A simple, physically motivated model of sea-level contributions from the Greenland Ice Sheet in response to temperature changes

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Abstract

Sea level could rise by several meters over the next centuries. The Greenland Ice Sheet could be an important contributor to future sea-level rise, because of its large volume and its high sensitivity to surface air temperature increases. Frameworks for the integrated climate risk management often require fast, simplified treatments of sea-level rise, in particular for estimating the risks associated with low probabilities but potentially high impacts. State-of-the-art ice sheet models provide important insights, but are often computationally too demanding to evaluate tail-area risks. Here we present SIMPLE, a physically motivated model of the Greenland ice sheet in response to temperature changes. SIMPLE can skillfully reproduce the results from a three-dimensional ice sheet model and outperforms existing simple models, after similar calibration. We anticipate that SIMPLE will be calibrated to

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other ice sheet models and can provide a fast approximation (emulator) for such models in studies that require many model evaluations.

Keywords: Greenland ice sheet, semi-empirical model, sea level change, emulator, scenario, geo-engineering

1. Introduction

The response of the Greenland Ice Sheet (GIS) to anthropogenic climate change is an important, but deeply uncertain factor determining future sea-level change (Meehl et al., 2007; Church et al., 2013; Moore et al., 2013, 2010; Applegate and Keller, 2015). Total melting of the GIS would lead to an increase in global mean sea level of approximately 7.4 meters (Bamber et al., 2013). The time needed for complete melting of the GIS is, however, deeply uncertain. The lowest published estimate of the time required for total loss of the GIS is 300 yr (Lenton et al., 2008), but most likely this time scale strongly depends on the magnitude of the temperature forcing (Applegate et al., 2015b; Robinson et al., 2012). Some have argued that geo-engineering can help the ice sheets to regrow (Moore et al., 2010; Irvine et al., 2012). Yet, regrowth of the GIS may be considerably slower than the melting (Applegate et al., 2015b; Applegate and Keller, 2015). Anticipating future sea-level rise requires a careful consideration of the relevant sources of uncertainty (e.g. Lempert et al., 2012).

Many applications require fast treatments of sea-level rise, including Integrated Assessment Models (IAMs; e.g. Nordhaus, 2008) and decision support frameworks such as (Many Objective) Robust Decision Making (RDM, MORDM; Lempert et al., 2003; Hall et al., 2012; Hadka et al., 2015). IAMs

21 represent the coupled economic-climate system, which could be strongly af-
22 fected by sea-level rise (e.g. Yohe and Schlesinger, 1998). RDM identifies
23 decisions that produce good outcomes over a wide range of potential futures.
24 Integrated assessments of the risks and strategies associated with climate
25 change and sea-level rise often require many (10^3 - 10^6) model runs (e.g. McIn-
26 erness et al., 2011; Lempert et al., 2012), although integrated assessment can
27 also refer to a more adaptive process actively involving stakeholders (Jake-
28 man and Letcher, 2003).

29 The integrated assessments of local risks and adaptation strategies re-
30 quire treatments that resolve the different components of sea-level rise sep-
31 arately (Slangen et al., 2012), because the individual components may have
32 quite different time scales. For example, small glaciers respond to tempera-
33 ture increases with a characteristic time scale of decades (Oerlemans, 2005),
34 whereas the Greenland ice sheet has a time scale that can vary over two
35 orders of magnitude depending on the forcing temperature (Applegate et al.,
36 2015b). Moreover, the different components of sea-level rise have very dif-
37 ferent ‘fingerprints’ (distinct spatial patterns of the response; see Mitrovica
38 et al., 2011). In particular, large mass losses from an ice sheet cause near
39 field sea-level fall (e.g. Vermeersen and Sabadini, 1999; Mitrovica et al., 2009),
40 even though the global mean sea level rises.

41 Three-dimensional models of the Greenland Ice Sheet capture many of
42 the relevant feedbacks in the ice sheet system, but can often be too compu-
43 tationally expensive to include in integrated assessments. Ice sheet models
44 track the change in mass of the ice sheet during each time step as a func-
45 tion of mass gain due to snowfall and mass loss due to melting and calving.

