Effect of numerical diffusion on the water mass transformation in eddy-resolving models

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Abstract

This study investigates the effect of numerical diffusion associated with advection schemes on water mass transformation in an eddy-resolving model. The effect of numerical diffusion is evaluated as a residual between the total water mass transformation and the explicit water mass transformation: the former is calculated as the sum of meridional streamfunction and the temporal change rate of an isopycnal surface depth, and the latter is directly calculated with the use of the tendency equation of density. This method is used for investigating a dependency of numerical diffusion on explicit diffusivity. It is found that idealized channel experiments are categorized into three regimes according to a magnitude of explicit diffusivity: numerical diffusion, transitional, and explicit diffusion regimes. The numerical diffusion regime is defined as the regime where explicit diffusion changes do not significantly impact the solution. The magnitude of numerical diffusion is independent of the explicit diffusivity there. In the transitional regime, explicit (numerical) diffusion works more (less) with higher explicit diffusivity. Explicit and numerical diffusions are comparably important there. The explicit diffusion becomes significantly large and the numerical diffusion is almost negligible in

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the explicit diffusion regime. The total diffusion effect on water mass transformation there is considerably larger than those in the two other regimes.

Two experiments are conducted with a Southern Ocean model under a realistic configuration. These belong to the numerical diffusion and transitional regimes. The model becomes a little too diffusive in the latter experiment. This result and results of channel experiments indicate that it is not an adequate option for a realistic Southern Ocean simulation that we adopt a diffusion coefficient in the explicit diffusion regime in order to reduce levels of numerical diffusion. It indicates that numerical diffusion is inevitable for eddy-resolving models with horizontal resolution around 0.1 degree and we must adequately evaluate its effect on model results when analyzing outputs of such high resolution models. The method proposed in the current study for assessing numerical diffusion will be useful for investigating an eddying ocean with numerical models.

Keywords: Eddy-resolving model, The Southern Ocean, Water mass transformation, Numerical diffusion

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1. Introduction

High resolution modeling has become a widely available method to investigate oceanic phenomena. Numerical errors generally depend on grid spacing, so they are expected to be reduced by employing higher resolution. However, numerical errors depend also on other factors, and high resolution does not automatically guarantee reduction of errors. Using idealized simulations, Griffies et al. (2000) make a caveat about the influence of spurious mixing in high resolution modeling: large velocities resolved by models with high

48 resolution induce large numerical diffusion. Lee et al. (2002) show their eddy-
 49 permitting model actually suffers from that problem although they employ a
 50 relatively high-accuracy third-order advection algorithm. They suggest that
 51 high-resolution ocean models require less diffusive advection schemes. Hill
 52 et al. (2012) indicate that a very high-accuracy advection scheme developed
 53 by Prather (1986) involves little spurious diapycnal mixing in an interior
 54 region away from model boundaries with the use of an idealized channel
 55 model. However, it is still an open question whether such a state-of-the-art
 56 scheme reduces levels of numerical errors enough for a realistic ocean mod-
 57 eling. Urakawa & Hasumi (2012) (UH hereafter) indicate that their eddy-
 58 resolving Southern Ocean modeling is still not free from these numerical
 59 errors although their model adopts Prather’s (1986) scheme. This problem
 60 should be further investigated with the use of high-resolution models.

61 Eddy resolving models usually adopt a horizontal biharmonic diffusion
 62 scheme as a way to parameterize mixing by sub-grid scale motions. Its dif-
 63 fusivity coefficient is, however, physically ¹ unknown and ranges widely from
 64 zero to $10^{10} \text{ m}^4 \text{ s}^{-1}$ among eddy-resolving models of preceding studies which
 65 have horizontal resolution similar to that of the model in UH (Masumoto
 66 et al., 2004; Lee et al., 2007; Tanaka & Hasumi, 2008a,b). It is anticipated
 67 that the stronger sub-grid scale eddy diffusivity results in the smoother dis-
 68 tributions of tracers. An advection scheme tends to cause less numerical

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¹Strictly speaking, the use of biharmonic diffusion is not physically justified. It is commonly used in high-resolution models in order to dissipate tracer variances in the small scale and prevent models to blow up. However, there must be a “physically” adequate value for its coefficient as long as it tries to represent mixing by sub-grid scale processes.

69 errors in that case. Reproducibility of models is also expected to depend on
70 the explicit diffusivity². Therefore, we need to know an adequate choice for
71 the explicit diffusivity and the effect of numerical errors on a model’s solution
72 in eddy resolving modeling. We try to address this problem by conducting
73 eddy resolving simulations of the Southern Ocean.

74 The current study shows that mixing induced by numerical errors asso-
75 ciated with an advection scheme works in the same way as an explicit hor-
76 izontal biharmonic diffusion. So, it is called “numerical diffusion” hereafter
77 although numerical diffusion is not the sole source of numerical errors to be
78 exact. This study also reveals that numerical diffusion inevitably becomes
79 comparable to or larger than explicit diffusion even with a very high-accuracy
80 advection scheme when the explicit diffusivity is adjusted to reproduce realis-
81 tic stratification of the Southern Ocean, especially near the abyss, as long as
82 we use a eddy resolving model with horizontal resolution around 0.1 degree.
83 So, we must adequately evaluate it when analyzing outputs of such a high-
84 resolution model. Hill et al. (2012) and Ilıcak et al. (2012) utilize a temporal
85 change of minimum potential energy to quantify the magnitude of numerical
86 diffusion. This method gives us effective diffusivity (diffusivity induced by
87 both explicit and numerical diffusions) but cannot isolate numerical diffusion
88 in experiments with external forcings and/or explicit diffusions. Burchard &
89 Rennau (2008) propose another method in which the magnitude of numerical
90 diffusion is evaluated as the difference of decay rate between the advected

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²Explicit diffusivity denotes diffusion coefficients for explicit diffusion schemes. Explicit diffusion means diffusions explicitly implemented in a model. In other words, they are diffusions which are not numerical diffusion associated with an advection scheme.

91 square of a tracer and the square of an advected tracer. It is compared to the
92 turbulent mean tracer variance decay rate calculated with explicit diffusivity
93 and spatial gradients of the tracer. This decay rate defines the magnitude
94 of explicit diffusion. We can compare these two decay rates with each other
95 and tell which of explicit and numerical diffusions is the larger. However,
96 the tracer variance decay does not clearly show how diffusions quantitatively
97 affect a model's solution such as a density distribution and a circulation. The
98 main subject of the current study is to develop a way to quantitatively inves-
99 tigate the effect of numerical diffusion associated with an advection scheme
100 on a tracer field.

101 We utilize the idea of water mass transformation, which has been de-
102 veloped by many studies (e.g., Walin, 1982). The water mass is one of the
103 basic concepts in physical oceanography. It is important to know how wa-
104 ter masses change their property when studying diabatic circulation. More-
105 over, this approach enables us to investigate the way how numerical diffusion
106 quantitatively affects the density field in the ocean. We discuss a relationship
107 between water mass transformation and meridional overturning circulation
108 as is done by Lee et al. (2002) and propose a way to diagnose the effect of
109 numerical diffusion on water mass transformation rate there.

110 The remainder of this paper is composed of the following sections. Sec-
111 tion 2 shows an experimental design of this study. Because an eddy-resolving
112 model of the Southern Ocean requires a lot of computational resource, it is
113 difficult to carry out a sensitivity study on explicit horizontal diffusivity
114 with it. We conduct only two experiments with different explicit horizon-
115 tal diffusion coefficients in the current study. Results of these experiments

are complemented by idealized channel experiments. Section 3 introduces methodology for analysis in the current study. Sections 4 and 5 show results of numerical experiments with the Southern Ocean model and the idealized channel model in the current study, respectively. Section 6 gives conclusions and discussions.

