Relationships between permeability, porosity and effective stress for low-permeability sedimentary rock

Jiangtao Zheng ^{a, b}, Liange Zheng ^b, Hui-Hai Liu ^c, Yang Ju ^{a, b, d*}

^a State Key Laboratory of Coal Resources and Safe Mining, China University of Mining & Technology, Beijing 100083, China

^b Earth Sciences Division, Lawrence Berkeley National Laboratory, Berkeley, CA 94720, USA

^c Aramco Research Center, Houston, TX 77084, USA

^d State Key Laboratory for Geomechanics and Deep Underground Engineering, China University of Mining & Technology, Xuzhou 221006, China

* Corresponding author.

Tel: +86 10 62331490

E-mail address: juy@cumtb.edu.cn or yju@lbl.gov

Abstract

As the effective stress increases, low-permeability rock undergoes fairly small porosity changes, but significant decrease in the permeability. Empirical relationships based on laboratory-measured data, typically exponential or power laws, have been proposed to describe the stress-permeability, stress-porosity, and permeability-porosity relationships. However, these approximations yield poor fitting in low effective stress ranges, or unreasonable prediction for certain effective stresses. In this study, we develop a series of theoretical models for the essential relationships among the porosity, permeability and the effective stresses for low-permeability sedimentary rock, based on the concept of Two-Part Hooke Model (TPHM). The TPHM conceptualizes an intact rock into a soft part and a hard part, which comply with the natural-strain-based and engineering-strain-based Hooke's law, respectively. The derived relationships are validated by the experimental data from the literature. The comparisons show that the theoretical predictions agree well with the experimental results. The soft-part, comprising of only a small portion of the rock body, is responsible for the significant permeability reduction in low stress levels. The high stress-sensitivity of permeability is mainly attributed to the micro-crack (soft-part) closure in the intact rock.

Keywords: Stress-dependent relationship, Low-permeability sedimentary rock, Two-part Hooke's model, Cubic law

1. Introduction

The stress-dependence of porosity and permeability for low-permeability sedimentary rock is important for various engineering applications, such as fossil fuel exploitation [1-8], CO₂ geological sequestration [9, 10], coal mining safety [11-13], modeling fluid percolation and pore pressure evolution in the crust [14-17], and nuclear waste disposal [18, 19]. In oil and gas exploitation industry, the unconventional reservoirs, such as tight oil/gas formations, typically refer to those have in-situ permeability less than 0.1 mD ($1 \text{ mD} = 10^{-15} \text{ m}^2$) [20, 21]. In this paper low-permeability sedimentary rock refers to those with permeability less than 0.1 mD under reservoir conditions. Nowadays, hydraulic fracturing is widely used to recover oil and natural gas from tight sandstones and shale gas reservoirs, which usually show highly stress-sensitive mechanical and/or hydraulic properties. Knowledge of the dependence of such properties on stress is critical for productivity estimation and recovery method design [3, 6]. Geologic carbon sequestration (GCS) has been considered as one of the effective method for mitigating the global climate change. The permeability evolution with effective stress of the caprock, usually low-permeability formations, is vital for predicting its retarding effects of the upward CO₂ migration and thus the safety of the GCS site [10, 22]. Similarly, low-permeability formations such as clay rocks have long been considered as host rock for the disposal of radioactive nuclear waste. The stress-dependence of porosity and permeability in these formations, especially in an excavation damaged zone (EDZ), is of great importance for the performance assessment of the disposal site [19].

The permeability and porosity changes with effective stress are complex for low-permeability rock. Particularly, a significant drop of permeability is usually observed for low-permeability rock in a low-effective stress range and accompanied with a small porosity decrease. In contrast, the permeability and porosity decrease with increasing effective stress are usually in a predictive manner for high-permeability rock [14, 16, 23]. It is hard to provide a physically robust explanation for the hydraulic behavior of low-permeability rock using the current models.

With an increase in effective stress, low-permeability rock undergoes fairly small

porosity changes, typically less than 10% of the porosity value under zero stress conditions [6, 24-27]. Some empirical relationships, usually exponential laws, have been proposed between porosity and effective stress based on laboratory-measured data [28-32].

In contrast, significant permeability decreases have been observed with relatively small increases in effective stress. For example, with the effective stress increases from zero to around 15 MPa, the permeability of this type of rock usually decreases more than one order of magnitude [1, 6, 14-16, 24, 33, 34]. Such a phenomenon is referred to stress-sensitive rock permeability at low effective stress levels. It should be noted that the artificial cracks or dilation cracks during the sample preparation may play a non-negligible role in the permeability drop with the effective stress increase [4]. However, the stress-sensitivity also exists in the unloading cycle of stress-dependent permeability tests [14, 35], in which most of the artificial cracks have been closed by the previous loading. Furthermore, as pointed out by Mclatchie et al. [1] and Vairogs et al. [2], for tight sandstones, the lower the reference permeability (routinely tested permeability), the greater the permeability reduction (in terms of percentage) under increasing effective stress. Some empirical relationships between permeability and effective stresses were proposed based on laboratory-measured data. Representative relationships include exponential laws [16, 36-38] and power laws [14, 15, 30, 39]. However, the exponential laws yield poor fitting in low effective stress ranges and the power laws give unreasonable prediction for certain effective stress values.

