A refined integro-surface energy-based model for vibration of magnetically actuated double-nanowire-systems carrying electric current

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Abstract

A novel surface energy-based model is developed to examine more precisely vibrations of current-carrying double-nanowire-systems immersed in a longitudinal magnetic field. Using Biot-Savart and Lorentz laws, a more refined version of interwire interactional magnetic forces is presented. By employing Rayleigh beam theory, the equations of motion are derived. In fact, these are coupled integro-differential equations which are more accurate with respect to those of the previously developed models. For simply supported and clamped nanosystems, governing equations are analyzed via assumed mode method. The effects of interwire distance, slenderness ratio, electric current, magnetic field strength, and surface effect on the fundamental frequency are addressed carefully. The obtained results display the importance of exploiting the refined model for vibration analysis of nanosystems with low interwire distance, high electric current, and high magnetic field strength.

Keywords: Transverse vibrations; Double-nanowire-systems; Direct electric current; Longitudinal magnetic field; Surface energy effect; Assumed mode method.

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1. Introduction

The assembly of nanowires (NWs) into nanodevices and nanoscale circuits brings about diverse applications in nanoelectronics [1, 2], nanogenerators [3–5], nanophotonics [6–8], and nano-sized sensors [9–11]. For most of these purposes, electric currents would pass through the NWs and they would interact dynamically due to the resulted magnetic fields. As a result, realizing true vibrations of group of current-carrying NWs helps us to design them optimally for the desired functionality, serviceability, and safety. A preliminary step towards this goal is to examine vibrations of double current-carrying nanowires (DCCNWs) as the basic building block of ensembles of nanowires. To generalize the problem, a longitudinal magnetic field is also considered and it is noted that magnetically actuated NWs carrying current could also have potential applications in the upcoming nanoscale magnetometers. Thereby, this paper deals on vibrations of magnetically actuated DCCNWs.

For an individual current-carrying NW (CCNW) subjected to a longitudinal magnetic field, any lateral vibration of the nanostructure leads to exertion of a magnetic force in which could endanger its stability depends on the magnitudes of the electric current, strength of the magnetic field, and geometry properties of the NW [12–14]. For magnetically actuated DCCNWs, the scenario of vibrations and instabilities would be completely different from that of the individual NWs. When an electric current passes through a double-nanowire-system, each nanowire is actuated by the generated magnetic field of its adjacent wire. The induced magnetic fields at the locations of the deformed NWs could be evaluated using Biot-Savart’s law whereas the exerted magnetic forces on them are commonly computed via Lorentz’s law. In the previous explorations [15, 16], vibrations and dynamic instability of lengthy DCCNWs with and without application of magnetic field were studied. Since the length of the NWs was large enough, their bending rigidities were provoked and the exerted magnetic field could be easily evaluated based on the infinite length assumption. These simplifications in the suggested models would not be satisfactory when examination of vibrations of a finite length magnetically affected DCCNWs is of concern. One of the chief

objectives of this study is to develop a more sophisticated model to take into account both bending rigidity in the equations of motion and the NWs' finite length in the formulations of the interwire dynamic magnetic forces.

As the dimensions of the solid structure decrease, the ratio of the surface area to the bulk volume increases. For nanoscale structures, the surface energy is high enough that its effect cannot be ignored in their total potential energy. The classical continuum theory cannot explain such a phenomenon since it is basically developed for mechanical behavior of macroscale structures which the surface energy can be safely excluded from their formulations. Gurtin and Murdoch [17–19] suggested an advanced elasticity theory to incorporate the surface energy into the equations of motion of the solid nanostructures. In constructing their model, it was displayed that the surface is a very thin layer with negligible thickness that adhered tightly to the underlying bulk zone. Therefore, the displacements and the resulting strain fields within the surface layer are the same as those of the bulk at its vicinity; however, the constitutive relations of the surface layer are basically different from those of the bulk as displayed by the Gurtin-Murdoch model [17–19]. Until now, the surface elasticity theory of Gurtin-Murdoch has been extensively implemented to inspect statics [20–25], vibrations [26–34], buckling and postbuckling [35–41] behaviors of nanoscale beams and nanowires.

In this article, free transverse vibrations of DCCNWs in the presence of a longitudinal magnetic field are going to be investigated numerically. Using Biot-Savart law, approximate and exact magnetic forces on the deformed nanowires carrying current are evaluated. By employing Gurtin-Murdoch elasticity theory in conjunction with the Rayleigh beam theory, the problem is formulated and analyzed via assumed mode method (AMM). For both simply supported and fully clamped ends, the capability of the approximate model in predicting the results of the exact model is displayed. Subsequently, the roles of the interwire distance, slenderness ratio, electric current, magnetic field strength, and surface energy effect on the free vibration behavior of the nanosystem are addressed. The suggested model based on

the exact formulations would provide useful insights for more accurate dynamic analysis of vertically aligned CCNWs.

2. Statement of the problem

Consider a system of double nanowires with length l_b and the interwire distance d which is used to carry electric current I_0 (see Fig. 1). The nanosystem is subjected to a longitudinal magnetic field of the form: $\mathbf{B} = B_0 \mathbf{e}_{x_1}$ or $\mathbf{B} = B_0 \mathbf{e}_{x_2}$ where B_0 denotes the strength of the magnetic field. The Cartesian coordinate systems associated with the nanowires have been demonstrated in Fig. 1. Both electric current and magnetic field strength do not vary with time. Herein, we are interested in exploring free vibrations of such a nanosystem in the context of the surface elasticity theory of Gurtin-Murdoch. To this end, the interactional dynamic magnetic forces on the deformed nanowires should be accurately evaluated.

3. The interwire interactional magnetic forces: Approximate vs. exact formulations

The electric current passes through the j th deformed CCNW is given by [12–14]:

$$\mathbf{I}_j = I_0 \left(\mathbf{e}_{x_j} + \frac{\partial u_{y_j}}{\partial x_j} \mathbf{e}_{y_j} + \frac{\partial u_{z_j}}{\partial x_j} \mathbf{e}_{z_j} \right); \quad j = 1, 2, \quad (1)$$

where u_{y_j} and u_{z_j} denote the transverse displacements of the j th CCNW along the y axis and the z axis, respectively. Each CCNW is actuated by the externally applied longitudinal magnetic field as well as by the magnetic field of its adjacent deformed CCNW. The total magnetic field at the location of the j th CCNW is calculated using the Biot-Savart law:

$$\mathbf{B}_j = B_0 \mathbf{e}_{x_j} + \frac{\mu_0}{4\pi} \int_0^{l_b} \frac{\mathbf{I}_{3-j} \times \mathbf{r}_{3-j}}{\|\mathbf{r}_{3-j}\|^3} ds_j, \quad (2)$$

where $\mu_0 = 4\pi \times 10^{-7} \frac{\text{Tm}}{\text{A}}$ is the permeability of free space, and $\|\cdot\|$ is the norm sign, and the position vector $\mathbf{r}_j$; $j = 1, 2$ is provided by: $\mathbf{r}_j = (-1)^{j-1}[(x_2 - x_1)\mathbf{e}_{x_1} + (d + \Delta u_y)\mathbf{e}_{y_1} + \Delta u_z\mathbf{e}_{z_1}]$ where $\Delta u_y = u_{y_2} - u_{y_1}$ and $\Delta u_z = u_{z_2} - u_{z_1}$. By employing the Lorentz's law, the magnetic force per unit length of the j th CCNW is calculated by: $\mathbf{f}_{m_j} = \mathbf{I}_j \times \mathbf{B}_j$; $j = 1, 2$.

3.1. An approximation formulation

Let's assume that the length of both CCNWs are large. Using Eq. (2), the magnetic fields at the location of deformed CCNWs take the following form:

$$\mathbf{B}_1 = \left(B_0 - \frac{\mu_0 I_0}{2\pi} \frac{\Delta w \frac{\partial v_2}{\partial x_2} - (d + \Delta v) \frac{\partial w_2}{\partial x_2}}{(d + \Delta v)^2 + (\Delta w)^2} \right) \mathbf{e}_{x_1} + \left(\frac{\mu_0 I_0}{2\pi} \frac{\Delta w}{(d + \Delta v)^2 + (\Delta w)^2} \right) \mathbf{e}_{y_1} - \left(\frac{\mu_0 I_0}{2\pi} \frac{d + \Delta v}{(d + \Delta v)^2 + (\Delta w)^2} \right) \mathbf{e}_{z_1}, \quad (3a)$$

$$\mathbf{B}_2 = \left(B_0 + \frac{\mu_0 I_0}{2\pi} \frac{\Delta w \frac{\partial v_1}{\partial x_1} - (d + \Delta v) \frac{\partial w_1}{\partial x_1}}{(d + \Delta v)^2 + (\Delta w)^2} \right) \mathbf{e}_{x_2} - \left(\frac{\mu_0 I_0}{2\pi} \frac{\Delta w}{(d + \Delta v)^2 + (\Delta w)^2} \right) \mathbf{e}_{y_2} + \left(\frac{\mu_0 I_0}{2\pi} \frac{d + \Delta v}{(d + \Delta v)^2 + (\Delta w)^2} \right) \mathbf{e}_{z_2}. \quad (3b)$$