2. Experimental design

The model employed for this study is COCO, version 4 (Hasumi, 2006). This model is based on the primitive equations under the hydrostatic and Boussinesq approximations with explicit free surface. It is formulated on a general orthogonal curvilinear horizontal coordinate and the geopotential height vertical coordinate. This model is run under two different configurations, which is briefly summarized in Table 1. One is called the Southern Ocean model and the other is called the channel model, hereafter. Both models have horizontal resolution of $1/8^\circ$ in longitude and $1/12^\circ$ in latitude. An enstrophy-conserving scheme (Ishizaki & Motoi, 1999) and a biharmonic friction with Smagorinsky-like viscosity ($C=2.5$; Griffies & Hallberg, 2000) are employed for momentum equations. The models adopt the second-order moment (SOM) scheme developed by Prather (1986) with the flux limiter of Morales Masqueda & Holloway (2006), which is known to cause less numerical diffusion than conventionally used ones (Morales Masqueda & Holloway, 2006). Vertical diffusion coefficient is $5 \times 10^{-5} \text{ m}^2 \text{ s}^{-1}$. A horizontal biharmonic diffusion scheme is used there and its coefficient differs among experiments described below. Specific configurations for the two models are shown in following sections, respectively.

140 *2.1. The Southern Ocean model*

141 The model domain is the Southern Ocean from 75°S to 20°S. The northern
142 and southern limits of the model domain are solid boundaries. A realistic
143 bathymetry based on GEBCO (IOC, IHO and BODC, 2003) is used, and
144 the coastline around Antarctica is modified with the use of MODIS Mosaic
145 of Antarctica (Haran et al., 2005). Ice shelves and grounded icebergs in this
146 model are treated as land. The model has 81 levels in the vertical direction,
147 and the grid spacing is 50 m for the upper 2000 m and linearly increased
148 to 100 m toward the bottom. Two different biharmonic horizontal diffusion
149 coefficients are used in this study: $1 \times 10^{10} \text{ m}^4 \text{ s}^{-1}$ and $5 \times 10^7 \text{ m}^4 \text{ s}^{-1}$. These
150 experiments are hereafter called HIGH and LOW, respectively. The former
151 experiment coincides with the experiment of UH.

152 The model is spun up for 1 year from a state of rest. During this spin-
153 up period, potential temperature and salinity are restored to WOCE global
154 hydrographic climatology (WGHC; Gouretski & Koltermann, 2004) from top
155 to bottom with a restoring time of 30 days. Then, the model is forced for 10
156 years by monthly climatological average of Röske’s (2001) wind forcing. Sea
157 surface temperature and salinity are restored to the monthly climatological
158 values of World Ocean Atlas 2001 (WOA’01; Stephens et al., 2002; Boyer
159 et al., 2002) with a restoring time of 50 days. At the northern boundary,
160 potential temperature and salinity from 23°S to 20°S are restored to WGHC
161 from top to bottom with a restoring time of 30 days. Dense shelf water
162 (DSW) on the Antarctic continental shelf is prescribed by restoring potential
163 temperature and salinity in the area where the depth of the sea bottom is
164 shallower than 1000 m depth to the monthly climatological average of a

165 high-resolution East Antarctic (EAA) model (Kusahara et al., 2010, 2011).
 166 A restoring time is 30 days except for 5 days at the southern boundary of
 167 the model domain. A set of 3-day interval snapshots for the last 1 year is
 168 analyzed.

169 2.2. Idealized channel model

170 This model has a rectangular and flat bottom basin. The model size
 171 is 20° in longitude, 20° in latitude from 50°S to 30°S and 4000 m deep.
 172 Vertical resolution is 50 m. Periodic boundary conditions are adopted in the
 173 zonal direction. The initial condition of potential temperature and salinity is
 174 zonally uniform. This initial state is taken from WGHC at 49°E in longitude
 175 and from 50°S to 30°S in latitude. Undefined values in WGHC are linearly
 176 interpolated with values of surrounding water. The initial meridional velocity
 177 is zero and the zonal velocity is calculated with the use of $u = -\frac{1}{f\rho_0}\frac{\partial p}{\partial y}$, where
 178 ρ_0 is a fixed reference density value (10^3 kg m^{-3}).

179 Two kinds of experiments are conducted with this model. One is a set
 180 of experiments where the model is integrated for 1 year without external
 181 forcings. These are called the short-run experiments hereafter. A linear
 182 equation of state (EOS) is also used in the former experiment as well as
 183 the nonlinear EOS. Horizontal biharmonic diffusion coefficients used there
 184 range from zero to $2 \times 10^{11} \text{ m}^4 \text{ s}^{-1}$. The model is integrated for 40 years
 185 without explicit horizontal diffusion in the other kind of experiment and the
 186 last 1 year is analyzed. This experiment is called the long-run experiment.
 187 It is driven by a sinusoidal zonal wind, whose stress is 0 at the northern
 188 and southern boundaries and 0.1 N m^{-2} at 40°S . Sea surface temperature
 189 and salinity are restored to those of initial values with a restoring time of

Table 1: Configurations of experiments. NL and LI in the EOS column means nonlinear and linear, respectively.

Model	Integration period	Horizontal diffusivity [m ⁴ s ⁻¹]	Vertical diffusivity [m ⁴ s ⁻¹]	EOS	External forcings
The Southern Ocean	11 years	10 ¹⁰ and 5 × 10 ⁷	5 × 10 ⁻⁴	NL	On
Channel (short-run)	1 year	0–2 × 10 ¹¹	5 × 10 ⁻⁴	NL and LI	Off
Channel (long-run)	40 years	0	5 × 10 ⁻⁴	NL	On

50 days. We analyze a set of 1-day interval snapshots for each experiment.
Note that the results do not qualitatively and quantitatively change with the
use of a set of 5-day interval snapshots.

3. A method for diagnosing water mass transformations induced by numerical mixing

We use the potential density referenced to 2000 dbar (σ_2) in analysis of
experiments with the nonlinear EOS. “Density” denotes the potential density
hereafter. Let us consider an arbitrary isopycnal surface S_σ whose density is
equal to σ . The following relationship holds there:

$$\frac{\partial}{\partial t}V(y, \sigma) = \Omega(y, \sigma) - \psi(y, \sigma), \quad (1)$$

where $V(y, \sigma)$ describes volume contained in the layer below S_σ and to the
south of y . The left hand side of Eq. 1 is calculated from the difference
between the initial and final states of the analyzed 1 year period. $\Omega(y, \sigma)$
and $\psi(y, \sigma)$ describe a volume transport across S_σ to the south of y and a
transport across y of water below S_σ , respectively. Water masses are defined

Table 2: Water mass definitions in this study. Density ranges are defined with the use of layer-average potential temperature and salinity at the hydrographics sections shown by Sloyan & Rintoul (2001) (See their Table 2).

Water masses	σ_2
Surface/thermocline water (STW)	– 34.70
Subantarctic Mode Water (SAMW)	34.70 – 35.75
Antarctic Intermediate Water (AAIW)	35.75 – 36.30
Upper Circumpolar Deep Water (UCDW)	36.30 – 36.86
Lower Circumpolar Deep Water (LCDW)	36.86 – 36.98
Antarctic Bottom Water (AABW)	36.98 –

by density in this study (Table 2). Thus, $\Omega(y, \sigma)$ denotes a water mass trans-
formation rate in the current study. A positive value denotes transformation
toward a denser water mass. $\psi(y, \sigma)$ is called a meridional streamfunction
here. It is an annual average of meridional streamfunctions at all snap-
shots which are calculated with the use of potential temperature, salinity
and meridional velocity. $\Omega(y, \sigma)$ is also calculated as an annual average of
water mass transformation rates at all snapshots which are calculated with
the method described below. In a steady state, these water mass transfor-
mation rate and meridional streamfunction coincide with each other.

