Variable Star Signature Classification using Slotted Symbolic Markov Modeling

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Abstract

With the advent of digital astronomy, new benefits and new challenges have been presented to the modern day astronomer. No longer can the astronomer rely on manual processing, instead the profession as a whole has begun to adopt more advanced computational means. This paper focuses on the construction and application of a novel time-domain signature extraction methodology and the development of a supporting supervised pattern classification algorithm for the identification of variable stars. A methodology for the reduction of stellar variable observations (time-domain data) into a novel feature space representation is introduced. The methodology presented will be referred to as Slotted Symbolic Markov Modeling (SSMM) and has a number of advantages which will be demonstrated to be beneficial; specifically to the supervised classification of stellar variables. It will be shown that the methodology outperformed a baseline standard methodology on a standardized set of stellar light curve data. The performance on a set of data derived from the LINEAR dataset will also be shown.

Keywords: stellar variability, supervised classification, Markov Modeling, time-domain analysis

1. Introduction

With the advent of digital astronomy, new benefits and new challenges have been presented to the modern day astronomer. While data is captured in a more efficient and accurate manor using digital means, the efficiency of data retrieval has led to an overload of scientific data for processing and storage. This means that more stars, in more detail are captured per night; but increasing data capture begets exponentially increasing data processing. Database management, digital signal processing, automated image reduction and statistical

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analysis of data have all made their way to the forefront of tools for the modern astronomer. Astro-statistics and astro-informatics are fields which focus on the application and development of these tools to help aid in the processing of large scale astronomical data resources.

A methodology for the reduction of stellar variable observations (time-domain data) into a novel feature space representation is introduced. The proposed methodology, referred to as Slotted Symbolic Markov Modeling (SSMM), has a number of advantages over other classification approaches for stellar variables (to be detailed below). The remainder of this paper is structured as follows. First, the data, prior efforts, and challenges uniquely associated to classification of stars via stellar variability is reviewed. Second, the novel methodology, SSMM, is outlined including the feature space and signal conditioning methods used to extract the unique time-domain signatures. Third, a set of classifiers (random forest/bagged decisions tree, k-nearest neighbor, and Parzen window classifier) is trained and tested on the extracted feature space using both a standardized stellar variability dataset and the LINEAR dataset. Fourth, performance statistics are generated for each classifier and a comparing and contrasting of the methods is discussed. Lastly, an anomaly detection algorithm is generated using the so called one-class Parzen Window Classifier and the LINEAR dataset. The result will be the demonstration of the SSMM methodology as being a competitive feature space reduction technique, for usage in supervised classification algorithms.

This paper focuses on the construction and application of a novel time-domain signature extraction methodology and the development of a supporting supervised pattern classification algorithm for the identification of variable stars. Given the reduction of a survey of stars into a standard feature space, the problem of using prior patterns to identify new observed patterns can be addressed via classification algorithms. These methods have two large advantages over manual-classification procedures: the rate at which new data is processed is dependent only on the computational processing power available and the performance of a supervised classification algorithm is quantifiable and consistent.

1.1. *Related Work*

The idea of constructing a supervised classification algorithm for stellar classification is not unique to this paper [12]. Methods pursued include the construction of a detector to determine variability [2], the design of random forests for the detection of photometric redshifts in spectra [7], the detection of transient events [10] and the development of machine-assisted discovery of astronomical parameter relationships [20]. Deboscher [9] explored several classification techniques for the supervised classification of variable stars, quantitatively comparing the performance in terms of computational speed and performance. Likewise, other efforts have focused on comparing speed and robustness of various methods [4, 43, 42]. These methods span both different classifiers and different spectral regimes, including IR surveys [1, 36], RF surveys [47] and optical [51]. Methods for automated supervised classification include procedures such as: direct para-

metric analysis [65], fully automated neural networking [44, 45] and Bayesian classification [15].

The majority of these studies rely on periodicity domain feature space reductions. Debosscher [9] and Templeton [63] review a number of feature spaces and a number of efforts to reduce the time domain data, most of which implement Fourier techniques, primarily the Lomb-Scargle (L-S) Method [34, 55], to estimate the primary periodicity [15, 40, 51, 37, 8]. Lomb-Scargle is favored because of the flexibility it provides with respect to observed datasets; when sample rates are irregular and drop outs are common in the data being observed. Long et al. [35] advance L-S even further, introducing multi-band (multidimensional) generalized L-S, allowing the algorithm to take advantage of information across filters, in cases where multi-channel time-domain data is available. There have also been efforts to estimate frequency using techniques other than L-S such as the Correntropy Kernelized Periodogram, [23] or MUlti SSignal Classifier [60].