By considering: $\frac{d + \Delta v}{(d + \Delta v)^2 + (\Delta w)^2} \approx \frac{1}{d} \left(1 - \frac{\Delta v}{d} + \left(\frac{\Delta v}{d} \right)^2 \right)$ and $\frac{\Delta w}{(d + \Delta v)^2 + (\Delta w)^2} \approx \frac{\Delta w}{d^2} \left(1 - \left(\frac{\Delta w}{d} \right)^2 \right)$, and then substituting these relations into Eqs. (3a) and (3b) through implementing the Lorentz's formulas, the exerted magnetic force on the j th laterally deformed CCNWs is evaluated as:

$$\mathbf{f}_{m_j} = f_{m_{x_j}} \mathbf{e}_{x_j} + f_{m_{y_j}} \mathbf{e}_{y_j} + f_{m_{z_j}} \mathbf{e}_{z_j}; \quad j = 1, 2, \quad (4)$$

where

$$f_{m_{x_1}} \approx \frac{\mu_0 I_0^2}{2\pi d} \left[\frac{\partial v_1}{\partial x_1} \left(1 + \frac{\Delta v}{d} + \left(\frac{\Delta v}{d} \right)^2 \right) - \frac{\partial w_2}{\partial x_2} \left(\frac{\Delta w}{d} \right) \left(1 - \left(\frac{\Delta w}{d} \right)^2 \right) \right], \quad (5a)$$

$$f_{m_{y_1}} \approx B_0 I_0 \frac{\partial w_1}{\partial x_1} - \frac{\mu_0 I_0^2}{2\pi d} \left[\frac{\partial v_1}{\partial x_1} \frac{\partial w_1}{\partial x_1} \left(\frac{\Delta w}{d} \right) \left(1 - \left(\frac{\Delta w}{d} \right)^2 \right) + \left(1 + \left(\frac{\partial w_1}{\partial x_1} \right)^2 \right) \left(1 + \frac{\Delta v}{d} + \left(\frac{\Delta v}{d} \right)^2 \right) \right], \quad (5b)$$

$$f_{m_{z_1}} \approx -B_0 I_0 \frac{\partial v_1}{\partial x_1} + \frac{\mu_0 I_0^2}{2\pi d} \left[\frac{\partial v_1}{\partial x_1} \frac{\partial w_1}{\partial x_1} \left(1 + \frac{\Delta v}{d} + \left(\frac{\Delta v}{d} \right)^2 \right) + \left(1 + \left(\frac{\partial v_1}{\partial x_1} \right)^2 \right) \left(\frac{\Delta w}{d} \right) \left(1 - \left(\frac{\Delta w}{d} \right)^2 \right) \right], \quad (5c)$$

and

$$f_{m_{x2}} \approx \frac{\mu_0 I_0^2}{2\pi d} \left[\frac{\partial v_2}{\partial x_2} \left(1 - \frac{\Delta v}{d} + \left(\frac{\Delta v}{d} \right)^2 \right) + \frac{\partial w_2}{\partial x_2} \left(\frac{\Delta w}{d} \right) \left(1 - \left(\frac{\Delta w}{d} \right)^2 \right) \right], \quad (6a)$$

$$f_{m_{y2}} \approx B_0 I_0 \frac{\partial w_2}{\partial x_2} + \frac{\mu_0 I_0^2}{2\pi d} \left[\frac{\partial v_2}{\partial x_2} \frac{\partial w_2}{\partial x_2} \left(\frac{\Delta w}{d} \right) \left(1 - \left(\frac{\Delta w}{d} \right)^2 \right) - \left(1 + \left(\frac{\partial w_2}{\partial x_2} \right)^2 \right) \left(1 - \frac{\Delta v}{d} + \left(\frac{\Delta v}{d} \right)^2 \right) \right], \quad (6b)$$

$$f_{m_{z2}} \approx -B_0 I_0 \frac{\partial v_2}{\partial x_2} + \frac{\mu_0 I_0^2}{2\pi d} \left[\frac{\partial v_2}{\partial x_2} \frac{\partial w_2}{\partial x_2} \left(1 - \frac{\Delta v}{d} + \left(\frac{\Delta v}{d} \right)^2 \right) - \left(1 + \left(\frac{\partial v_2}{\partial x_2} \right)^2 \right) \left(\frac{\Delta w}{d} \right) \left(1 - \left(\frac{\Delta w}{d} \right)^2 \right) \right]. \quad (6c)$$

Through neglecting the products of partial derivatives of deflections as well as the square of ratio of relative deflections to the interwire distance, the magnetic forces applied on the deformed CCNWs are simplified to:

$$\begin{aligned} \mathbf{f}_{m_1} = & \frac{\mu_0 I_0^2}{2\pi d} \frac{\partial v_1}{\partial x_1} \mathbf{e}_{x_1} + \left(B_0 I_0 \frac{\partial w_1}{\partial x_1} + \frac{\mu_0 I_0^2}{2\pi d} \left(1 - \frac{\Delta v}{d} \right) \right) \mathbf{e}_{y_1} \\ & + \left(-B_0 I_0 \frac{\partial v_1}{\partial x_1} + \frac{\mu_0 I_0^2}{2\pi d} \left(\frac{\Delta w}{d} \right) \right) \mathbf{e}_{z_1}, \end{aligned} \quad (7a)$$

$$\begin{aligned} \mathbf{f}_{m_2} = & \frac{\mu_0 I_0^2}{2\pi d} \frac{\partial v_2}{\partial x_2} \mathbf{e}_{x_2} + \left(B_0 I_0 \frac{\partial w_2}{\partial x_2} - \frac{\mu_0 I_0^2}{2\pi d} \left(1 - \frac{\Delta v}{d} \right) \right) \mathbf{e}_{y_2} \\ & + \left(-B_0 I_0 \frac{\partial v_2}{\partial x_2} - \frac{\mu_0 I_0^2}{2\pi d} \left(\frac{\Delta w}{d} \right) \right) \mathbf{e}_{z_2}. \end{aligned} \quad (7b)$$

By ignoring the static terms of magnetic forces in Eqs. (7a) and (7b), the dynamic magnetic forces that exert laterally on the CCNWs of the nanosystem are evaluated by:

$$\mathbf{f}_{m_1} = \left(B_0 I_0 \frac{\partial w_1}{\partial x_1} - \frac{\mu_0 I_0^2}{2\pi d^2} (v_2 - v_1) \right) \mathbf{e}_{y_1} + \left(-B_0 I_0 \frac{\partial v_1}{\partial x_1} + \frac{\mu_0 I_0^2}{2\pi d^2} (w_2 - w_1) \right) \mathbf{e}_{z_1}, \quad (8a)$$

$$\mathbf{f}_{m_2} = \left(B_0 I_0 \frac{\partial w_2}{\partial x_2} + \frac{\mu_0 I_0^2}{2\pi d^2} (v_2 - v_1) \right) \mathbf{e}_{y_2} - \left(B_0 I_0 \frac{\partial v_2}{\partial x_2} + \frac{\mu_0 I_0^2}{2\pi d^2} (w_2 - w_1) \right) \mathbf{e}_{z_2}. \quad (8b)$$

3.2. An exact formulation

By introducing Eq. (1) to Eqs. (2a) and (2b), the magnetic fields at the neighboring of the CCNWs are exactly calculated as:

$$\begin{aligned} \mathbf{B}_1 = & \left(B_0 - \frac{\mu_0 I_0}{4\pi} \int_0^{l_b} \frac{\Delta w \frac{\partial v_2}{\partial x_2} - (d + \Delta v) \frac{\partial w_2}{\partial x_2}}{((x_2 - x_1)^2 + (d + \Delta v)^2 + (\Delta w)^2)^{\frac{3}{2}}} dx_2 \right) \mathbf{e}_{x_1} - \\ & \left(\frac{\mu_0 I_0}{4\pi} \int_0^{l_b} \frac{(x_2 - x_1) \frac{\partial w_2}{\partial x_2} - \Delta w}{((x_2 - x_1)^2 + (d - \Delta v)^2 + (\Delta w)^2)^{\frac{3}{2}}} dx_2 \right) \mathbf{e}_{y_1} - \\ & \left(\frac{\mu_0 I_0}{4\pi} \int_0^{l_b} \frac{(d + \Delta v) - (x_2 - x_1) \frac{\partial v_2}{\partial x_2}}{((x_2 - x_1)^2 + (d + \Delta v)^2 + (\Delta w)^2)^{\frac{3}{2}}} dx_1 \right) \mathbf{e}_{z_1}, \end{aligned} \quad (9)$$

and

$$\begin{aligned} \mathbf{B}_2 = & \left(B_0 + \frac{\mu_0 I_0}{4\pi} \int_0^{l_b} \frac{\Delta w \frac{\partial v_1}{\partial x_1} - (d + \Delta v) \frac{\partial w_1}{\partial x_1}}{((x_2 - x_1)^2 + (d + \Delta v)^2 + (\Delta w)^2)^{\frac{3}{2}}} dx_1 \right) \mathbf{e}_{x_2} + \\ & \left(\frac{\mu_0 I_0}{4\pi} \int_0^{l_b} \frac{(x_2 - x_1) \frac{\partial w_1}{\partial x_1} - \Delta w}{((x_2 - x_1)^2 + (d + \Delta v)^2 + (\Delta w)^2)^{\frac{3}{2}}} dx_1 \right) \mathbf{e}_{y_2} + \\ & \left(\frac{\mu_0 I_0}{4\pi} \int_0^{l_b} \frac{(d + \Delta v) - (x_2 - x_1) \frac{\partial v_1}{\partial x_1}}{((x_2 - x_1)^2 + (d + \Delta v)^2 + (\Delta w)^2)^{\frac{3}{2}}} dx_1 \right) \mathbf{e}_{z_2}. \end{aligned} \quad (10)$$