$\Omega(y, \sigma)$ can be calculated from the thermodynamic equations. The tem-
poral and spatial derivatives of density, σ , are described as

$$\frac{\partial \sigma}{\partial t} = \alpha^* \frac{\partial \theta}{\partial t} + \beta^* \frac{\partial s}{\partial t}, \quad (2)$$

$$\nabla \sigma = \alpha^* \nabla \theta + \beta^* \nabla s, \quad (3)$$

where θ and s denote potential temperature and salinity, and α^* and β^* are

220 defined as: $\alpha^* \equiv \partial\sigma/\partial\theta$ and $\beta^* \equiv \partial\sigma/\partial s$. Note that $\nabla = (\partial/\partial x, \partial/\partial y, \partial/\partial z)$.
 221 Advection-diffusion equations for potential temperature and salinity in a
 222 model are described as:

$$223 \quad \frac{D\theta}{Dt} = \mathcal{D}(\theta) + q_\theta, \quad (4)$$

$$224 \quad \frac{Ds}{Dt} = \mathcal{D}(s) + q_s, \quad (5)$$

226 where D/Dt denotes the material derivative and $\mathcal{D}(\phi)$ is the contribution of
 227 diffusion³ to the tendency of ϕ . q_θ and q_s are source/sink terms of potential
 228 temperature and salinity, respectively. They include effects of sea surface
 229 and lateral boundary conditions, and convective adjustment in the current
 230 study. Note that $\mathcal{D}(\phi)$ includes the effect of numerical diffusion associated
 231 with an advection scheme. The advection-diffusion equation for density can
 232 be derived as:

$$233 \quad \frac{D\sigma}{Dt} = \alpha^* \mathcal{D}(\theta) + \beta^* \mathcal{D}(s) + \alpha^* q_\theta + \beta^* q_s$$

$$234 \quad = \mathcal{D}(\sigma) + \mathcal{C} + \alpha^* q_\theta + \beta^* q_s \quad (6)$$

236 The contribution of diffusion terms in the first line is decomposed into the
 237 diffusion term of density ($\mathcal{D}(\sigma)$) and the cabbeling⁴ term ($\mathcal{C}$) in the second
 238 line (e.g., Urakawa & Hasumi, 2009). The effects of diffusion and cabbeling

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³It is known that the biharmonic diffusion could cause an anti-diffusive tracer flux (Delhez & Deleersnijder, 2007). However, UH confirm that such an anti-diffusive process is scarcely found in our model. So, we use the term “diffusion” here and hereafter in the current study.

⁴Cabbeling is calculated as follows: $\mathcal{C} = -\nabla\alpha \cdot \mathcal{F}(\theta) - \nabla\beta \cdot \mathcal{F}(s)$, where $\mathcal{F}(\phi)$ denotes a diffusive flux of an arbitrary tracer ϕ .

239 can be decomposed into the explicit ones $(\mathcal{D}_{exp}, \mathcal{C}_{exp})$ and numerical ones
 240 $(\mathcal{D}_{num}, \mathcal{C}_{num})$. Then, the tendency equation for density can be rewritten as:

$$241 \quad \frac{D\sigma}{Dt} = \mathcal{D}_{exp}(\sigma) + \mathcal{D}_{num}(\sigma) + \mathcal{C}_{exp} + \mathcal{C}_{num} + \alpha^* q_\theta + \beta^* q_s. \quad (7)$$

242 The material derivative of density on the left hand side can be described as:

$$243 \quad \frac{D\sigma}{Dt} = \omega_\sigma \nabla_\perp \sigma, \quad (8)$$

244 where ω_σ is the diapycnal velocity across S_σ and $\nabla_\perp \sigma$ is the gradient of
 245 density in the direction normal to S_σ . The water mass transformation rate
 246 Ω can be calculated by integrating ω_σ over S_σ :

$$247 \quad \Omega = \iint_{S_\sigma} (\mathcal{D}_{exp}(\sigma) + \mathcal{D}_{num}(\sigma) + \mathcal{C}_{exp} + \mathcal{C}_{num} + \alpha^* q_\theta + \beta^* q_s) (\nabla_\perp \sigma)^{-1} dS. \quad (9)$$

248 Ω is a function of y and σ but its description is omitted hereafter in this
 249 section. The right hand side of this equation can be transformed with the
 250 use of a generalized form of Leibnitz theorem as (Iudicone et al., 2008):

$$251 \quad \Omega = \frac{\partial}{\partial \sigma} \iiint_V (\mathcal{D}_{exp}(\sigma) + \mathcal{D}_{num}(\sigma) + \mathcal{C}_{exp} + \mathcal{C}_{num} + \alpha^* q_\theta + \beta^* q_s) dV \\ 252 \quad = \Omega_{DE} + \Omega_{DN} + \Omega_{CE} + \Omega_{CN} + \Omega_Q. \quad (10) \\ 253$$

254 The first to fourth terms in the second line correspond to those in the paren-
 255 theses of the first line in the same order and Ω_Q corresponds to the remaining
 256 two terms.

257 We can easily calculate every term in the second line of Eq. 10 except
 258 for water mass transformation rates due to numerical diffusion (Ω_{DN} and
 259 Ω_{CN}) with the use of representations in the model. Numerical cabbeling

term Ω_{CN} can be directly estimated with the use of the method in Appendix A. Utilizing Eq. 1, we can indirectly diagnose Ω_{DN} by the following equation:

$$\Omega_{DN} = \psi + \frac{\partial V}{\partial t} - \Omega_{DE} - \Omega_{CE} - \Omega_{CN} - \Omega_Q. \quad (11)$$

Appendix B shows discrete representations of these terms.

Water masses tend to move not along an isopycnal surface but along a neutral surface. Water mass transformation rates diagnosed with the use of density does not perfectly represent what actually goes in a model and contains errors. This might affect conclusions of this study. We also conduct the idealized experiments with a linear EOS as already noted. Unique density surfaces can be defined there and do not suffer from the errors noted above. We confirm that our conclusions do not change there quantitatively and qualitatively.

4. Results of the Southern Ocean model

4.1. Reproducibility of the experiments HIGH and LOW

The meridional streamfunction ψ shows three major cells in the Southern Ocean: a subtropical cell ($\sigma_2 < 35$), a subpolar cell ($35 < \sigma_2 < 36.7$) and a deep cell ($36.7 < \sigma_2$) (Fig. 1). The subpolar cell is associated with the NADW formation and the deep cell is associated with the AABW formation. The spatial pattern and the magnitude of the subtropical cell differ little between the experiments HIGH and LOW. The magnitude of the subpolar cell is stronger in the experiment HIGH, whereas that of the deep cell is weaker therein.

This meridional streamfunction in each experiment largely differs from the net water mass transformation rate Ω , which is calculated as a sum of ψ

Figure 1: ψ in (a) the experiment HIGH and (b) the experiment LOW. The contour interval is 4 Sv. Positive for counter-clockwise.

and the volume inflation term $\partial V/\partial t$ (Fig. 2). The difference is especially large in the dense water mass classes ($\sigma_2 > 36.8$). There is another clockwise cell between the deep cell and the ocean bottom. This cell means that the bottom water is modified into lighter water at the rate of about 40 and 25 Sv in the experiments HIGH and LOW, respectively.

Ω seemingly indicates a poleward transport of the densest water at the corresponding rate. This results from the fact that the meridional streamfunction can not strictly be defined in a non-steady state as already discussed by Marsh et al. (2000). ψ shows the correct meridional volume transport because it is calculated from the meridional velocity, but inferred diapycnal volume transports are generally incorrect. Likewise, Ω offers the right information about the diapycnal volume transport, but inferred meridional transports are incorrect. For example, the net water mass transformation in the experiment HIGH (LOW) shows that about 30 (9) Sv of dense wa-

298 ter is diapycnally transported toward the lighter water mass layer across the
 299 isopycnal surface of $37 \sigma_2$ to the south of 30°S . It does not mean that the cor-
 300 responding amount of the bottom water ($\sigma > 37$) is meridionally transported
 301 into the Southern Ocean at 30°S in our model. ψ shows that over 4 (8) Sv of
 302 the bottom water is transported northward at 30°S in the experiment HIGH
 303 (LOW) (Fig. 1). There is a large difference between them which amounts to
 304 about 34 (17) Sv in the experiment HIGH (LOW). Such a difference between
 305 ψ and Ω shows that the model is in a non-steady state. This is due to the
 306 short integration time. However, it is not an appropriate option to extend
 307 the model integration and achieve a steady state in order to fill this gap. As
 308 discussed in UH, this is not feasible from the standpoint of available numer-
 309 ical resources and leads to unrealistic distributions of potential temperature
 310 and salinity in this model. The experiments with the current model aim
 311 to quantitatively evaluate water mass transformation rates due to numerical
 312 diffusion in the realistic eddy-resolving simulations of the Southern Ocean.
 313 The realistic active tracer distributions and eddy activities are important
 314 for the current purpose. It is better to be based on such a non-steady but
 315 close-to-observed state in this study.

