the baroclinic eddies case produce
 662 velocities that deform the grid as the fluid adjusts from large temperature
 663 gradients in y . This is similar to the application of z-tilde to the lock ex-
 664 change, discussed in Section 3.2. The baroclinic eddies test case is not near
 665 equilibrium, and is not an appropriate application of z-tilde.

666 3.6. Evaluation of Advection Methods and Cell Shape

667 The rate of mixing is strongly sensitive to the tracer advection method.
 668 Here we use the 4 km resolution baroclinic eddies test case to evaluate mixing
 669 as a function of the order of the advection method. This is motivated by

670 results in Figure 12, where MPAS-Ocean values for $dRPE/dt$ on a hexagon
 671 mesh are uniformly lower than POP, MOM, and MITgcm.

672 The tests were conducted within the framework of the flux-corrected
 673 transport (FCT) scheme, which is described for structured grids in Zale-
 674 sak (1979) and Wang et al. (2009), and used in the development of transport
 675 schemes on unstructured grids in Skamarock and Gassmann (2011). The
 676 tracer flux at each cell edge, $F = uh\phi$, is composed of the velocity normal to
 677 the edge, u , the vertical thickness, h , and a value for the tracer, ϕ , at that
 678 edge. In FCT, the flux

$$F = F_L + r(F_H - F_L) \quad (33)$$

679 is the sum of a low-order flux, F_L , and a flux correction term (Wang et al.,
 680 2009, eqn. 8). The weighting coefficient r varies between 0 and 1 and is
 681 chosen to preserve monotonicity (i.e., no new minimums or maximums cre-
 682 ated in the tracer values). The low-order flux is chosen to be a monotonic
 683 method which, as a result, is diffusive; the high order flux, F_H , while pre-
 684 ferred for its accuracy, is not a monotonic scheme. The standard FCT option
 685 in MPAS-Ocean uses a first-order upwind scheme for F_L and a third-order
 686 reconstruction of ϕ to the edge for F_H in both the horizontal and verti-
 687 cal. If monotonicity has been preserved by the high-order scheme, then the
 688 algorithm sets $r = 1$ and only the high-order flux is used. Conversely, if
 689 F_H violates monotonicity, r is adjusted to add a portion of first-order up-
 690 wind to the total flux. The scheme is formally first-order accurate due to
 691 the first-order contribution, but in practice produces smaller errors, as the
 692 leading-order truncation error term is much smaller than for second-order
 693 schemes (Skamarock and Gassmann, 2011).

694 Figure 13 shows a clear progression to lower $dRPE/dt$ values with higher-
 695 order advection. To vary the order of the FCT algorithm for this test, we
 696 set

$$F_H = \alpha F_1 + (1 - \alpha) F_2 \quad (34)$$

697 where F_1 is the flux from first-order upwind advection, and F_2 uses second-
 698 order tracer interpolation to the edge. The lowest order tested is pure first
 699 order, with $r = 0$. All other tests use FCT, where r is chosen by the algorithm
 700 at each edge to preserve monotonicity. As α is reduced from 1 to 0, the high
 701 order flux is varied from first to second order, and $dRPE/dt$ is reduced in
 702 a regular fashion. Increasing from second to third-order tracer interpolation
 703 for F_H produces nearly the same $dRPE/dt$ curve.

704 In these tests, the tracer advection schemes for MITgcm was 7th-order
705 monotonicity preserving advection; MOM used the multi-dimensional piece-
706 wise parabolic method (PPM), which is third order and includes a mono-
707 tonicity constraint (Ilicak et al., 2012); and POP was run with 3rd-order
708 upwind, which is not strictly monotonic. These three models produce nearly
709 identical $dRPE/dt$ values, despite differences in advection methods and or-
710 ders. MPAS-Ocean, with its standard FCT algorithm, is a factor of 5-10
711 lower in $dRPE/dt$, but can be brought to similar values by diminishing the
712 order just slightly (by setting $\alpha = 0.05$).

