Hydrodynamics of gas-solids flow in a bubbling fluidized bed with immersed vertical U-tube banks

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Abstract

We apply a two-fluid model (TFM) from the open-source code Multiphase Flow with Interphase eXchanges (MFIx) to investigate hydrodynamics in a gas-solids fluidized bed with immersed vertical tubes. The cut-cell method implemented in MFIx is used to fully resolve the flow around vertical U-tube banks. Simulations are performed in a bed diameter of 0.145 m with square and triangular tube arrangements, for inlet gas velocities of $U_0/U_{mf} = 2.3, 4.5$ and 6.8 . Simulation results are compared with experimental results from the literature and show very good agreement for the bubble size. The efficiency of vertical tubes in reducing bubble size depends upon inlet gas velocity and tube arrangement. Reduction in bubble size is due to the vertical tubes preventing bubble coalescence and promoting bubble splitting. In-bed vertical tubes result in uniform distribution of bubbles within the bed with increase in bubble frequency. The bubble frequency is higher within the bed for square tube arrangements. For a bed with vertical tubes, the bubble shape is generally elongated, which results in high bubble rise velocity. Axial solid velocity and solids circulation patterns are significantly affected by the vertical tubes, where triangular tube arrangements rarely show any solids circulating zone.

Keywords: fluidized beds, vertical tubes, two-fluid model, hydrodynamics, bubble.

1. Introduction

Gas-Solids fluidized beds are widely used in industrial operation due to their large contacting area between gas and solids, which enhances chemical reactions, heat and mass transfer. The performance of a fluidized bed is primarily governed by the formation of gas bubbles and their distribution, facilitating rapid solids mixing and high heat transfer rates to immersed surfaces. Formation and propagations of these gas bubbles in a fluidized bed is a fairly complicated process influenced by many factors such as properties of gas and particulate phase, bed operating conditions, reactor geometry and presence of internal objects. Many experimental and numerical studies [1-6] are available in the literature to understand complex hydrodynamics in fluidized beds. However these studies are mainly for 2D or 3D bed without taking into account the presence of in-bed tubes. Presence of in-bed tubes has several advantages. Initially those tubes were introduced to prevent slugging in the bed, where bubble size can reach the bed diameter. Almstedt and Jin et al. [7, 8] suggested that in-bed tube geometries offer a fairly well-defined bubble path up through the bed, as the bubbles grow to a limited size in free passage between the tubes. If there are many such free bubble paths in a wide bed, the bed as a whole will not show a slugging behavior. In many industrial fluidized beds, submerged tubes (i.e heating or cooling tubes) are used to enhance heat and mass transfer. In addition to acting as a heat exchanger these submerged tubes also alter the hydrodynamics significantly. It is well known that presence of these tubes affects the bubble growth significantly, such as preventing them from coalescence and forcing large bubbles to split [8, 9]. The splitting of bubbles results in redistribution into emulsion phase and a homogenous distribution of bubbles across the reactors [10, 11]. Thus fundamental understanding of bubble behavior in fluidized beds containing in-bed tubes has significant practical importance for the design and scale-up of these reactors

[12, 13]. However, complex hydrodynamics due to continuous interactions of the dense tubes with rising bubbles are not yet fully understood and therefore require detailed investigation.

Hydrodynamics studies on the effects of design configurations, such as the orientation and arrangement of tubes, tube diameter, tube spacing and the distance between the tube bank and gas distributor have been reviewed [8, 14]. Many experimental studies are available to study the effect of submerged tubes on hydrodynamics, and heat transfer characteristics. Majorities of the studies [15-19] are for in-bed arrays of horizontal tubes only. Horizontal tubes are found to be effective for splitting bubbles into daughter bubbles, but show relatively poor heat transfer coefficient at the bottom and top of the horizontal tubes due to formation of stagnant zones [8]. Under some circumstances, horizontal tubes cause daughter bubbles to recombine, thus nullifying the beneficial effect of splitting [20]. Formation of dead zone at the top and bottom of horizontally aligned tubes is a major industrial problem in operating fluidized beds. To overcome such a problem vertical tubes alignment are preferred over horizontal tubes. Vertical alignment of tubes provides high heat transfer rate and low erosion. Moreover vertical tubes divide the bed into number of parallel longitudinal compartments standing side by side, simplifying the scale-up [8, 9]. A significant drawback of vertical tubes compared to freely bubbling fluidized beds is their reduction of axial and lateral solids mixing, such that particle segregation is promoted and proper fluidization becomes difficult due to channeling [10, 21, 22]. However these can be prevented by choosing proper tube diameter and spacing, where spacing between adjacent tubes of at least 30 particle diameters is effective [8, 9].

There has been limited work published in the literature for in-bed vertical tubes, and this has limited applications to design and scale-up purposes. Detailed investigation on bubble characteristics and solids motion in fluidized beds with vertical tube are not reported yet. This may be due to limitations of experimental measurement techniques, which becomes difficult to perform with vertical tubes inside the beds. Rudisuli et al. [11, 23] studied small scale fluidized beds with vertical tubes, using pressure fluctuation measurements and optical probe measurements to extract bubble characteristics. They [11, 23]

1 studied different vertical tube configurations at different inlet gas velocities. They found significant
2 reduction in bubble size at higher gas velocities, and for narrow tube spacing, when compared to bed
3 without tubes. For the same flow condition, they also observed different bubble sizes, when different
4 measurement techniques (pressure fluctuation measurements and optical probe) were used. They have
5 shown encouraging results on bubble size and frequency. To a certain extent they succeeded to investigate
6 bubble rise velocity using optical probe measurement. Unfortunately, the measurement techniques they
7 applied have some limitations, and are unable to capture the bubble shape factor and solid flow patterns.

8 Almost all studies available in the literature regarding the effect of horizontal/vertical tubes focus on
9 experimental measurements. Only few numerical studies [24-27] report the effect of tubes inside fluidized
10 beds. Even these studies are mainly limited to 2D system with horizontal tubes, and mainly deal with
11 microscopic tube-bed heat transfer coefficients. To our knowledge no numerical study is available to
12 show the effect of vertical tubes on bubble characteristics and solids motion inside fluidized beds. This is
13 possibly due to the high computational cost and difficulty to represent banks of tubes using structured
14 grids. Although it is fairly reasonable to study a two-dimensional cross section of the bed when horizontal
15 tubes are present, numerical simulations with vertical tubes must be performed with three-dimensional
16 geometries. Fortunately, with increasing computational power and recent development in computational
17 algorithm to resolve internal objects, it has become possible to conduct three-dimensional simulations of
18 complex geometry systems. Therefore in this work we take the advantage of cut-cell method [28, 29] in
19 MFIX, to investigate hydrodynamics of 3D fluidized beds containing vertical U-tubes banks. The
20 motivation of this study arises from carbon capture units, which has been principally designed to contain
21 number of vertical U-tube banks. This study will provide proper understanding on the hydrodynamics in
22 such units. On the other hand comparison with experimental results and finding in the literature will show
23 the reliability of CFD models to study such complex geometries.