46 The treatments of ice flow used in ice sheet models fall along a continuum
47 (Kirchner et al., 2011) from the shallow ice approximation (e.g. SICOPOLIS
48 Greve, 1997) to full-Stokes treatments (e.g. Elmer/Ice; Seddik et al., 2012),
49 with hybrid models (e.g. Pollard and DeConto, 2012) falling in between.
50 Full-Stokes models include all the stress components acting within the ice
51 body, but have high computational costs, and their projections still include
52 uncertainties due to imperfectly-known model input parameters, processes,
53 and boundary conditions (Moore et al., 2013; Applegate et al., 2015b).

54 The high number of runs required for the integrated assessment of cli-
55 mate risks is typically computationally infeasible using the state-of-the-art
56 three-dimensional ice sheet models. And, even with simpler one-dimensional
57 models (e.g. GLISTEN, Haqq-Misra et al. (2012)) this task may prove too
58 computationally demanding (Figure 1). In response, there is an active area
59 of research that develops simpler, faster approximations of the complex sys-
60 tems (e.g. Oerlemans, 2003). Other approaches to reduce the computational
61 burden include the reduction of the climatological input (e.g. Bakker et al.,
62 2011) or the strategic subsetting of the relevant scenarios (e.g. Ntegeka et al.,
63 2014; Van den Hurk et al., 2014).

64 Simplified treatments of the Greenland ice sheet include emulators (sim-
65 ple models of complex model output) of simulations with the state-of-the-art
66 ice-sheet models. For example, Meehl et al. (2007; see also Irvine et al.
67 (2012)) emulated the rate of mass loss from the GIS as a second-order poly-
68 nomial function of temperature, whereas Bindshadler et al. (2013) used a
69 linear combination of different forcings. These approaches are useful for inter-
70 polating the results from complex model simulations. They neglect, however,

71 the dependence of the change rate to ice volume and are therefore less suited
72 to project large changes.

73 Semi-empirical models may be better suited to guide a probabilistic as-
74 sessment of potential large changes. Semi-empirical models are simple, but
75 mechanistically motivated models of which the complexity is balanced to
76 the availability of observational data to calibrate the model (Moore et al.,
77 2013, e.g.). Recently, Applegate et al. (2015b) proposed a semi-empirical
78 model that addresses both the ice sheet’s potential sensitivity to ice volume
79 changes and to temperature. The model expands on the delayed linear con-
80 cept (e.g. used for global sea-level projections by Grinsted et al., 2010), in
81 which the ice volume V [m sle (meter sea-level equivalent)] is assumed to
82 change exponentially towards a temperature-dependent equilibrium volume
83 $V_{eq} = f(T)$ with a temperature-dependent timescale $\tau = g(T)$. A similar
84 concept has been applied in the latest versions of the Integrated Assessment
85 Models DICE and RICE (Dynamic/Regional Integrated model of Climate
86 and the Economy) (Nordhaus, 2010; Nordhaus and Sztorc, 2013).

87 Here, we introduce a computationally efficient, physically motivated model
88 SIMPLE (Simple Ice-sheet Model for Projecting Large Ensembles). SIMPLE
89 can provide quite fast and arguably skillful emulations of the GIS response
90 as simulated by the shallow ice approximation, three-dimensional ice sheet
91 model SICOPOLIS for a wide range of warming and cooling temperature
92 scenarios (Greenland surface temperatures T between 0-12 °C relative to
93 1976-2005; Applegate and Keller, 2015), while its computational efficiency
94 enables large ensembles ($> 10^7$) within reasonable time, even on a single
95 computer core.

96 SIMPLE expands on the work of Applegate et al. (2015b) and Nordhaus
 97 (2010). Rather than an exponential decay towards an equilibrium volume
 98 (≥ 0 m sle), SIMPLE assumes that *initially* the ice mass will exponentially
 99 decay towards a ‘virtual equilibrium’ $V_{v.eq}$ that may be lower than zero for
 100 high temperatures. The difference between the current volume and ‘virtual
 101 equilibrium’ ($\Delta V_{v.eq} = V - V_{v.eq}$) is interpreted as a measure for the ‘im-
 102 balance’ of the ice sheet that is roughly determined by snow accumulation,
 103 melt water runoff and dynamic ice flow. This concept allows the ‘imbalance’
 104 that drives the ice volume change $\dot{V}$, to increase with temperature, even for
 105 temperatures that will (eventually) lead to an ice-free state $V_{eq} \approx 0$.