713 Our ability to increase mixing by decreasing the order of the advection
714 method does not necessarily single out the FCT algorithm as the cause of
715 lower spurious mixing in MPAS. After all, there are many differences amongst
716 these models. MPAS-Ocean’s hexagon horizontal grid cell shape is unique
717 among these models and may be responsible for the reduced mixing; POP,
718 MOM, and MITgcm all use quadrilateral grids. To test this hypothesis,
719 the baroclinic eddies test case was repeated with MPAS-Ocean on a quadri-
720 lateral grid. The advection method is FCT with $F_H = F_2$. Mixing data
721 from MPAS-Ocean on quadrilaterals is closer to the other quadrilateral mod-
722 els than MPAS-Ocean on hexagons (Figure 12), particularly at higher grid
723 Reynolds numbers. Reduced spurious mixing on hexagon grids may be due
724 to the representation of velocity on C-grid cells. Each cell expresses velocity
725 as six vector components pointing in three directions (three unique degrees
726 of freedom per cell), which is a more detailed representation of a continuous
727 velocity field than four vector components pointing in two directions, as in
728 a quadrilateral grid (two unique degrees of freedom per cell). Due to their
729 geometry, hexagons pack 12–15% more grid cells into the same domain as
730 quadrilaterals. This slight increase in resolution is not enough to explain the
731 observed difference in $dRPE/dt$ between hexagon and quadrilateral grids.

732 4. Global simulations

733 We now proceed to the global real-world simulations. Two tests were
734 conducted: a ten-year spin-up from rest to test the z-tilde scheme, and a
735 ten-year spin-up followed by an unforced spin-down, in order to measure
736 RPE and compare to Ilicak et al. (2012). All simulations use a z-star vertical
737 coordinate, and some spin-up simulations use z-tilde in addition to z-star (see
738 Section 2.1). The horizontal mesh is quasi-uniform over the globe, with simu-
739 lations performed at nominal cell widths of 120, 60, 30, and 15 km. The mesh

740 is based on Voronoi tessellations with uniform mesh density, which results in
 741 cells that are almost all hexagons. Initial distributions of potential temper-
 742 ature and salinity are obtained from the annual mean WOCE climatology
 743 (Gouretski and Koltermann, 2004). For the spin-up simulations, monthly
 744 mean wind stress is obtained from “Normal Year” forcing data from the Co-
 745 ordinated Ocean Reference Experiment (CORE, Large and Yeager (2004)),
 746 and surface temperature and salinity are restored to the initial condition with
 747 a time scale of 30 days, as in Ringler et al. (2013). Surface restoring is differ-
 748 ent from standard CORE experiments, which prescribe the atmospheric state
 749 and use bulk formulas (Griffies et al., 2009). Configuration settings similar to
 750 those presented in Ringler et al. (2013), and include the following: 40 vertical
 751 layers, ranging in thickness from 10 m near the surface to 250 m in the deep
 752 ocean; quadratic bottom drag with a coefficient of 0.001; nonlinear equation
 753 of state by Jackett and McDougall (1995); monotonic flux-corrected tracer
 754 transport (Skamarock and Gassmann, 2011), with third order reconstruction
 755 of the tracer values at the cell edges; split-explicit timestepping with 40 sub-
 756 cycles to step through $2\Delta t$, and long baroclinic time steps (Δt) of 3000, 2400,
 757 1200, and 600 seconds for the 120, 60, 30, and 15 km resolutions, respectively.
 758 The horizontal turbulence closure is a simple biharmonic viscosity ($-\nabla^4$) op-
 759 erator with coefficients of $\nu_{4h} = \nu_0(\Delta x/\Delta x_0)^3$ where $\nu_0 = 5 \times 10^{10} \text{ m}^4 \text{ s}^{-1}$
 760 and $\Delta x_0 = 15 \text{ km}$ cells. Zero horizontal diffusion is applied to the tracers.
 761 The vertical mixing is Richardson number-based, with background viscos-
 762 ity and diffusion of $10^{-4} \text{ m}^2 \text{ s}^{-1}$ and $10^{-5} \text{ m}^2 \text{ s}^{-1}$, respectively. Richardson
 763 number-based vertical tracer diffusion is on during the spin-up, and off for
 764 the spin-down. Constant vertical diffusion is a variable parameter for the
 765 spin-down simulations. See the MPAS-Ocean User’s Guide (Jacobsen et al.,
 766 2013) for more details on how these settings are applied.