Using Lorentz's formula in view of Eqs. (9) and (10), the components of the magnetic forces on the CCNWs immersed in a longitudinal magnetic field are obtained:

$$f_{m_{x_1}} = -\frac{\mu_0 I_0^2}{4\pi} \int_0^{l_b} \frac{(d + \Delta v) \frac{\partial v_1}{\partial x_1} + (x_2 - x_1) \left(\frac{\partial v_1}{\partial x_1} \frac{\partial v_2}{\partial x_2} + \frac{\partial w_1}{\partial x_1} \frac{\partial w_2}{\partial x_2} \right) + \Delta w \frac{\partial w_1}{\partial x_1}}{((x_2 - x_1)^2 + (d + \Delta v)^2 + (\Delta w)^2)^{\frac{3}{2}}} dx_2, \quad (11a)$$

$$f_{m_{y_1}} = B_0 I_0 \frac{\partial w_1}{\partial x_1} - \frac{\mu_0 I_0^2}{4\pi} \int_0^{l_b} \frac{\Delta w \frac{\partial w_1}{\partial x_1} \frac{\partial v_2}{\partial x_2} + (d + \Delta v) \left(1 + \frac{\partial w_1}{\partial x_1} \frac{\partial w_2}{\partial x_2} \right) + (x_2 - x_1) \frac{\partial v_2}{\partial x_2}}{((x_2 - x_1)^2 + (d + \Delta v)^2 + (\Delta w)^2)^{\frac{3}{2}}} dx_2, \quad (11b)$$

$$f_{m_{z_1}} = -B_0 I_0 \frac{\partial v_1}{\partial x_1} - \frac{\mu_0 I_0^2}{4\pi} \int_0^{l_b} \frac{(x_2 - x_1) \frac{\partial w_2}{\partial x_2} + \Delta w \left(1 + \frac{\partial v_1}{\partial x_1} \frac{\partial v_2}{\partial x_2} \right) + (d + \Delta v) \frac{\partial w_1}{\partial x_1} \frac{\partial v_2}{\partial x_2}}{((x_2 - x_1)^2 + (d + \Delta v)^2 + (\Delta w)^2)^{\frac{3}{2}}} dx_2, \quad (11c)$$

$$f_{m_{x_2}} = \frac{\mu_0 I_0^2}{4\pi} \int_0^{l_b} \frac{(d + \Delta v) \frac{\partial v_2}{\partial x_2} - (x_2 - x_1) \left(\frac{\partial v_1}{\partial x_1} \frac{\partial v_2}{\partial x_2} + \frac{\partial w_1}{\partial x_1} \frac{\partial w_2}{\partial x_2} \right) - \Delta w \frac{\partial w_2}{\partial x_2}}{\left((x_2 - x_1)^2 + (d + \Delta v)^2 + (\Delta w)^2 \right)^{\frac{3}{2}}} dx_1, \quad (11d)$$

$$f_{m_{y_2}} = B_0 I_0 \frac{\partial w_2}{\partial x_2} + \frac{\mu_0 I_0^2}{4\pi} \int_0^{l_b} \frac{\Delta w \frac{\partial v_1}{\partial x_1} \frac{\partial w_2}{\partial x_2} - (d + \Delta v) \left(1 + \frac{\partial w_1}{\partial x_1} \frac{\partial w_2}{\partial x_2} \right) + (x_2 - x_1) \frac{\partial v_1}{\partial x_1}}{\left((x_2 - x_1)^2 + (d + \Delta v)^2 + (\Delta w)^2 \right)^{\frac{3}{2}}} dx_1, \quad (11e)$$

$$f_{m_{z_2}} = -B_0 I_0 \frac{\partial v_2}{\partial x_2} + \frac{\mu_0 I_0^2}{4\pi} \int_0^{l_b} \frac{(x_2 - x_1) \frac{\partial w_1}{\partial x_1} - \Delta w \left(1 + \frac{\partial v_1}{\partial x_1} \frac{\partial v_2}{\partial x_2} \right) + (d + \Delta v) \frac{\partial w_1}{\partial x_1} \frac{\partial v_2}{\partial x_2}}{\left((x_2 - x_1)^2 + (d + \Delta v)^2 + (\Delta w)^2 \right)^{\frac{3}{2}}} dx_1. \quad (11f)$$

One can rationally assume that: $\Delta v \ll d$ and $\Delta w \ll d$. By ignoring the products of the first derivatives of displacements, and excluding the static terms, the dynamical magnetic forces on the deformed CCNWs are evaluated as follows:

$$\begin{aligned} \mathbf{f}_{m_1} = & \left(\frac{\mu_0 I_0^2}{4\pi} \int_0^{l_b} \frac{d}{\left((x_2 - x_1)^2 + d^2 \right)^{\frac{3}{2}}} \frac{\partial v_1}{\partial x_1} dx_2 \right) \mathbf{e}_{x_1} + \\ & \left(B_0 I_0 \frac{\partial w_1}{\partial x_1} - \frac{\mu_0 I_0^2}{4\pi} \int_0^{l_b} \frac{(x_2 - x_1) \frac{\partial v_2}{\partial x_2} - \Delta v}{\left((x_2 - x_1)^2 + d^2 \right)^{\frac{3}{2}}} dx_2 \right) \mathbf{e}_{y_1} + \\ & \left(-B_0 I_0 \frac{\partial v_1}{\partial x_1} - \frac{\mu_0 I_0^2}{4\pi} \int_0^{l_b} \frac{(x_2 - x_1) \frac{\partial w_2}{\partial x_2} - \Delta w}{\left((x_2 - x_1)^2 + d^2 \right)^{\frac{3}{2}}} dx_2 \right) \mathbf{e}_{z_1}, \end{aligned} \quad (12a)$$

$$\begin{aligned} \mathbf{f}_{m_2} = & \left(\frac{\mu_0 I_0^2}{4\pi} \int_0^{l_b} \frac{d}{\left((x_2 - x_1)^2 + d^2 \right)^{\frac{3}{2}}} \frac{\partial v_2}{\partial x_2} dx_1 \right) \mathbf{e}_{x_2} + \\ & \left(B_0 I_0 \frac{\partial w_2}{\partial x_2} + \frac{\mu_0 I_0^2}{4\pi} \int_0^{l_b} \frac{(x_2 - x_1) \frac{\partial v_1}{\partial x_1} - \Delta v}{\left((x_2 - x_1)^2 + d^2 \right)^{\frac{3}{2}}} dx_1 \right) \mathbf{e}_{y_2} + \\ & \left(-B_0 I_0 \frac{\partial v_2}{\partial x_2} + \frac{\mu_0 I_0^2}{4\pi} \int_0^{l_b} \frac{(x_2 - x_1) \frac{\partial w_1}{\partial x_1} - \Delta w}{\left((x_2 - x_1)^2 + d^2 \right)^{\frac{3}{2}}} dx_1 \right) \mathbf{e}_{z_2}. \end{aligned} \quad (12b)$$

4. Establishment of a novel integro-surface energy-based model

4.1. The constitutive relations of the surface layer and the bulk

Using theory of elasticity of Gurtin-Murdoch [17–19], the constitutive relations of the surface layer are given by:

$$\begin{aligned}\tau_{\alpha\beta} &= \tau_0 \delta_{\alpha\beta} + \tau_0 \frac{\partial u_\alpha}{\partial \beta} + 2(\mu_0 - \tau_0) \epsilon_{\alpha\beta} + (\lambda_0 + \tau_0) \epsilon_{\gamma\gamma} \delta_{\alpha\beta}; \quad \alpha, \beta, \gamma = x, y, z, \\ \tau_{x\alpha} &= \tau_0 \frac{\partial u_\alpha}{\partial x}; \quad \alpha = y, z,\end{aligned}\tag{13}$$

where $\tau_{\alpha\beta}$ is the surface stress tensor, $\epsilon_{\alpha\beta}$ is the surface strain tensor, u_α is the displacement along the α direction, λ_0 and μ_0 are the Lamé's constants of the surface layer, τ_0 is the residual surface stress under unconstrained conditions, and $\delta_{\alpha\beta}$ denotes the Kronecker delta. By employing the Rayleigh beam theory, the displacements of the m th nanowire are provided by: $u_{x_m} = -\left(z_m \frac{\partial w_m(x_m, t)}{\partial x_m} + y_m \frac{\partial v_m(x_m, t)}{\partial x_m}\right)$, $u_{y_m} = v_m(x_m, t)$, $u_{z_m} = w_m(x_m, t)$. By introducing these displacements to Eq. (13), the only existing stresses of the m th NW's surface layer would be:

$$\tau_{xx_m} = \tau_0 - (\lambda_0 + 2\mu_0) \left(z_m \frac{\partial^2 w_m}{\partial x_m^2} + y_m \frac{\partial^2 v_m}{\partial x_m^2} \right), \quad \tau_{xy_m} = n_{y_m} \tau_0 \frac{\partial v_m}{\partial x_m}, \quad \tau_{xz_m} = n_{z_m} \tau_0 \frac{\partial w_m}{\partial x_m},\tag{14}$$

where τ_{xx_m} is the longitudinal normal stress of the surface layer, τ_{xy_m} and τ_{xz_m} are the shear stresses, and n_{y_m} and n_{z_m} are the components of the unit outward normal vector of the surface layer of the m th NW.

To construct more accurate constitutive relations for the bulk, it is assumed that the stress components σ_{yy_m} and σ_{zz_m} within the bulk would vary linearly between their corresponding surface values. By exploiting the generalized Hook's law, the longitudinal stress of the bulk of the m th NW would be expressed by:

$$\begin{aligned}\sigma_{xx_m} &= -z_m \left(\left(E_b - \frac{2\tau_0 \nu_b}{D_0} \right) \frac{\partial^2 w_m}{\partial x_m^2} + \frac{2\rho_0 \nu_b}{D_0} \frac{\partial^2 w_m}{\partial t^2} \right) \\ &\quad - y_m \left(\left(E_b - \frac{2\tau_0 \nu_b}{D_0} \right) \frac{\partial^2 v_m}{\partial x_m^2} + \frac{2\rho_0 \nu_b}{D_0} \frac{\partial^2 v_m}{\partial t^2} \right),\end{aligned}\tag{15}$$

where ν_b is the Poisson's ratio of the bulk, E_b is the elastic modulus of the bulk, ρ_0 is the mass per unit area of the surface, and D_0 represents the diameter of the CCNWs,