To describe the relationship between the significant permeability drop and insignificant porosity reduction under effective stress has been a great challenge for scientists over a long time. Up to now, empirical relationships, usually in the form of a power law, were used to relate permeability and porosity under effective stress based on laboratory-measured data. For example, Dong et al. [14] systematically measured the porosity and permeability change with increasing effective stress. The relationship between permeability and porosity was represented by a power law, i.e. $k/k_0 = (\phi/\phi_0)^m$, where ϕ and ϕ_0 are the porosity under the current stress state and ambient conditions, respectively; k and k_0 are the permeability under the current stress state and ambient conditions, respectively; m is a material constant named the porosity sensitivity exponent of permeability. The underlying assumption in relating permeability change to the total porosity change is that the reduction of the total volume of pore space is the only driver for the permeability decrease—an assumption that leads to some

problems with these relationships, one of which is the abnormally high value of m . As known, if the flow path is largely controlled by slot-like micro-crack networks (as has been postulated by many researchers [6, 11, 14, 16, 25, 26, 38-42]), the permeability change with the crack aperture reduction (reflected by porosity reduction) should approximately obey the “cubic law” [42, 43] which implies that value of m should be around 3. However, the calculated values of m based on the experimental data for low-permeability samples were much higher than 3, with m value up to 70.17 [14, 16]. The extremely high exponent in the current power law for the relationship between permeability and porosity suggests that relating permeability changes to the total porosity changes is not a valid assumption. In this paper, we therefore attempt to establish relationships that are more physically reasonable for low-permeability rock.

This study is based on the concept of the Two-part Hooke’s model (TPHM), which is a macroscopic model that deals with mechanisms of micro-mechanics in a phenomenological manner [44, 45]. Natural rock, which contains different mineral compositions, pores and micro-cracks, are inherently heterogeneous and will experience non-uniform deformation under uniform stress. This has been demonstrated by both experimental observations and theoretical analyses. For instance, Zimmerman [46] analyzed the compressibility of sandstones from a micromechanical point of view and pointed out that the pore structure is the main reason that the compressibility of sandstone changes with stress. Similarly, Jaeger et al. [47] stated the crack-like voids in a porous rock samples are responsible for the observed nonlinear deformation in low stress range. Distinguishing the influences of different types of pore structure (stiff and compliant porosity), was also employed by Shapiro et al. [48] for better interpretation of the rock physical behavior. More recently, Liu et al. [44, 45] conceptually divided the rock body into “soft” part and “hard” part and pointed out that the natural strain (volume change divided by rock volume at the current stress state), rather than the engineering strain (volume change divided by the unstressed rock volume), should be employed in Hooke’s law for accurately modeling the elastic deformation, unless the two strains are essentially identical (as they might be for small mechanical deformations in the “hard” part). Based on this concept, a series of constitutive relations between stress and a variety of hydro-mechanical rock properties were derived, e.g., stress-dependent rock bulk compressibility, pore compressibility, rock porosity and fracture aperture [44, 45, 49]. The relationships derived using TPHM are consistent with those revealed from micromechanical point of view s, albeit

with different physical origins [44]. These relationships based on TPHM represent the experimental data in literature very well. However, the TPHM-based stress-dependent permeability relationship for a rock matrix is not investigated yet in the literature. In this paper, we intend to formulate the stress-dependence of permeability based on the concept of TPHM. The derived relationships explain well the permeability stress-sensitive phenomena in the low effective stress range. The cornerstone of our development is the recognition of the fact that porosities from the soft and hard parts have different contributions to the permeability change with stresses. As demonstrated in the late part of this article, the soft part, while only a small portion of the low-permeability rock, plays a critical role in the stress-dependent permeability relationship.

2. Existing relationships for stress-dependent rock properties

The stress-dependent rock mechanical and/or hydraulic properties have been extensively studied, due to their importance in engineering applications [8, 50-54]. In this section, we discuss the existing empirical relationships for describing the stress-dependence of rock porosity and permeability. Also presented are relationships between rock permeability and porosity under different effective stresses. Note that the effective stress refers to the confining pressure minus the pore pressure in this paper.

2.1 Relationship between porosity and effective stress

The influence of effective stress on porosity for low-permeability sedimentary rock is relatively small [25, 26]. The relationship between porosity and effective stress, albeit with a slightly different form, can be described by exponential function as shown in the following equations.

After studying the porosity of shale samples from various depths, Athy [28] presented the compaction induced porosity-depth relationship using an exponential equation:

$$\phi = \phi_0 \exp(-bx) \quad (1)$$

where ϕ is porosity at depth x , ϕ_0 is the porosity of surface sample, and b is a material constant. This relationship could also be applied to sandstones [29]. Because the equivalent effective stress can be deduced from burial depth, Shi and Wang [30] reformat Eq. (1) to relate the porosity with effective stress:

$$\phi = \phi_0 \exp(-\beta\sigma) \quad (2)$$

where σ is effective stress, and β is a material constant.

Based on laboratory experiments on sandstone by Davis and Davis [31], Rutqvist et al. [32] proposed a modified empirical stress-porosity expression which added an additional parameter called “residual porosity” for better describing the porosity change with effective stress:

$$\phi = \phi_r + (\phi_0 - \phi_r) \exp(-\beta\sigma) \quad (3)$$

where ϕ_r is the residual porosity at high effective stress. With the presence of residual porosity, Eq. (3) can better characterize the relatively large degree of stress dependency on porosity in the low effective-stress ranges.

2.2 Relationship between permeability and effective stress

It has been recognized that the effective stress has a large influence on rock permeability, especially on low-permeability sedimentary rock. Many empirical relationships have been put forward to describe the permeability changes with effective stress. In general, these relationships can be divided into exponential law and power law.

2.2.1 Empirical exponential law for permeability and effective stress

The exponential law for describing stress-dependent permeability takes the form of [16]:

$$k = k_0 \exp[-\gamma(\sigma - \sigma_0)] \quad (4)$$

where k is the permeability at the effective stress σ , k_0 is the permeability at ambient stress σ_0 , and γ is the stress sensitivity coefficient.