The assumption of the light curve being periodic, or even that the functionality of the signal being represented in the limited Fourier space that Lomb-Scargle uses, has been shown [39, 2] to result in biases and other challenges when used for signature identification purposes. Supervised classification algorithms implementing these frequency estimation algorithms do so to generate an estimate of primary frequency used to fold all observations resulting in a plot of magnitude vs. phase, something Deb and Singh [8] refer to as “reconstruction”. After some interpolation to place the magnitude vs. phase plots on similar regularly sampled scales, the new folded time series can be directly compared (1-to-1) with known folded time series. Comparisons can be performed via distance metric [60], correlation [46], further feature space reduction [9] or more novel methods [24]. It should be noted that the family of stars with the label “stellar variable” is a large and diverse population: eclipsing binaries, irregularly pulsating variables, nova (stars in outburst), multi-model variables, and many others are frequently processed using the described methods despite the underlying stellar variability functionality not naturally lending itself to Fourier decomposition and the associated assumptions that accompany the said decomposition. Indeed this is why Szatmary et al. [59], Barclay et al. [2], Palaversa et al. [39] and others suggest using other decomposition methods such as discrete wavelet transformations, which have been shown to be powerful in the effort to decompose a time series into the time-frequency (phase) space for analysis [64, 5, 53]. It is noted that the digital signal processing possibilities beyond Fourier domain analysis time series comparison and wavelet transformation are too numerous to outline here; however the near complete review by Fulcher et al. [17] is highly recommended.

2. Slotted Symbolic Markov Modeling

The discussion of the algorithm designed encompasses the analysis, reduction and classification of data. The *a priori* distribution of class labels are roughly evenly distributed for both studies, therefore the approach uses a multi-class

classifier. Should the class labels with additional data become unbalanced, other approaches are possible [52]. Data specific challenges, associated with astronomical time series observations, have been identified as needing to be addressed as part of the algorithm design.

2.1. Algorithm Design

Stellar variable time series data can roughly be described as passively observed time series snippets, extracted from what is a contiguous signal (star shine) over multiple nights or sets of observations. The time series signature have the potential to change over time, and new observations allow for the increased opportunity for an unstable signature over the long term. Astronomical time series data is also frequently irregular, i.e., there is often no associated fixed Δt over the whole of the data that is consistent with the observation. Even when there is a consistent observation rate, this rate is often broken up because of observational constraints. The stellar variable moniker covers a wide variety of variable types: stationary (consistently repeating identical patterns), non-stationary (patterns that increase/decrease in frequency over time), non-regular variances (variances that change over the course of time, shape changes), as well as both Fourier and non-Fourier sequences/patterns. Pure time-domain signals do not lend themselves to signature identification and pattern matching, as their domain is infinite in terms of potential discrete data (dimensionality). Not only must a feature space representation be found, but the dimensionality should not increase with increasing data.

Based on these outlined data/domain specific challenges (continuous time series, irregular sampling, and varied signature representations) this paper will attempt to develop a feature space extraction methodology that will construct an analysis of stellar variables and characterize the shape of the periodic stellar variable signature. A number of methods have been demonstrated that fit this profile [19, 16, 17], however many of these methods focus on identifying a specific time series shape sequence in a long(er) continuous time series, and not necessarily on the differentiation between time series sequences. To address these domain specific challenges, the following methodology outline is implemented:

1. To address the irregular sampling rate, a slotting methodology is used [49]: Gaussian kernel window slotting with overlap. The slotting methodology is used to generate estimates of amplitudes at regularized points, with the result being a up-sampled conditioned waveform. This has been shown to be useful in the modeling and reconstruction of variability dynamics[48, 28, 23], and is similar to the methodologies used to perform Piecewise Aggregate Approximation [26].
2. To reduce the conditioned time series into a usable feature space, the amplitudes of the conditioned time series will be mapped to a discrete state space based on a standardized alphabet. The result is the state space representation of the time domain signal, and is similar to the methodologies used to perform Symbolic Aggregate Approximation [32].

3. The state space transitions are then modeled as a first order Markov Chain Ge and Smyth [18], and the state transition probability matrix (Markov Matrix) is generated, a procedure unique to this study. It will be shown that a mapping of the transitions from observation to observation will provide an accurate and flexible characterization of the stellar variability signature.

The Markov Matrix is unfolded into a vector, and is the signature pattern (feature vector) used in the classification of time-domain signals for this study. It should be noted, if the underlying signature was too sparsely sampled the feature space transform would not capture the shapes or features of interest, as it would be true with any time-domain transform (i.e., Fourier methods will not capture frequency content over the Nyquist). Many of the transforms that are commonly used combat the problems of irregular samples or low sample density, make additional assumptions such as the underlying waveform shape (e.g. Box-fitting Least Squares) or the frequency of occurrence (based on physical parameters). Many of these assumptions are oriented towards specific target detection; as such, the false alarm and missed detection rates are tied specifically to those assumptions. A collection and comparison of these frequency sampling methods and the associated assumptions can be found in Graham et al. [21].

2.2. Slotting (Irregular Sampling)

Each waveform is conditioned using the slotting re-sampling methodology for irregularly sampled waveforms outlined in Rehfeld et al. [49]. The slotting procedure acts as follows:

1. a set of evenly spaced (in time) windows, with size w are generated, the windows can be overlapping or adjacent
2. within each window (slot) are observed samples, the difference in time between the observation point and the center of the window is computed
3. a gaussian kernel with width s weights the contribution of the observation, a estimate of the amplitude for the new point is generated from the weighted amplitudes of the contributing observations.