106 Section 2 introduces SIMPLE and the GIS simulations used for its de-
 107 velopment. Subsequently, we assess and compare SIMPLE to other simple
 108 representations of the GIS behaviour in section 3. Section 4 discusses SIM-
 109 PLE.

110 **2. Methods and data**

111 *2.1. The SIMPLE model*

112 SIMPLE estimates the ice volume change $\dot{V}$ [m sle/yr] from the volume
 113 V [m sle] and annual mean surface temperature at Greenland T [°C] relative
 114 to 1976-2005 by applying an exponential decay function

$$\dot{V} = -\frac{1}{\tau}(\Delta V_{v.eq}). \quad (1)$$

115 Here, the difference between the ice volume and ‘virtual equilibrium’
 116 $\Delta V_{v.eq} = V - V_{v.eq}$ parameterizes the imbalance between mass gain due to
 117 accumulation and mass loss due to melting and calving. We refer to ‘virtual’

118 equilibrium because this measure may be lower than zero for high values (see
 119 Figure 2). We assume that *initially* the ice mass will exponentially decay
 120 towards $V_{v.eq}$ (dashed, red line), but for low ice volumes close to zero this
 121 relation is obviously not valid.

122 Both the timescale τ [yr] and the ‘virtual equilibrium’ $V_{v.eq}$ are linear
 123 functions of temperature T ,

$$V_{v.eq} = aT + b, \text{ and} \quad (2)$$

$$\frac{1}{\tau} = \alpha T + \beta, \quad (3)$$

where a , b , α and β are regression coefficients. Numerical integration is used to estimate the transient GIS response to a temperature trajectory. The ice volume is prevented from reading negative values by

$$V(t_i) = \max \left(0, V(t_{i-1}) + (t_i - t_{i-1})\dot{V}(t_i) \right), \quad (4)$$

124 where t_i is time [yr] at time step i .

125 2.2. Alternative simplified treatments

126 We compare SIMPLE’s ability to reproduce multiple three-dimensional
 127 ice-sheet simulations with three alternative simplified representations: i) a
 128 second-order polynomial (quadratic function), as applied by IPCC’s AR4
 129 (Meehl et al., 2007), ii) the ‘delayed linear’ concept as often applied to project
 130 global sea-level rise (e.g. Moore et al., 2010), iii) and the representation as
 131 applied in the integrated assessment model DICE (Nordhaus, 2010), ‘GIS-
 132 DICE’. These three models are discussed below.

133 *Quadratic function.* This emulator describes the ice volume change $\dot{V}$ as a
 134 quadratic function of temperature T ,

$$\dot{V} = a_q + b_q T + c_q T^2. \quad (5)$$

135 This approach ignores the dependence of the ice volume change $\dot{V}$ on
 136 ice volume V ; SIMPLE simplifies to a second order polynomial (quadratic)
 137 function if V is assumed to be constant in time. This means that for every
 138 V the coefficients a_q , b_q and c_q of equation 5 can be exactly estimated from
 139 the SIMPLE coefficients a , b , α and β (equations 1-3).

140 *Delayed linear concept.* The delayed linear concept is also a simplified version
 141 of SIMPLE in which the time-scale τ_d is assumed constant,

$$\dot{V} = -\frac{1}{\tau_d}(V - a_d T + b_d). \quad (6)$$

142 *GIS-DICE.* In contrast to the quadratic function and the delayed linear con-
 143 cept, GIS-DICE is slightly more complex than SIMPLE. Like, SIMPLE and
 144 the delayed linear concept, GIS-DICE also applies an exponential decay func-
 145 tion. Yet, rather than to the ‘virtual equilibrium’ the volume V is assumed
 146 to exponentially decay towards a physically plausible equilibrium V_{eq} ,