In the study of generation and maintenance of pore pressure excess in the crust, David et al. [16] conducted stress-dependent permeability experiments for five sandstones with effective stress from 3 MPa to about 600 MPa using water as the test fluid. As indicated by the authors [16], the permeability reduction caused by mechanical compaction (effective stress ranging from 3 MPa to the “critical pressure”, i.e. pore collapse occurs pressure) could be approximated using Eq. (4). This relationship was suitable for the relatively high stress part of the focused effective stress range. However, the permeability of two out of five sandstones exhibits high stress-sensitivity at low effective stress range (0-15 MPa), which argued by David et al. [16] was related to micro-cracks closure. Equation (4) cannot reflect the significant

permeability decrease at this effective stress range. Evans [38] measured the stress-dependent permeability of intact core, fault core, and damaged zone core of granite rock with effective stresses ranging from 2.07–46.89 MPa using nitrogen as the tested fluid. An exponential equation, similar to Eq. (4), was used to fit the stress-dependent permeability data. Again, the fitted curves of permeability at both relative low and high effective stress range would underestimated the permeability in the relative low effective stresses. The fitted stress-sensitive coefficient for intact core was much higher than the other two, which, as the authors speculated, was due to the micro-crack closure in the intact rock.

The major problem of the exponential law for permeability-effective stress is that it yields poor fittings in low effective stress ranges. When the measured permeability spans large ranges of effective stress, it is very hard to find one generally accepted permeability-stress function to fit all the data. Evans et al. [38] seemed to be able to fit their data with one equation, but because their data (for intact rock) is only for low effective stress.

2.2.2 Empirical power law for permeability and effective stress

A simple power law described by Eq. (5) has also been used to fit the measured permeability with effective stress [15, 30, 35] :

$$k = a\sigma^{-b} \quad (5)$$

where a and b are material constants, σ is effective stress.

Ghabezloo et al. [15] studied the effective stress law of permeability for limestone with effective porosity (connected porosity) between 13.4% and 17.6% using water as the test fluid. The authors used Eq. (5) to describe the variation in permeability with the effective stress ranging around 1 MPa to 8 MPa. However, as indicated by the authors, this power law cannot be accepted as a general stress-permeability relationship, due to the infinite predicted permeability value at zero effective stress. Shi and Wang [30] also use Eq. (5) to describe the relationship between effective stress and the permeability of fault gouge measured by Morrow et al. [35]. The tests measured the permeability changes in clay-rich, non-clay, and pure-clay artificial gouges under an effective stress ranging from 5 MPa to 200 MPa using distilled water as the test fluid.

A different form of power law was employed by Kwon et al. [55] to fit the measured permeability of illite-rich shale with different effective stresses ranging from 3 MPa to 12 MPa

and using 1M NaCl solution as the test fluid:

$$k = k_0[1 - (\sigma/P_1)^m]^3 \quad (6)$$

where k_0 is the reference permeability at zero effective pressure, σ is effective stress, P_1 is the effective modulus of the equivalent asperities, and m is a constant between 0 and 1. The Eq. (6) has the advantage that it relates permeability with the equivalent fracture aperture, which provided a possible explanation for the permeability decrease with the fracture compaction caused by the effective stress through the “cubic law” [42]. However, the flaw of the equation is also obvious, in that it gives negative k when σ is larger than P_1 . The value of P_1 is only 19.3 ± 1.6 MPa for shale, based on the fitting of the measured data [55]. Jones and Owens [6] also used a similar relationship to describe permeability reduction with increasing effective stress for tight sandstone. A comprehensive review of stress-dependent permeability based on the equivalent fracture models could be found in Ostensen [56]. The common flaw of this type of relationship is that above a certain effective stress, the predicted permeability gives negative values.

In summary, while the power law can provide a reasonable description of the permeability-stress relationship at relatively low effective stress ranges, its flaw is quite obvious—the Eq. (5) yields infinite permeability at zero effective stress, and the Eq. (6) yields negative permeability when effective stress is larger than a certain value.

2.3 Relationship between permeability and porosity

Porosity acts as a bridge that links permeability changes and effective stresses when assuming the mechanical compaction of the flow path as the main factor in causing permeability reduction. Several previous attempts include Sisavath et al. [57] who studied the decrease of hydraulic conductivity of thin elliptic and tapered cracks due to the applied stresses or pore pressures. Doyen [58] studied the variation of permeability with porosity for Fontainebleau sandstone. Specifically, permeability and porosity were related with the characteristics lengths of the porous structure. As indicated, the porosity was controlled by pore chambers whereas the permeability was controlled by throat size in the porous structure. Kilmer et al [34] carried out the permeability measurements under loading and unloading cycles of confining pressure. As speculated by the authors, the pressure sensitivity of low-permeability sandstones depends on details of pore structure. Researchers have tried to

construct permeability-porosity relationships in describing the mechanical compaction. It is generally considered that the relationship between permeability and porosity of a rock undergoing mechanical compaction could be expressed by a power law:

$$k/k_0 = (\phi/\phi_0)^m \quad (7)$$

where k_0 and ϕ_0 are permeability and porosity under ambient stress state, and m is a material constant named the porosity sensitivity exponent.

David et al. [16] used Eq. (7) to describe permeability and porosity data under different effective stresses for five types of sandstones, with exponent value m ranging from 4.6 to 25.4. They also reviewed a batch of published data on the permeability-porosity relationship for different geomaterials, with m varying from 1.11 to 25.4. Ghabezloo et al. [28] measured the permeability-porosity relationship for oil-well cement paste using a single transient method which allows the porosity and permeability data to be measured in a single test. As analyzed by Ghabezloo et al. [28], the permeability-porosity relationships obeyed a power law identical with Eq. (7), with m value equal to 11. Dong et al. [14] conducted a series of permeability and porosity tests under different effective stresses. Eq. (7) was also employed in describing the permeability-porosity relationship, with exponent m value ranging from 7.00 to 70.17 for silty-shale samples.

As shown above, the value of exponent m for most low-permeability rock is very large, which means that a small change in porosity (due to the applied stress) causes a huge decrease in permeability. This was first noticed by David et al. [16] that the theoretical models which used low values of m would underestimate the permeability reduction induced by compaction for low-permeability rock. Dong et al. [14] observed even higher value of m , and postulated that it was caused by the effect of micro-crack closure. This was further supported by the experimental results presented by Byrnes [26], Brower and Morrow [59] and Smith et al. [60]. If permeability changes are largely determined by slot-like micro-cracks that connect large pores to form a flow network, then the permeability-porosity relationship should be similar with the “cubic law” [42, 49], which was derived for a fractured medium. However, the fact that an exponent m up to 70 has to be used to fit the measured data suggests that relating permeability changes to total porosity is fundamentally flawed.