For this implementation, an overlapping slot (75% overlap) was used, meaning that the window width is larger than the distance between slot centers. This methodology is effectively Kernel Smoothing with Slotting [30], with a slot width of w . This w is optimized, via cross-validation of the data; antedotely however, median sample rate of the waveforms is often a best estimate; as such the rate would capture at least one point in each slot when applied to continuous observation data. Initial testing (ANOVA) demonstrated that the misclassification rate varied little with changes in w about the median.

Let the set of $\{y(t_n)\}_{n=1}^N$ samples, where $t_1 < t_2 < t_3 < \dots < t_N$ and there are N samples, be the initial time series dataset. The observed time series data is standardized (subtract the mean, divide by the standard deviation), and then the slotting procedure is applied. If $x[i] \leftarrow y(t_i)_{i=1}^N$, then the algorithm to generate the slotted time domain data is given in Algorithm 1.

Algorithm 1 Gaussian Kernel Slotting

```

1: procedure GAUSSIANKERNELSLOTTING( $x[i], t[i], w, \lambda$ )
2:
3:    $x_{prime}[i] \leftarrow (x[i] - \text{mean}(x[i])) / \text{std}(x[i])$        $\triangleright$  Standardize Amplitudes
4:    $t[i] \leftarrow t[i] - \min(t[i])$                            $\triangleright$  Start at Time Origin
5:    $\text{slotCenters} \leftarrow 0 : \frac{w}{4} : \max(t[i]) + w$            $\triangleright$  Make Slot Locations
6:    $\text{timeSeriesSets} = []$                                      $\triangleright$  Initialize Time Series Sets
7:    $\text{slotSet} = []$                                             $\triangleright$  Make an Empty Slot Set
8:
9:   while  $i < \text{length}(\text{slotCenters})$  do                     $\triangleright$  Compute Slots
10:     $\text{idx} \leftarrow \text{all } t \text{ in interval } [\text{slotCenters} - w, \text{slotCenters} + w]$ 
11:     $\text{inSlotX} \leftarrow x[\text{idx}]$ 
12:     $\text{inSlotT} \leftarrow t[\text{idx}]$ 
13:
14:    if  $\text{inSlot}$  is empty then                                   $\triangleright$  There is a Gap
15:      if  $\text{slotSet}$  is empty then                                 $\triangleright$  Move to Where Data is
16:         $\text{currentPt} \leftarrow \text{find next } t > \text{slotCenters} + w$ 
17:         $i \leftarrow \text{find last } \text{slotCenters} < t[\text{currentPt}]$ 
18:      else                                                     $\triangleright$  Store the Slotted Estimates
19:        add  $\text{slotSet}$  to structure  $\text{timeSeriesSets}$ 
20:         $\text{slotSet} \leftarrow []$ 
21:      end if
22:    else
23:       $\text{weights} \leftarrow \exp(-((\text{inSlot} - \text{slotCenters})^2 * \lambda))$ 
24:       $\text{meanAmp} \leftarrow \text{sum}(\text{weights} * \text{inSlotX}) / \text{sum}(\text{weights})$ 
25:      add  $\text{meanAmp}$  to the current  $\text{slotSet}$ 
26:    end if
27:     $i++$ 
28:  end while
29: end procedure

```

The time series, with irregular sampling and large gaps is conditioned by the Gaussian slotting method. Gaps in the waveform are defined as regions where a slot contains no observations. Continuous observations (segments) are the set of observations between the gaps. This results in a irregularly sampled waveform potentially resulting in a set of regularly sampled waveforms for various lengths depending on the original survey parameters. This conditioning is also similar to the Piecewise Aggregation Approximation[31, 26]. Instead of down-sampling the time domain datasets as PAA does however, the data is up-sampled using the slotting methodology. This is necessary because of the sparsity of the time domain sampling of astronomical data.

2.3. State Space Representation

If it is assumed that the conditioned standardized waveform segments have an amplitude distribution that approximates a Gaussian distribution (which

they won't, but that is irrelevant to the effort), then using a methodology similar to Symbolic Aggregate Approximation [32, 33] methodologies, an alphabet (state space) is defined based on our assumptions as an alphabet extending between $\pm 2\sigma$ and will encompass 95% of the amplitudes observed. This need not always be the case, but the advantage of the standardization of the waveform is that, with some degree of confidence the information from the waveform is contained roughly between $\pm 2\sigma$. Figure 1 demonstrates a eight state translation; the alphabet will be significantly more resolved then this for astronomical waveforms.

The resolution of the alphabet granularity is to be determined via cross-validation to determine an optimal resolution for a given survey. The set of state transitions, the transformation of the conditioned signal, is used to populate a transition probability matrix or first order Markov Matrix.

2.4. Transition Probability Matrix (Markov Matrix)

The transition state frequencies are estimated for signal measured between empty slots, transitions are not evaluated between day-night periods, or between slews (changes in observation directions during a night) and only evaluated for continuous observations. Each continuous set of conditioned waveforms (with Slotting and State Approximation applied) is used to populate the empty matrix P , with dimensions equal to $r \times r$, where r is the number of states, is built. The matrix is populated using the following rules:

- N_{ij} is the number of observation pairs $x[t]$ and $x[t+1]$ with $x[t]$ is state s_i and $x[t+1]$ in state r_j
- N_i is the number of observation pairs $x[t]$ and $x[t+1]$ with $x[t]$ in state s_i and $x[t+1]$ in any one of the states $j = 1, \dots, r$

The now populated matrix P is a transition frequency matrix, with each row i representing a frequency distribution (histogram) of transitions out of the state s_i . The transition probability matrix is approximated by converting the elements of P by approximating the transition probabilities using $P_{ij} = N_{ij}/N_i$. The resulting matrix is often described as a first order Markov Matrix [54]. State changes are based on only the observation-to-observation amplitude changes; the matrix is a representation of the linearly interpolated sequence [18]. Furthermore, the matrix is unpacked similar to image analysis methods into a feature space vector, with dimensions depend on the resolution and bounds of the states. The algorithm to process the time-domain conditioned data is given in Algorithm 2.