24 Discrete element modeling (DEM) and two-fluid modeling (TFM) are two common modeling approaches
25 to study fluidized beds. In DEM, the gas phase is considered as the continuous phase whereas the particle

trajectories are obtained by integrating Newton's equation of motion. Such model quickly becomes computationally too expensive because of large number of particle interactions. Therefore TFM is preferred over DEM, treating gas and solids phases as fully interpenetrating continua, and particle-particle interaction is taken into account using kinetic theory of granular flow. Many studies available in the literature have used TFM continuum approach to predict hydrodynamics successfully. In this work we used TFM from open-source code MFIX, incorporating state of art cut-cell method to resolve internal vertical U-tubes.

This paper is organized as follows. First, a general description of TFM is given in section 2 and the cut-cell method used to accommodate vertical U-tube bundles is then introduced in section 3. Details on bed geometry and simulation setting are given in section 4. Details on post-processing of simulation data to obtain bubble characteristics and solids motion are given in section 5. Subsequently, section 6 comprises the results and discussions including experimental comparison from Rudisuli et al. 2012a and finally the conclusions are drawn.

2. Two-Fluid Model

The two-fluid continuum model describes both the gas phase and the solids phase as fully interpenetrating continua using a generalized form of the Navier–Stokes equations. The gas and solid phases are coupled through the interphase momentum transfer coefficient in the momentum equations. The Gidaspow drag force model [30] is used in this work. A summary of conservation equations are given in Table 1. Kinetic theory of granular flow [30, 31] has been used to take into account the particle-particle interaction explicitly using granular temperature. Instead of the more detailed partial differential equation formulation, a simple algebraic approximation of the granular energy equation is used (Table 1 eq. T1-3), simplified by neglecting the convection and diffusion components of granular energy equation. This is a necessary choice to allow using the Cartesian grid cut-cell method of MFIX [29] to model vertical tubes inside fluidized beds and the cylindrical domain itself on Cartesian staggered grids. Bakshi et. al. [32] used an algebraic approximation for granular energy to perform simulations for bubbling beds and found good agreement with experimental results. Furthermore references [27, 33] have used the cut-cell feature

implemented in MFIX with an algebraic approximation for granular energy to model a horizontal tube bundle in a bubbling bed. Their simulation results show reasonably good agreement with experimental measurements of bubble frequency and bubble phase fraction, suggesting that the algebraic approximation is an acceptable simplification. Further details of governing equations solved in MFIX are summarized in reference [34]. Details on theory and numerical techniques can be found at <https://mfix.netl.doe.gov> [35, 36].

3. Cartesian Cut-Cell method

In the cut-cell method, a Cartesian grid is used to discretize the computational domain while the boundary cells are truncated so that they conform to the boundary surface. This method retains the advantages of a Cartesian grid formulation in the standard, non-boundary cells and additional treatment is required only for the boundary cells. Hence, it is very suitable for adoption in legacy Cartesian grid CFD solvers to represent complex geometries. Details about the Cartesian grid cut-cell method can be found in Kirkpatrick [37] and the MFIX implementation is described in references [28, 29].

To provide an overview, the following three steps are involved in the implementation and utilization of the cut-cell technique. First, as a pre-processing step, the geometry is defined as a collection of quadric surfaces which are intersected with the background Cartesian grid. Computational cells are truncated to conform to the shape of the boundaries and newly formed cut-cell volumes and face areas are updated. Velocity nodes are moved to the center of cut-cell faces. During computation of the flow field, fluxes are evaluated based on the new geometry and the location of velocity nodes. An additional term, arising from the contribution of the cut face shear stress, is included in the computation. Non-orthogonality correction terms are included in the computation of velocity gradients. Finally, post-processing tools have been modified to visualize the shape of cut-cells. Fig.1 (a) and (b) illustrate a typical computational grid near a curved boundary, along with velocity nodes. The thick solid line represents the wall boundary and three types of cells are shown: (1) standard (uncut) cells; (2) cut-cells that require special treatment to incorporate the presence of the solid wall; and (3) blocked cells that are excluded from computations since they are located outside the active computational domain. A no-slip or free-slip velocity boundary condition can be applied for each phase at the wall. For more details on implementation and incorporation of U-tube shapes readers are referred to Dietiker [28].

4. Simulation Settings

An experimental set-up described by Rudisuli et al. [23] is considered in this work. They performed experiments on a 0.145 m diameter fluidized beds using aluminum oxide particles, for inlet gas velocities of $U_0/U_{mf} = 2.3, 4.5$ and 6.8 , and a variety of vertical U-tubes configurations inside the bed. They measured pressure fluctuation time series at several heights within the bubbling bed using a piezoelectric pressure transducer. They obtained bubble size by means of the spectral data decomposition of pressure fluctuation measurements. Corresponding details for our simulation settings are given in Tables 2 and 3. The U_{mf} calculated from drag force model is found to be 0.042 m/s, which is close to the measured value of 0.041 m/s from Rudisuli et al. [23]. Therefore in this work we took U_{mf} value of 0.041 m/s as mentioned in Table 3. We performed simulations for two different tube configurations, i.e. square and triangular arrangements with the tubes 15 mm in diameter. We made this choice because these configurations have the same tube diameter and tube spacing but different tube arrangements. Experimental results show that these two arrangements are efficient in reducing bubble size. As their names suggest, the square tube arrangement illustrates an in-line tube position forming squares, whereas the triangular tube arrangement forms equilateral triangle with the tubes. The square tube arrangement results in large empty space near the column wall, which is filled by vertical auxiliary tubes in the experiments, and we followed the same arrangements for this work. For details on tube arrangements, the reader is referred to [23]. Fig. 2 (a) and (b) shows the geometry created for simulations with square and triangular tube arrangements respectively. Details about the geometry and computational grids are presented in Table 3. Details of TFM simulation settings can be found in Table 2. A non-slip boundary condition was applied for both gas and solid phases on the tube surface and the side walls. A uniform gas inflow from the bottom and a pressure outflow at the top exit were specified. The second order discretization Superbee scheme was used for solving the governing equations. MFIX uses a variable time step which is automatically adjusted to maintain stability and reduce run time. Typically, the maximum time step achieved in our simulations was in the order of 10^{-4} s. Three operating conditions corresponding to $U_0/U_{mf} = 2.3, 4.5$ and 6.8 were simulated. Here the superficial gas velocity was determined based on the cross-sectional area at the inlet. Please note that we did not take into account the effect of reduced area due to immersed tubes on the superficial gas velocity. The reduced area is nearly the same for both square and triangular tube arrangements. We performed simulations for isothermal and non-reactive flow for a

total time of 20.0 s. Time-averaging was performed after neglecting the first second of simulation time to avoid inclusion of start-up effects. The data sampling frequency is 125 frames per second.