Using total porosity in Eq. (7) is essentially assuming that all pores, both large pores and micro-cracks, have equal contribution to permeability, which runs contrary to the fact that

micro-cracks are actually the decisive factor of pores connectivity for low-permeability rock. Another fact is that the micro-cracks are generally more deformable, which means that they experience relatively large deformation at low effective stress range. Thus, the impropriety of homogenizing the contribution of different types of pores to permeability is more pronounced for low-permeability rock under low effective stress. When the rock is subject to compaction, the deformation of micro-cracks, although insignificant in term of volume, is critical for permeability changes. It is therefore necessary to separate the contribution of micro-cracks and large pores, which can be done via the concept of TPHM. The derivation of TPHM is described in detail in the following section.

3. A brief description of the TPHM

The TPHM proposed by Liu et al. [44] is briefly described here for completeness. Liu et al. [44] suggested that the true strain, rather than the engineering strain, should be used in Hooke's law for accurately modeling elastic deformation of rock, unless the two strains are essentially identical (as they might be for small mechanical deformations). In terms of volumetric strain, the true strain refers to volume change divided by rock volume at the current stress state, and the engineering strain refers to volume change divided by the unstressed rock volume. (In the literature of rock mechanics and other related scientific fields, however, engineering strain is now used exclusively.) Liu et al. [44] further proposed that natural rocks are inherently heterogeneous, and thus different varieties of Hooke's law should be applied within ranges having significantly different stress-strain behavior. They conceptually divide the rock body into two parts, hard and soft, and hypothesize that the soft part obeys the natural-strain-based Hooke's law, while the hard part approximately follows the engineering-strain based Hooke's law for mathematical convenience, because its deformation is small (true strain is practically identical to the engineering strain when the deformation is small). This conceptualization can be represented by the hypothesized composite spring system shown in Fig. 1. These two springs are subject to the same stress, but follow different variations on Hooke's law. To be consistent with previous work, the subscripts 0, e, and t denote the unstressed state, the hard part and the soft part, respectively. For the soft part, Hooke's law could be expressed using the true strain:

$$d\sigma = K_t d\varepsilon_{V,t} = -K_t \frac{dV_t}{V_t} \quad (8)$$

where K_t is the elastic modulus of the soft part, and V_t is the volume of the soft part. Integrating Eq. (8) and the initial condition $V_t = V_{0,t}$ for $\sigma = 0$, we have:

$$V_t = V_{0,t} \exp\left(-\frac{\sigma}{K_t}\right) \quad (9)$$

For the hard part, using the engineering strain, we have:

$$d\sigma = K_e d\varepsilon_{V,e} = -K_e \frac{dV_e}{V_{0,e}} \quad (10)$$

where K_e is the elastic modulus of the hard part, and V_e is the volume of the hard part. Integrating Eq. (9) and the initial condition $V_e = V_{0,e}$ for $\sigma = 0$, we have:

$$V_e = V_{0,e} \left(1 - \frac{\sigma}{K_e}\right) \quad (11)$$

By combining Eqs. (9) and (11), the TPHM based stress-strain relationship under a hydrostatic state can be expressed by:

$$\varepsilon = -\frac{dV}{V_0} = -\frac{dV_e + dV_t}{V_0} = \gamma_e \frac{d\sigma}{K_e} + \gamma_t \exp\left(-\frac{\sigma}{K_t}\right) \frac{d\sigma}{K_t} \quad (12)$$

$$\gamma_t = \frac{V_{0,t}}{V_0} \quad (13)$$

$$\gamma_e = 1 - \gamma_t \quad (14)$$

where V_0 is the rock bulk volume under an unstressed state. Eqs. (12)-(14) together comprise the basic stress-strain relationship within the context of TPHM.

With the basic stress-strain relationship available, a number of rock mechanical and/or hydraulic properties subjected to elastic deformation were developed. For example, Liu et al. [44] derived the stress-dependent bulk rock compressibility, pore compressibility, rock porosity and fracture aperture based on the TPHM. The derived relationships were compared with a variety of laboratory experimental data which showed a great consistency. Furthermore, they argued that the proposed relationships are robust for engineering applications because the parameter values estimated from different kinds of measurements are quite consistent for similar types of rocks. Zhao and Liu [45] extended the stress-dependent relationships for porous rock under elastic and anisotropic conditions. Liu et al. [49] focused on the normal-stress dependent fracture hydraulic properties. The derived relationship between stress and

fracture permeability (or related aperture and closure) based on the TPHM showed a satisfactory agreement with the datasets obtained from the literature. Li et al. [61] incorporated the TPHM based relationships into a geomechanical simulator to study the coupled hydro-mechanical behavior of the Opalinus Clay in responding to tunnel excavations. The simulation results obtained using the TPHM-based constitutive relationships are more consistent with the field observations than the results using the traditional constitutive relationships.