767 4.1. Global spin-up

768 The z-tilde scheme was tested with a global quasi-uniform resolution of
 769 60 km. Simulations begin from rest, and each layer has constant thickness
 770 in the horizontal. The filter time scale τ_{Dlf} was varied from 0.1 to 10 days,
 771 and all cases used $\tau_{hhf} = 5$ days and $\kappa_{hhf} = 0$. Over the ten-year spin-up,
 772 average horizontal kinetic energy increases over the first five years and then
 773 levels off. All simulations have similar circulation patterns (not shown) and
 774 average kinetic energy within 7% of each other (Figure 14a).

775 The filter time scale is the main control on the frequency filtered coor-
 776 dinate system. When $\tau_{Dlf} = 0.1$ day, all baroclinic divergence is in the low

777 frequency part, so the high frequency thickness is nearly zero. This simu-
778 lation behaves the same way as when z -tilde is off (compare to z -level and
779 z -star in Figure 14b). As the filter time scale increases, more thickness oscil-
780 lations are included in the high frequency thickness. This reduces the verti-
781 cal cross-coordinate transport (Figure 14b). Specifically, the time-averaged
782 root mean squared vertical cross-coordinate transport is reduced from $2.3e$ -
783 5 m/s to $1.75e$ -5 m/s, a reduction of 24%. Snapshots show that vertical
784 cross-coordinate transport produced by internal waves along the equator is
785 significantly reduced by the addition of z -tilde (Figure 15). Note that z -tilde
786 does not suppress internal wave activity or vertical motion. It simply allows
787 the grid to follow internal waves, or any fast vertical motion, in a Lagrangian
788 fashion. These figures show that z -tilde reduces transport across grid cell
789 interfaces in the vertical. Vertical motion of the fluid is not suppressed, as
790 it is the sum of cross-coordinate transport and grid motion. Z -tilde results
791 in the reduction of unphysical mixing caused by internal waves, as shown in
792 the internal wave test case (Section 3.4).

793 The extra computation required for the frequency filtered coordinate sys-
794 tem is in solving the two additional prognostic equations (12 and 13), adding
795 high frequency thickness to the desired ALE thickness, and then altering the
796 desired thickness to be within minimum and maximum constraints (7). Com-
797 pute times for the forward model increased by 4% per time step for the global
798 60 km mesh. Timing excludes i/o and analysis calculations, and was tested
799 on typical configurations of 512 processors. A larger problem is that the
800 addition of frequency-filtering often imposes a stricter time step for stability,
801 usually half the standard time step when $\tau_{Dlf} = 5$ days and a quarter when
802 $\tau_{Dlf} = 10$ days. This is a significant hindrance to the use of frequency-filtered
803 vertical coordinates, and a problem that we are working to improve.

804 4.2. Global spin-down

805 The second set of global simulations begin after year ten of the spin-
806 up simulations in the previous section. Wind forcing and temperature and
807 salinity restoring are turned off, and the simulations are continued for 70 days
808 to assess the mixing characteristics of the model. The turbulence closure for
809 the momentum equation remains the same as the forced case, and may vary
810 amongst models. MPAS-Ocean used biharmonic viscosity in the horizontal
811 and Richardson-number based momentum mixing in the vertical. The POP
812 turbulence closures for momentum included anisotropic horizontal viscosity
813 with standard settings, KPP (Large et al., 1994) in the vertical for spin-up,

814 and constant ($\nu_v = 2.5 \times 10^{-5} \text{ m}^2 \text{ s}^{-1}$) for spin-down. POP used Gent-McWilliams
815 (Gent and McWilliams, 1990) as a tracer turbulence closure for spin-up. For
816 all models, tracer diffusion during spin-down is zero in the horizontal and
817 constant in the vertical, with a coefficient varying from $\kappa_v = 0$ to $\kappa_v = 10^{-5}$
818 $\text{m}^2 \text{ s}^{-1}$ (Ilicak et al., 2012).