4.2. Development of equations of motion of magnetically actuated DCCNWs

By taking an infinitesimal element of a magnetically actuated CCNW, depicting the free body diagram of such an element, and applying the Newton's second law, the transverse equations of motion of the nanosystem as a function of the bending moments and the shear forces within the CCNWs are obtained as follows:

$$\frac{\partial M_{b_{z1}}}{\partial x_1} + \int_S y_1 \frac{\partial \tau_{xx1}}{\partial x_1} dS - Q_{b_{y1}} = \int_{A_b} \rho_b \frac{\partial^2 u_{x1}}{\partial t^2} y_1 dA + \int_S \rho_0 \frac{\partial^2 u_{x1}}{\partial t^2} y_1 dS, \quad (16a)$$

$$-\frac{\partial Q_{b_{y1}}}{\partial x} - \int_S \frac{\partial \tau_{xy1}}{\partial x} n_{y1} dS - T \frac{\partial^2 v_1}{\partial x_1^2} = f_{m_{y1}} - \int_{A_b} \rho_b \frac{\partial^2 u_{y1}}{\partial t^2} dA - \int_S \rho_0 \frac{\partial^2 u_{y1}}{\partial t^2} n_{y1} dS, \quad (16b)$$

$$\frac{\partial M_{b_{z1}}}{\partial x_1} + \int_S z_1 \frac{\partial \tau_{xx1}}{\partial x_1} dS - Q_{b_{z1}} = \int_{A_b} \rho_b \frac{\partial^2 u_{x1}}{\partial t^2} z_1 dA + \int_S \rho_0 \frac{\partial^2 u_{x1}}{\partial t^2} z_1 dS, \quad (16c)$$

$$-\frac{\partial Q_{b_{z1}}}{\partial x} - \int_S \frac{\partial \tau_{xz1}}{\partial x} n_{z1} dS - T \frac{\partial^2 w_1}{\partial x_1^2} = f_{m_{z1}} - \int_{A_b} \rho_b \frac{\partial^2 u_{z1}}{\partial t^2} dA - \int_S \rho_0 \frac{\partial^2 u_{z1}}{\partial t^2} n_{z1} dS, \quad (16d)$$

$$\frac{\partial M_{b_{z2}}}{\partial x_2} + \int_S y_2 \frac{\partial \tau_{xx2}}{\partial x_2} dS - Q_{b_{y2}} = \int_{A_b} \rho_b \frac{\partial^2 u_{x2}}{\partial t^2} y_2 dA + \int_S \rho_0 \frac{\partial^2 u_{x2}}{\partial t^2} y_2 dS, \quad (16e)$$

$$-\frac{\partial Q_{b_{y2}}}{\partial x} - \int_S \frac{\partial \tau_{xy2}}{\partial x} n_{y2} dS - T \frac{\partial^2 v_2}{\partial x_2^2} = f_{m_{y2}} - \int_{A_b} \rho_b \frac{\partial^2 u_{y2}}{\partial t^2} dA - \int_S \rho_0 \frac{\partial^2 u_{y2}}{\partial t^2} n_{y2} dS, \quad (16f)$$

$$\frac{\partial M_{b_{y2}}}{\partial x_2} + \int_S z_2 \frac{\partial \tau_{xx2}}{\partial x_2} dS - Q_{b_{z2}} = \int_{A_b} \rho_b \frac{\partial^2 u_{x2}}{\partial t^2} z_2 dA + \int_S \rho_0 \frac{\partial^2 u_{x2}}{\partial t^2} z_2 dS, \quad (16g)$$

$$-\frac{\partial Q_{b_{z2}}}{\partial x} - \int_S \frac{\partial \tau_{xz2}}{\partial x} n_{z2} dS - T \frac{\partial^2 w_2}{\partial x_2^2} = f_{m_{z2}} - \int_{A_b} \rho_b \frac{\partial^2 u_{z2}}{\partial t^2} dA - \int_S \rho_0 \frac{\partial^2 u_{z2}}{\partial t^2} n_{z2} dS, \quad (16h)$$

where $d\mathcal{S}$ is the length of an infinitesimal segment of the perimeter of the cross-section of CCNWs, T is the initial tensile force within the CCNWs, $M_{b_{y_m}}/M_{b_{z_m}}$ and $Q_{b_{y_m}}/Q_{b_{z_m}}$ in order represent the bending moments and the shear forces of the bulk of m th CCNW pertinent to the y_m and z_m axes which are stated by:

$$M_{b_{y_m}} = \int_A z_m \sigma_{xx_m} dA = - \left(E_b I_b - \frac{2\tau_0 \nu_b I_b}{D_0} \right) \frac{\partial^2 w_m}{\partial x_m^2} - \frac{2\nu_b I_b \tau_0}{D_0} \frac{\partial^2 w_m}{\partial t^2}, \quad (17a)$$

$$M_{b_{z_m}} = \int_A y_m \sigma_{xx_m} dA = - \left(E_b I_b - \frac{2\tau_0 \nu_b I_b}{D_0} \right) \frac{\partial^2 v_m}{\partial x_m^2} - \frac{2\nu_b I_b \tau_0}{D_0} \frac{\partial^2 v_m}{\partial t^2}, \quad (17b)$$

By combining Eqs. (16a), (16c), (16e), and (16g) with Eqs. (16b), (16d), (16f), and (16h), respectively, and introducing Eqs. (17a) and (17b) to the resulting relations:

$$\begin{aligned} & (\rho_b A_b + \rho_0 S_0^*) \frac{\partial^2 v_1}{\partial t^2} - \left(\rho_b I_b + \rho_0 I_0^* - \frac{2\nu_b I_b \rho_0}{D_0} \right) \frac{\partial^4 v_1}{\partial t^2 \partial x_1^2} + \\ & \left[E_b I_b + (\lambda_0 + 2\mu_0) I_0^* - \frac{2\nu_b I_b \tau_0}{D_0} \right] \frac{\partial^4 v_1}{\partial x_1^4} - (T + \tau_0 S^*) \frac{\partial^2 v_1}{\partial x_1^2} = f_{m_{y1}}, \end{aligned} \quad (18a)$$

$$\begin{aligned} & (\rho_b A_b + \rho_0 S_0^*) \frac{\partial^2 w_1}{\partial t^2} - \left(\rho_b I_b + \rho_0 I_0^* - \frac{2\nu_b I_b \rho_0}{D_0} \right) \frac{\partial^4 w_1}{\partial t^2 \partial x_1^2} + \\ & \left[E_b I_b + (\lambda_0 + 2\mu_0) I_0^* - \frac{2\nu_b I_b \tau_0}{D_0} \right] \frac{\partial^4 w_1}{\partial x_1^4} - (T + \tau_0 S^*) \frac{\partial^2 w_1}{\partial x_1^2} = f_{m_{z1}}, \end{aligned} \quad (18b)$$

$$\begin{aligned} & (\rho_b A_b + \rho_0 S_0^*) \frac{\partial^2 v_2}{\partial t^2} - \left(\rho_b I_b + \rho_0 I_0^* - \frac{2\nu_b I_b \rho_0}{D_0} \right) \frac{\partial^4 v_2}{\partial t^2 \partial x_2^2} + \\ & \left[E_b I_b + (\lambda_0 + 2\mu_0) I_0^* - \frac{2\nu_b I_b \tau_0}{D_0} \right] \frac{\partial^4 v_2}{\partial x_2^4} - (T + \tau_0 S^*) \frac{\partial^2 v_2}{\partial x_2^2} = f_{m_{y2}}, \end{aligned} \quad (18c)$$

$$\begin{aligned} & (\rho_b A_b + \rho_0 S_0^*) \frac{\partial^2 w_2}{\partial t^2} - \left(\rho_b I_b + \rho_0 I_0^* - \frac{2\nu_b I_b \rho_0}{D_0} \right) \frac{\partial^4 w_2}{\partial t^2 \partial x_2^2} + \\ & \left[E_b I_b + (\lambda_0 + 2\mu_0) I_0^* - \frac{2\nu_b I_b \tau_0}{D_0} \right] \frac{\partial^4 w_2}{\partial x_2^4} - (T + \tau_0 S^*) \frac{\partial^2 w_2}{\partial x_2^2} = f_{m_{z2}}, \end{aligned} \quad (18d)$$

where $I_0^* = \int_{\mathcal{S}} z^2 d\mathcal{S}$ and $S^* = \int_{\mathcal{S}} n_z^2 d\mathcal{S}$. Eqs. (18a)-(18d) furnish us regarding transverse equations of motion of magnetically actuated DCCNWs in terms of interactional interwire magnetic forces. Based on the newly developed approximate and exact magnetic forces in section 3, the approximate and exact versions of the governing equations are established in the following parts.

4.2.1. An approximate model

By substituting Eqs. (8a) and (8b) into Eqs. (18a)-(18d), the approximate form of the governing equations as a function of transverse displacements of the CCNWs is extracted as follows:

$$\begin{aligned}
& (\rho_b A_b + \rho_0 S_0^*) \frac{\partial^2 v_1}{\partial t^2} - \left(\rho_b I_b + \rho_0 I_0^* - \frac{2\nu_b I_b \rho_0}{D_0} \right) \frac{\partial^4 v_1}{\partial t^2 \partial x_1^2} + \\
& \left[E_b I_b + (\lambda_0 + 2\mu_0) I_0^* - \frac{2\nu_b I_b \tau_0}{D_0} \right] \frac{\partial^4 v_1}{\partial x_1^4} - (T + \tau_0 S^*) \frac{\partial^2 v_1}{\partial x_1^2} + \\
& -B_0 I_0 \frac{\partial w_1}{\partial x_1} + \frac{\mu_0 I_0^2}{2\pi d^2} (v_2 - v_1) = 0,
\end{aligned} \tag{19a}$$

$$\begin{aligned}
& (\rho_b A_b + \rho_0 S_0^*) \frac{\partial^2 w_1}{\partial t^2} - \left(\rho_b I_b + \rho_0 I_0^* - \frac{2\nu_b I_b \rho_0}{D_0} \right) \frac{\partial^4 w_1}{\partial t^2 \partial x_1^2} + \\
& \left[E_b I_b + (\lambda_0 + 2\mu_0) I_0^* - \frac{2\nu_b I_b \tau_0}{D_0} \right] \frac{\partial^4 w_1}{\partial x_1^4} - (T + \tau_0 S^*) \frac{\partial^2 w_1}{\partial x_1^2} + \\
& B_0 I_0 \frac{\partial v_1}{\partial x_1} - \frac{\mu_0 I_0^2}{2\pi d^2} (w_2 - w_1) = 0,
\end{aligned} \tag{19b}$$

$$\begin{aligned}
& (\rho_b A_b + \rho_0 S_0^*) \frac{\partial^2 v_2}{\partial t^2} - \left(\rho_b I_b + \rho_0 I_0^* - \frac{2\nu_b I_b \rho_0}{D_0} \right) \frac{\partial^4 v_2}{\partial t^2 \partial x_2^2} + \\
& \left[E_b I_b + (\lambda_0 + 2\mu_0) I_0^* - \frac{2\nu_b I_b \tau_0}{D_0} \right] \frac{\partial^4 v_2}{\partial x_2^4} - (T + \tau_0 S^*) \frac{\partial^2 v_2}{\partial x_2^2} + \\
& B_0 I_0 \frac{\partial w_2}{\partial x_2} - \frac{\mu_0 I_0^2}{2\pi d^2} (v_2 - v_1) = 0,
\end{aligned} \tag{19c}$$

$$\begin{aligned}
& (\rho_b A_b + \rho_0 S_0^*) \frac{\partial^2 w_2}{\partial t^2} - \left(\rho_b I_b + \rho_0 I_0^* - \frac{2\nu_b I_b \rho_0}{D_0} \right) \frac{\partial^4 w_2}{\partial t^2 \partial x_2^2} + \\
& \left[E_b I_b + (\lambda_0 + 2\mu_0) I_0^* - \frac{2\nu_b I_b \tau_0}{D_0} \right] \frac{\partial^4 w_2}{\partial x_2^4} - (T + \tau_0 S^*) \frac{\partial^2 w_2}{\partial x_2^2} - \\
& B_0 I_0 \frac{\partial v_2}{\partial x_2} + \frac{\mu_0 I_0^2}{2\pi d^2} (w_2 - w_1) = 0.
\end{aligned} \tag{19d}$$