Algorithm 2 Markov Matrix Generation

```

procedure MARKOVMATRIXGENERATION(timeSeriesSets, s)
2:   markovMatrix = []
   for i := 1 to length of timeSeriesSets do
4:     markovMatrixPrime ← []
     currentSlotSet ← markovMatrixPrime[j]
6:     for k := 1 to length of currentSlotSet do
       idxIn ← find state containing currentSlotSet[k − 1]
       idxOut ← find state containing currentSlotSet[k]
       markovMatrixPrime[idxIn, idxOut] ++
10:    end for
    markovMatrix ← markovMatrix + markovMatrixPrime
12:  end for
  Ni = sum along row of markovMatrix
14:  for j := 1 to length of s do
    if Ni ≠ 0 then
16:      markovMatrix[j, j] ←  $\frac{N_{ij}}{N_i}$       ▷ Estimate Markov Matrix
    end if
18:  end for
end procedure

```

The resulting Markov Matrix is unpacked into a feature vector given by:

$$\mathbf{P}_i = \begin{bmatrix} p_{11} & p_{12} & \dots & p_{1r} \\ p_{21} & p_{22} & \dots & \dots \\ \vdots & \vdots & \ddots & \vdots \\ p_{r1} & p_{r2} & \dots & p_{rr} \end{bmatrix} \Rightarrow x_i = [p_{11} \quad p_{12} \quad \dots \quad p_{21} \quad \dots \quad p_{rr}] \quad (1)$$

Where $\mathbf{P}_i$ is the Markov Chain of the i^{th} input training set, and x_i is the i^{th} input unfolded training pattern.

When using the Markov matrix representation, the resolution of the state set needs to be small to avoid loss of information resulting from over generalization. However, if the state resolution is too small the sparsity of the transition matrix will result in a shape signature that is too dependent on noise and the “individualness” of specific waveform to be of any use. Thus additional processing is necessary for further analysis; even a small set of states (12 x 12) will result in a feature vector with high dimensionality (144 dimensions). While a window and overlap size is assumed for the slotting to address the irregular sampling of the time series data, there are two adjustable features associated with this analysis: the kernel width associated with the slotting and the state space (alphabet) resolution. It is apparent that a range of resolutions and kernel width need to be tested to determine best performance given a generic supervised classifier.

2.5. Feature Space Reduction (ECVA)

For these purposes a rapid initial classification algorithm, General Quadratic Discriminate Analysis [13], was implemented to estimate the mis-classification rate (wrong decisions/total decisions). To reduce the large, sparse, feature vector resulting from the unpacking of the Markov Matrix we applied a supervised dimensionality reduction technique commonly referred to as canonical variate analysis (ECVA) [38]. The methodology for ECVA has roots in principle component analysis (PCA). PCA is a procedure performed on large multidimensional datasets with the intent of rotating what is a set of possibly correlated dimensions into a set of linearly uncorrelated variables [56]. The transformation results in a dataset, where the first principle component (dimension) has the largest possible variance. PCA is an unsupervised methodology, i.e. a priori known labels for the data being processed is not taken into consideration, thus a reduction in feature dimensionality and while it maximizes the variance it might not maximize the linear separability of the class space. In contrast to PCA, Canonical Variate Analysis does take class labels into considerations. The variation between groups is maximized resulting in a transformation that benefits the goal of separating classes. Given a set of data $\mathbf{x}$ with: g different classes, n_i observations of each class, and $r \times r$ dimensions in each observation; following Johnson et al. [25], the within-group and between-group covariance matrix is defined as:

$$\mathbf{S}_{within} = \frac{1}{n-g} \sum_{i=1}^g \sum_{j=1}^{n_i} (\mathbf{x}_{ij} - \bar{\mathbf{x}}_{ij})(\mathbf{x}_{ij} - \bar{\mathbf{x}}_i)' \quad (2)$$

$$\mathbf{S}_{between} = \frac{1}{g-1} \sum_{i=1}^g n_i (\mathbf{x}_i - \bar{\mathbf{x}})(\mathbf{x}_i - \bar{\mathbf{x}})' \quad (3)$$

where $n = \sum_{i=1}^g n_i$, $\bar{\mathbf{x}}_i = \frac{1}{n_i} \sum_{j=1}^{n_i} \mathbf{x}_{ij}$, and $\bar{\mathbf{x}} = \frac{1}{n} \sum_{j=1}^{n_i} n_i \mathbf{x}_i$. CVA attempts to maximize the function:

$$J(\mathbf{w}) = \frac{\mathbf{w}' \mathbf{S}_{between} \mathbf{w}}{\mathbf{w}' \mathbf{S}_{within} \mathbf{w}} \quad (4)$$

Which is solvable so long as $\mathbf{S}_{within}$ is non-singular, which need not be the case, especially when analyzing multi-collinear data. When the case arises that the dimensions of the observed patterns are multi-collinear additional considerations need to be made. Nørgaard et al. [38] outlines a methodology for handling these cases in CVA. Partial least squares analysis, PLS2 [66], is used to solve the above linear equation, resulting in an estimate of $\mathbf{w}$, and given that, an estimate of the canonical variates (the reduced dimension set). The application of ECVA to our vectorized Markov Matrices results in a reduced feature space of dimension $g-1$.