4. Stress-dependent relationships for rock porosity and permeability based on the TPHM

The current consensus is that the mechanical compaction caused permeability drop in the low stress range is mainly due to the closure of slot-like micro-cracks. As pointed out by Liu et al. [44, 49], some portion of the pores and micro-cracks in a rock body could be subjected to significant deformation due to their geometrical heterogeneity [34, 57, 62, 63]. Byrnes et al. [25, 26] found that the average pore-throat size in low-permeability sandstones under in-situ stress decreased by as much as 50% to 70% (e.g., from 1 μm to 0.5–0.3 μm) compared with that under no confining stress, whereas, the porosity under in-situ stress were just several percent less than that under no confining stress. As indicated by the Byrnes et al. [25, 26], the thin and sheet-like interconnecting throats, although constituting only a small portion of the porosity, make a significant contribution to permeability reduction. Brower and Morrow [59] compared the pore cast micrographs of a pair of tight gas sands: one was put under ambient condition and the other under a hydrostatic stress of 34.5 MPa. In the latter one, many of the interconnected micro-cracks had either closed or experienced large deformation as a result of confining pressure. This is a direct evidence that the stress sensitivity of permeability for tight gas sands was caused by the closure of high aspect ratio micro-cracks. The authors also pointed out that these micro-cracks only occupied a small portion (less than one-tenth) of the total porosity. Another evidence of micro-cracks closure under confining pressure is provided by Xu et al. and Smith et al. [60]. They observed a rapid increase in P-wave velocity as effective stress increases for tight gas sand, which they postulated was caused by the closure of the compliant micro-cracks in the rock matrix. As concluded from previous work, more attention should be paid to the deformation of the more compliant part of the rock body (e.g., micro-cracks), since they act as critical fluid-flow paths that determine hydraulic properties [4]. However, most previous studies [14, 15, 30, 64] assumed homogeneity of the

rock sample and ignored the likelihood that the different parts of the rock body have different mechanical responses under stress and thus affect permeability differently. The division of hard and soft parts in THPM provides us with a framework to consider such heterogeneity development of stress-dependent permeability and porosity relationships. The derivations based on the TPHM are given below.

4.1 Stress-dependence of porosity

As discussed above, we conceptualize the pore structures into two parts. The hard part experiences small deformation, whereas the soft part undergoes relatively large deformation. The aforementioned micro-cracks correspond to the soft part, while the rest of the pores and rock matrix belong to the hard part. Using the same notations as in Section 3, the porosity change could be defined as:

$$d\phi \approx \frac{dV^p}{V_0} = \frac{dV_t^p + dV_e^p}{V_0} = \frac{dV_t + dV_e^p}{V_0} \quad (15)$$

where the superscript p refers to pore space. The soft part of rock body V_t is considered to be a portion of pore volume, which leads to $V_t = V_t^p$. Liu et al. [44] indicated that, for the purpose of calculating porosity, the changes in total rock volume V could be ignored, and its value could be approximated with the unstressed volume V_0 . With this assumption, and following the same procedure used to derive Eq. (12), one can obtain:

$$d\phi = -\phi_{e,0}C_e d\sigma - \frac{\gamma_t}{K_t} \exp\left(-\frac{\sigma}{K_t}\right) d\sigma \quad (16)$$

$$\phi_{e,0} = \phi_0 - \gamma_t \quad (17)$$

where C_e is the compressibility for the hard fraction of pore volume.

Integrating Eq. (16) and using $\phi = \phi_0$ for $\sigma = 0$ gives [44],

$$\phi = \phi_{e,0}(1 - C_e\sigma) + \gamma_{t,0} \exp\left(-\frac{\sigma}{K_t}\right) \quad (18)$$

$$\phi_e = \phi_{e,0}(1 - C_e\sigma) \quad (19)$$

$$\phi_t = \gamma_{t,0} \exp\left(-\frac{\sigma}{K_t}\right) \quad (20)$$

As expressed by Eqs. (18)–(20), the porosity of a rock sample is divided into two parts. The hard part porosity, i.e., $\phi_e = \phi_{e,0}(1 - C_e\sigma)$, has a linear relationship with the effective

stress. The soft part porosity, i.e., $\phi_t = \gamma_{t,0} \exp\left(-\frac{\sigma}{K_t}\right)$, changes exponentially with effective stress. The soft part porosity corresponds to those more compliant, slot-like micro-cracks, which experiences relatively large deformation under low effective stress. The two porosities experience different types of reduction with increasing effective stress. Specifically, the soft part is important in the low-stress range while negligible in the high stress range because K_t is relatively small (i.e., $\exp\left(-\frac{\sigma}{K_t}\right) \approx 0$ when $\sigma \gg K_t$).

4.2 Stress-dependence of permeability

The division of total porosity into soft part and hard part, which obeys different relationships with effective stress, allows us to consider their respective effect on the total permeability.

The permeability changes in relatively high effective stress ranges is controlled by the hard part with the consideration that the soft part porosity could be neglected in this stress range (micro-crack closure). Mathematically, the stress-dependent permeability contributed by the hard part (referred to as “hard part permeability” hereafter) could be empirically given as:

$$k_e = k_{e,0} \exp[\beta(\phi_e - \phi_{e,0})] = k_{e,0} \exp[-\beta C_e \phi_{e,0} \sigma] \quad (21)$$

where ϕ_e and k_e are the stress-dependent hard part porosity and permeability, β is a constant that represents a stress sensitive coefficient. Eq. (21) is consistent with most of the experimental observations [16, 38, 55, 65], such that logarithm of permeability changes linearly with the effective stress within the relatively high effective stress range, as discussed in Section 2.2.1.

The stress-sensitive permeability drops in the low effective stress range are mainly due to the deformation of the soft part (micro-crack) porosity, which experienced relatively large deformation, despite the fact that the soft part porosity makes up only a small portion of the total pore volume. The permeability contributed by the soft part, referred to as the “soft part permeability” hereafter, could be considered as the total permeability minus the hard part permeability:

$$k_t = k - k_e \quad (22)$$

where k is the total permeability and k_t is the soft part permeability.

Mathematically, we postulate the relationship between soft part porosity and soft part

permeability as:

$$k_t = \alpha \phi_t^m \quad (23)$$

where ϕ_t, k_t are the soft part porosity and permeability, respectively, and α and m are material constants. Combining Eqs. (20)-(23) yields the total permeability as:

$$k = k_{e,0} \exp[-\beta C_e \phi_{e,0} \sigma] + \alpha \left[\gamma_t \exp\left(-\frac{\sigma}{K_t}\right) \right]^m \quad (24)$$

As mentioned before, the soft part corresponds to those thin, slot-like micro-cracks. If such a hypothesis holds, the permeability change caused by the soft part deformation should obey the “cubic law” [42, 43, 55]. In other words, the soft part permeability should be well related with the soft part porosity through a relationship similar to the “cubic law”, i.e., m in Eq. (23), depending on how micro-cracks are connected with each other and with pores, should be not very far from 3. In the next section, we will evaluate the equations proposed here using the experimental observations collected from the literature.