$$\dot{V} = -\frac{1}{\tau}(V - V_{eq}), \quad (7)$$

147 where V_{eq} and τ are (non-linear) functions of T ,

$$V_{eq} = \max(0, a_D T + b_D) \quad (8)$$

$$\frac{1}{\tau} = \alpha_D T + \beta_D T^2. \quad (9)$$

148 *2.3. Temperature scenarios and SICOPOLIS simulations*

149 We calibrated SIMPLE using a large set of simulations previously per-
150 formed with the ice sheet model SICOPOLIS (SIMulation COde for POLy-
151 thermal Ice Sheets; see Greve, 1997). SICOPOLIS is a three-dimensional
152 ice sheet model that applies the computationally efficient shallow ice approx-
153 imation. This makes it attractive for studies that require long simulations
154 such as ice sheet reconstructions, perturbed parameter ensembles or long-
155 term projections (Applegate et al., 2012). The shallow ice approximation is,
156 however, less suitable for short-term projections (Larour et al., 2012) because
157 it does not represent the movement of rapid components such as ice streams
158 and ice shelves (e.g. Kirchner et al., 2011).

159 The applied SICOPOLIS simulations include one ‘spinup run’ (Applegate
160 et al., 2012); eight ‘equilibration runs’ (Applegate et al., 2015b) and twenty
161 ‘geo-engineering runs’ (Applegate and Keller, 2015). A single parameter
162 combination was used in all of these SICOPOLIS runs, corresponding to
163 simulation #29 from Applegate et al. (2012). This ‘best’ simulation achieved
164 a modern ice volume at the end of the spinup run (described below) that is
165 closest to the estimated modern ice volume (about 7.4 m, Bamber et al.,
166 2013).

167 *I) Spinup run.* The spinup simulations run from 125,000 years ago to present
168 (Applegate et al., 2012). Modern-day ice thickness and bedrock elevation grid
169 are applied as initial condition and the model is forced by paleo-records for
170 temperature and sea-level and precipitation derived from the temperature
171 record.

172 *II) Equilibration runs.* Applegate et al. (2015b) extended the spinup simula-
 173 tions of the 27 ‘best’ members of the 100-member perturbed parameter en-
 174 semble by forcing SICOPOLIS by eight instantaneous temperature increases
 175 ($T = 0, 1, 2, 3, 4.5, 6, 9$, and 12°C relative to the average T from 1976
 176 to 2005) and let it equilibrate for at least 60,000 years. We used the eight
 177 equilibration simulations with the single ‘best’ member.

178 *III) Geo-engineering scenarios.* Applegate and Keller (2015) used the single
 179 ‘best’ member to extend the spinup simulations by 20 geo-engineered temper-
 180 ature trajectories (Figure 3). Those T trajectories were obtained by adjust-
 181 ing a base scenario (black line, Figure 3) derived from a simulation with the
 182 intermediate-complexity climate model CLIMBER-3a (Schewe et al., 2011)
 183 following the Representative Concentration Pathway RCP8.5 (Moss et al.,
 184 2010). The scenarios distinguish two geo-engineering strategies that start
 185 at ten different years (2025, 2075, ..., 2475). In the ‘temperature stabiliza-
 186 tion’ scenarios, the base scenario is leveled off at the temperature anomaly
 187 reached at that year (colored dots, Figure 3). In the ‘gradual draw-down’, the
 188 temperature anomaly is rapidly forced down to zero at a rate of $0.1^\circ\text{C}/\text{yr}$.

189 *2.4. Emulation of SICOPOLIS: calibration and cross-validation*

190 We calibrate the parameters of SIMPLE and the other emulators we com-
 191 pare SIMPLE to by minimizing the RMSE (Root Mean Squared Error) of the
 192 emulated change $\dot{V}_{\text{em}}$ in response to T and V_{SIC} as simulated with SICOPOLIS.
 193 We do this on the basis of the complete set of geo-engineering scenarios
 194 and several cross-validation subsets (Table 1, see subsection 3.3). The cross-
 195 validation provides insight into the model’s sensitivity to ‘over-fitting’. Good

196 cross-validation results may increase the confidence in simulated response
 197 outside the calibrated range, i.e. predictive ability (see Rao and Wu, 2001).