3. Implementation of Methodology

3.1. Datasets

Two datasets are addressed here, the first is the STARLIGHT dataset from the UCR time series database, the second is published data from the LINEAR survey. The UCR time series dataset is used to base line the time-domain dataset feature extraction methodology proposed, it is compared to the results published on the UCR website. The UCR time series data contains only time domain data that has already been folded and put into magnitude phase space, no photometric data from either SDSS or 2MASS, nor star identifications for these data, could be recovered, and only three class types are provided which are not defined besides by number. The second dataset, the LINEAR survey, provides an example of a modern large scale astronomical survey, contains time-domain data that has not been folded or otherwise manipulated, is already associated with SDSS and 2MASS photometric values, and has five identified stellar variable types. For each dataset, the state space resolution and the kernel widths for the slotting methods will be optimized using 5-fold cross-validation. The performances of three classifiers on only the time-domain dataset for the UCR data, and on the mixture of time-domain data and color data for the LINEAR survey, are estimated using 5-fold cross-validation and testing. The performances of the classifiers will be compared. Finally an anomaly detection algorithm will be trained and tested, for the LINEAR dataset.

3.2. Pattern Classification Algorithm

The training set is used for 5-Fold cross-validation, and a set of three classification algorithms are tested [22, 13]: k Nearest Neighbor (k-NN), Parzen Window Classifier (PWC) and Random Forest (RF). Cross-validation is used to determine optimal classification parameters (e.g., kernel width) for each of the classification algorithms. The first three algorithms implemented were designed by the authors in MATLAB, based on Duda et al. [13] and Hastie et al. [22] (k-NN and PWC) algorithm outlines. Request for the implemented code should be made to the authors directly.

3.2.1. k-NN

The k nearest neighbor algorithm is a non-parametric classification method; it uses a voting scheme based on an initial training set to determine the estimated label. For a given new observation, the L_2 Euclidean distance is found between the new observation and all points in the training set. The distances are sorted, and the k closest training sample labels are used to determine the new observed sample estimated label (majority rule). Cross-validation is used to find an optimal k value, where k is any integer greater than zero.

3.2.2. PWC

Parzen windows classification is a technique for non-parametric density estimation, which is also used for classification [41, 13]. Using a given kernel

function, the technique approximates a given training set distribution via a linear combination of kernels centered on the observed points. As the PWC algorithm (much like a k-NN) does not require a training phase, as the data points are used explicitly to infer a decision space. Rather than choosing the k nearest neighbors of a test point and labeling the test point with the weighted majority of its neighbor's votes, one can consider all points in the voting scheme and assign their weight by means of the kernel function. With Gaussian kernels, the weight decreases exponentially with the square of the distance, so far away points are practically irrelevant. Cross-validation is necessary however, to determine an optimal value of h , the “width” of the radial basis function (or whatever kernel is being used).

3.2.3. Random Forest Classifier

To generate the random forest classifier, the TreeBagger algorithm in MATLAB is implemented. The algorithm generates n decision trees on the provided training sample. The n decision trees operate on any new observed pattern, and the decision made by each tree are conglomerated together (majority rule) to generate a combined estimated label. To generate Breiman's 'random forest' algorithm [6], the value NVarToSample is provided a value (other than 'all') and a random set of variables is used to generate the decision trees; see the MATLAB TreeBagger documentation for more information.

3.3. Comparison to Standard Set (UCR)

The UCR time domain datasets are used to basis classification methodologies [27]. The UCR time domain datasets [46], are derived from a set of Cepheid, RR Lyrae, and Eclipsing Binary Stars. The time-domain datasets have been phased (folded) via the primary period and smoothed using the SUPER-SMOOTHER algorithm [50] by the Protopapas study prior to being provided to the UCR database. The waveforms received from UCR are amplitude as a function of phase; the SUPERSMOOTHER algorithm was also used [46] to produce regular samples (in the amplitude vs. phase space). The sub-groups of each of the three classes are combined together in the UCR data (i.e., RRab + RRc = RR), similarly the data is taken from two different studies (OGLE and MACHO). A plot of the phased light curves is given in Figure 2.

Class analysis is a secondary effort when applying the methodology outlined to the UCR dataset, the primary concern is a demonstration of performance of the supervised classification methodology with respect to the baseline performance reported by UCR implementing a simple waveform nearest neighbor algorithm.