316 4.2. *Water mass transformation by explicit and numerical diffusions*

317 We separately calculate water mass transformation rates induced by ex-
 318 plicit and numerical diffusions. UH show that explicit horizontal biharmonic
 319 diffusion in our model leads to strong diapycnal diffusion in the experiment
 320 HIGH (Fig. 3a). It modifies a significant amount of LCDW and AABW into
 321 lighter waters. On the other hand, there is hardly any explicit horizontal
 322 diffusion in experiment LOW (Fig. 3b). Note that the contour interval is

Figure 2: The same as Fig. 1 but Ω . The contour interval is 5 Sv.

different between Fig. 3a and b. Its magnitude is less than 1 Sv in most of the density range and the negative peak value is about 1.3 Sv found in the AABW class.

We diagnose the water mass transformation rate induced by numerical diffusion (Ω_{DN}) with the use of Eq. 11. Figure 4 shows water mass transformation induced by the total horizontal diffusion⁵, Ω_D^H , which is the sum of Ω_{DN} and the horizontal part of Ω_{DE} (Ω_{DE}^H). In the experiment HIGH, the pattern of this water mass transformation differs little from that of Ω_{DE}^H but its magnitude slightly increases in the dense LCDW and AABW classes. The negative peak value is about -47 Sv for Ω_{DE}^H and about -53 Sv for Ω_D^H . Explicit

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⁵ Ω_{DN} cannot be decomposed into its horizontal and vertical parts. As is discussed in UH, however, the transformation rate due to numerical diffusion is quite similar to that due to horizontal diffusion. So, we believe that numerical diffusion occurs in the horizontal plane.

Figure 3: The same as Fig. 1 but Ω_{DE}^H . The contour interval is 5 Sv for (a) and 1 Sv for (b).

horizontal diffusion dominates in water mass transformation but numerical diffusion is not negligible in the experiment HIGH. On the other hand, Ω_D^H is considerably dissimilar to Ω_{DE}^H in the experiment LOW. There appears strong modification to lighter waters in the dense LCDW and AABW classes as found in the experiment HIGH. It indicates that the numerical diffusion well imitates the explicit diffusion in the experiment LOW. Generally speaking, numerical diffusion could be anti-diffusive. Such up-gradient fluxes of active tracers lead to negative cabbeling effect on the water mass transformation, that is to say modification to lighter waters by cabbeling. However, the numerical cabbeling scarcely shows such a negative cabbeling effect in our model (not shown). The SOM scheme is known to hardly cause an anti-diffusive flux (Morales Masqueda & Holloway, 2006).

The patterns of the water mass transformation due to the total diffusion

Figure 4: The same as Fig. 1 but the sum of Ω_{DN} and Ω_{DE}^H . The contour interval is 5 Sv.

in experiments HIGH and LOW are very similar. However, the magnitudes significantly differ from each other. UH discuss that strong modification of dense water in the experiment HIGH, which amounts up to 53 Sv, is too large for a realistic simulation of the Southern Ocean. The experiment LOW also shows the strong modification but it is quite moderate. UH show that this modification in the experiment LOW does not seem unrealistic when the insufficient dense water formation in the model is taken into account. The experiment LOW looks more realistic, whereas the experiment HIGH seems a little too diffusive.

Our two experiments HIGH and LOW seem to belong to two different regimes in terms of explicit horizontal diffusivity: one is a regime where explicit diffusion dominates but numerical diffusion is not negligible, and the other is a regime where numerical diffusion dominates and explicit diffusion is almost negligible. Water mass transformation induced by numerical diffu-

360 sion as well as explicit diffusion seems to significantly depend on an explicit
 361 horizontal diffusion coefficient. Such a dependency has not well been investi-
 362 gated by preceding studies. It is difficult to carry out a sensitivity study on
 363 explicit horizontal diffusivity with the use of our eddy-resolving model of the
 364 Southern Ocean because of its high numerical cost. So, we utilize an ideal-
 365 ized channel model and conduct numerical experiments with various explicit
 366 horizontal diffusion coefficients.

367 **5. Results of the channel model**

368 Figures 5a and b show the initial state of potential temperature and
 369 salinity for the channel model, respectively. A front exists around 40°S in
 370 both temperature and salinity profiles. Isothermal and isohaline surfaces are
 371 steeply inclined in the shallow region above 1000 m depth. Density also has
 372 a strong front around 42°S (Fig. 5c), although salinity inversely contributes
 373 to it. Strong baroclinicity of this initial condition leads to instability which
 374 generates mesoscale eddies in our channel model (Fig. 5d). These eddies
 375 meridionally transport heat and salinity and restratify the steep isopycnal
 376 front found in the initial state (Fig. 5e). The meridional streamfunction
 377 shows an anticlockwise circulation which pushes the isopycnal surface down
 378 in the south and pulls them up in the north (Fig. 5e). This streamfunction is
 379 not along the density surfaces because our short-run experiments are not in
 380 a steady state. Note that this does not mean that the corresponding amount
 381 of water masses crosses the isopycnal surfaces as mentioned above. Only one
 382 case is shown in Fig. 5 but such flattening of isopycnal surfaces is also found
 383 in all other cases.

Figure 5: Initial conditions for (a) potential temperature, (b) salinity and (c) σ_2 for channel experiments. The lower left panel (d) shows a snapshot of the experiment with a nonlinear EOS and without explicit horizontal diffusion and external forcings (one of short-run experiments) at 180 day. The shade denotes the sea surface height and arrows denote the horizontal velocity of the top layer. The lower right panel (e) shows the zonal average of density at the final state and the 20 annual average of meridional streamfunction (gray line).

384 5.1. Water mass transformation induced by numerical diffusion

385 Here, we show water mass transformation rates in the short-run exper-
 386 iments. Meridional velocities are zero at the northern and southern solid
 387 boundaries. A meridional streamfunction becomes zero at $y = 30^\circ S$. The
 388 water mass transformation rate due to numerical diffusion (Ω_{DN}) there can
 389 be diagnosed with the following equation.

$$390 \quad \Omega_{DN} = \frac{\partial V}{\partial t} - \Omega_{DE} - \Omega_{CE} - \Omega_{CN} - \Omega_Q \quad (12)$$

$$391 \quad = \frac{\partial V}{\partial t} - \Omega' \quad (13)$$

393 where Ω_Q denotes the transformation rate due to convective adjustment,
 394 which is almost equal to zero.

395 The volume inflation term ($\partial V/\partial t$) is comparable in magnitude to the
 396 sum of water mass transformation terms on the right hand of Eq. 12 (Ω') for
 397 $36.4 < \sigma_2 < 36.9$ in the experiment without explicit horizontal diffusion (Fig.
 398 6a). It means that Ω_{DN} , which is calculated as their difference, is almost zero
 399 there. Two vertical lines in Fig. 6 describe densities at 1000 and 3000 m
 400 depth in the virtual state where all isopycnal surfaces are adiabatically flat-
 401 tened, respectively. We define the region between these two lines as interior
 402 region. This result shows that numerical diffusion associated with the SOM
 403 scheme works little in the most part of the interior region compared with in
 404 the upper and lower boundary regions. Large numerical diffusion near the
 405 sea surface would result from the strong activity of mesoscale eddies. Nu-
 406 merical diffusion is also strong in the high density range ($\sigma_2 > 36.9$), which
 407 corresponds to the region around the sea bottom.