198 ‘L1O’ consists of 20 subsets that leave one scenario out for calibration
 199 and compares the calibrated model to the SICOPOLIS results from the one
 200 left-out scenario. ‘L4O’ consists of five subsets leaving 4 scenarios out, based
 201 on the starting year of the geo-engineering and ‘L5O’ exists of four subsets
 202 leaving 5 scenarios out.

Table 1: Scenarios that are left out of the different cross-validation subsets. ‘L1O’, ‘L4O’ and ‘L5O’ refer to the different cross-validation strategies that leave 1, 4 and 5 scenarios out. ‘st’ and ‘dd’ respectively refer to the temperature ‘stabilization’ and ‘draw-down’ geo-engineering strategies and the first column refers to the starting year of the geo-engineering.

		2025	2075	2125	2175	2225	2275	2325	2375	2425	2475
L10	temperature stabilization	1	1	1	1	1	1	1	1	1	1
	temperature draw-down	1	1	1	1	1	1	1	1	1	1
L40	temperature stabilization	4		4		4		4		4	
	temperature draw-down										
L50	temperature stabilization	5					5				
	temperature draw-down	5					5				

203 3. Results

204 3.1. ‘Virtual equilibrium’

205 The ice volume change $\dot{V}$ clearly increases with the distance to the ‘vir-
 206 tual equilibrium’ $\Delta V_{v.eq} = V - V_{v.eq}$, for all time steps of all simulations as

introduced in subsection 2.3 (Figure 4). SIMPLE applies the concept of ‘virtual equilibrium’ volume that, in contrast to the equilibrium, is not bounded by a zero volume. This allows for an increasing imbalance ($\Delta V_{v.eq}$) with temperature, also for all temperatures that will (eventually) lead to an ice-free state, i.e. $V_{eq} \approx 0$ m sle. In case of equal $\Delta V_{v.eq}$, the ice volume change $\dot{V}$ is generally lower for higher ice volume V (dark blue). Yet, for ice volumes V close to or lower than 1 meter sle (bright green), the ice volume change is much lower because reduced ice sheet surface available for melting and reduced efficiency of melt water runoff and ice flow.

3.2. Time-scale

The SICOPOLIS runs show a clear dependency of the time-scale τ on temperature (Figure 5, see also Applegate et al. (2015b)). This observation explains the curvature of the relation between $\Delta V_{v.eq}$ and $\dot{V}$ (Figure 4), in that $\Delta V_{v.eq}$ is largely determined by temperature. The ‘exponential change rate’ $\frac{1}{\tau} = -\frac{\dot{V}}{\Delta V_{v.eq}}$ increases with temperature. For the T range 0-12°C, this relation is well approximated by equation 3.

3.3. Transient response

SIMPLE reproduces the SICOPOLIS simulated transient GIS response to the geo-engineering scenarios more tightly than the other three simple models that we tested (Figure 6). The quadratic function ($\dot{V} = a_q + bqT + c_qT^2$) ignores the dependence on ice volume V . In the delayed linear concept as used for global sea-level projections (e.g. Moore et al., 2010; Grinsted et al., 2010), the ice volume change $\dot{V}$ does depend on volume V , but the constant time-scale τ results in a decay rate biased to the average. This constant

time-scale results, for example, in too high regrowth rates in the temperature draw-down geoengineering scenarios (second row of panels).

The GIS model used in DICE (Nordhaus, 2010) largely solves this problem by describing the change rate as a quadratic function of temperature ($\frac{1}{\tau} = \alpha_D T + \beta_D T^2$). However, this model’s use of an equilibrium volume that is bounded by zero ($V_{eq} = \max(0, a_D T + b_D)$) causes an underestimation of the imbalance of the ice sheet at high temperatures, resulting in too low ice volume change rates. This discrepancy is most pronounced in case of relatively low ice volumes (Figure 6, red colors). SIMPLE addresses this problem by applying the distance to the ‘virtual equilibrium’ $\Delta V_{v.eq}$ as a measure of the imbalance. This enables an improved reproduction of the simulated GIS behavior (lower panels) even though SIMPLE is based on simpler algorithmic methods than DICE-GIS.