3.3.1. Analysis

The folded waveforms are treated identical to the unfolded waveforms in terms of the processing presented. Values of phase were generated to accommodate the slotting technique, thereby allowing the functionally developed to be used for both amplitude vs. time (LINEAR) as well as amplitude vs. phase

(UCR). The slotting, State Space Representation, Markov Matrix and ECVA flow is implemented exactly the same. As there are only three classes in the dataset, the ECVA algorithm results in a dimensionality of only two ($g - 1$). There is no accompanying color data with the time-domain data, so only the time-domain data will be focused on for this analysis. The resulting ECVA plot is presented in Figure 3.

Each classifier is then trained only on the ECVA reduced time-domain feature space. The resulting optimization analysis, based on the 5-fold cross-validation is presented in Figures 4a, 4b and 4c.

Depending on the methodology used, cross-validation estimates a misclassification error of $< 10\%$. The UCR website reports the following error estimates for this dataset, note that all methods reported use direct distance to generate a feature space (direct comparison of curves): 1-NN Euclidean Distance (15.1%), 1-NN Best Warping Window DTW (9.5%) and 1-NN DTW, no warping window (9.3%). For a more detailed comparison, the confusion matrix for each of the optimized classifiers is presented in Tables 1a, 1b, and 1c.

3.3.2. Discussion

The SSMM methodology presented does no worse than the 1-NN presented by Keogh et al. [27] and appears to provide some increase in performance. The procedure described operates on folded data as well as unfolded data and does not need time-warping for alignment of the waveform, demonstrating the flexibility of the method. The procedure not only separated out the classes outlined, but in addition found additional clusters of similarity in the dataset. Whether these clusters correspond to the sub-groupings reported by the original generating source (RRab and RRc, etc.) is not known, as object identification is not provided by the UCR dataset.

3.4. Application to New Set (LINEAR)

For the analysis of the proposed algorithm design, the LINEAR dataset is parsed into training, cross-validation and test sets on time series data from the LINEAR survey that has been verified, and for which accurate photometric values are available [57, 39]. From the starting sample of 7194 LINEAR variables, a clean sample of 6146 time series datasets and their associated photometric values were used for classification. Stellar class type is limited further to the top five most populous classes: RR lyr(ab), RR lyr (c), Delta Scuti / SX Phe, Contact Binaries and Algol-Like Stars with 2 Minima; resulting in a set of 6086 observations. The distribution of stellar classes is presented in Table 2.

3.4.1. Non-Variable Artificial Data

In support of the supervised classification algorithm, artificial datasets have been generated and introduced into the training/testing set. These artificial datasets are representation of stars with-out variability. This introduction of artificial data is done for the same reasons the training of the anomaly detection algorithm is performed:

- The LINEAR dataset implemented only represents five of the most populous variable star types, while at least 23 stellar variable types are known [51], thus the class space defined by the classes is incomplete.
- Even if the class space was complete, studies such as Debosscher [9], Dubath et al. [12] have all shown that many stellar variable populations are under-sampled.
- Similarly, many of the studies focus on stellar variables only, and do not include non-variable stars. While filters are often applied to separate variable and non-variable stars (Chi-Squared specifically), these are not necessarily perfect methods for removing non-variable populations, and could result in an increase in false alarms.

This artificial time series is generated with a Gaussian Random amplitude distribution. In addition to the time-domain information randomly generated, photometric information is also generated. The photometric measurements used to classify the stars are used to generate empirical distributions (histograms) of each of the feature vectors. These histograms are turned into cumulative distribution functions (CDFs). The artificially generated photometric patterns are generated via sampling from these generated empirical distribution functions. Sampling is performed via the Inverse Transform method [29]. These artificial datasets are treated identical in processing to the other observed waveforms.

3.4.2. Time Domain and Color Feature Space

In addition to the time domain data, color data is obtainable for the LINEAR dataset, resulting from the efforts of large photometric surveys such as SDSS and 2MASS. These additional features are merged with the reduced time domain feature space, resulting in an overall feature space. For this study, the optical SDSS filters (*ugriz*) and the IR filters (*JK*) are used to generate the color features: $u - g$, $g - i$, $i - K$ and $J - K$. The color magnitudes are corrected for the ISM extinction using $E(B - V)$ from the SFD maps and the extinction curve shape from Berry et al. [3]. In addition to these color features, bulk time domain statistics are also generated: $\log P$ is the log of the primary period derived from the Fourier domain space, $magMed$ is the median LINEAR magnitude, $ampl$, $skew$ and $kurt$ are the amplitude, skewness and kurtosis for the observed light curve distribution. These additional features will be included for the analysis of the LINEAR dataset. See electronic supplement (Combined LINEAR Features, Extra-Figure-CombinedLINEARFeatures.fig) for a plot matrix of the combined feature space.

3.4.3. Analysis

It is assumed that the state space resolution that minimizes the mis-classification rate using QDA, will likewise minimize the mis-classification rate using any of the other classification algorithms. The slot width was taken to be 0.015 and a kernel spread of 0.01 was used. Using the optimal amplitude state space resolution (0.03), a three dimensional plot (the first three ECVA parameters)

is constructed; see the electronic supplement for the associated movie (ECVA Feature LINEAR Movie, ExtendedCanonicalVariates.mp4). Figure 5 is a plot of the first two extended canonical variates:

Based on the merged feature space, the optimal parameters for the kNN, PWC and Random Forest Classifier are generated. The cross-validation optimization figures for each are presented in Figures 6a, 6b and 6c respectively.