5. Post-processing

Post-processing of 3D simulation data is crucial to calculate bubble properties such as bubble size and bubble rise velocity. It becomes further challenging when bubbles continuously interact with vertical tubes because the bubble definition may not be straight forward. From 3D simulation animations, we have seen that for a given tube configuration bubbles can rise along the vertical tube. To perform actual calculation on bubble properties and solids motion, we choose a measurement plane inside the fluid bed, where no or few number of tubes intersects that plane. Typically this plane is located in the center of the bed, between and parallel to the vertical tubes. We feel this the best possible measurement plane to illustrate the effect of vertical tubes on the bed hydrodynamics. To calculate the bubble characteristics, the gas volume fraction (voidage) is used. Voidage data is available in every computational cell within the measurement plane. A threshold voidage value of 0.80 is used to define the bubble boundary, as suggested by many authors [38]. Verma et al. [39] have reported bubble behavior using three different approaches to define the bubble in a 3D domain as follows: the bubble defined by the 2D bubble area in a plane inclined vertically to the bubble; a plane oriented horizontally to the bubble; and using the equivalent bubble size from the full 3D bubble volume. They compared results from these three bubble definitions in beds of different diameters. They found that both the 2D and 3D bubble definition are in good agreement with literature correlations for bed diameters less than 0.30 m. However for bed diameters greater than 0.30 m, bubble definition from 2D planes should be used with caution. Since in our simulations the bed diameter is 0.15 m, bubble properties are extracted from a 2D vertical measurement plane as given by equation (1) where N_b represents the number of predicted bubbles and $A_{b,i}$ is the area of an individual bubble. The number of predicted bubbles N_b is used to obtain the average equivalent bubble diameter.

$$\text{Equivalent bubble diameter} = \frac{1}{N_b} \sum_{i=1}^{N_b} \sqrt{\frac{4A_{b,i}}{\pi}} \quad (1)$$

In the work of Rudisuli et al. [23] bubble size is evaluated from PFM by means of spectral data decomposition, which provides a characteristic length scale proportional to the bubble size. The proportionality constant to obtain equivalent bubble size from PFM have been discussed in Rudisuli et al. [40]. In this work we compare actual bubble properties as given in Rudisuli et al. [23].

The bubble shape is characterized by the bubble aspect ratio given by ratio of maximum vertical length to the horizontal length of the bubble. The bubble distribution is the number of bubbles detected in each frame i.e. number of bubbles per frame.

The bubble rise velocity is calculated using a bubble tracking method [39] that considers individual bubble motion. In this approach, the displacement of the centroid of each bubble is traced at a given time frequency (125 fps). The net movement of the bubble centroid with time provides the bubble rise velocity. The averaged bubble rise velocity is obtained from an arithmetic average of individual bubble velocities.

Solids flow characteristics are compared for solids circulation patterns and solids axial velocity in the measurement plane. Equation 2 shows the expression for time-averaged axial solids velocity.

$$u_{axial,s}^{avg} = \frac{\sum \phi_s u_{axial,s}}{\sum \phi_s} \quad (2)$$

Where ϕ_s is the solids concentration and $u_{axial,s}$ is the axial solids velocity.

6. Results and Discussion

In this section we investigate the effect of vertical tubes on bubble characteristics and solids motion. The bubble characteristics are based on the equivalent bubble diameter, bubble distribution, bubble aspect ratio and the bubble rise velocity. The solids motion are investigated and compared on the basis of solids circulation patterns and the magnitude of axial solids rise velocity. Definition of these parameters has been discussed in section 5. Our discussion will be divided into a grid size dependency test presented in Section 6.1, a comparison of simulation with experimental results to show reliability of the TFM in

Section 6.2, and a detailed comparison of the effect of various configurations of vertical tubes in Section 6.3.

6.1. Grid independence test

It is known that numerical solutions are grid dependent, where solutions obtained on grids that are too coarse might not be accurate. Adequately refined grid simulations are required to obtain acceptable solution accuracy. However, simulations on overly fine grids will require very small time steps that will make them computationally too expensive for studying a large domain. The proper choice of grid size should resolve gas-solids flow structures on the smallest relevant microscopic length and time scales [41]. Andrews et al. [42] reported that a grid size less than $10d_p$ is required to capture the flow behavior accurately. This shows that gas-solids flow simulations are sensitive to grid size-to-particle diameter ratio. As a test of grid dependency, we performed the simulation for a triangular tube arrangement with grid sizes of 1.5, 2.0 and 2.5 mm ($5.2d_p$, $7.0d_p$ and $8.65d_p$ respectively). The grids were uniformly sized in all three directions. Fig. 3 (a) and (b) compare equivalent bubble diameter and bubble rise velocity on selected grid sizes. Fig. 3(a) shows that the equivalent bubble diameter predicted on coarse grid is slightly larger than with the finer mesh. Similarly, an increase in bubble rise velocity is observed as the mesh is coarsened in Fig. 3(b). This is caused by the fact that small bubbles (less than few grid sizes) are not well resolved on the coarser grid. This is clear from Fig 4(a) where a more bubbles are predicted on fine grid throughout the height of the bed and the number of small bubbles increases as the grid size is decreased (Fig. 4(b)). We also found that excessive refinement leads to convergence difficulties; therefore the uniform grid size of 1.5 mm ($5.2d_p$) is used for the remainder of this work.