3.4. Cross-validation

The four simple models differ in terms of structure and complexity. One robust and simple approach to assess the appropriateness of the model structure and complexity is to analyze the cross-validation errors (e.g. Schoups et al., 2008). We quantified the ability of the model to mimic the simulated transient GIS response on the basis of the RMSE, applying three different cross-validation strategies. Other common measures to evaluate model performance are, in our particular case, hard to interpret. We do not consider the fraction explained variance because the SICOPOLIS runs we wish to emulate show largely monotonic decreasing trends, which would give too much weight to the scenarios that do show a clear change of direction. Also autocorrelation of the emulator error is hard to interpret since we are dealing

256 with a strongly autocorrelated process and model. A serially correlated error
257 would rapidly accumulate to large errors.

258 SIMPLE outperforms the other tested simple models in terms of cross-
259 validation RMSE (Figure 7). The figure shows the RMSE as a function
260 of evaluation length of the transient scenarios for the four simple models
261 (colors) trained on the complete set of scenarios (dots), and three cross-
262 validation strategies (L1O: triangles, L4O: squares, L5O: crosses). SIMPLE
263 and GIS-DICE outperform the quadratic function and delayed linear concept
264 by about a factor 4, regardless of the year until which the future emulation
265 is evaluated. SIMPLE has a 20-30% lower RMSE than GIS-DICE.

266 In general, the different cross-validation strategies provide very similar
267 RMSE estimates as when all scenarios are used to calibrate the different
268 models. GIS-DICE, however, shows considerably higher RMSE for the L5O
269 cross-validation, indicating a higher sensitivity to the training set. This
270 difference in RMSE given the cross-validation data sets may be the result
271 of the slightly more complex structure of GIS-DICE compared to SIMPLE.

272 4. Discussion

273 SIMPLE is a computationally very efficient (see figure 1) 4-parameter,
274 physically motivated model that describes the GIS volume change $\dot{V}$ as a
275 linear function of current volume V and surface temperature T forcing. SIM-
276 PLE displays very similar change direction-dependent behavior as SICOPOLIS
277 (figure 8) and SIMPLE skillfully emulates the transient GIS response to
278 a wide range of temperature forcings as simulated by the three-dimensional
279 ice sheet model SICOPOLIS (Figure 6). It may be utilized to evaluate simu-

280 lations with complex process-based ice sheet models like earlier done on the
 281 basis of the delayed linear concept (Applegate et al., 2015b). Yet, it is es-
 282 pecially designed to allow for the large numbers of simulations necessary for
 283 integrated assessments, Many-Objective Robust Decision Making (Kasprzyk
 284 et al., 2013; Singh et al., 2015) and the exploration of model parameter un-
 285 certainty.

286 One might argue that SIMPLE has been developed and validated on the
 287 basis of simulations with the shallow ice approximation model SICOPOLIS,
 288 that neglects important stress components and therefore is unable to repre-
 289 sent faster components. Yet, the incorporation of higher order physics may
 290 not necessarily change the overall GIS response by much (Applegate et al.,
 291 2015b). It can be argued that the relatively high efficiency of SICOPOLIS is
 292 beneficial for some analysis. For example, this feature allowed several studies
 293 (e.g. Applegate et al., 2012, 2015b; Applegate and Keller, 2015) to produce
 294 a very large ensemble of GIS simulations. The use of large ensembles can be
 295 important to test a model over wide range of forcings.

296 Nevertheless, these higher-order components may affect the ice sheet re-
 297 sponse and it is worthwhile to calibrate (and test) SIMPLE using runs from
 298 more complete, process-based models. This additional calibration may iden-
 299 tify the need to adjust equations 2 and 3 to estimate $V_{v.eq}$ and τ . This can be
 300 done following a similar approach as presented in this paper by simply adjust-
 301 ing the presented linear equations 2 and 3. More drastic changes to SIMPLE
 302 are not necessarily required since the ignored processes by SICOPOLIS are
 303 also largely controlled by the Greenland temperature.

304 Overall, we conclude that SIMPLE is a physically motivated model that

305 performs well and is suited for many applications.