To explore free vibration of the nanosystem at hand in a more general context, the following dimensionless parameters are taken into account:

$$\begin{aligned}\bar{v}_m &= \frac{v_m}{l_b}, \quad \bar{w}_m = \frac{w_m}{l_b}, \quad \xi_m = \frac{x_m}{l_b}, \quad \tau = \frac{t}{l_b^2} \sqrt{\frac{E_b I_b}{\rho_b A_b}}, \quad \lambda = \frac{l_b}{r_b}, \\ \bar{T} &= \frac{T l_b^2}{E_b I_b}, \quad \bar{f}_0 = \frac{B_0 I_0 l_b^3}{E_b I_b}, \quad \bar{f}_1 = \frac{\mu_0 I_0^2 l_b^4}{2\pi d^2 E_b I_b}, \quad \chi_1 = \frac{\rho_0 S_0^*}{\rho_b A_b}, \\ \chi_2 &= \frac{\rho_0 I_0^* - \frac{2\nu_b I_b \rho_0}{D_0}}{\rho_b I_b}, \quad \chi_3 = \frac{(\lambda_0 + 2\mu_0) I_0^* - \frac{2\nu_b I_b \tau_0}{D_0}}{E_b I_b}, \quad \chi_4 = \frac{\tau_0 S_0^* l_b^2}{E_b I_b},\end{aligned}\tag{20}$$

by introducing Eq. (20) to Eqs. (19a)-(19d), the dimensionless approximate governing equations of magnetically actuated DCCNWs are obtained as:

$$\begin{aligned}(1 + \chi_1) \frac{\partial^2 \bar{v}_1}{\partial \tau^2} - \lambda^{-2} (1 + \chi_2) \frac{\partial^4 \bar{v}_1}{\partial \tau^2 \partial \xi_1^2} + \\ (1 + \chi_3) \frac{\partial^4 \bar{v}_1}{\partial \xi_1^4} - (\bar{T} + \chi_4) \frac{\partial^2 \bar{v}_1}{\partial \xi_1^2} - \bar{f}_0 \frac{\partial \bar{w}_1}{\partial \xi_1} + \bar{f}_1 (\bar{v}_2 - \bar{v}_1) = 0,\end{aligned}\tag{21a}$$

$$\begin{aligned}(1 + \chi_1) \frac{\partial^2 \bar{w}_1}{\partial \tau^2} - \lambda^{-2} (1 + \chi_2) \frac{\partial^4 \bar{w}_1}{\partial \tau^2 \partial \xi_1^2} + \\ (1 + \chi_3) \frac{\partial^4 \bar{w}_1}{\partial \xi_1^4} - (\bar{T} + \chi_4) \frac{\partial^2 \bar{w}_1}{\partial \xi_1^2} + \bar{f}_0 \frac{\partial \bar{v}_1}{\partial \xi_1} - \bar{f}_1 (\bar{w}_2 - \bar{w}_1) = 0,\end{aligned}\tag{21b}$$

$$\begin{aligned}(1 + \chi_1) \frac{\partial^2 \bar{v}_2}{\partial \tau^2} - \lambda^{-2} (1 + \chi_2) \frac{\partial^4 \bar{v}_2}{\partial \tau^2 \partial \xi_2^2} + \\ (1 + \chi_3) \frac{\partial^4 \bar{v}_2}{\partial \xi_2^4} - (\bar{T} + \chi_4) \frac{\partial^2 \bar{v}_2}{\partial \xi_2^2} - \bar{f}_0 \frac{\partial \bar{w}_2}{\partial \xi_2} - \bar{f}_1 (\bar{v}_2 - \bar{v}_1) = 0,\end{aligned}\tag{21c}$$

$$\begin{aligned}(1 + \chi_1) \frac{\partial^2 \bar{w}_2}{\partial \tau^2} - \lambda^{-2} (1 + \chi_2) \frac{\partial^4 \bar{w}_2}{\partial \tau^2 \partial \xi_2^2} + \\ (1 + \chi_3) \frac{\partial^4 \bar{w}_2}{\partial \xi_2^4} - (\bar{T} + \chi_4) \frac{\partial^2 \bar{w}_2}{\partial \xi_2^2} + \bar{f}_0 \frac{\partial \bar{v}_2}{\partial \xi_2} + \bar{f}_1 (\bar{w}_2 - \bar{w}_1) = 0.\end{aligned}\tag{21d}$$

It should be noticed that through setting $\chi_m = 0$; $i = 1, 2, 3, 4$, Eqs. (21a)-(21d) are reduced to the equations of motion of doubly parallel current-carrying macrowires in the presence of a longitudinal magnetic field. For very lengthy nanowires (i.e., in the absence of bending rigidity and rotational inertia), the dynamic response of the nanosystem under arbitrary initial conditions was investigated by Kiani [15, 16]. The explicit expressions of the frequencies

and their corresponding vibration modes were obtained. The conditions lead to the dynamic instability of the nanosystem were also explained and discussed.

4.2.2. An exact model

By substituting Eqs. (12a) and (12b) into Eqs. (18a)-(18d), the more refined equations of motion of the magnetically actuated nanosystem are provided in the following form:

$$\begin{aligned}
& (\rho_b A_b + \rho_0 S_0^*) \frac{\partial^2 v_1}{\partial t^2} - \left(\rho_b I_b + \rho_0 I_0^* - \frac{2\nu_b I_b \rho_0}{D_0} \right) \frac{\partial^4 v_1}{\partial t^2 \partial x_1^2} + \\
& \left[E_b I_b + (\lambda_0 + 2\mu_0) I_0^* - \frac{2\nu_b I_b \tau_0}{D_0} \right] \frac{\partial^4 v_1}{\partial x_1^4} - (T + \tau_0 S^*) \frac{\partial^2 v_1}{\partial x_1^2} - \\
& B_0 I_0 \frac{\partial w_1}{\partial x_1} + \frac{\mu_0 I_0^2}{4\pi} \int_0^{l_b} \frac{(x_2 - x_1) \frac{\partial v_2}{\partial x_2} - (v_2 - v_1)}{((x_2 - x_1)^2 + d^2)^{\frac{3}{2}}} dx_2 = 0,
\end{aligned} \tag{22a}$$

$$\begin{aligned}
& (\rho_b A_b + \rho_0 S_0^*) \frac{\partial^2 w_1}{\partial t^2} - \left(\rho_b I_b + \rho_0 I_0^* - \frac{2\nu_b I_b \rho_0}{D_0} \right) \frac{\partial^4 w_1}{\partial t^2 \partial x_1^2} + \\
& \left[E_b I_b + (\lambda_0 + 2\mu_0) I_0^* - \frac{2\nu_b I_b \tau_0}{D_0} \right] \frac{\partial^4 w_1}{\partial x_1^4} - (T + \tau_0 S^*) \frac{\partial^2 w_1}{\partial x_1^2} + \\
& B_0 I_0 \frac{\partial v_1}{\partial x_1} + \frac{\mu_0 I_0^2}{4\pi} \int_0^{l_b} \frac{(x_2 - x_1) \frac{\partial w_2}{\partial x_2} - (w_2 - w_1)}{((x_2 - x_1)^2 + d^2)^{\frac{3}{2}}} dx_2 = 0,
\end{aligned} \tag{22b}$$

$$\begin{aligned}
& (\rho_b A_b + \rho_0 S_0^*) \frac{\partial^2 v_2}{\partial t^2} - \left(\rho_b I_b + \rho_0 I_0^* - \frac{2\nu_b I_b \rho_0}{D_0} \right) \frac{\partial^4 v_2}{\partial t^2 \partial x_2^2} + \\
& \left[E_b I_b + (\lambda_0 + 2\mu_0) I_0^* - \frac{2\nu_b I_b \tau_0}{D_0} \right] \frac{\partial^4 v_2}{\partial x_2^4} - (T + \tau_0 S^*) \frac{\partial^2 v_2}{\partial x_2^2} - \\
& B_0 I_0 \frac{\partial w_2}{\partial x_2} - \frac{\mu_0 I_0^2}{4\pi} \int_0^{l_b} \frac{(x_2 - x_1) \frac{\partial v_1}{\partial x_1} - (v_2 - v_1)}{((x_2 - x_1)^2 + d^2)^{\frac{3}{2}}} dx_1 = 0,
\end{aligned} \tag{22c}$$

$$\begin{aligned}
& (\rho_b A_b + \rho_0 S_0^*) \frac{\partial^2 w_2}{\partial t^2} - \left(\rho_b I_b + \rho_0 I_0^* - \frac{2\nu_b I_b \rho_0}{D_0} \right) \frac{\partial^4 w_2}{\partial t^2 \partial x_2^2} + \\
& \left[E_b I_b + (\lambda_0 + 2\mu_0) I_0^* - \frac{2\nu_b I_b \tau_0}{D_0} \right] \frac{\partial^4 w_2}{\partial x_2^4} - (T + \tau_0 S^*) \frac{\partial^2 w_2}{\partial x_2^2} + \\
& B_0 I_0 \frac{\partial v_2}{\partial x_2} - \frac{\mu_0 I_0^2}{4\pi} \int_0^{l_b} \frac{(x_2 - x_1) \frac{\partial w_1}{\partial x_1} - (w_2 - w_1)}{((x_2 - x_1)^2 + d^2)^{\frac{3}{2}}} dx_1 = 0.
\end{aligned} \tag{22d}$$