6.2. Comparison between simulations and experimental results

In this section, we compare our simulation results with the experimental work of Rudisuli et al. [23]. Details on simulation settings and experimental conditions are given in Tables 2 and 3. Fig. 5 (a), (b) and (c) show comparison of equivalent bubble diameter for inlet gas velocities of $U_0/U_{mf} = 2.3, 4.5$ and 6.8 , respectively. In these figures, we will first focus on the experimental comparison for a bed without tubes. For the bed without tubes, the simulation and experimental results are in good agreement for the higher

inlet gas velocities of $U_0/U_{mf} = 4.5$ and 6.8 , except in the top section of the bed where bubble size measured in the experiment is lower. This is the region of bubble eruption into the free board region, where pressure fluctuation gives an artifact in bubble size due to bursting of bubble, as mentioned by Rudisuli et al. [23]. The bubble size compared at $U_0/U_{mf} = 4.5$ and 6.8 (Fig. 5(b) and (c)) is in very good agreement with the bubble size correlation from Werther [43]. For low inlet gas velocity of $U_0/U_{mf} = 2.3$ (Fig. 5(a)) simulation results show large differences, when compared with experimental measurements and the literature correlation of Werther [43]. Where experimental results are under predicted and literature correlation of Werther [43] shows slightly over predictions. Perhaps in the experiments, pressure waves generated at the lower inlet gas velocity may not be strong enough to be detected accurately by pressure sensors, resulting in inaccurate bubble size measurements. It explicitly depends upon configuration/strength of pressure sensor used; therefore more detail analysis of pressure fluctuation measurement is required at low inlet gas velocity. Furthermore literature correlation of Werther [43] is obtained from limited range of $(U_0 - U_{mf})$ i.e. $0.05 - 0.30$ m/s, therefore should be used with caution [40].

Considering the effect of in-bed vertical tubes in Fig. 5 (a), (b) and (c), a resultant reduction in bubble size is observed for all three inlet gas velocities. Substantial reduction in bubble size is seen at the high inlet gas velocity of $U_0/U_{mf} = 6.8$ (Fig. 5(c)). This shows that there is a significant influence of gas velocity on the ability of vertical tubes to reduce bubble size. Qualitatively this is consistent with the experimental results. Rudisuli et al. [23] observed the same bubble size for both square and triangular tube arrangements; however simulations predict bubble sizes with a small sensitivity to the tube arrangement. One potential reason for this difference is the fact that different methods were used to determine bubble size in the simulations and experiments. We used a single vertical plane between the tubes to determine bubble size, whereas pressure fluctuation measurements (PFM) in the experiments give an averaged value over the full horizontal cross-sectional area. Also note that we did not take into account the reduction area due to tubes, which can be another source of discrepancy. Nonetheless, experimental results and simulation predictions are in fairly close agreement at higher inlet gas velocities of $U_0/U_{mf} = 4.5$ and 6.8 . For low inlet gas velocity of $U_0/U_{mf} = 2.3$, the bubble size measurement from experiments shows a lower trend, this behavior is due to the same reason as explained that the pressure waves generated at the lower inlet gas velocity may not be strong enough to be detected accurately by pressure sensors.

6.3. Effect of vertical tube arrangement

In this section we focus on comparison of square and triangular tube arrangements. As a reference these two tube configurations are also compared with bubbling bed without tubes with same operating conditions.

6.3.1. General bubble characteristics

Fig. 6 shows instantaneous 3D bubble contours in a bed without and with vertical tubes. When there are no vertical tubes, bubbles grow in size due to coalescence and move towards the center of the bed. In the presence of vertical tubes, bubbles are comparatively smaller and distributed throughout the bed. As discussed in the literature [8, 44], vertical tubes prevent coalescence and promote bubble splitting. This phenomenon was observed in our simulation animations. Fig. 6 (b) and (c) show that bubbles take the preferred path along the tubes. Actual calculations on bubble properties have been discussed in section 6.3.2. Fig. 7 shows time-averaged porosity plot within the measurement plane. This time-averaged porosity indicates the preferred bubble trajectories in the bed. The time-averaged porosity plot for the bed without tubes (Fig. 7 (a)) indicates that bubbles originate near the wall and move towards the center of the bed, probably driven by lower resistance offered in the center. For the bed with vertical tubes in (Fig. 7 (b) and (c)), higher void fraction is predicted along the tube, indicating rising bubbles in contact with the tubes. The trajectories of rising bubbles with and without vertical tubes are also measured by Ozawa et al. [44], and are consistent with our simulation results. Similarly, the time-averaged gas velocity magnitude (Fig. 8) is predicted to be higher along the vertical tubes inside the bed. The variation in velocity with different tube arrangements can be seen in Fig. 8, where the square tube arrangement shows higher gas velocities in the measurement plane as compared to the triangular tube arrangement. At this stage, the argument could be that the rising bubble enclosed vertical tubes completely within the square arrangement. The cross-sectional view of instantaneous gas volume fraction in Fig. 9 shows that vertical tubes can be enclosed by larger bubble. We will discuss such observation in detail by comparing actual bubble properties in the following sections.

6.3.2. Bubble size and their distribution

It is shown in Fig. 5 that both the square and the triangular tube arrangements act to reduce bubble size. This reduction in bubble size is approximately 20, 50 and 60% for corresponding inlet gas velocities of $U_0/U_{mf} = 2.3, 4.5$ and 6.8 , respectively. With vertical tubes, bubbles do not show significant growth as they rise through vertical tubes, with the rate of growth being linear (See Fig. 5(a) and (b)). Fig. 10 shows predicted bubble size as a function of lateral position averaged throughout the height of the bed. For a given inlet gas velocity, the bubble size is larger in the center for a bed without tubes. This results from the bubbles forming at the bottom near the wall and then moving towards the bed center and coalescing as they rise [45]. Vertical tubes prevent coalescence and promote bubble splitting. This is seen in Fig. 10 which shows an almost uniform average bubble size across the bed diameter as predicted by the model. Splitting of bubble is evident from Fig. 11, where bubble distribution (calculated as the ratio of number of bubble detected to the number of frame analyzed) is plotted as a function of axial position. For vertical tube, a large number of bubbles are predicted throughout the height irrespective of inlet gas velocity. Significantly more bubbles are predicted in the bottom section of the bed. This is due to the fact that U-shape bank prevents bubble coalescence at the initial stage as the bubble grows. For inlet gas velocities of $U_0/U_{mf} = 2.3$ and 4.5 (Fig. 11 (a) and (b) respectively), a similar bubble distribution is predicted higher in the bed, above $h = 0.25$ m, for both square and triangular tube arrangements. However, in the bottom section of the bed (below 0.25 m), the square tube arrangement predicts more bubbles. This shows that the square tube arrangement is efficient in preventing coalescence of small bubbles in the lower bed region, because the tubes create parallel chambers for the bubble to rise. At a higher gas velocity of $U_0/U_{mf} = 6.5$, (Fig. 11(c)) more bubbles are predicted for the triangular tube arrangement in the upper section of the bed. This indicates that a triangular tube arrangement promotes bubble splitting due to the staggered alignment of the tubes, compared to inline alignment in the square arrangement. This is also a reason to observe lower average bubble size with triangular tube arrangement (See Fig. 5).