Testing was performed on a pre-partitioned set, separate from the training and cross-validation populations. The transformation applied to the training and cross-validation data were also applied to the testing data. After optimal parameters have been found for both the resolution of the Markov Model and the classification algorithms, the testing set is used to estimate the confusion matrix. A confusion matrix is generated, True Labels are shown on the left column and Estimated Label are shown on the top row (Tables 3a, 3b and 3c).

Further analysis was performed comparing the classification capability of a supervised classifier with only the SSMM features (post-ECVA analysis), with only the traditional feature spaces and a classifier with all three feature spaces (photometric data, frequency and time statistics, and SSMM) to show the SSMM relative performance (Table : 4).

Comparable performance is obtained just using the SSMM feature space compared to the color and frequency space, and for PWC, a small increase is obtained when the features are combined.

3.4.4. Anomaly Detection

In addition to the pattern classification algorithm outlined, the procedure outlined includes the construction of an anomaly detector. The pattern classification algorithm presented as part of this analysis, partition the entire decision space based on the known class type provided in the LINEAR dataset. For many supervised classifier algorithms, and indeed those presented here, there is no consideration for deviations of patterns beyond the training set observed, i.e. absolute distance from population centers. All of the algorithms investigated consider relative distances, i.e. is the new pattern P closer to the class center of B or A? Thus, despite that an anomalous pattern is observed by a new survey, the classifier will attempt to estimate a label for the observed star based on the labels it knows. To address this concern, a one-class anomaly detection algorithm is implemented.

Anomaly Detection and Novelty Detection methods are descriptions of similar processes with the same intent, i.e., the detection of new observations outside of the class space established by training. These methods have been proposed for stellar variable implementations prior to this analysis [46]. Tax [61] and Tax and Muller [62] outline the implementation of a number of classifiers for One-Class (OC) classification, i.e., novel or anomaly detection. Here, the PWC algorithm (described earlier) is transformed into an OC anomaly detection algorithm. The result is the “lassoing” or dynamic encompassing of known data patterns. The lasso boundary represents the division between known (previously observed) regions of feature space and unknown (not-previously observed) regions. New patterns observed with feature vectors occurring in this unknown

region are considered anomalies or patterns without support, and the estimated labels returned from the supervised classification algorithms should be questioned, despite the associated posterior probability of the label estimate. This paper implements the DD Toolbox designed by Tax and implements the PR toolbox [14]. The resulting error curve generated from the cross-validation of the PWC-OC algorithm resembles a threshold model (probit), the point which minimizes the error and minimizes the kernel width is found (Figure 7).

This point (minimization of error and kernel width) is the optimal kernel width (2.5). Estimated mis-classification rate of the detector is determined via evaluation of the testing set and found to be 0.067%.

3.4.5. Discussion

Given only time series data (no photometric data, frequency data, or time domain statistics), for the classes and the LINEAR observations made (resolution of amplitude and frequency rate of observations) a $\sim 2\%$ mis-classification rate with the various, more general, classifiers. Kernel width of the slots used to account for irregular sampling and state space resolution are major factors in performance, as mentioned we have assumed a slotting with that is a function of the survey (median of the continuous sample rate). This has been found to work optimally for the LINEAR dataset, however cross-validation could be performed with other dataset to determine if addition optimization is possible. With the addition of photometric data, the mis-classification rate is reduced by another $\sim 1\%$, and results in a nearly separable class space, depending on the methodology used to determine the estimated class. An anomaly detection algorithm is trained and tested on the time series data and photometric data. An expected mis-classification rate, with both the anomaly detection algorithm and supervised classification algorithm, of $\sim 0.07\%$ is found.

4. Conclusions

The Slotted Symbolic Markov Modeling (SSMM) methodology developed has been able to generate a feature space which separates variable stars by class (supervised classification). This methodology has the benefit of being able to accommodate irregular sampling rates, dropouts and some degree of time-domain variance. It also provides a fairly simple methodology for feature space generation, necessary for classification. One of the major advantages of the methodology used is that a signature pattern (the transition state model) is generated and updated with new observations. The transition frequency matrix for each star is accumulated, given new observations, and the probability transition matrix is re-estimated. The methodology's ability to perform is based on the input data sampling rate, photometric error and most importantly the uniqueness of the time-domain patterns expressed by variable stars of interest.

The analysis presented has demonstrated the SSMM methodology performance is comparable to the UCR baseline performance analysis, if not slightly better. In addition, the translation of the feature space has demonstrated that

the original suggestion of three classes might not be correct; a number of additional clusters are revealed as are some potential mis-classifications in the training set. The performance of four separate classifiers trained on the UCR dataset is examined. It has been shown that the methodology presented is comparable to direct distance methods (UCR base line). It is also shown that the methodology presented is more flexible. The LINEAR dataset provides more opportunity to demonstrate the proposed methodology. The larger class space, unevenly sampled data with dropouts and color data all provide additional challenges to be addressed. After optimization, the mis-classification rate is roughly $\sim 1\%$, depending on the classifier implemented. An anomaly detection algorithm is trained and tested on the time series data and color data as well, the combined algorithm has an expected mis-classification rate of $\sim 0.07\%$. The effort represents the construction of a supervised classification algorithm.