Finally, by introducing the dimensionless parameters in Eq. (20) to Eqs. (22a)-(22d), the dimensionless equations of motion of magnetically actuated DCCNWs based on the proposed refined model are extracted as:

$$(1 + \chi_1) \frac{\partial^2 \bar{v}_1}{\partial \tau^2} - \lambda^{-2} (1 + \chi_2) \frac{\partial^4 \bar{v}_1}{\partial \tau^2 \partial \xi_1^2} + (1 + \chi_3) \frac{\partial^4 \bar{v}_1}{\partial \xi_1^4} - (\bar{T} + \chi_4) \frac{\partial^2 \bar{v}_1}{\partial \xi_1^2} - \bar{f}_0 \frac{\partial \bar{v}_1}{\partial \xi_1} + \frac{\bar{f}_1 \bar{d}^2}{2} \int_0^1 \frac{(\xi_2 - \xi_1) \frac{\partial \bar{v}_2}{\partial \xi_2} - (\bar{v}_2 - \bar{v}_1)}{\left((\xi_2 - \xi_1)^2 + \bar{d}^2 \right)^{\frac{3}{2}}} d\xi_2 = 0, \quad (23a)$$

$$(1 + \chi_1) \frac{\partial^2 \bar{w}_1}{\partial \tau^2} - \lambda^{-2} (1 + \chi_2) \frac{\partial^4 \bar{w}_1}{\partial \tau^2 \partial \xi_1^2} + (1 + \chi_3) \frac{\partial^4 \bar{w}_1}{\partial \xi_1^4} - (\bar{T} + \chi_4) \frac{\partial^2 \bar{w}_1}{\partial \xi_1^2} + \bar{f}_0 \frac{\partial \bar{w}_1}{\partial \xi_1} + \frac{\bar{f}_1 \bar{d}^2}{2} \int_0^1 \frac{(\xi_2 - \xi_1) \frac{\partial \bar{w}_2}{\partial \xi_2} - (\bar{w}_2 - \bar{w}_1)}{\left((\xi_2 - \xi_1)^2 + \bar{d}^2 \right)^{\frac{3}{2}}} d\xi_2 = 0, \quad (23b)$$

$$(1 + \chi_1) \frac{\partial^2 \bar{v}_2}{\partial \tau^2} - \lambda^{-2} (1 + \chi_2) \frac{\partial^4 \bar{v}_2}{\partial \tau^2 \partial \xi_2^2} + (1 + \chi_3) \frac{\partial^4 \bar{v}_2}{\partial \xi_2^4} - (\bar{T} + \chi_4) \frac{\partial^2 \bar{v}_2}{\partial \xi_2^2} - \bar{f}_0 \frac{\partial \bar{v}_2}{\partial \xi_2} - \frac{\bar{f}_1 \bar{d}^2}{2} \int_0^1 \frac{(\xi_2 - \xi_1) \frac{\partial \bar{v}_1}{\partial \xi_1} - (\bar{v}_2 - \bar{v}_1)}{\left((\xi_2 - \xi_1)^2 + \bar{d}^2 \right)^{\frac{3}{2}}} d\xi_1 = 0, \quad (23c)$$

$$(1 + \chi_1) \frac{\partial^2 \bar{w}_2}{\partial \tau^2} - \lambda^{-2} (1 + \chi_2) \frac{\partial^4 \bar{w}_2}{\partial \tau^2 \partial \xi_2^2} + (1 + \chi_3) \frac{\partial^4 \bar{w}_2}{\partial \xi_2^4} - (\bar{T} + \chi_4) \frac{\partial^2 \bar{w}_2}{\partial \xi_2^2} + \bar{f}_0 \frac{\partial \bar{w}_2}{\partial \xi_2} - \frac{\bar{f}_1 \bar{d}^2}{2} \int_0^1 \frac{(\xi_2 - \xi_1) \frac{\partial \bar{w}_1}{\partial \xi_1} - (\bar{w}_2 - \bar{w}_1)}{\left((\xi_2 - \xi_1)^2 + \bar{d}^2 \right)^{\frac{3}{2}}} d\xi_1 = 0. \quad (23d)$$

Eqs. (23a)-(23d) represent four coupled integro-partial differential equations (IPDE) that should be appropriately solved. In the following part, free vibration of the nanosystem are examined by employing AMM.

4.3. Dynamic analysis of the magnetically affected nanosystem via AMM

4.3.1. Free transverse vibration of the nanosystem using the proposed approximate model

Let express the dimensionless transverse displacements of the magnetically actuated nanosystem as follows:

$$\bar{v}_m(\xi_m, \tau) = \sum_{i=1}^{NM_m} \phi_i^{v_m}(\xi_m) \bar{v}_{m_i}(\tau), \quad \bar{w}_m(\xi_m, \tau) = \sum_{i=1}^{NM_m} \phi_i^{w_m}(\xi_m) \bar{w}_{m_i}(\tau); \quad m = 1, 2, \quad (24)$$

where NM_m is the number of considered modes pertinent to the m th NW, $\phi_i^{v_m}(\xi)$ and $\phi_i^{w_m}(\xi)$ are the i th mode shape functions associated with $\bar{v}_m$ and $\bar{w}_m$, $\bar{v}_{m_i}(\tau)$ and $\bar{w}_{m_i}(\tau)$ are their corresponding time-dependent parameters. Now we premultiply both sides of Eqs. (21a)-(21d) in order by $\delta\bar{v}_1$, $\delta\bar{w}_1$, $\delta\bar{v}_2$, and $\delta\bar{w}_2$, where δ is the variational sign. By taking the integral from the resulting expressions, and taking the required integration by parts, it is obtained:

$$\begin{aligned} & \begin{bmatrix} [\bar{\mathbf{M}}_b]^{v_1 v_1} & [\bar{\mathbf{M}}_b]^{v_1 w_1} & [\bar{\mathbf{M}}_b]^{v_1 v_2} & [\bar{\mathbf{M}}_b]^{v_1 w_2} \\ [\bar{\mathbf{M}}_b]^{w_1 v_1} & [\bar{\mathbf{M}}_b]^{w_1 w_1} & [\bar{\mathbf{M}}_b]^{w_1 v_2} & [\bar{\mathbf{M}}_b]^{w_1 w_2} \\ [\bar{\mathbf{M}}_b]^{v_2 v_1} & [\bar{\mathbf{M}}_b]^{v_2 w_1} & [\bar{\mathbf{M}}_b]^{v_2 v_2} & [\bar{\mathbf{M}}_b]^{v_2 w_2} \\ [\bar{\mathbf{M}}_b]^{w_2 v_1} & [\bar{\mathbf{M}}_b]^{w_2 w_1} & [\bar{\mathbf{M}}_b]^{w_2 v_2} & [\bar{\mathbf{M}}_b]^{w_2 w_2} \end{bmatrix} \begin{Bmatrix} \frac{d^2 \bar{\mathbf{v}}_1}{d\tau^2} \\ \frac{d^2 \bar{\mathbf{w}}_1}{d\tau^2} \\ \frac{d^2 \bar{\mathbf{v}}_2}{d\tau^2} \\ \frac{d^2 \bar{\mathbf{w}}_2}{d\tau^2} \end{Bmatrix} + \\ & \begin{bmatrix} [\bar{\mathbf{K}}_b]^{v_1 v_1} & [\bar{\mathbf{K}}_b]^{v_1 w_1} & [\bar{\mathbf{K}}_b]^{v_1 v_2} & [\bar{\mathbf{K}}_b]^{v_1 w_2} \\ [\bar{\mathbf{K}}_b]^{w_1 v_1} & [\bar{\mathbf{K}}_b]^{w_1 w_1} & [\bar{\mathbf{K}}_b]^{w_1 v_2} & [\bar{\mathbf{K}}_b]^{w_1 w_2} \\ [\bar{\mathbf{K}}_b]^{v_2 v_1} & [\bar{\mathbf{K}}_b]^{v_2 w_1} & [\bar{\mathbf{K}}_b]^{v_2 v_2} & [\bar{\mathbf{K}}_b]^{v_2 w_2} \\ [\bar{\mathbf{K}}_b]^{w_2 v_1} & [\bar{\mathbf{K}}_b]^{w_2 w_1} & [\bar{\mathbf{K}}_b]^{w_2 v_2} & [\bar{\mathbf{K}}_b]^{w_2 w_2} \end{bmatrix} \begin{Bmatrix} \bar{\mathbf{v}}_1 \\ \bar{\mathbf{w}}_1 \\ \bar{\mathbf{v}}_2 \\ \bar{\mathbf{w}}_2 \end{Bmatrix} = \begin{Bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{Bmatrix}, \end{aligned} \quad (25)$$

where the nonzero submatrices in Eq. (25) are defined by:

$$[\bar{\mathbf{M}}_b]_{ij}^{v_m v_m} = \int_0^1 \left((1 + \chi_1) \phi_i^{v_m} \phi_j^{v_m} + \lambda^{-2} (1 + \chi_2) \frac{d\phi_i^{v_m}}{d\xi_m} \frac{d\phi_j^{v_m}}{d\xi_m} \right) d\xi_m, \quad (26a)$$

$$[\bar{\mathbf{M}}_b]_{ij}^{w_m w_m} = \int_0^1 \left((1 + \chi_1) \phi_i^{w_m} \phi_j^{w_m} + \lambda^{-2} (1 + \chi_2) \frac{d\phi_i^{w_m}}{d\xi_m} \frac{d\phi_j^{w_m}}{d\xi_m} \right) d\xi_m, \quad (26b)$$