408 Comparison of Ω_{DN} among the experiments with different diffusivity

Figure 6: (a) Annual averages of the volume inflation term (green) and the water mass transformation rates due to explicit diffusion, explicit cabbeling, numerical cabbeling and convective adjustment (red) and numerical diffusion (blue) in the experiment with a non-linear EOS and without explicit horizontal diffusion. (b) The water mass transformation rate due to numerical diffusion. Five colored lines denote the experiments with a horizontal diffusion coefficient of 10^8 (cyan), 10^9 (green), 10^{10} (yellow) and $10^{11} \text{ m}^4 \text{ s}^{-1}$ (red) and the experiment without explicit horizontal diffusion (blue), respectively. Positive value denotes transformation toward the denser water mass. Two vertical dashed lines denote densities at 1000 and 3000 m depth in the state where all density surfaces are adiabatically flattened, respectively.

409 shows that a larger diffusion coefficient leads to less numerical diffusion (Fig.
 410 6b). This is remarkable in the high density range. Numerical diffusion is
 411 especially large in the three experiments with lower diffusivity in Fig. 6b
 412 and its magnitude is almost the same among them. It becomes significantly
 413 smaller with higher diffusivity (10^{10} and $10^{11}\text{m}^4\text{s}^{-1}$). Explicit diffusivity in
 414 the experiment LOW belongs to the former group and that in the experi-
 415 ment HIGH belongs to the latter. The results of the channel experiments are
 416 consistent with those of the Southern Ocean model. The volume inflation
 417 term $(\partial V/\partial t)$ differs little among these five experiments. This means that
 418 the change of Ω' ($= \Omega_{DE} + \Omega_{CE} + \Omega_{CN} + \Omega_Q$) corresponds to the reduction of
 419 Ω_{DN} with higher diffusivity. The change of Ω_{DE} dominates that of Ω' (not
 420 shown). Higher explicit diffusivity leads to larger Ω_{DE} which corresponds to
 421 smaller Ω_{DN} .

422 These analyses are conducted based on the density σ_2 . The use of single
 423 density more or less accompanies analytical errors (Iudicone et al., 2008).
 424 The conclusion should be validated by the experiments with a linear EOS.
 425 The extra sensitivity study is conducted with the following EOS:

$$426 \quad \sigma_L = \rho - 1000 = -0.2\theta + 0.8s. \quad (14)$$

427 The pattern of density distribution at the initial and final states is similar
 428 to that of Fig. 5c and e but the magnitude is smaller by around 9.5 kg m^{-3}
 429 below the 1000m depth. Errors in diagnosing the numerical cabbeling effect
 430 can be excluded in these experiments. The water mass transformation due

431 to numerical diffusion is calculated with the following equation.

$$432 \quad \Omega_{DN} = \frac{\partial V}{\partial t} - \Omega_{DE} - \Omega_Q \quad (15)$$

$$433 \quad = \frac{\partial V}{\partial t} - \Omega', \quad (16)$$

434

435 where $\Omega' = \Omega_{DE} + \Omega_Q$ in the linear experiments.

436 Figure 7a shows $\partial V/\partial t$, Ω' and Ω_{DN} in the experiment without explicit
 437 horizontal diffusion. The volume inflation term and Ω' are almost the same
 438 in magnitude in the most part of the ocean interior, which means small nu-
 439 merical diffusion. Ω_{DN} contributes a lot to water mass transformation in
 440 both boundary regions and its magnitude decreases with increase in an ex-
 441 plicit horizontal diffusion coefficient especially near the ocean bottom (Fig.
 442 7b). By contrast, explicit diffusion becomes larger with increase in explicit
 443 horizontal diffusivity (Fig. 7c). These five experiments are also divided into
 444 two groups: high diffusivity group ($10^{10}, 10^{11} \text{m}^4 \text{s}^{-1}$) in which explicit diffu-
 445 sion dominates and low diffusivity group ($0, 10^8, 10^9 \text{m}^4 \text{s}^{-1}$) where numerical
 446 diffusion dominates.

447 Figure 8 shows absolute values of Ω_{DE} and Ω_{DN} at $\sigma_L = 27.725$, where the
 448 transformation rates due to explicit and numerical diffusions most strongly
 449 appear, plotted against the explicit horizontal diffusivity (κ). $|\Omega_{DN}|$ amounts
 450 to about 1.1 Sv in the experiment without explicit horizontal diffusion. Its
 451 magnitude varies little among the experiments with low diffusivity ($\kappa \leq 10^9$)
 452 and decreases with rise of explicit diffusivity. Of course, $|\Omega_{DE}|$ increases with
 453 the explicit diffusivity. The vertical component accounts for about 0.3 Sv of
 454 $|\Omega_{DE}|$ and differs little among the all experiments. The horizontal component
 455 is very small or even almost zero in the low diffusivity experiments ($\kappa \leq 10^9$).

Figure 7: (a, b) The same as Fig. 6 but in the experiments with a linear EOS and without explicit horizontal diffusion. (c) The same as Fig. 6b but the water mass transformation rates induced by explicit diffusion in the experiments with a linear EOS.

Figure 8: Absolute values of water mass transformation rates induced by explicit diffusion (red), numerical diffusion (blue) and total diffusions (both explicit and numerical diffusions; black solid) across the isopycnal surface $27.725 \sigma_L$. The black dashed line denotes the vertical component of Ω_{DE} . The absolute value of Ω_{DN} is about 1.1 Sv in the experiment without horizontal diffusion.

456 Explicit diffusion dominates when κ exceeds $10^{10} \text{m}^4 \text{s}^{-1}$.

457 There seems to be three regimes as for the effect of diffusions. One is a
 458 “numerical diffusion” regime where the explicit horizontal diffusion scarcely
 459 works and the numerical diffusion dominantly accounts for water mass trans-
 460 formation due to diffusion. The second is a “transitional” regime where both
 461 the explicit and numerical diffusions comparably contribute. The last one
 462 is an “explicit diffusion” regime in which the explicit diffusion dominates in
 463 water mass transformation due to diffusion. Boundaries among them exist
 464 around $\kappa = 10^9$ and 10^{10} for the isopycnal surface of $27.725 \sigma_L$, respectively.
 465 The experiment LOW with the Southern Ocean model belongs to the numer-
 466 ical diffusion regime and the experiment HIGH falls between the transitional
 467 and explicit diffusion regimes. This classification into three regimes are con-
 468 sistent with the results of the Southern Ocean model that numerical diffusion

Figure 9: The same as Fig. 6a but in the long-run experiment. The black dashed denotes Ω_{DN} for the experiment shown in Fig. 6a (the blue line there).

dominates in the experiment LOW and explicit diffusion dominates in the experiment HIGH.

5.2. Deep stratification and numerical diffusion

Hill et al. (2012) also conduct channel experiments under an idealized configuration with the use of SOM scheme and find that numerical diffusion is negligibly small in the interior region. They also find that numerical diffusion is strong near the boundaries. This study is consistent with their study. On the other hand, the diffusivity near the abyss is obviously smaller than that near the sea surface in their study (see their Fig. 4). In contrast, Ω_{DN} in this study is larger near the sea bottom than near the sea surface. Water mass transformation induced by diffusions depends on diffusivity and spatial gradients of density. Because stratification is clearly weaker near the abyss, it is supposed that the diffusivity of numerical diffusion near the sea bottom is much larger than that near the sea surface in this study.

This strong numerical diffusion near the abyss plays an important role in simulating the Southern Ocean as already discussed and deserves further

485 investigations. The difference between these two studies might result from a
 486 difference of the integration period. Hill et al. (2012) drive their model with
 487 external forcings until the model is spun up to statistical equilibrium. So,
 488 we drive our channel model for 40 years with surface winds and buoyancy
 489 fluxes and analyze the last 1 year. The explicit horizontal diffusivity is set
 490 to zero. Ω_{DN} in the interior region is small as well as that in the short-
 491 run experiment (blue and black dashed lines in Fig. 9, respectively). It is
 492 comparable or even larger than that in the short-run experiment near the
 493 sea surface. This is because thermal and haline fronts near the surface are
 494 maintained by damping sea surface temperature and salinity to climatology
 495 and this leads to a stronger activity of eddies. On the other hand, Ω_{DN}
 496 near the ocean bottom significantly decreases in the long-run experiment.
 497 Its magnitude is comparable with that in the interior.