5. Verification of the proposed stress-dependent relationships

Here we evaluate the proposed relationships using the experimental observations provided by Dong et al. [14]. Actually, there are a number of stress-dependent permeability data for low-permeability rock in the literature [1-5, 13, 24, 33, 34, 39, 55, 66, 67]. However, those that include stress-dependent porosity data are few. To the best of our knowledge, these data sets can be found in Wyble [68], Jones and Owens [6], Yale and Nur [69], David et al. [16], Mohiuddin et al. [36], and Dong et al. [14]. In the selection of the experimental data sets we used to evaluate the proposed relationships, several criteria were considered. First of all, the samples and the measurement procedures had to be well documented. Second, we preferred that the permeability be measured by gas rather than liquid, because gas (e.g., nitrogen and helium) is more chemically inert than liquid (e.g., pure water and NaCl solution). Third, the samples collected from wells were preferred over outcrop rock samples, since the relationships of interest are mostly for underground engineering applications. Fourth, the samples for permeability tests should be carefully treated to minimize the occurrence of artificial cracks during the preparation and operation of the experiment. By the same consideration, tests with unloading data in addition with loading data are preferred. Other selection considerations include sufficient data density and microscopic pore structure analysis. Given these

considerations, we chose the data sets provided by Dong et al. [14] to evaluate the proposed relationships.

5.1 Description of experiments by Dong et al. [14]

Rock samples from depths of 900–1,235 m were collected from a deep drilling project (Taiwan Chelungpu fault Drilling Project, TCDP-A) in the Western Foothills of Taiwan [14]. Only the relatively homogeneous cores were selected, and cores with interbedded layers were discarded. The samples were carefully prepared to reduce the occurrence of obvious micro-cracks during sample preparation. A series of stress-dependent porosity and permeability tests for dry rock samples were conducted on an integrated porosity/permeability measurement system, i.e. YOYK2. A steady state flow method was employed to assess the stress-dependent permeability while stress-dependent porosity was measured by a gas expansion method. Nitrogen was used as the test fluid for both tests. The experiments were conducted by first gradually increasing (loading) the confining pressure P_c from 3 to 5 MPa, then to 20 MPa (in 5 MPa increments), and finally to 120 MPa (in 10 MPa increments). P_c was then gradually reduced (unloading) back to 3 MPa in the reverse order. The average pore pressures P_p were relatively low, i.e., 0.13–1.40 MPa for the permeability measurement and 0.3–1.41 MPa for the porosity measurement. The effective stress is defined as the difference between the P_c and P_p . Tested samples included Pliocene to Pleistocene fine-grained sandstone and silty-shale. The sandstone samples had relative high permeability, exceeding the definition of low-permeability rock. In this study, we focused on the stress-dependent hydraulic properties of the silty-shale. The basic geophysical properties of the samples are listed in Table 1. In addition, scanning electron microscope (SEM) was employed to observe the microstructure in the silty-shale rock samples. Micro-cracks could be clearly identified in the SEM images which argued by Dong et al [14] was responsible for the significant permeability reduction under a low effective stress.

5.2 Matching experiment data using TPHM based relationships

Since the porosity under zero effective stress ($\phi_{e,0}$) was not given due to the limitation of experimental method, we simply used the porosity at the lowest effective stress $\phi_{e,1}$ to substitute $\phi_{e,0}$. The subscript 1 hereafter refers to the lowest effective stress state. Eq. (18) is therefore revised as:

$$\phi = \phi_{e,1}(1 - C_e\Delta\sigma) + \gamma_{t,1}\exp\left(-\frac{\Delta\sigma}{K_t}\right) \quad (25)$$

where $\Delta\sigma = \sigma - \sigma_1$, and $\gamma_{t,1}$ was the soft part fraction at the lowest effective stress.

Similarly, the Eq. (24) was revised as:

$$k = k_{e,1}\exp[-\beta C_e\phi_{e,1}\Delta\sigma] + \alpha \left[\gamma_{t,1}\exp\left(-\frac{\Delta\sigma}{K_t}\right) \right]^m \quad (26)$$

The matching process of stress-dependent porosity and permeability for the loading part of sample R255_sec2 was used to illustrate the determination of parameters in Eqs. (25) and (26). As shown in Fig. 2(a), the porosity-effective stress relationship can be well represented by a straight line for the relatively high effective stress range. In this study, we chose the effective stress roughly range 35 MPa-120 MPa as the relatively high effective stress range. The slope of the straight line was used to determine $(-\phi_{e,1}C_e)$, because the second term on the right hand side of Eq. (25) is negligible for high effective stress values. The value of $\phi_{e,1}$ was determined by extrapolating the straight line at σ_1 , considering that the straight line (red line in Fig. 2(a)) represents the first term on the right hand side of Eq. (25). The measured porosity ϕ_1 was equal to $\phi_{e,1} + \gamma_{t,1}$ as implied by Eq. (25). The above procedure allowed for direct determination of values for $\phi_{e,1}$, C_e , and $\gamma_{t,1}$. The remaining parameter K_t could be estimated using the porosity data at relatively low effective stress, based on Eq. (25). The effective stress under roughly 15 MPa was chosen as the relatively low effective stress range.

Figure 2(b) shows the permeability-effective stress relationship with permeability plotted in logarithm. A linear relationship exists for $\log(k_e)$ versus σ , as shown by the red straight line for the relatively high effective stress range. In this effective stress range, the contribution of the soft part to permeability is neglected, because the second term on the right hand side of Eq. (26) is negligible. The slope of the straight line in the relatively high stress range yields the value of $(-\beta C_e\phi_{e,1})$, which leads to the determination of β as $\phi_{e,1}$ and C_e are known by fitting the porosity-stress data. Parameter $k_{e,1}$, which is the hard part permeability at σ_1 , can be obtained by the extrapolating the straight line at σ_1 .