819 Mixing is a strong function of vertical tracer diffusion; for all models,
820 $dRPE/dt$ increases with κ_v at a fixed resolution (Figure 16). At lower reso-
821 lutions, POP displays the highest mixing, MOM the least, and MPAS-Ocean
822 is between the two. Both POP and MOM use nominally one-degree quadri-
823 lateral grids with typical horizontal grid length of 100 km. For a particular
824 κ_v all the low-resolution simulations report $dRPE/dt$ values within a factor
825 of two, and so appear to be in good agreement overall.

826 As one proceeds to higher resolution, MPAS-Ocean simulations have less
827 mixing. This is similar to the behavior of the baroclinic eddy case at a fixed
828 horizontal viscosity (Figure 12). Ilicak et al. (2012) found the opposite: their
829 MOM $1/4^\circ$ spin-down produced higher $dRPE/dt$ than the one-degree case
830 due to mixing by the eddies. In order to better understand these differences,
831 we conducted a sensitivity study of $dRPE/dt$ versus grid Reynolds number
832 (Figure 17). In a typical series of simulations with varying resolution, the
833 biharmonic viscosity is chosen to scale with $(\Delta x)^3$, so that the grid Reynolds
834 number is nearly constant (see equation 19). This is the case for the MPAS-
835 Ocean simulations in Figure 16 with $\kappa_v = 0$, which appear as the middle
836 points in Figure 17, with grid Reynolds numbers between 5 and 10. (Small
837 differences between these two series are due to monthly versus annual mean
838 forcing during the spin-up in Figures 16 and 17, respectively.) The results
839 in Figure 17 all used $\kappa_v = 0$, were spun-up for ten years with a standard ν_h
840 value for each resolution, and then continued for one additional year of spin-
841 up with the tested ν_h values before the 70 day spin-down experiment. At
842 the lowest grid Reynolds number, the fluid is sufficiently viscous that spurious
843 mixing, as measured by $dRPE/dt$, is not resolution dependant. For more
844 typical values of viscosity, the spurious mixing decreases monotonically with
845 increasing resolution at a fixed grid Reynolds number. With this information,
846 a user may choose the value of numerical mixing they are willing to live with,
847 and then read off the required resolution for a given grid Reynolds number.

848 The addition of $\tilde{z}$ did not cause any appreciable reduction in $dRPE/dt$
849 in the global spin-down experiments. The $\tilde{z}$ coordinate is only appro-
850 priate for well-equilibrated simulations. The spin-down simulations begin by
851 turning off wind forcing and surface restoring at year ten of spun-up simula-

852 tions, causing large transient flows to readjust to the new forcing conditions.
853 As shown with the lock exchange test case (Section 3.2), transient flows that
854 occur at similar or faster time-scales than the frequency filter (τ_{Dlf}) result in
855 simulations with similar or higher values of $dRPE/dt$.

856 5. Conclusions

857 In the development of a new ocean-climate model, verification of model
858 behavior against observations and standard test cases is a critical part of the
859 development process. Published results of model intercomparisons give model
860 developers and users confidence in the proper functioning of the model, and
861 an understanding of what model components need further improvement. The
862 best place to start is comparison of idealized test cases with simple initial
863 conditions and very few parameters, so that individual processes may be
864 isolated and assessed. In the end, an ocean-climate model must be evaluated
865 in real-world simulations if it is to fulfill its goal of decadal and century-
866 long climate predictions. In this work we evaluate the vertical coordinate
867 of MPAS-Ocean using numerous configurations ranging from idealized to
868 realistic.