498 The magnitude of numerical diffusion depends on velocity and the spa-
 499 tial gradient of the tracer. Figure 10a shows annual averages of horizontally
 500 integrated mean and eddy kinetic energy (MKE and EKE) of the long-run
 501 experiment and the short-run experiment without explicit horizontal diffu-
 502 sion. Because the wind stress imposed in the long-run experiment supplies
 503 a large amount of kinetic energy, both MKE and EKE exceed those in the
 504 short-run experiment, respectively. However, this does not necessarily mean
 505 larger numerical diffusion in the long-run experiment than in the short-run
 506 experiment. For example, uniform tracer distributions will result in zero
 507 Ω_{DN} . In fact, there are no water masses denser than $37\sigma_2$ in the long-run
 508 experiment (Fig. 9). Such dense water masses are not produced in this area
 509 (at 49°E and from 50°S to 30°S). They are modified into lighter water masses

510 by strong numerical diffusion near the bottom and disappears after the long
511 integration period.

512   The strong numerical diffusion also results in smoother gradients of po-
513 tential temperature and salinity near the bottom (above the isopycnal sur-
514 face of $37 \sigma_2$) in the long-run experiment. The horizontal density gradient
515 there is smaller than that in the short-run experiment by a factor of about
516 5 (Fig. 10b). It is also smaller than the density gradient in the experiment
517 LOW which also suffers from large numerical diffusion near the bottom. The
518 deep stratification is an important key of the numerical diffusion around the
519 abyss. The dense bottom water formation at high latitudes leads to density
520 gradients near the bottom in the real ocean. The lack of this bottom wa-
521 ter supply results in weak deep stratification and small numerical diffusion
522 in the long-run experiment. Although the experiment LOW and the short-
523 run experiment also have the problem of insufficient dense water formation,
524 they retain realistic distributions of potential temperature and salinity due
525 to their short integration period. The density gradient in the short-run ex-
526 periment is close to that in the experiment LOW around 4000 m depth. We
527 believe that the short-run experiment better reproduces what occurs near
528 the bottom in the real ocean than our long-run experiment. As already dis-
529 cussed, applying high explicit diffusivity for less numerical diffusion leads to
530 unrealistically diffusive deep stratification. These results indicate that nu-
531 merical diffusion must be significantly large near the bottom as long as the
532 near bottom stratification is realistically simulated.

Figure 10: (a) Annual averages of horizontally integrated MKE (solid) and EKE (dashed) in the long-run experiment (blue) and the short-run experiment without explicit horizontal diffusion (red). (b) Absolute values of the horizontal σ_2 gradient ($|\nabla_h \sigma_2|$) in the short-run experiment without explicit horizontal diffusivity (red), the long-run experiment (blue) and the experiment LOW (purple). They are annually and horizontally averaged. The gradient in the experiment LOW is calculated with data in the Southern Ocean (south to 30°S). Unit is $\text{kg m}^{-3} \text{ km}^{-1}$.

533 6. Conclusions and discussions

534 This study investigates the effect of numerical diffusion from the view-
535 point of water mass transformation. We conduct a set of idealized exper-
536 iments with a channel model. Numerical integration starts from zonally
537 uniform distributions of potential temperature and salinity, which are taken
538 from WGHC at 49°E and from 50°S to 30°S. The model is run for 1 year
539 without external forcings. These experiments show that numerical diffusion
540 is small in the ocean interior region. On the other hand, the experiment
541 without explicit diffusion shows that numerical diffusion is significantly large
542 near the sea surface and especially around the sea bottom. Because of its
543 short integration period, the model retains strong initial stratification near

544 the abyss, which leads to large water mass transformation due to numerical
545 diffusion there. Such a situation has not been focused on in earlier studies.

546 The sensitivity study against the explicit horizontal diffusivity shows that
547 the magnitude of these numerical diffusion effects tends to become small with
548 increase in the explicit diffusivity. Of course, explicit diffusion tends to be
549 strong with increase in the explicit diffusivity. There are three regimes as to
550 the effects of diffusions. One is the numerical diffusion regime, where explicit
551 diffusion hardly works. Numerical diffusion associated with an advection
552 scheme dominantly diffuses fine structures created by mesoscale eddies there,
553 and its magnitude varies little with explicit horizontal diffusivity. So, the
554 water mass transformation rate due to total diffusion remains almost constant
555 there. Another regime is the transitional regime, where both explicit and
556 numerical diffusions are important. Increase in explicit horizontal diffusivity
557 leads to increase (decrease) in the explicit (numerical) diffusion there. The
558 last regime is the explicit diffusion regime. The total diffusion effect there
559 becomes significantly larger than those in the other two regimes. Explicit
560 diffusion dominates in this total diffusion effect and numerical diffusion is
561 almost negligible there. Some readers might doubt that the small effect of
562 numerical diffusion results from cancellation between positive and negative
563 water mass transformation rates induced by numerical diffusion because it is
564 not sign-definite. This study also investigates a sensitivity of the cabbeling
565 effect, which is positive definite⁶, on explicit horizontal diffusivity (Appendix

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⁶Strictly speaking, the cabbeling effect induced by the biharmonic diffusion is not necessarily positive definite (Delhez & Deleersnijder, 2007). As already noted in the previous footnote, however, it is confirmed that the effect of such an anti-diffusive process is small

566 C). It shows consistent results with those noted above and suggests that the
567 small numerical diffusion does not result from cancellation between positive
568 and negative transformation rates.

569 The Southern Ocean model would not suffer from numerical diffusion if
570 we use an explicit horizontal diffusion coefficient which belongs to the ex-
571 plicit diffusion regime. However, our two experiments under the realistic
572 configuration show that it is not a better option for the Southern Ocean
573 modeling. The two experiments, LOW and HIGH, belong to the numerical
574 diffusion and the transitional regimes, respectively. Results of both experi-
575 ments are consistent with those of channel experiments. That is, numerical
576 diffusion dominates in the experiment LOW and works little in the experi-
577 ment HIGH. The pattern of the water mass transformation due to the total
578 diffusion resembles each other, which indicates that the numerical diffusion
579 well duplicates the explicit diffusion. The total diffusion is especially large in
580 the high density range, which corresponds to the region near the bottom, in
581 both experiments. This leads to reduction in the volume of AABW by trans-
582 forming it to the lighter water mass. Although inadequate DSW formation
583 around Antarctica is mainly blamed for this AABW loss, UH discuss that the
584 water mass transformation due to the total diffusion might be too diffusive
585 in the experiment HIGH for a realistic simulation of the Southern Ocean. It
586 is naturally expected that diffusivity in the explicit diffusion regime would
587 also be too diffusive for our eddy-resolving model of the Southern Ocean.

588 We cannot find the most adequate diffusivity for the Southern Ocean

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and its cabbeling effect is positive definite.

589 models with horizontal resolution around 0.1 degree from the results of the
 590 current study. However, it seems robust that an explicit diffusion coefficient
 591 in the explicit diffusion regime is inadequate for a realistic simulation of the
 592 Southern Ocean with such models. This indicates that such models are not
 593 free from numerical diffusion associated with advection schemes even if the
 594 models adopt high-accuracy advection schemes such as the SOM scheme.
 595 So, we must adequately diagnose its effect when analyzing outputs of such
 596 high-resolution models. The current study proposes a method to evaluate
 597 numerical diffusion in terms of water mass transformation, which has been
 598 used in this study to reach the conclusion noted above. Water mass trans-
 599 formation due to numerical diffusion (including cabbeling) can be evaluated
 600 as the difference between the net water mass transformation ($= \psi + \partial V / \partial t$)
 601 and the explicit water mass transformation which can be directly calculated
 602 ($\Omega_{DE} + \Omega_{CE} + \Omega_Q$ in Eq. 10). Numerical cabbeling (Ω_{CN}) can be estimated
 603 with the use of the method proposed by UH (also shown in Appendix A).
 604 Subtracting Ω_{CN} from the numerical diffusion effect calculated above, we
 605 can get the water mass transformation due to numerical diffusion of density
 606 (Ω_{DN}). This method is a useful tool for investigating an eddying ocean with
 607 a numerical model.