$$[\bar{\mathbf{K}}_b]_{ij}^{v_m v_m} = \int_0^1 \left((1 + \chi_3) \frac{d^2 \phi_i^{v_m}}{d\xi_m^2} \frac{d^2 \phi_j^{v_m}}{d\xi_m^2} + (\bar{T} + \chi_4) \frac{d\phi_i^{v_m}}{d\xi_m} \frac{d\phi_j^{v_m}}{d\xi_m} - \bar{f}_1 \phi_i^{v_m} \phi_j^{v_m} \right) d\xi_m, \quad (26c)$$

$$[\bar{\mathbf{K}}_b]_{ij}^{v_m w_m} = - \int_0^1 \bar{f}_0 \phi_i^{v_m} \frac{d\phi_j^{w_m}}{d\xi_m} d\xi_m, \quad (26d)$$

$$[\bar{\mathbf{K}}_b]_{ij}^{v_m v_n} = \int_0^1 \bar{f}_1 \phi_i^{v_m} \phi_j^{v_n} d\xi_n; \quad m \neq n, \quad (26e)$$

$$[\overline{\mathbf{K}}_b]_{ij}^{w_m v_m} = \int_0^1 \overline{f}_0 \phi_i^{w_m} \frac{d\phi_j^{v_m}}{d\xi_m} d\xi_m, \quad (26f)$$

$$[\overline{\mathbf{K}}_b]_{ij}^{w_m w_m} = \int_0^1 \left((1 + \chi_3) \frac{d^2 \phi_i^{w_m}}{d\xi_m^2} \frac{d^2 \phi_j^{w_m}}{d\xi_m^2} + (\overline{T} + \chi_4) \frac{d\phi_i^{w_m}}{d\xi_m} \frac{d\phi_j^{w_m}}{d\xi_m} + \overline{f}_1 \phi_i^{w_m} \phi_j^{w_m} \right) d\xi_m, \quad (26g)$$

$$[\overline{\mathbf{K}}_b]_{ij}^{w_m w_n} = - \int_0^1 \overline{f}_1 \phi_i^{w_m} \phi_j^{w_n} d\xi_n; \quad m \neq n, \quad (26h)$$

$$\overline{\mathbf{v}}_m(\tau) = \langle \overline{v}_{i_1}(\tau), \overline{v}_{i_2}(\tau), \dots, \overline{v}_{i_{NM_m}}(\tau) \rangle^T, \quad (26i)$$

$$\overline{\mathbf{w}}_m(\tau) = \langle \overline{w}_{i_1}(\tau), \overline{w}_{i_2}(\tau), \dots, \overline{w}_{i_{NM_m}}(\tau) \rangle^T, \quad (26j)$$

where $m, n=1,2$. For frequency analysis of the problem, let: $\overline{\mathbf{v}}_m(\tau) = \overline{\mathbf{v}}_{m0} e^{i\varpi\tau}$ and $\overline{\mathbf{w}}_m(\tau) = \overline{\mathbf{w}}_{m0} e^{i\varpi\tau}$ where ϖ represents the dimensionless frequency, $i = \sqrt{-1}$, and $\overline{\mathbf{v}}_{m0}$ and $\overline{\mathbf{w}}_{m0}$ are the dimensionless amplitude vectors. By substituting these expressions into Eq. (25), and by solving the resulting set of eigenvalue equations, the eigenvalues and their corresponding mode shapes could be readily evaluated.

4.3.2. Free transverse vibration of the nanosystem using the proposed exact model

To construct the discretized version of the governing equations via the suggested exact formulations, we premultiply both sides of Eqs. (23a)-(23d) by $\delta\overline{v}_1$, $\delta\overline{w}_1$, $\delta\overline{v}_2$, and $\delta\overline{w}_2$, respectively. By integration of the resulting statements over the dimensionless domains of the NWs, taking the required integration by parts, and introducing Eq. (24) to the obtained expressions, a set of ordinary equations of motion similar to those in Eq. (25) is derived. All dimensionless mass submatrices and some stiffness submatrices are identical to those of the approximate model whereas most of the dimensionless stiffness submatrices are modified as follows:

$$\begin{aligned} [\overline{\mathbf{K}}_b]_{ij}^{v_m v_m} = & \int_0^1 \left((1 + \chi_3) \frac{d^2 \phi_i^{v_m}}{d\xi_m^2} \frac{d^2 \phi_j^{v_m}}{d\xi_m^2} + (\overline{T} + \chi_4) \frac{d\phi_i^{v_m}}{d\xi_m} \frac{d\phi_j^{v_m}}{d\xi_m} \right) d\xi_m \\ & + \frac{\overline{f}_1 \overline{d}^2}{2} \int_0^1 \int_0^1 \frac{\phi_i^{v_m}(\xi_m) \phi_j^{v_m}(\xi_m)}{\left((\xi_2 - \xi_1)^2 + \overline{d}^2 \right)^{\frac{3}{2}}} d\xi_2 d\xi_1, \end{aligned} \quad (27a)$$

$$[\bar{\mathbf{K}}_b]_{ij}^{v_m v_n} = -\frac{\bar{f}_1 \bar{d}^2}{2} \int_0^1 \int_0^1 \frac{\phi_i^{v_m} \left((\xi_m - \xi_n) \frac{d\phi_j^{v_n}}{d\xi_n} + \phi_j^{v_n} \right)}{\left((\xi_2 - \xi_1)^2 + \bar{d}^2 \right)^{\frac{3}{2}}} d\xi_m d\xi_n; \quad m \neq n, \quad (27b)$$

$$\begin{aligned} [\bar{\mathbf{K}}_b]_{ij}^{w_m w_m} = & \int_0^1 \left((1 + \chi_3) \frac{d^2 \phi_i^{w_m}}{d\xi_m^2} \frac{d^2 \phi_j^{w_m}}{d\xi_m^2} + (\bar{T} + \chi_4) \frac{d\phi_i^{w_m}}{d\xi_m} \frac{d\phi_j^{w_m}}{d\xi_m} \right) d\xi_m + \\ & + \frac{\bar{f}_1 \bar{d}^2}{2} \int_0^1 \int_0^1 \frac{\phi_i^{w_m}(\xi_m) \phi_j^{w_m}(\xi_m)}{\left((\xi_2 - \xi_1)^2 + \bar{d}^2 \right)^{\frac{3}{2}}} d\xi_2 d\xi_1, \end{aligned} \quad (27c)$$

$$[\bar{\mathbf{K}}_b]_{ij}^{w_m w_n} = -\frac{\bar{f}_1 \bar{d}^2}{2} \int_0^1 \int_0^1 \frac{\phi_i^{w_m} \left((\xi_m - \xi_n) \frac{d\phi_j^{w_n}}{d\xi_n} + \phi_j^{w_n} \right)}{\left((\xi_2 - \xi_1)^2 + \bar{d}^2 \right)^{\frac{3}{2}}} d\xi_m d\xi_n; \quad m \neq n, \quad (27d)$$

where $m, n=1,2$. By following the given procedure in the previous part, the frequencies and the vibration modes of the nanosystem based on the newly developed exact model could be easily obtained.

5. Results and discussion

In this part, the capabilities of the approximate model in predicting the natural frequencies of the nanosystem based on the exact model are explained. Additionally, the role of the surface energy effect on the obtained results is displayed for different levels of the electric current, magnetic field strength, and slenderness ratio. For this purpose, consider DCC-NWs made from Au with the following bulk and surface properties: $E_b=37 \times 10^9$, $\nu_b=0.26$, $\rho_b=19300$ kg/m³, $\tau_0=1.4$ N/m, $E_0=-3.6$ N/m, $\lambda_0=3.6$ N/m, and $\rho_0=1.5 \times 10^{-7}$ kg/m². Free vibration of the nanosystem for two kinds of end conditions are of interest: nanowires with simply supported (SS) ends and clamped-clamped (CC) condition. In numerical analysis of the problem, the following admissible mode shapes are taken into account:

$$\text{SS condition: } \phi_i^{v_m}(\xi) = \phi_i^{w_m}(\xi) = \sin(i\pi\xi); \quad m = 1, 2, \quad (28a)$$

$$\begin{aligned}
&\text{CC condition: } \phi_i^{v_m}(\xi) = \phi_i^{w_m}(\xi) = \\
&\sin(e_i\xi)/(\sin(e_i) - \sinh(e_i)) + \cos(e_i\xi)/(\cos(e_i) - \cosh(e_i)) \\
&- \sinh(e_i\xi)/(\sin(e_i) - \sinh(e_i)) - \cosh(e_i\xi)/(\cos(e_i) - \cosh(e_i)),
\end{aligned} \tag{28b}$$

where

$$e_1 = 4.730, \quad e_2 = 7.853, \quad e_i = \frac{(2i+1)\pi}{2}; \quad i \geq 3. \tag{29}$$

5.1. The capabilities of the proposed exact model

In the previous works [12, 13], the NWs' length was large enough that the approximate model could be reasonably implemented for transverse vibrations of the nanosystem. Because of this fact, the bending rigidity of the NWs was also ignored. Herein, a more accurate model is developed by including the bending rigidity of both the bulk and the surface layer. It implies that the previously developed models for the problem of current-carrying double-nanowire-systems immersed in a longitudinal magnetic field is basically modified accounting for both the bending rigidity and the interwire magnetic forces. Therefore, it is expected that the given model would expand our view to some unseen realms of the previous models. For this purpose, the obtained results by the exact model are compared with those of the approximate model.