Yates et al. [20] claimed that at a certain distance from each other, vertical tubes break up bubbles into daughter bubbles. Considering this phenomena, from Fig. 11(a) and (b) it can be seen that above the height of 0.20 m larger bubbles split into two daughter bubbles for both tubes arrangements at low and moderate inlet gas velocities. However, for the largest inlet gas velocity, Fig.11(c) shows that square arrangement is effectively splitting bubble into two daughter bubbles and triangular arrangement into

three. This further illustrates that the efficiency of vertical tube to split bubble not only depends on inlet gas velocity but also on their arrangement.

Fig. 12 shows bubble distribution as a function of equivalent bubble diameter. In the model results, the number of small bubbles in the bed is significantly greater for the beds with vertical tubes when compared to the bed with no tubes. At a gas velocity of $U_0/U_{mf} = 6.8$ (Fig. 12(c)), the triangular tube arrangement shows significantly more small bubbles due to splitting of larger bubbles; this result is consistent with Fig. 11 (c). The number of larger bubbles is similar for both tube arrangements indicating that bubble size is unaffected if it is sufficiently large compared to the tube spacing.

6.3.3. Bubble aspect ratio

Vertical tubes not only affect bubble size and bubble distribution, but also the bubble shape. The shape of the bubble can be estimated from the bubble aspect ratio, defined as ratio of vertical length to the horizontal length of the bubble. Fig. 13 shows bubble aspect ratio as a function of equivalent bubble diameter. When there are no tubes, bubbles are nearly spherical in shape with a bubble aspect ratio close to one but become slightly more elongated as the fluidization velocity increases. The presence of vertical tubes shows a significant effect on the bubble shape over the range of inlet gas velocities modeled as seen in Fig 13(a), (b) and (c). The larger bubble aspect ratio shows that bubbles elongate significantly under the influence of vertical tubes. This is because the initial effect of vertical tubes is to squeeze and deform bubbles to fit the space between the tubes. A similar observation on bubble shape impact is measured by Asegehegn et al. [13] when they measured bubble properties between arrays of horizontal tubes. We have not yet found experimental data available on the effect of vertical tubes on the bubble shape. The increase in bubble aspect ratio is also attributed to splitting bubbles in horizontal direction by vertical tubes. This splitting of bubble in horizontal direction reduces their horizontal length, resulting in larger bubble aspect ratio. It is interesting to note that triangular tube arrangements shows considerable difference in bubble shape factor when compared with square tube arrangement. The reason for this corresponds to the tube alignment. The triangular tube arrangement occupies more number of principal tubes, thus creating narrow spacing throughout the cross section. This narrow spacing split and squeezed bubble significantly resulting in higher bubble aspect ratio. For inlet gas velocity of $U_0/U_{mf} = 6.8$ (Fig. 13(c)), same sized

1 bubble have lower bubble aspect ratio i.e. bubble is less elongated, when compared to inlet gas velocity of
2 $U_0/U_{mf} = 2.3$ (Fig. 13(a)). This is because at very high gas velocity bubbles are trapped and rise along the
3 tube.

4 Overall it can be concluded that the square tube arrangement prevents coalescence of small bubbles and
5 the triangular tube arrangement promotes splitting of large bubbles - depending on the fluidizing
6 velocities. If the bubble is larger than tube spacing, it encloses the vertical tube as it rises.

7 **6.3.4. Bubble rise velocity**

8 The bubble rise velocity is an important property to characterize fluidization, as it directly influences the
9 bubble phase residence time. Grace and Harrison, [9] reported that vertical tubes significantly affect
10 bubble rise velocity. They concluded that bubbles travel faster when there are vertical tubes inside the
11 bed. Similarly references [20, 46] showed that bubble rise velocity increases with an increase in the
12 number of vertical tubes. Rudisuli et al. [11] found no clear trend in bubble rise velocities for vertical
13 tubes. They observed trends in bubble rise velocity for varying inlet gas velocity at two different heights
14 in the column. Since these findings are contrary to other literature studies, they suggested examining
15 thoroughly the effect of vertical tubes on bubble rise velocity. In this work we use simulations to
16 investigate the effect of vertical tubes on bubble rise velocity..

17 Fig. 14 shows average bubble rise velocity as a function of height from the bottom for different inlet gas
18 velocities. For low inlet gas velocities of $U_0/U_{mf} = 2.3$ and 4.5 i.e. in Fig. 14 (a) and (b), bubble rise
19 velocity shows an increasing trend in the presence of tubes, below the height of 0.3 m. Above this height
20 bubble rise velocity is higher for bed without tubes. For inlet gas velocity of $U_0/U_{mf} = 6.5$ i.e. Fig 14(c)
21 shows that the bubble rise velocity is lower for the bed with the tubes, specifically above the height of
22 0.10 m. Rudisuli et al. [11] also studied trends in bubble rise velocity, depending upon gas flow rate and
23 height from the bottom. For a square tube arrangement at $U_0/U_{mf} = 2.3$, they found a decrease in bubble
24 rise velocity at height of 0.243 m and 0.45 m (note that they have only two measurement locations)

1 compared to a bed without tubes. Our simulation results are consistent with their findings for an inlet gas
2 velocity of $U_0/U_{mf} = 2.3$. For $U_0/U_{mf} = 4.5$, Rudisuli et al. [11] found that bubble rise velocity increases at
3 both measured heights. Simulation results are consistent with their observation only for the height of
4 0.245 m. For $U_0/U_{mf} = 6.8$ they found bubble rise velocity decreases at 0.243 m and increase at 0.450 m.
5 Simulation results are consistent at a height of 0.45 only. The possible discrepancy in the results could be
6 due to limitations of the optical probe measurement technique. The optical probe measurement gives only
7 local information depending upon the location of optical probe, whereas in our simulation we considered
8 averaging of all the bubbles at the given height, to determine average bubble rise velocity.