608 Ilicak et al. (2012) show that applying high viscosity significantly reduces
 609 numerical diffusion. Of course, it is desirable to reduce its effect because it
 610 is difficult to analyze as is easily seen from the discussion above. However,
 611 high viscosity leads to reduction of eddy kinetic energy (EKE). Thoppil et al.
 612 (2011) show that EKE in a model with horizontal resolution of $1/12.5^\circ$ is
 613 deficient and accounts for only about 80 % of observational estimates. So,

614 simply applying high viscosity might not be an appropriate option for a
615 realistic simulation of the ocean.

616 Thoppil et al. (2011) also find that EKE increases to the level consis-
617 tent with observation by doubling horizontal resolution. Numerical diffusion
618 must have a direct relationship with other parameters such as grid spacing
619 and time step sizes along with scales of velocities and tracer gradients that
620 are discussed in the current study. However, it is doubtful that such finer
621 resolution could lead to reduction of numerical diffusion. Higher resolution
622 results in representing finer distributions of tracers and more rapid velocity
623 fields, which lead to stronger numerical diffusion. If the model's resolution is
624 high enough that resolved velocity and length scales do not differ from those
625 of sub-grid scale processes, increase of resolution would lead to reduction of
626 numerical diffusion. Such resolution will be at least finer than sub-mesoscale.
627 It will be important in the regime of models with such finer resolutions to
628 pursue the relationship between numerical diffusion and grid spacing and
629 time step sizes, which cannot be found in the current study because of single
630 horizontal resolution and time step size of the experiments. For the time
631 being, however, such a high resolution modeling seems not to be feasible for
632 studies of the ocean general circulation. So, we need to properly evaluate the
633 numerical diffusion when analyzing outputs of high resolution models.

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647 **Appendix A. A method for diagnosing the effect of numerical** 648 **cabbeling**

649 The advection-diffusion equations for potential temperature and salinity
650 in a model are described as:

$$651 \quad \frac{\partial \theta}{\partial t} = -\mathcal{A}(\theta) + \mathcal{D}(\theta), \quad (\text{A.1})$$

$$652 \quad \frac{\partial s}{\partial t} = -\mathcal{A}(s) + \mathcal{D}(s), \quad (\text{A.2})$$

654 where $\mathcal{A}(\phi)$ is the advection term for an arbitrary tracer ϕ in a model, and
655 the source/sink terms of potential temperature and salinity (q_θ and q_s in Eqs.
656 4 and 5 are neglected). This term can be decomposed into the contributions
657 from true advection and numerical diffusion⁷ as:

$$658 \quad -\mathcal{A}(\phi) = -\mathbf{u} \cdot \nabla \phi + \nabla \cdot \mathbf{F}(\phi), \quad (\text{A.3})$$

---

⁷The term “numerical errors” might be more appropriate for this effect but the word
“numerical diffusion” is used here and also in the main text.

659 where $\mathbf{F}(\phi)$ denotes the flux of ϕ due to numerical diffusion. The contribution
 660 of these advection terms to the tendency of density can be described as:

$$\begin{aligned}
 661 \quad -\alpha^* \mathcal{A}(\theta) - \beta^* \mathcal{A}(s) &= -\mathbf{u} \cdot (\alpha^* \nabla \theta + \beta^* \nabla s) + \alpha^* \nabla \cdot \mathbf{F}(\theta) + \beta^* \nabla \cdot \mathbf{F}(s) \\
 662 \quad &= -\mathbf{u} \cdot \nabla \sigma + \alpha^* \nabla \cdot \mathbf{F}(\theta) + \beta^* \nabla \cdot \mathbf{F}(s). \quad (\text{A.4}) \\
 663
 \end{aligned}$$

664 The last two terms in the second line can be decomposed into the density
 665 diffusion term ($\nabla \cdot \mathbf{F}'(\sigma)$) and the cabbeling term ($\mathcal{C}_{num}$) as in the case of the
 666 physical diffusion.

$$\begin{aligned}
 667 \quad \alpha^* \nabla \cdot \mathbf{F}(\theta) + \beta^* \nabla \cdot \mathbf{F}(s) &= \nabla \cdot [\alpha^* \mathbf{F}(\theta) + \beta^* \mathbf{F}(s)] - [\nabla \alpha^* \cdot \mathbf{F}(\theta) + \nabla \beta^* \cdot \mathbf{F}(s)] \\
 668 \quad &= \nabla \cdot \mathbf{F}'(\sigma) + \mathcal{C}_{num}. \quad (\text{A.5})
 \end{aligned}$$

670 Here, we describe the sum of true advection and numerical diffusion terms
 671 of density as follows:

$$672 \quad -\mathbf{u} \cdot \nabla \sigma + \nabla \cdot \mathbf{F}'(\sigma) = -\mathcal{A}'(\sigma). \quad (\text{A.6})$$

673 The advection-diffusion equation for density can be rewritten as:

$$\begin{aligned}
 674 \quad \frac{\partial \sigma}{\partial t} &= -\alpha^* \mathcal{A}(\theta) - \beta^* \mathcal{A}(s) + \mathcal{D}(\sigma) + \mathcal{C} \\
 675 \quad &= -\mathcal{A}'(\sigma) + \mathcal{C}_{num} + \mathcal{D}(\sigma) + \mathcal{C}. \quad (\text{A.7}) \\
 676
 \end{aligned}$$

677 Therefore, the cabbeling effect associated with the numerical diffusion can
 678 be diagnosed by:

$$679 \quad \mathcal{C}_{num} = \mathcal{A}'(\sigma) - [\alpha^* \mathcal{A}(\theta) + \beta^* \mathcal{A}(s)]. \quad (\text{A.8})$$

680 By substituting this term in Eq. 10, we can diagnose the cabbeling effect on
 681 the water mass transformation.

682 The way to evaluate the advection term of density $\mathcal{A}'(\sigma)$ has not been
 683 clarified. We adopt a way to calculate advected density ($\sigma_a = \sigma(\theta_a, s_a)$) by
 684 substituting advected potential temperature and salinity (θ_a and s_a) into the
 685 EOS. Here, we describe the true advected potential temperature and salinity
 686 as θ_t and s_t . There holds the following relationships between θ_a and θ_t , and
 687 s_a and s_t :

$$688 \quad \theta_a = \theta_t + \delta\theta, \quad (\text{A.9})$$

$$689 \quad s_a = s_t + \delta s, \quad (\text{A.10})$$

691 where $\delta\theta$ and δs denote differences between the two advected variables. Note
 692 that terms in the right hand sides of these equations are all unknown. We
 693 can describe the tracer flux due to numerical diffusion in the Eq. A.3 as
 694 follows.

$$695 \quad \mathbf{F}(\theta) = -\mathbf{u} \cdot \delta\theta, \quad (\text{A.11})$$

$$696 \quad \mathbf{F}(s) = -\mathbf{u} \cdot \delta s. \quad (\text{A.12})$$

698 Then, $\mathcal{A}'(\sigma)$ in Eq. A.6 can be described as follows.

$$699 \quad \begin{aligned} \mathcal{A}'(\sigma) &= \mathbf{u} \cdot \nabla \sigma_t - \nabla \cdot [\alpha^* \mathbf{F}(\theta) + \beta^* \mathbf{F}(s)] \\ &= \mathbf{u} \cdot \nabla (\sigma_t + \alpha^* \delta\theta + \beta^* \delta s). \end{aligned} \quad (\text{A.13})$$

702 This equation contains three unknown variables (σ_t , $\delta\theta$ and δs). On the other
 703 hand, the Taylor series of the true advected density ($\sigma_t = \sigma(\theta_t, s_t)$) around
 704 (σ_a, s_a) is

$$705 \quad \sigma_t = \sigma_a - \alpha^*(\theta_a, s_a) \delta\theta - \beta^*(\theta_a, s_a) \delta s + \mathcal{O}(\delta^2), \quad (\text{A.14})$$

706 where δ denotes a scale of $\delta\theta$ and δs . We can approximately evaluate $\mathcal{A}'(\sigma)$
 707 with the errors that are quadratic in the scale δ as follows.

$$708 \quad \mathcal{A}'(\sigma) \sim \mathbf{u} \cdot \nabla \sigma_a. \quad (\text{A.15})$$

709 Note that we assume that the difference between α^* (β^*) in Eq. A.13 and
 710 $\alpha^*(\theta_a, s_a)$ ($\beta^*(\theta_a, s_a)$) is negligible.