Once the hard part permeability is known, the soft part permeability is calculated by the total permeability minus the hard part permeability. Then, the soft part permeability is related to the soft part porosity at corresponding stress with $k_t = \alpha\phi_t^m$, which yields the exponent m . The soft part porosity and permeability are plotted in Fig. 2(c) in a double logarithm

coordination and the m value is marked out on the figure.

Experimental data matched with Eqs. (25) and (26) for the four silty-shale samples are shown in Figs. 2–5. A close match between Eqs. (25) and (26) and the experimental data confirms the validity of these TPHM based porosity-stress and permeability-stress relationships. As shown in the porosity-stress figures, the soft part mainly accounts for the nonlinear porosity reduction with effective stress in the low stress range. The porosity reduction in this range resulted mainly from the deformation of slot-like micro-cracks. In addition, as shown in the permeability-effective stress figures, the soft part accounts for most of the permeability reduction in the low effective stress range. The soft part porosity, which occupied only a small portion of the total pore volume, acted as critical flow paths that connected the hard part porosity. Thus, its deformation significantly affected the permeability. Using the concept of dividing the rock body into hard and soft parts, the stress-dependence of rock porosity and permeability has a reasonable explanation, especially the stress-sensitivity of permeability at low effective stress.

The determined value of $\Phi_{e,1}$, C_e , $\gamma_{t,1}$, K_t , $k_{e,1}$, β , α , m and its coefficient of determination R^2 are listed in Table 2. As listed, the soft part porosity at lowest effective stress $\gamma_{t,1}$ was only a small portion of the total porosity (from 0.07% to 0.90%). The elastic modulus of the soft part (from 2.83 to 14.81 MPa) was significantly smaller than the general rock bulk modulus. The values of $k_{e,1}$ were significantly smaller than the total permeability, suggesting that the soft part makes the major contribution to the total permeability at low effective stress range. In addition, the soft part permeability is nicely correlated to the soft part porosity by a power law with average exponent m values of 2.28 for loading cycle and 2.08 for unloading cycle that are not very far from 3 (corresponding to cubic law) and therefore supports the hypothesis that the soft part permeability is controlled by the slot-like micro-cracks. It should be noted that the choices of stress ranges for hard part and soft part have some effect on the fitting parameters, but quite modest. The average fitted m values by other stress ranges choices are also around 3.

5.3 Match of Klinkenberg corrected permeability data using TPHM based relationships

The permeability measured using gas as the test fluid is generally higher than when using liquid due to the Klinkenberg gas slippage effect [70]. As discussed in Tanikawa and

Shimamoto [71], the Klinkenberg effect is important when the permeability is lower than 10^{-18} m^2 and when little pore-pressure difference applied on the different sides of the test samples in the experiment. The following equations, suggested by Tanikawa and Shimamoto [71] and Dong et al. [14], are used in this study to correct the measured permeability data.

$$k_g = k_l[1 + (b/P_{av})] \quad (27)$$

where k_g is the measured gas permeability, k_l is the corrected permeability using Klinkenberg correction, and P_{av} is the average pore pressure which could be calculated using [14]:

$$P_{av} = 2(P_u^2 + P_u P_d + P_d^2)/3(P_u + P_d) \quad (28)$$

where P_u denotes upper-end pore pressure, P_d denotes the lower-end pore pressure which is assumed at atmospheric pressure. b is the Klinkenberg slip factor and can be obtained using the following equation based on the experimental data [71]:

$$b = 0.15 \times k_l^{-0.37} \quad (29)$$

The same procedure is used to determine the parameters in the permeability-stress relationship for Klinkenberg corrected permeability. The matching results for the Klinkenberg corrected permeability are shown in Figs. 6-9. The determined value of $k_{e,1}$, β , α , m and the coefficient of determination R^2 are listed in Table 3.

In general, the Klinkenberg corrected permeability k_l is smaller than the gas measured permeability k_g . After the Klinkenberg correction, the $k_{e,1}$ value decreases, and the stress sensitivity coefficient β for the hard part increases. In addition, the porosity sensitivity parameter for soft part m increases slightly (values of 2.41 for cycle process and 2.20 for unloading cycle), which means that the Klinkenberg corrected permeability drops slightly quicker with the porosity decrease at low stress range. The coefficient of determination R^2 almost remains the same.

6. Concluding remarks

The stress-dependent hydromechanical properties of low-permeability sedimentary rock are important for engineering applications. Accurate description of stress-dependence of rock porosity and permeability are fundamental for studying the coupled hydro-mechanical process of the underground rock.

A series of rock stress-porosity and stress-permeability relationships is developed based

on the concept of TPHM and tested using the experimental data provided by Dong et al [14]. Based on the concept that different varieties of Hooke's law are needed to be employed for different parts of a rock body, we can more accurately describe the elastic deformation of the rock body. Conceptually dividing rock bodies into hard and soft parts provides an opportunity to relate rock hydraulic properties with their complex mechanical responses under stress. The overall hydraulic and mechanical change under stress has contributions from both the hard and soft part crossing all stress range, although the contribution of the soft part at relatively high stress range is so small and can be neglected. The less compressible pores from hard part, albeit accounting for most part of the rock porosity, is not the primary factor that influences the rock permeability change with effective stress. One plausible explanation is that hard part voids are scattered and needed to be interconnected before acting as effective transport channels. The soft part plays the role that connects the hard part for low-permeability rock. It has relatively low elastic modulus and experiences relatively large deformation even within a low effective stress ranges. Although the soft part only occupies a small portion of the rock body, it is important when considering the hydraulic properties of low-permeability sedimentary rock. Specifically, the soft part refers to the slot-like micro-cracks which act as critical pathways that connected others. The large degree deformation of the soft part is the main reason for significant permeability reduction in the low effective stress range. The relationship between the soft part porosity and permeability approximately obeys a cubic law, further demonstrating that the soft part is composed of more deformable slot-like micro-cracks.