9 When comparing bubble rise velocity for square and triangular tube arrangement, the latter one shows
10 higher bubble rise velocity at $U_0/U_{mf} = 2.3$ and 4.5, and lower at $U_0/U_{mf} = 6.5$. The potential reason is that
11 at low inlet gas velocities bubble size is comparable to the tube spacing, therefore considerable squeeze
12 occurs between the tubes and bubbles rise faster. At higher gas velocities, bubble sizes are large enough
13 that they enclose the tube and rise along the tube walls. Grace and Harrison, [9] claimed that if the ratio of
14 bubble size to tube diameter is greater than 5, several bubbles will enclose the vertical tube and rise along
15 their wall. In another work of Rudisuli et al. [11], faster bubble rise velocity was observed between the
16 tubes as compared to velocities along the tube. As a result of the squeezing of bubble between the tubes,
17 the centroid of bubble moves a longer distance than uniform size bubble. Elongation of bubbles is
18 apparent in the bubble aspect ratio results shown in Fig. 13. For $U_0/U_{mf} = 6.5$, bubble rise velocity is
19 much higher in the bed without tubes because bubbles formed at a high inlet gas velocity are much larger
20 (see Fig. 5(c)) and are expected to rise faster.

21 Fig. 15 (a), (b) and (c) shows bubble rise velocity as a function of equivalent bubble diameter for inlet
22 gas velocities of $U_0/U_{mf} = 2.3$, 4.5 and 6.8 respectively. These figures show that bubbles of the same size
23 rise with different velocities, where bubbles travel faster in the bed with tubes. The reason is given in the
24 literature [9, 46]: the bubble is elongated and follows preferential path along the vertical tubes. Yates et
25 al. [20] indicates that bubble rise velocity can become twice as fast as bubble of the same volume in the

absence of the tubes. Our simulation predicts this behavior for inlet gas velocity of $U_0/U_{mf} = 2.3$ and $U_0/U_{mf} = 4.5$. For $U_0/U_{mf} = 6.8$ bubble rise velocity is approximately 50% higher than in the absence of tubes. This shows that bubble rise velocity in the bed with tubes depends upon fluidizing gas velocity. The bubbles are rising relatively faster in the triangular tube arrangement. This observation is consistent with Fig. 13 (a) and (b).

6.3.5. Solids flow characteristics

Solids circulation pattern in the bubbling bed is another important factor which affects the fluidization quality. Solids particles are dragged up by the bubbles in their wakes and, by continuity particle will move downwards in the regions with no or few bubbles. Researchers [1, 39, 47] have investigated extensively the solid circulation patterns for bubbling beds without tubes. The impact of horizontal tubes on solids circulation pattern have been investigated [24], but to our knowledge no investigations of solids circulations for in-bed vertical tubes have been reported. Rudisuli et al. (2012a) [11] proposed a description of solid motion/circulation from analogy of bubble motion for the bed with vertical tubes. However actual validation or confirmation on the solids motion in the bed with vertical tubes has not been reported. Therefore it becomes of practical importance to investigate fluidized bed solids motion as affected by vertical tubes. Fig. 16 (a), (b) and (c) show time-averaged solids circulation pattern within the measurement plane for $U_0/U_{mf} = 4.5$ representing bed without tube, square and triangular arrangement respectively. For clarity only reasonable numbers of vectors are shown, and lateral and axial lengths in these figures are on different scale. For the triangular tube arrangement in Fig. 16(c), velocity vectors in the region $x > 0.12$ m are ignored due to clustering of surrounding tubes in that region. Note that vertical dotted line in these figures represent the location of vertical tubes, where magenta colored line represent tubes located on the other side of the measurement plane. Fig. 16(a) shows that in the absence of tubes, solids circulation forms distinct vortices within the bed. In the bottom section of the bed below 0.1 m, solids are descending in the center and ascending near to the wall. The predicted behavior above the

height of 0.1 m shows the opposite trend. Solids circulation zones are formed in the middle and top of the bed, with dominant upward solids motion in the bed center. High velocity solids at the bed surface indicate the splashing of particles into the freeboard region due to abrupt eruption of the bubble. These predictions are consistent with the literature [1, 47]. In the bed with vertical tubes (Fig. 16 (b) and (c)), this splashing of solids into freeboard regions is gentle, due to vertical tubes promoting bubble eruption phenomena to occur slowly. Solids circulation patterns in the bed with vertical tubes are different compared to beds without tubes, especially in the bottom section of the bed. In the bottom section of the bed, solids circulation vortices disappear and solids move upward in the center. This is due to presence of U-shape banks, preventing solid circulation. Elongated solids circulation vortices are formed due to vertical tubes, with solids rising along the tubes. Fig. 16(b) indicates that solids are circulating in small, parallel compartments for the square tube arrangement, whereas in the triangular arrangement these circulating zones are missing (Fig. 16(c)). Radial motion of particles is less pronounced for vertical tubes, preventing radial mixing. Authors [8, 44] also claimed that vertical tubes did not allow radial mixing in fluidized beds. For vertical tubes, solids are continuously descending near the wall of the fluidized bed. Very similar solids flow patterns are seen for inlet gas velocities of $U_0/U_{mf} = 2.3$ and 6.8 (Figures not shown).

Fig. 17 and 18 (a), (b) and (c) show the magnitude of axial solids velocity at heights of 0.18, 0.34 and 0.51 m from the bottom at fluidizing velocities of $U_0/U_{mf} = 2.3$ and 4.5. A clear trend for the bed without tubes is seen with high solids velocity in the bed center and low solids velocity away from the bed center at all the reported heights. At the reported bed heights, downward motion of solids takes place near the walls. Fig. 17(b) and 18(b) show the solids axial velocity profile for the square tube arrangement, where higher solids velocities lie in the region between the tubes. This shows that collections of 4 tubes (i.e. the square arrangement) form a column of solids that act as a separate, small 3D bed. Descending of particles is seen near the bed walls due to more free space available due to the presence of only auxiliary tube of relatively small tube diameter (see Table 3). For the triangular tube arrangement data shown in Fig. 17(c),

1 zig-zag motion of solids is predicted, rising high in the region formed by the triangular vertical tube
2 arrangement. The magnitude of solids velocities is nearly the same at these three heights for vertical
3 tubes. Experimental work would be very valuable to investigate solids velocities for in-bed vertical tubes
4 and useful to validate the simulations. Nevertheless simulations results provide an understanding on
5 solids motion.