869 The overall conclusion of these tests is that the MPAS-Ocean ALE verti-
870 cal coordinate can correctly represent z-level, z-star (with and without partial
871 bottom cells), z-tilde, isopycnal and sigma coordinates with mixing rates, as
872 measured by $dRPE/dt$, that are commensurate with existing ocean models.
873 In addition, a newly implemented z-star coordinate in POP was shown to pro-
874 duce nearly identical results as the standard z-level formulation, as expected.
875 The test problems fully specify the domain, initial conditions, viscosity, and
876 diffusion, so differences in mixing rates must be due to model numerics such
877 as the horizontal and vertical coordinates, discretization methods, derivative
878 operators, time-stepping schemes, and advection schemes.

879 MPAS-Ocean has much lower spurious mixing than other models in three-
880 dimensional domains, which are the baroclinic eddies and high-resolution
881 global spin-down cases. This result, while encouraging, must be thoroughly
882 vetted to be trusted and understood. Two features of MPAS-Ocean stand
883 out as different from the other ocean models tested: a horizontal grid based
884 on Voronoi tessellations (mostly hexagons), and the flux-corrected transport
885 (FCT) tracer advection scheme with interpolations designed for unstructured
886 grids. MPAS-Ocean simulations carried out on hexagon and quadrilateral
887 grids show that spurious mixing is lower on hexagon grids, particularly at

888 high grid Reynolds numbers. Another set of simulations tested the tracer
889 advection algorithm, and showed that a small reduction in the order of ac-
890 curacy produces $dRPE/dt$ values similar to the other models. In addition,
891 identical domain set-up and analysis tools used for MPAS-Ocean were used
892 for POP in the baroclinic eddies and global spin-down cases; the POP results
893 were nearly identical to MOM and MITgcm. We conclude that both the ad-
894 vection scheme and hexagon-type grids play a role in the reduced spurious
895 mixing by MPAS-Ocean. In two-dimensional (x - z) domains MPAS-Ocean
896 produced spurious mixing similar to or higher than other models, which in-
897 dicates that the advection schemes and horizontal grid type plays a stronger
898 role in reducing mixing in fully three-dimensional domains.

899 The frequency-filtered scheme (z-tilde) reduces vertical transport across
900 layer interfaces. This occurs in a consistent way as a function of the filter
901 time scale: in the internal wave test case by over a factor of ten, and the
902 global spin-up case by 24%. This leads to a reduction in spurious mixing
903 caused by internal waves. The z-tilde scheme requires solving two additional
904 prognostic equations, which adds 4% to the compute time of each time step.
905 In the internal gravity wave test case the standard time step could be used,
906 but in global simulations with z-tilde the time step had to be reduced to half
907 its standard value when $\tau_{Dif} = 5$ days. We hope to improve the time step,
908 but this issue could be a hindrance for z-tilde in practical applications.

909 It was shown that z-tilde is not appropriate in simulations with large, fast
910 transient flows, such as the lock exchange, overflow, and baroclinic eddies test
911 cases, and the global spin-down. In these cases the grid follows the fast flows
912 in a Lagrangian manner, but becomes too distorted, so that mixing is not
913 reduced from a z-level or z-star coordinate. The z-tilde scheme is effective
914 at reducing mixing for small, fast perturbations, such as those caused by
915 internal gravity waves.

916 Development of MPAS-Ocean began in 2010, and progressed to its first
917 public release in 2013. The comparisons against long-standing ocean models
918 presented here have been an intrinsic part of model development, verification,
919 and improvement. In particular, these results show that the MPAS-Ocean
920 vertical coordinate is robust, and produces spurious mixing that is similar to
921 or less than other ocean models.

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930 Appendix A. The POP z-star formulation

931 The new POP vertical coordinate may be run in z-level or z-star mode.
 932 It is not a full ALE implementation, which requires a prognostic thickness
 933 equation, as in (2), but diagnoses the thickness based on the sea surface
 934 height (SSH),

$$h_k = dz_k \left(1 + \eta \frac{W_k}{\sum_{k'=1}^{kmax} dz_{k'} W_{k'}} \right), \quad (\text{A.1})$$

935 where the weights W_k are described in Table 3, η is the SSH, and dz_k is the
 936 original thickness of the z-level model.