306 **Author contributions**

307 K. Keller and P.J. Applegate initiated the study. K. Keller and A.M.R.
308 Bakker designed the research. P.J. Applegate processed the SICOPOLIS
309 model output and A.M.R. Bakker coded the simple models, carried out the
310 model emulations, processed the model output, made the figures, and wrote
311 the first draft of the manuscript.

312 **Acknowledgements**

313 This work was partially supported by the National Science Foundation
314 through the Network for Sustainable Climate Risk Management (SCRiM)
315 under NSF cooperative agreement GEO-1240507 and the Penn State center
316 for Climate Risk Management.

317 **Appendix A. Additional figures**

318 **Appendix B. R-function of SIMPLE**

```
319 # SIMPLE.R #####  
320 #  
321 # SIMPLE (Simple Ice-sheet Model for Projecting Large Ensembles) is  
322 # a simple physically motivated model of the Greenland ice-sheet that  
323 # estimates the annual change in volume (Vdot) [m sle/yr] from  
324 # volume (Vt) [m sle] and surface temperature [degC relative to 1976-2005].  
325 #
```



```

326 # (Bakker, A.M.R. and P.J. Applegate, and K. Keller, 2016. "A simple,
327 # physically motivated model of sea-level contributions from the
328 # Greenland Ice Sheet in response to temperature changes")
329 #
330 # variables needed
331 # params: numeric vector containing 'a', 'b', 'alpha' and 'beta'
332 #         default values from Bakker et al. (2016)
333 # Tt:      numeric vector or matrix containing annual mean temperature
334 # Vt:      numeric vector or matrix containing current ice volume
335 # VO:      numeric with initial ice volume [m sle]
336 #
337 # Copyright (C) 2016 Alexander Bakker
338 #
339 # SIMPLE.R is free software: you can redistribute it and/or modify
340 # it under the terms of the GNU Lesser General Public License as published by
341 # the Free Software Foundation, either version 3 of the License, or (at your
342 # option) any later version.
343 #
344 # SIMPLE.R is distributed in the hope that it will be useful, but
345 # WITHOUT ANY WARRANTY; without even the implied warranty of MERCHANTABILITY
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347 # License for more details.
348 #
349 # You should have received a copy of the GNU Lesser General Public License
350 # along with the SIMPLE code. If not, see <http://www.gnu.org/licenses/

```

```

351 #####
352
353
354 # Volume change from Volume and Temperature
355 SIMPLE_Vdot <- function(
356   params = c(-0.827, 7.242, 1.630e-4, 2.845e-05), Tt, Vt) {
357   # Extract the parameters from the vector params.
358   a      = params[1]
359   b      = params[2]
360   alpha = params[3]
361   beta  = params[4]
362
363   # model
364   Veq <- a * Tt + b           # 'virtual' equilibrium
365   tau <- 1 / (alpha * Tt + beta) # time scale
366   Vdot <- -(Vt-Veq)/tau      # annual change in volume
367
368   return(Vdot)
369 }
370
371 # Transient volume response from Temperature and initial volume
372 SIMPLE_traject = function(params = c(-0.827, 7.242, 1.630e-4, 2.845e-05), V0, Tt)
373   # Extract the parameters from the vector params.
374   a      = params[1]
375   b      = params[2]

```

```

376   alpha = params[3]
377   beta  = params[4]
378
379   nt    <- length(Tt)           # number of time steps
380   Vt    <- rep(NA, nt)          # create volume vector
381   Vt[1] <- V0                   # assign volume first entry
382
383   Veq <- a * Tt + b              # 'virtual' equilibrium volume
384   tau <- 1 / (alpha * Tt + beta) # timescale
385
386   # Step through time, calculating the ice volume <Vt> for each time step <i>
387   for (i in 2:nt) {
388     Vt[i] <- max( 0, Vt[i-1] + -(Vt[i-1]-Veq[i]) / tau[i] )
389   }
390
391   return(Vt)
392 }