6 **7. Conclusions**

7 The influence of vertical tubes on bubble characteristics and solids motion in a fluidized bed has been
8 investigated using the MFIX two-fluid model. Square and triangular tube arrangements have been
9 compared to the bed without tubes for a range of inlet gas velocities of $U_0/U_{mf} = 2.3, 4.5$ and 6.8 . The
10 vertical tubes influence the hydrodynamics of fluidized beds significantly. A decrease in equivalent
11 bubble diameter and a uniform distribution of bubble are seen for the bed with vertical tubes at all three
12 inlet gas flow conditions. A considerable reduction in bubble sizes is seen for in-bed vertical tubes at
13 $U_0/U_{mf} = 6.8$. Simulation results show that the square tube arrangement forms longitudinal, parallel
14 chambers that prevent bubble coalescence. Since triangular tubes are in a staggered arrangement, they
15 promote bubble splitting. Due to splitting and squeezing of bubbles between the tubes their shapes change
16 significantly, becoming more elongated. The elongated bubble of the same size has a higher bubble rise
17 velocity in the bed with the vertical tubes. Differences in solids circulation patterns are very distinct for
18 the three bed configurations. Solids circulating zones are predicted for the square arrangement, whereas
19 circulating zones are not seen for the triangular tube arrangement. Significant solids motion is rarely seen
20 in the radial direction because the vertical tubes prevent lateral solids motion. Higher axial solid velocity
21 is predicted in the center for the bed without tubes whereas the beds with tubes show higher axial velocity
22 between the tubes. This indicates that vertical tubes divide the bed into smaller, parallel chambers for gas-
23 solids to flow. Therefore, use of vertical tubes in fluidized beds show an advantage for bed scale up.

1 A comparison of simulation results with experimental data shows good agreement. Evaluations of various
2 bubble characteristics are consistent with results reported in the literature. Further experimental work is
3 important for further validation of simulation result and to improve understanding of solids motion in the
4 presence of the vertical tubes. Nevertheless this study provides valuable insight into bubble and solids
5 behavior and the effects of in-bed, vertical tubes.

6 **Nomenclature**

7	g	gravitational acceleration, [m/s^2]
8	g_0	radial distribution function, [-]
9	d_p	diameter of particle, [m]
10	p	pressure, [Pa]
11	v	velocity [m/s]
12	t	time, [s]
13	e_{pp}	coefficient of restitutions, [-]
14	U_{mf}	minimum fluidization velocity, [m/s]
15	U_0	superficial velocity, [m/s]
16	N_b	Number of bubbles
17	A_b	Area occupied by the bubble
18	Greek symbols	
19	ϕ	volume fraction, [-]
20	ρ	density, [kg/m^3]
21	θ	pseudo particles temperature, [m^2/s^2]
22	τ	stress tensor, [Pa]
23	Subscripts/Superscript	
24	s	solid phase

1	g	gas phase
2	p	particle
3	mf	minimum fluidization
4	b	bubble
5	avg	averaged quantity

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22 those of the United States Government or any agency thereof.

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Table 1: Summary of TFM governing equations.

Continuity equation: (T1-1)

$$\frac{\partial(\phi_g \rho_g)}{\partial t} + \nabla \cdot (\phi_g \rho_g \mathbf{v}_g) = 0$$

$$\frac{\partial(\phi_s \rho_s)}{\partial t} + \nabla \cdot (\phi_s \rho_s \mathbf{v}_s) = 0$$

Momentum equation: (T1-2)

$$\begin{aligned} \frac{\partial(\phi_g \rho_g \mathbf{v}_g)}{\partial t} + \nabla \cdot (\phi_g \rho_g \mathbf{v}_g \mathbf{v}_g) &= -\phi_g \nabla p_g - \nabla \cdot (\phi_g \bar{\bar{\tau}}_g) - \beta_{g:s} (\mathbf{v}_g - \mathbf{v}_s) + \phi_g \rho_g \mathbf{g} \\ \frac{\partial(\phi_s \rho_s \mathbf{v}_s)}{\partial t} + \nabla \cdot (\phi_s \rho_s \mathbf{v}_s \mathbf{v}_s) &= -\phi_s \nabla p_g - \nabla p_s - \nabla \cdot (\phi_s \bar{\bar{\tau}}_s) + \beta_{g:s} (\mathbf{v}_g - \mathbf{v}_s) + \phi_s \rho_s \mathbf{g} \end{aligned}$$

Granular energy conservation (algebraic formulation): (T1-3)

$$\theta_s = \left[\frac{-K_1 \phi_s tr(D_s) + \sqrt{[K_1 tr(D_s) \phi_s]^2 + 4K_4 \phi_s [K_2 tr^2(D_s) + 2K_3(D_s : D_s)]}}{2\phi_s K_4} \right]^2$$

$$\text{Where } D_s = \frac{1}{2} \left[\nabla \mathbf{v}_s + (\nabla \mathbf{v}_s)^T \right],$$

$$K_1 = 2(1 + e_{pp})\rho_s g_0, \quad K_2 = \frac{4d_p(1 + e_{pp})\rho_s \phi_s g_0}{3\sqrt{\pi}} - \frac{2}{3}K_3,$$

$$K_3 = \frac{d_p \rho_s}{2} \left\{ \frac{\sqrt{\pi}}{3(3 - e_{pp})} \left[0.5(1 + 3e_{pp}) + 0.4(1 + 3e_{pp})(3e_{pp} - 1)\phi_s g_0 \right] + \frac{8\phi_s g_0(1 + e_{pp})}{5\sqrt{\pi}} \right\},$$

$$K_4 = \frac{12(1 - e_{pp}^2)\rho_s g_0}{d_p \sqrt{\pi}}$$

Gidaspow drag correlation:

$$\beta_{g,s} = \begin{cases} \frac{3}{4} C_D \frac{\rho_g \phi_g \phi_s |v_g - v_s|}{d_p} \phi_s^{-2.65} & \phi_g \geq 0.8 \\ \frac{150\phi_s(1 - \phi_g)\mu_g}{\phi_g d_p^2} + \frac{1.75\rho_g \phi_s |v_g - v_s|}{d_p} & \phi_g < 0.8 \end{cases}$$

$$C_D = \begin{cases} \frac{24}{\text{Re}(1 + 0.15\text{Re}^{0.687})} & \text{Re} < 1000 \\ 0.44 & \text{Re} \geq 1000 \end{cases}$$

$$\text{Re} = \frac{\rho_g \phi_g |v_g - v_s| d_p}{\mu_g}$$

Table 2: Material properties and operating conditions for the TFM simulation

Properties	Values
Particle type	Aluminum oxide
Particle density	1350 kg/m ³
Particle diameter	289 μm
Coefficient of restitution	0.90
Minimum fluidizing velocity(U _{mf})	0.041 m/s
Superficial velocity at inlet (U ₀)	2.3U _{mf} , 4.5U _{mf} , 6.8U _{mf}