937 Here we refer to the POP reference manual (Smith et al., 2010), hereafter
 938 POP. The z-level form of the momentum equation requires the addition of
 939 the ∇z_{mid} term for tilted layers, shown in (1). This has the form

$$\left(-\frac{\rho g}{\rho_0} \delta_x \bar{z}_{mid}^y, -\frac{\rho g}{\rho_0} \delta_y \bar{z}_{mid}^x \right), \quad (\text{A.2})$$

940 added to the right-hand sides of POP eqns 3.21 and 3.22.

941 The tracer equation becomes thickness weighted, so that POP eqn 3.11
 942 is now

$$\frac{\partial}{\partial t} h\varphi + dz \mathcal{L}_T(\varphi) = dz \mathcal{D}_H(\varphi) + dz \mathcal{D}_V(\varphi) + dz \mathcal{F}_w(\varphi), \quad (\text{A.3})$$

943 where the tracer advection, horizontal diffusion, vertical diffusion, and fresh-
 944 water input remain unchanged (POP eqns 3.13, 3.14, 3.18, 3.19).

945 In the time stepping, the barotropic step remains untouched. The baro-
 946 clinic time-stepping will change by factors of thickness in many places. Pre-
 947 viously in z-level coordinates, a scaling factor was included to account for
 948 SSH in the top layer only (POP eqn 3.12),

$$\xi = \frac{\delta_{k1}}{dz_1} \eta. \quad (\text{A.4})$$

949 The tracer equation is now thickness-weighted, so that the factor $(1 + \xi^n)$
 950 may be replaced by h^n in the predictor/corrector time stepping scheme, and
 951 likewise for $n + 1$ and $n - 1$ (POP equations 4.1–4.15).

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Figure 2: Lock exchange test case: Normalized resting potential energy (a) for several models using a horizontal viscosity $\nu_h = 0.01$ and (b) for MPAS-Ocean using z-star and varying horizontal viscosity; (c) rate of change of RPE at 17 hours using the ν_h values in (b).

Figure 3: Lock exchange test case: Rate of change of RPE at 17 hours as a function of the frequency filter time scale, τ_{Dlf} , using the z-tilde vertical coordinate. These simulations used values of $\nu_h = 100\text{m}^2/\text{s}$, and $\kappa_{hhf} = 0$. The $dRPE/dt$ value for z-level and z-star are identical (red line), and are not a function of τ_{Dlf} . The two series begin with z-level (blue) or isopycnal (black) initial conditions (IC), and use $\tau_{hhf} = 30$ days and $\tau_{hhf} = 0$, respectively.

Figure 4: Overflow test case: Images from MPAS-Ocean at three hours (left column) and six hours (right column) for z-star (a-d), isopycnal (e-h) and sigma vertical coordinates (i-l). Two cases of horizontal viscosity are shown, $\nu_h = 10^{-2}$ and $\nu_h = 10^3 \text{ m}^2 \text{ s}^{-1}$, corresponding to grid Reynolds numbers of $Re_\Delta = 1.5e5$ and $Re_\Delta = 1.5$.

Figure 5: Overflow test case: Rate of change of RPE, measured from 13–37 hours. MPAS-Ocean z-level results (not shown) are identical to the z-star results.

Figure 6: Internal wave test case: Temperature at 200 days with $\nu_h = 0.01$ and $Re_\Delta = 5e4$, using (a) z -star and (b) z -tilde with $\tau_{Dlf} = 10$ days. Black lines show positions of grid cell boundaries.

Figure 7: Internal wave test case: Normalized RPE (a) using several models with horizontal viscosity $\nu_h = 0.01$, and (b) using MPAS-Ocean with z-star and varying horizontal viscosities; (c) rate of change of RPE, averaged from day 10 to 100, using the ν_h values in (b). Legend entries with τ_{Dlf} in (c) indicate the z-tilde coordinate. As the filter time scale (τ_{Dlf}) is increased from 0.1 to 100 days, $dRPE/dt$ decreases, showing reduced vertical mixing. Z-level results overlay those for z-star, and are not shown.