711 **Appendix B. Discrete representations**

712 Figure B.11 shows a schematic of the model grid system. A black point
 713 denotes a tracer point. The number of grids in the x, y and z directions
 714 are NX , NY and NZ , respectively. The volume inflation term ($\partial V/\partial t$) is
 715 calculated from profiles of potential temperature and salinity at the initial
 716 ($t = 0$) and final ($t = T$) states. A volume of each grid is defined as $v_{i,j,k} \equiv$
 717 $\Delta x \Delta y \Delta z$. We consider N isopycnal layers whose boundaries are described
 718 as σ_n ($0 \leq n \leq N$). We can calculate the volume below the isopycnal surface
 719 S_{σ_n} , whose density is σ_n , and from the southern boundary of the model to
 720 the grid of $j = ny$ as follows.

$$721 \quad V_{ny+\frac{1}{2},n,t} = \sum_{k=1}^{NZ} \sum_{j=1}^{ny} \sum_{i=1}^{NX} v_{i,j,k} U(\sigma_{i,j,k,t} - \sigma_n), \quad (\text{B.1})$$

722 where

$$723 \quad U(x) = \begin{cases} 1 & \text{if } x > 0, \\ 0 & \text{if } x \leq 0. \end{cases} \quad (\text{B.2})$$

724 The volume inflation term is diagnosed as

$$725 \quad \left. \frac{\partial V}{\partial t} \right|_{ny+\frac{1}{2},n} = \frac{V_{ny+\frac{1}{2},n,T} - V_{ny+\frac{1}{2},n,0}}{T}. \quad (\text{B.3})$$

Figure B.11: A schematics for model grids.

726 ψ is defined as an annual average of meridional streamfunctions at all
 727 snapshots. It is calculated as follows.

$$728 \quad \psi_{ny+\frac{1}{2},n} = -\frac{1}{NT} \sum_{m=1}^{NT} \sum_{k=1}^{NZ} \sum_{j=1}^{ny} \sum_{i=1}^{NX} F_{i,j+\frac{1}{2},k,t_m}^v U \left(\sigma_{i,j+\frac{1}{2},k,t_m} - \sigma_n \right), \quad (\text{B.4})$$

729 where NT is the number of snapshots, $F_{i,j+\frac{1}{2},k,t_m}^v$ is the northward volume
 730 transport across the northern boundary of the grid (i, j, k) at $t = t_m$ and
 731 $\sigma_{i,j+\frac{1}{2},k,t_m} = (\sigma_{i,j,k,t_m} + \sigma_{i,j+1,k,t_m})/2$.

732 Ω is also calculated as an annual average of water mass transformation
 733 rates at all snapshots. For the ease of introduction, we consider an advection-
 734 diffusion equation of density without numerical diffusion, cabbeling and other
 735 source/sink terms. There remains a convergence/divergence term of explicit
 736 diffusion fluxes across the lateral boundaries of the grid box, $\mathcal{D}_{i,j,k,t_m}(\sigma)$. We
 737 can calculate the water mass transformation rate induced by explicit diffusion
 738 with the use of Eq. 10. The volume integration part is described as follows.

$$739 \quad G_{ny+\frac{1}{2},n,t_m} = \sum_{k=1}^{NZ} \sum_{j=1}^{ny} \sum_{i=1}^{NX} \mathcal{D}_{i,j,k,t_m}(\sigma) U(\sigma_n - \sigma_{i,j,k,t_m}). \quad (\text{B.5})$$

740 So, the water mass transformation rate can be calculated as

$$741 \quad \Omega_{ny+\frac{1}{2},n+\frac{1}{2}} = \frac{1}{NT} \sum_{m=1}^{NT} \frac{G_{ny+\frac{1}{2},n+1,t_m} - G_{ny+\frac{1}{2},n,t_m}}{\sigma_{n+1} - \sigma_n}. \quad (\text{B.6})$$

742 Since the water mass transformation rate is defined on different isopycnal
 743 surfaces from the volume inflation term and meridional streamfunction, the
 744 latter two need to be interpolated on the middle of the isopycnal surfaces
 745 on which the volume inflation term and the meridional streamfunction are
 746 calculated.

747 Each grid box belongs to only one density layer according to the density
 748 at its center in the current study. This is a simplest way to convert variables
 749 from a z -coordinate to a density-coordinate. Getzlaff et al. (2010) shortly
 750 discuss such a method of conversion and they adopt an improved method.
 751 We need to check whether evaluation of this term is sensitive to its method
 752 for calculation. Figure B.12 compares water mass transformation rates and
 753 $\partial V/\partial t$ calculated with the two methods. There is little difference between
 754 Ω_{DN} diagnosed with the use of two methods. The conclusion of the current
 755 study does not depend on the way to calculate water mass transformation
 756 rates, meridional overturning circulation and the volume inflation term.

757 **Appendix C. The cabbeling effect**

758 The cabbeling effect is another indicator of the magnitude of diffusions.
 759 The explicit cabbeling becomes larger and the numerical cabbeling becomes
 760 smaller with higher explicit diffusivity (Fig. C.13a and b). Figure C.13c
 761 shows the water mass transformation rates across the isopycnal surface 35.75
 762 σ_2 . The red and blue lines denote the effect of explicit cabbeling and numer-
 763 ical cabbeling (Ω_{CE} and Ω_{CN}), respectively. The black line shows the sum of
 764 both effects. The dashed line denotes the vertical component of Ω_{CE} . The
 765 numerical diffusion, transitional and explicit diffusion regimes are clearly

Figure B.12: The same as (a) Fig. 6a and (b) Fig. 9. $\Omega' = \Omega_F + \Omega_{DE} + \Omega_{CE} + \Omega_{CN} + \Omega_Q$ ($\Omega_F = 0$ in (a)). Ω_{DN} , $-\Omega'$ and $\partial V / \partial t$ calculated with the method proposed by Getzlaff et al. (2010) (GNO10) are also shown with dotted blue, red and green lines with points, respectively.

766 found there. Boundaries among these regimes are found around 10^9 and
 767 $10^{10} \text{ m}^4 \text{ s}^{-1}$, respectively. Ω_{CE} (Ω_{CN}) becomes large (small) with increase in
 768 explicit horizontal diffusivity. Ω_{CN} differs little for $\kappa < 10^9$. The same holds
 769 in almost all of other density surfaces as expected from Fig. C.13a and b.

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Figure C.13: (a, b) The same as Fig. 6b but the water mass transformation rates due to cabbeling associated with the explicit diffusion (Ω_{CE}) and due to cabbeling associated with numerical diffusion (Ω_{CN}), respectively. Note that intervals for the horizontal axis are uniform here. The black dashed lines here are same as those in Fig. 6. (c) Values of Ω_{CE} (red) and Ω_{CN} (blue) at $35.75 \sigma_2$ in the channel experiments. Sum of them is described by a black solid line. The transformation rate due to numerical cabbeling is 0.4 Sv in the experiment with no explicit diffusion coefficient. A black dashed line shows the water mass transformation rate due to cabbeling induced by vertical diffusion.

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