4.1. Future Research

Further research is outlined in three main focus topics: dataset improvement, methodology improvement, simulation/performance analysis. The limited dataset and class space used for this study is known. Future efforts will include a more complete class space, as well as more data to support under-represented class types. Specifically datasets such as the Catalina Real Time Transient Survey [11], will provide greater depth and completeness as a prelude to the data sets that will be available from the Panoramic Survey Telescope & Rapid Response System and the Large Synoptic Survey Telescope (LSST).

In addition to improving the underlying training data used, the methodology outline will also be researched to determine if more optimal methods are available. Exploring the effects of variable size state space for the translation could potentially yield performance improvements, as could a comparison of slotting methods (e.g. box slots vs. Gaussian slots vs. other kernels or weighting schemes). Likewise, implementations beyond supervised classification (e.g., unsupervised classification) were not explored as part of this analysis. How the feature space outlined in this analysis would lend itself to clustering or expectation-maximization algorithms is yet to be determined.

In a future paper, how sampling rates and photometric errors affect the ability to represent the underlying time-domain functionality using synthetic time-domain signals will be explored. Simulation of the expected time domain signals will allow for an estimation of performance of other spectral methods (DWT/DFT for irregular sampling), which will intern allow for and understanding of the benefits and drawbacks of each methodology, relative to both class type and observational conditions. This type of analysis would require the modeling and development of synthetic stellar variable functions to produce reasonable (and varied) time domain signature.

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5. Figures

Figure 1: Example State Space Representation

Figure 2: UCR Phased Light Curves. Classes are given by number only: 1 = Blue Line, 2 = Green Small Dashed Line, 3 = Red Big Dashed Line

Figure 3: ECVA reduced feature space using the UCR Star Light Curve Data

Figure 4: Classifier Optimization for UCR Data

(a) Nearest Neighbor Classifiers

(b) Parzen Window Classifier

(c) Random Forest

Figure 5: First two Extended Canonical Variates for the Time-Domain Feature Space

Figure 6: Classifier Optimization for LINEAR Data

(a) Nearest Neighbor classifiers

(b) Parzen window classifier

(c) Random Forest

Figure 7: OC-PWC Kernel Width Optimization for LINEAR Data

6. Tables

Table 1: Confusion Matrix for Classifiers Based on UCR Starlight Data

(a) 1-NN				(b) PWC			
True\Est	1	2	3	True\Est	1	2	3
1	0.86	0.003	0.13	1	0.82	0.003	0.18
2	0.0	0.99	0.008	2	0.00	0.97	0.035
3	0.031	0.002	0.97	3	0.16	0.004	0.84

(c) RF			
True\Est	1	2	3
1	0.91	0.003	0.082
2	0.0	0.99	0.005
3	0.004	0.0007	0.99

Table 2: Distribution of LINEAR Data Across Classes

Type	Count	Percent
Algol	287	4.7%
Contact Binary	1805	29.7%
Delta Scuti	68	1.1%
No Variablity	1000	16.4%
RRab	2189	36.0%
RRc	737	12.1%

Table 3: Confusion Matrix for Classifiers Based on UCR Starlight Data

(a) 1-NN

True\Est	Algol	Contact Binary	Delta Scuti	No Variation	RRab	RRc
Algol	0.76	0.20	0.0	0.0	0.0	0.04
Contact Binary	0.03	0.95	0.005	0.005	0.01	0.0
Delta Scuti	0.0	0.0	0.88	0.12	0.0	0.0
No Variation	0.0	0.0	0.01	0.99	0.0	0.0
RRab	0.0	0.005	0.0	0.0	0.95	0.045
RRc	0.0	0.03	0.0	0.0	0.14	0.83

(b) PWC

True\Est	Algol	Contact Binary	Delta Scuti	No Variation	RRab	RRc
Algol	0.97	0.01	0.0	0.0	0.02	0.0
Contact Binary	0.0	0.99	0.0	0.0	0.0	0.01
Delta Scuti	0.0	0.0	0.94	0.06	0.0	0.0
No Variation	0.0	0.0	0.0	1.0	0.0	0.0
RRab	0.0	0.01	0.0	0.0	0.99	0.0
RRc	0.0	0.01	0.0	0.0	0.0	0.99

(c) RF

True\Est	Algol	Contact Binary	Delta Scuti	No Variation	RRab	RRc
Algol	0.93	0.07	0.0	0.0	0.0	0.04
Contact Binary	0.0	0.99	0.0	0.0	0.0	0.0
Delta Scuti	0.0	0.0	0.94	0.0	0.0	0.06
No Variation	0.0	0.02	0.0	0.98	0.0	0.0
RRab	0.0	0.0	0.0	0.0	1.0	0.05
RRc	0.0	0.0	0.0	0.0	0.0	1.0

Table 4: Misclassification Rates of Feature Spaces from Testing Data

	1-NN	PWC	RF
All Features	0.01	0.01	0.01
Color and Frequency	0.01	0.02	0.01
SSMM Only	0.03	0.04	0.03

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