(b)

Figure 8: Internal wave test case: RMS of vertical cross-coordinate transport at the layer interface averaged from day 10 to 100, using MPAS-Ocean. As the filter time scale (τ_{Dlf}) is increased from 0.1 to 100 days, the vertical cross-coordinate transport between layers decreases. The top panel (a) shows the horizontal average for simulations with $\nu_h = 0.01$, and lower panel (b) shows domain average for simulations with $\nu_v = 0.01, 1, 15, 150$. Legend entries with τ_{Dlf} indicate the z -tilde coordinate.

Figure 9: Baroclinic eddies test case: MPAS-Ocean with z-star, temperature, top layer ($k=1$), using a resolution of $\Delta x = 1$ km. Horizontal viscosity values of $\nu_h = 200$, 20, and 1 are associated with grid Reynolds numbers of $Re_\Delta = 0.3$, 4, and 100.

Figure 10: Baroclinic eddies test case: POP model, showing temperature, as in Figure 9, for day 200. Simulations were conducted with both z-level and z-star. Shown are z-level in (a) and z-star in (b, c)

Figure 11: Baroclinic eddies test case: Normalized RPE for the 4km using MPAS-Ocean with z-star on a hexagon grid. Horizontal viscosity values of $\nu_h = 1, 5, 10, 20$, and 200 are associated with grid Reynolds numbers of $Re_\Delta = 350, 70, 35, 15$, and 1 .

Figure 12: Baroclinic eddies test case: Rate of change of RPE using the ν_h values in Figure 11 for grid cell resolutions of (a) 10 km, (b) 4 km, and (c) 1 km. Derivative data is averaged from day 1-320 for POP and MPAS-Ocean and 3-319 days for MITgcm and MOM. MPAS-Ocean was tested with regularly spaced hexagon (hex) and quadrilateral (quad) horizontal grids, both using z-star. MPAS-Ocean z-level simulations overlay z-star data and are not shown.

Figure 13: Baroclinic eddies test case: Comparison of mixing by varying the high-order flux component (F_H) within the flux-corrected transport (FCT) scheme in MPAS-Ocean with hexagon cells. The low-order flux component (F_L) is always first-order upwind advection. As the order of F_H is increased, $dRPE/dt$ decreases, up to the standard FCT scheme, which produces the least mixing. Other model data and timing is identical to Figure 12b.

Figure 14: Global spin-up case: domain-average horizontal kinetic energy, m^2s^{-2} (a) and vertical cross-coordinate transport, m/s (b) for the global 60km spin-up simulations using MPAS-Ocean, with varying values of the filter time scale (τ_{Dlf}). Legend entries with τ_{Dlf} indicates that z-tilde was used in addition to the z-star vertical coordinate.

Figure 15: Global spin-up case: z -tilde significantly reduces vertical cross-coordinate transport (m/s). Images are at 9.5 years and a depth of 644 m (a, b) and 3000 m (c,d) using a z -star vertical coordinate without z -tilde (a, c) and with z -tilde and $\tau_{Dlf} = 5$ days (b, d). Images show equatorial Pacific, extending from 20S to 30N latitude and 180W to 35W longitude. All simulations have a grid resolution of 60 km.

Figure 16: Global spin-down case: Rate of change of RPE for (a) lower and (b) higher resolutions, averaged over day 20 to 70. MPAS-Ocean cases use a z-star vertical coordinate (no cases use z-tilde). Grid Reynolds numbers are 5, 7.5, 8.5, and 9 for the 120 km, 60 km, 30 km, and 15 km resolutions, respectively, when $\kappa_v = 0$.

Figure 17: Global spin-down case: Rate of change of RPE versus grid Reynolds number for MPAS-Ocean simulations with zero vertical diffusion, averaged over day 20 to 70, using a z-star vertical coordinate. No cases use z-tilde. For a fixed resolution, the grid Reynolds number varies with the biharmonic viscosity coefficient. Squares highlight standard values of grid Reynolds numbers for MPAS-Ocean simulations, which are near one.