The focus of this paper is on proposing a physically reasonable relationship for stress-dependent permeability, based on a relatively simple and intuitive theoretical framework. Our preliminary evaluation produced encouraging results. However, further work is still needed to validate our theory, especially involving more comprehensive experimental data.

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FIGURES:

Fig.1. A composite spring system consisting of two springs. The hard and soft springs follow engineering-strain-based and natural-strain-based Hooke's law, respectively.

Fig.2. The matching result of the proposed relationships and the experimental test data for sample R255_sec2. (a) porosity-stress on loading stage. (b) permeability-stress on loading stage. (c) soft part permeability-soft part

porosity on loading stage. (d) porosity-stress on unloading stage. (e) permeability-stress on unloading stage. (f) soft part permeability-soft part porosity on unloading stage.

(a) (b)

(c)

(d) (e)

(f)

Fig.3. The matching result of the proposed relationships and the experimental test data for sample R287_sec1. (a) porosity-stress on loading stage. (b) permeability-stress on loading stage. (c) soft part permeability-soft part porosity on loading stage. (d) porosity-stress on unloading stage. (e) permeability-stress on unloading stage. (f) soft part permeability-soft part porosity on unloading stage.

(a)

(b)

Fig.4. The matching result of the proposed relationships and the experimental test data for sample R351_sec2. (a)

porosity-stress on loading stage. (b) permeability-stress on loading stage. (c) soft part permeability-soft part porosity on loading stage. (d) porosity-stress on unloading stage. (e) permeability-stress on unloading stage. (f) soft part permeability-soft part porosity on unloading stage.

(a)

(b)

(c)

Fig.5. The matching result of the proposed relationships and the experimental test data for sample R390_sec3. (a) porosity-stress on loading stage. (b) permeability-stress on loading stage. (c) soft part permeability-soft part porosity on loading stage. (d) porosity-stress on unloading stage. (e) permeability-stress on unloading stage. (f) soft part permeability-soft part porosity on unloading stage.

(c)

(d)

Fig. 6. The Klinkenberg corrected permeability matching result of the proposed relationships and the experimental test data for sample R255_sec2. (a) Stress-dependent permeability on loading stage. (b) soft part permeability-soft part porosity on loading stage. (c) Stress-dependent permeability on unloading stage. (d) soft part permeability-soft part porosity on unloading stage.

(a)

(b)

Fig.8. The Klinkenberg corrected permeability matching result of the proposed relationships and the experimental test data for sample R351_sec2. (a) Stress-dependent permeability on loading stage. (b) soft part permeability-soft part porosity on loading stage. (c) Stress-dependent permeability on unloading stage. (d) soft part permeability-soft part porosity on unloading stage.

Fig.9. The Klinkenberg corrected permeability matching result of the proposed relationships and the experimental test data for sample R390_sec3. (a) Stress-dependent permeability on loading stage. (b) soft part permeability-soft part porosity on loading stage. (c) Stress-dependent permeability on unloading stage. (d) soft part permeability-soft part porosity on unloading stage.

TALBLES:

Table 1. Basic geophysical properties of silty-shale samples for permeability and porosity measurements

Sample name	Depth (m)	Dry density (g/cm^3)	Rock type
R255_sec2	902.68	2.59	Silty-shale
R287_sec1	972.42	2.58	Silty-shale
R351_sec2	1114.33	2.59	Silty-shale
R390_sec3	1174.24	2.66	Silty-shale

Table 2. Fitted parameter values from the experimental data of Dong et al. [14].

Sample		$\phi_{e,1}$ (%)	σ_1 (MPa)	$\gamma_{t,1}$ (%)	C_e (MPa ⁻¹)	K_t (MPa)	$k_{e,1}$ (m ²)	β	α (m ²)	m	R ²
R255_sec2	Loading	9.73	4.00	0.07	8.75E-04	2.83	1.97E-19	0.61	1.67E-15	1.70	0.96
	Unloading	9.01	3.99	0.34	2.40E-04	4.84	1.51E-19	1.30	1.99E-17	2.41	0.79
R287_sec1	Loading	10.40	4.08	0.41	6.81E-04	14.81	9.54E-19	3.24	1.32E-16	2.34	1.00
	Unloading	9.75	4.07	0.36	1.48E-04	13.04	1.19E-19	2.53	4.83E-18	2.65	0.95
R351_sec2	Loading	8.75	1.62	0.49	6.97E-04	8.09	2.54E-19	2.24	2.61E-17	3.03	0.88
	Unloading	8.26	1.62	0.90	2.35E-04	6.67	1.00E-19	2.68	4.91E-19	1.53	0.96
R390_sec3	Loading	10.64	3.69	0.33	8.11E-04	8.74	3.66E-17	3.29	5.42E-15	2.04	1.00
	Unloading	9.87	3.68	0.77	2.12E-04	11.30	2.04E-18	1.48	1.38E-18	1.73	1.00

Table 3. Fitted parameter values for stress-dependent relationship of Klinkenberg corrected permeability

Sample		$k_{e,1}$ (m ²)	β	α (m ²)	m	R ²
R255_sec2	Loading	9.80×10^{-20}	0.76	1.83×10^{-15}	1.80	0.93
	Unloading	7.05×10^{-20}	1.64	1.71×10^{-17}	2.56	0.83
R287_sec1	Loading	6.75×10^{-19}	4.00	1.28×10^{-16}	2.45	0.97
	Unloading	5.22×10^{-20}	3.22	3.67×10^{-18}	2.82	0.97
R351_sec2	Loading	1.44×10^{-19}	2.83	2.39×10^{-17}	3.22	0.97
	Unloading	4.48×10^{-20}	3.45	3.30×10^{-19}	1.63	0.99
R390_sec3	Loading	2.85×10^{-17}	4.10	5.46×10^{-15}	2.16	1.00
	Unloading	7.85×10^{-19}	1.91	7.32×10^{-19}	1.78	0.89

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