```

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Figure 1: Approximate estimates of the computing time required for various numbers of model runs using a hierarchy of models (colored lines), assuming that only one processor is available to perform the calculations. Computing times for 125,000-yr simulations are shown because ice sheet models must be ‘spun up’ over at least this much time for every unique parameter combination (Rogozhina et al., 2011; Applegate et al., 2012; Haqq-Misra et al., 2012). The colored boxes mark the zones that achieve the minimum number of simulations required for Integrated Assessment studies ($> 10^5$, green; e.g. McInerney et al. (2011)) and for Multi-Objective Robust Decision Making (MORDM) ($> 10^6$, red; Kasprzyk et al. (2013); Singh et al. (2015)) within a computation time of one month. Three-dimensional ice sheet models such as the PSU-3D ice sheet model (Pollard and DeConto, 2012), which includes ‘hybrid’ ice dynamics, and SICOPOLIS (Greve, 1997), which is a Shallow-Ice Approximation model, are clearly too computationally expensive to achieve these numbers of simulations in a reasonable time. One-dimensional models such as GLISTEN (Haqq-Misra et al., 2012) may be more appropriate for IAM studies from a computational perspective; however, SIMPLE, which we describe here, is about two orders of magnitude faster than GLISTEN.

Figure 2: Response of Greenland Ice Sheet volume (lower panel) to moderate (ΔT , orange) and high ($2\Delta T$, red) instantaneous temperature increase (upper panel) from system that is initially in equilibrium $V = V_0$ as simulated by SIMPLE (equations 1-3).

Figure 3: Projected Greenland surface temperature for rcp8.5 (Schewe et al., 2011, black line) together with 20 geo-engineering scenarios. Upper panel, ‘temperature stabilization’ scenarios; lower panel, ‘gradual temperature draw-down’ scenarios. Different colors indicate different starting years (dots) for geo-engineering. Gray line, observed temperatures. Replotted from Applegate and Keller (2015).

Figure 4: Rate of ice volume change $\dot{V}$ as a function of the distance to the ‘virtual equilibrium’ ice volume $\Delta V_{v.eq} = V - V_{v.eq}$ derived from different simulations with SICOPOLIS; 125ka spinup run (Applegate et al., 2012), 8 equilibrium runs (Applegate et al., 2015b) and RCP8.5 and 20 geo-engineering runs (Applegate and Keller, 2015). Here, $V_{v.eq}$ is estimated with equation 2 applying $a = -1.62$ and $b = 5.56$, based on a linear regression through the equilibria as simulated by Applegate et al. (2015b). Colors refer to SICOPOLIS ice volume. The rate of mass loss clearly increases with distance to ‘virtual equilibrium’, whereas the growing rate seems always small. In similar figure (A.9) colors refer to model runs.

Figure 5: ‘Exponential change rate’ $1/\tau$ as a function of temperature T . Same simulations are used as for figure 4. For temperature anomalies between 0 and 12 °C, the exponential decay rate approximately linearly increases with temperature (in case ice volume is not yet equilibrated).

Figure 6: Simulated (SICOPOLIS, solid lines) and emulated (dashed lines) GIS response to 20 different geo-engineering scenarios (left panels: temperature stabilization, right panels: gradual temperature draw-down), where the colors refer to the starting year (dots, see Figure 3). The thick black line is the simulated response to the ‘base’ RCP8.5 scenario. SIMPLE (bottom row) reproduces SICOPOLIS slightly better than GIS-DICE (third row) and much better than the quadratic function (upper row) and the delayed linear concept (second row).

Figure 7: RMSE (Root Mean Squared Error) of emulated GIS volume as a function of evaluation time as emulated by SIMPLE and other emulators (colors) estimated by different cross-validation strategies (symbols). RMSE is evaluated against the SICOPOLIS geo-engineering scenarios (see figure 3) for different time spans into the future (horizontal axis). SIMPLE shows smallest RMSE and lowest sensitivity for calibration data.

Figure 8: Simulated (SICOPOLIS, solid lines) and emulated (SIMPLE, dashed lines) change path dependent behavior (sea-level contribution as a function of temperature). Transient response to Greenland surface temperature forcings according 10 geo-engineered temperature ‘draw-down’ scenarios (see figure 3, right panel). Colors refer to starting year (dots) geo-engineering.

Figure A.9: As figure 4, but colors refer to model simulations rather than to ice volume