Table 3: geometry and grid specification in the simulations

Properties	Without tubes	Sq. arrangement	Tri. arrangement
Column width (number of grids)	0.15 m (100)	0.15 m (100)	0.15 m (100)
Column depth (number of grids)	0.15 m (100)	0.15 m (100)	0.15 m (100)
Column height (number of grids)	0.96 m (640)	0.96 m (640)	0.96 m (640)

Bed diameter from cut-cells	0.145 m	0.145 m	0.145
Initial solid bed height	0.50 m	0.50 m	0.50 m
Initial solid volume fraction	0.58	0.58	0.58
Number of principal tubes (diameter)	-	16 (15 mm)	24 (15 mm)
Number of auxiliary tubes (diameter)	-	8 (12 mm)	2 (10 mm)

Fig. 1. Cut cell treatment on a staggered grid, (a) Scalar cut-cells with location of velocity nodes, (b) interpolation of velocity at the face of cut-cell.

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(a)

(b)

Fig. 2. 3D view of the column with vertical tube inside having (a) square tube arrangement and, (b) triangular tube arrangement.

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Fig. 3. Effect of grid size on (a) the equivalent bubble diameter, and (b) the bubble rise velocity, at different height from the bottom in the bed with triangular tube arrangement for $U_0/U_{mf} = 4.5$.

Fig. 4. Effect of grid size on bubble distribution as a function of (a) height from the bottom, and (b) the equivalent bubble diameter, for the bed with triangular tube arrangement operating at $U_0/U_{mf} = 4.5$.

Fig. 5. Equivalent bubble diameter as a function of height from the bottom comparing bed with and without tubes at operating inlet gas velocity of (a) $U_0/U_{mf} = 2.3$, (b) $U_0/U_{mf} = 4.5$, and (c) $U_0/U_{mf} = 6.8$, comparison with experimental results of Rudisuli et al. [23].

Fig. 6. Randomly selected instantaneous 3D bubble contours ($\phi_g = 0.80$) for (a) bed without tubes, (b) with tubes in a square arrangement, and (c) with tubes in a triangular arrangement, for inlet gas velocity of $U_0/U_{mf} = 4.5$.

Fig. 7. Time-averaged porosity plot in a measurement plane, in a bed (a) without tubes, (b) with tubes in square arrangement, and (c) with tubes in triangular arrangement, for inlet gas velocity of $U_0/U_{mf} = 4.5$.

Fig. 8. Time-averaged axial gas velocity in a vertical measurement plane for a bed (a) without tubes, (b) with tubes in a square arrangement, and (c) with tubes in a triangular arrangement for an inlet gas velocity of $U_0/U_{mf} = 4.5$.

Fig. 9. Randomly selected instantaneous snapshots of gas volume fraction in a horizontal cross section for a bed with square tube arrangement: (a), (b) and (c) represent gas volume fractions at different times at a height of 0.2 m from the bottom for $U_0/U_{mf} = 4.5$.

Fig. 10. Modeled equivalent bubble diameter as a function of lateral position averaged through the bed height, comparing bed with and without tubes at operating inlet gas velocity of (a) $U_0/U_{mf} = 2.3$, (b) $U_0/U_{mf} = 4.5$, and (c) $U_0/U_{mf} = 6.8$.

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Fig. 11. Bubble distribution as a function of height from the, comparing bed with and without tubes at operating inlet gas velocity of (a) $U_0/U_{mf} = 2.3$, (b) $U_0/U_{mf} = 4.5$, and (c) $U_0/U_{mf} = 6.8$.

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Fig. 12. Bubble distribution as a function of the equivalent bubble diameter, comparing bed with and without tubes at operating inlet gas velocity of (a) $U_0/U_{mf} = 2.3$, (b) $U_0/U_{mf} = 4.5$, and (c) $U_0/U_{mf} = 6.8$.

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5 Fig. 13. Bubble aspect ratio as a function of the equivalent bubble diameter, comparing bed with and

6 without tubes at operating inlet gas velocity of (a) $U_0/U_{mf} = 2.3$, (b) $U_0/U_{mf} = 4.5$, and (c) $U_0/U_{mf} = 6.8$.

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5 Fig. 14. Bubble rise velocity as a function of height from the bottom, comparing bed with and without

6 tubes at operating inlet gas velocity of (a) $U_0/U_{mf} = 2.3$, (b) $U_0/U_{mf} = 4.5$, and (c) $U_0/U_{mf} = 6.8$.

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5 Fig. 15. Bubble rise velocity as a function of equivalent bubble diameter, comparing bed with and without

6 tubes at operating inlet gas velocity of (a) $U_0/U_{mf} = 2.3$, (b) $U_0/U_{mf} = 4.5$, and (c) $U_0/U_{mf} = 6.8$.

Fig. 16. Time-averaged solids circulation pattern at $U_0/U_{mf} = 4.5$ for a bed (a) without tubes, (b) tubes with square arrangement, and (c) tubes with triangular arrangement. Vertical lines in (b) and (c) represent locations of vertical tubes surrounding the measurement plane. In (c) vertical lines with different colors represents tubes in front and back of the measurement plane, in (b) tubes in front and back plane are in inline so these lines overlap. The color bar represents the magnitude of the solids velocity in m/s.

Fig. 17. Time-averaged axial solids velocity at $U_0/U_{mf} = 2.3$ for bed (a) without tubes, (b) tubes with square arrangement, and (c) tubes with triangular arrangement, vertical line in (b) and (c) represent location of vertical tubes surrounding the measurement plane. In (c) vertical lines with different color

- 1 represents tubes in front and back of the measurement plane, in (b) tubes in front and back plane are in
- 2 inline so these line overlap.

1 Fig. 18. Time-averaged axial solids velocity at $U_0/U_{mf} = 4.5$ for bed (a) without tubes, (b) tubes with
2 square arrangement, and (c) tubes with triangular arrangement, vertical line in (b) and (c) represent
3 location of vertical tubes surrounding the measurement plane. In (c) vertical lines with different color
4 represents tubes in front and back of the measurement plane, in (b) tubes in front and back plane are in
5 inline so these line overlap.