In Figs. 2(a) and 2(b), the approximate and exact fundamental frequencies of the nanosystem in terms of the interwire distance have been plotted. The results are demonstrated for SS and CC nanosystems with three levels of the electric current whereas the strength of the applied longitudinal magnetic field and the slenderness ratio of the NWs are constant. As it is obvious in Fig. 2, both the approximate model and the exact model predict that the fundamental frequency would reduce as the interwire distance decreases. Commonly, the rate of reduction of the plots based on the approximate analysis is more apparent with respect to that of the exact model, particularly for higher levels of the electric currents. Based on a close survey of the plotted results, it is indicated that the relative discrepancies between the predicted fundamental frequencies by the approximate model and

those of the exact model would lessen as the interwire distance increases. Such a fact is attributed to this reason that the interwire interactional forces would diminish as the interwire distance grow, and the role of the longitudinal magnetic field on the vibration of the nanosystem becomes important. Additionally, the above-mentioned relative discrepancies would increase by increasing of the electric current. For instance, the approximate model can capture the predicted fundamental frequencies of the SS(CC) nanosystem carrying electric currents $\bar{I}=0.1, 1$, and 2 with relative errors lower than $0.03(0.007)$, $3.8(0.75)$, and $12.4(1.8)$ percent, respectively. This data also reveals that the discrepancies between the obtained results by the approximate model and those of the exact model for nanosystems with SS ends are generally lower with respect to the nanosystems with CC ends. Both models also display that the variation of the interwire distance is more influential on the variation of the fundamental frequency of SS nanosystems in compare to the CC nanosystems.

We are also interested in understanding that how variation of the slenderness ratio would influence on the efficiency of the approximate model. To this end, the predicted fundamental frequencies of the nanosystems with SS and CC ends as a function of the slenderness ratio have been plotted in Figs. 3(a) and 3(b) for three levels of the electric current. The dimensionless electric current and magnetic field strength are defined such that the variation of the slenderness ratio would have no influence on the variation of the electric current and the magnetic field strength. For a given electric current intensity and all considered boundary conditions, the relative discrepancies between the plotted results based on the exact model and those of the approximate model would grow as the slenderness ratio increases. Such an issue is more apparent for higher levels of the electric current, particularly in the case of SS boundary condition. For a nanosystem with SS(CC) boundary condition and $\lambda=30$, the above-mentioned discrepancies for the cases of $\bar{I}_0^*=0.1, 1$, and 2 in order are about $0.05(0.012)$, $6.2(1.56)$, and $24.3(3.94)$ percent. **In all demonstrated plots, the nanosystem based on the exact model is stiffer than that based on the approximate model. The main reason of this fact is that the approximate formulation of the Lorentz magnetic force**

has been constructed based on this important assumption that the length of the NWs is infinite. It means that the dynamic interactional forces between double current-carrying NWs are overestimated by the approximate model, and thereby, the transverse stiffness of the nanosystem would be underestimated with respect to the real situation. As a result, the predicted frequencies by the approximate model would be generally lower than those obtained via the exact model.

5.2. *The effect of the surface energy on free vibration of the nanosystem*

A very exciting study has been carried out to determine the role of the surface energy on vibrational behavior of the nanosystem. In Figs. 4(a) and 4(b), the plots of the fundamental frequency in terms of the electric current have been provided for a constant magnetic field strength. The results have been provided for both SS and CC nanosystems with three levels of the slenderness ratio (i.e., $\lambda=15$, 20, and 25). The obtained results by considering the surface energy effect and without that have been presented by the solid lines and dashed lines, respectively. Irrespective of the surface energy effect, the fundamental frequencies of the nanosystems with SS and CC ends reduce as the electric current increases. Such a reduction is more obvious for SS nanosystems with higher slenderness ratio. For the nanosystem with $\lambda=15$ under SS boundary condition, the predicted fundamental frequencies by considering the surface effect are overestimated by those predicted by the classical continuum theory. However, in the cases of $\lambda=20$ and 25, the obtained fundamental frequencies based on the surface elasticity theory are greater than those obtained via the classical continuum theory for the considered range of the electric current. Furthermore, for CC nanosystems and all considered slenderness ratios, the predicted fundamental frequencies by the surface elasticity theory are lower than those of the classical continuum theory. Such a complex behavior of the nanosystem due to the inclusion of the surface energy effect is mainly related to the positive or negative contributions of the Young's modulus (i.e., $E_0 = \lambda_0 + 2\mu_0$) and the residual surface stress of the surface layer into the transverse stiffness expressions. For

the considered gold nanowires, $\tau_0 > 0$ while $E_0 < 0$. It implies that the residual surface stress positively contributes to the lateral stiffness and thereby, the natural frequencies of the nanosystem increase while the elastic modulus of the surface layer reduces the lateral stiffness of the nanosystem (see Eqs. 28(c) and 28(g)). A combination of these effects leads to a decrease or an increase of the natural frequencies depends on the length (i.e., slenderness ratio) of the NWs and their boundary conditions (i.e., in which directly affects on the mode shapes). Regarding nanosystems with CC boundary conditions, a detailed investigation of the plotted results shows that the role of the surface energy on the vibration behavior of the nanosystem increases with the electric current. Such a fact is more obvious for more slender nanosystems; however, the surface effect becomes diminish as the slenderness ratio of the nanosystem increases. Concerning SS nanosystems, variation of the surface energy effect in terms of the electric current also depends on the slenderness ratio. Generally, the relative discrepancies between the fundamental frequencies based on the surface elasticity theory and those of the classical continuum theory would increase by increasing of the electric current. Similar to individual current-carrying nanowires in a longitudinal magnetic field, it is expected that the relative discrepancy between the above-mentioned theories reaches to its maximum value at the critical electric current (i.e., the electric current corresponds to the zero fundamental frequency).

In Figs. 5(a) and 5(b), the plots of the fundamental frequencies of SS and CC nanosystems as a function of the magnetic field strength are provided for three levels of the slenderness ratio. A close survey of the plotted results in Figs. 5(a) and 5(b) reveals that the effect of the magnetic field strength on the vibration behavior of the nanosystems is identical to that of the electric current in which comprehensively explained in the previous part. For the sake of brevity, such explanations are not given.

6. Conclusions

Free transverse vibrations of magnetically actuated DCCNWs are studied in the context of theory of surface elasticity of the Gurtin-Murdoch. Based on the Biot-Savart law and Lorentz law, two sets of formulations are developed: *approximate* and *exact* versions of the interactional magnetic forces on the CCNWs. By exploiting the Rayleigh beam theory, the equations of motion of the nanosystem are derived accounting for the surface energy effect. Based on the approximate magnetic force, the derived governing equations are four coupled PDEs whereas those obtained via the exact formulations are four coupled IPDEs. These equations are solved via AMM for nanosystems with simple and clamped ends. The capabilities of the approximate model in capturing the predicted fundamental frequency by the exact model are displayed. The influences of the slenderness ratio, interwire distance, electric current, magnetic field strength, and surface energy effect on the free vibration behavior of the nanosystem are investigated. The major obtained results are as:

1. By increasing of the electric current or magnetic field strength, the discrepancy between the predicted fundamental frequency by the exact model and that of the approximate model would grow. Additionally, the exact model predicts that the nanosystem would be commonly stiffer with respect to that based on the approximate model. Such a fact is more obvious for more slender nanosystems with lower interwire distances.

2. As the constraints of the nanosystem at its supports increase, the discrepancies between the results obtained based on the exact model and those of the approximate model would generally reduce. According to the demonstrated fundamental frequencies based on these models, these discrepancies for fully clamped nanosystems are apparently lower than those of the simply supported nanosystems.

3. Generally, the role of the surface energy effect on the free dynamic response of the nanosystem becomes more obvious as the electric current or magnetic field strength increases. This issue is more apparent for nanosystems with higher slenderness ratios. In fact, variation of the electric current or magnetic field strength has a more influence on the free vibration

of higher slender nanosystems.

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Captions of the figures:

Fig. 1. Schematic representation of magnetically actuated double nanowires carrying electric current.

Fig. 2. Fundamental frequency as a function of interwire distance for different levels of the electric current: (a) SS, (b) CC; ($\bar{B}_0=50$, $\lambda=15$, $\bar{f}_0 = \bar{B}_0 \bar{I}_0$, $\bar{f}_1 = \left(\frac{\bar{I}_0}{\bar{d}}\right)^2$; (---) approximate formulations, (—) exact formulations; ($\square$) $\bar{I}_0=0.1$, ($\circ$) $\bar{I}_0=1$, ($\triangle$) $\bar{I}_0=2$).

Fig. 3. Fundamental frequency as a function of slenderness ratio for different levels of the electric current: (a) SS, (b) CC; ($\lambda^*=30$, $\bar{B}_0^*=75$, $d=4r_0$; $\bar{I}_0 = \bar{I}_0^* \left(\frac{\lambda}{\lambda^*}\right)$, $\bar{B}_0 = \bar{B}_0^* \left(\frac{\lambda}{\lambda^*}\right)^2$; (---) approximate formulations, (—) exact formulations; ($\square$) $\bar{I}_0^*=0.1$, ($\circ$) $\bar{I}_0^*=1$, ($\triangle$) $\bar{I}_0^*=2$).

Fig. 4. Fundamental frequency as a function of electric current for different levels of the slenderness ratio: (a) SS, (b) CC; ($\bar{B}_0^*=50$, $d=5r_0$; $\bar{I}_0 = \bar{I}_0^* \left(\frac{\lambda}{\lambda^*}\right)$, $\bar{B}_0 = \bar{B}_0^* \left(\frac{\lambda}{\lambda^*}\right)^2$; (---) without surface effect, (—) with surface effect; ($\square$) $\lambda^*=15$, ($\circ$) $\lambda^*=20$, ($\triangle$) $\lambda^*=25$).

Fig. 5. Fundamental frequency as a function of longitudinal magnetic field strength for different levels of the slenderness ratio: (a) SS, (b) CC; ($\bar{I}_0^*=0.2$, $d=5r_0$; $\bar{I}_0 = \bar{I}_0^* \left(\frac{\lambda}{\lambda^*}\right)$, $\bar{B}_0 = \bar{B}_0^* \left(\frac{\lambda}{\lambda^*}\right)^2$; (---) without surface effect, (—) with surface effect; ($\square$) $\lambda^*=15$, ($\circ$) $\lambda^*=20$, ($\triangle$) $\lambda^*=25$).

Fig. 1.

Fig. 2.

Fig. 3.

Fig. 4.

Fig. 5.