Application of an Evolutionary Algorithm for Parameter Optimization in a Gully Erosion Model

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Abstract

Herein we demonstrate how to use model optimization to determine a set of best-fit parameters for a landform model simulating gully incision and headcut retreat. To achieve this result we employed the Covariance Matrix Adaptation Evolution Strategy (CMA-ES), an iterative process in which samples are created based on a distribution of parameter values that evolve over time to better fit an objective function. CMA-ES efficiently finds optimal parameters, even with high-dimensional objective functions that are non-convex, multimodal, and non-separable. We ran model instances in parallel on a high-performance cluster, and from hundreds of model runs we obtained the best parameter choices. This method is far superior to brute-force search algorithms, and has great potential for many applications in earth science modeling. We found that parameters representing boundary conditions tended to converge toward an optimal single value, whereas parameters controlling geomorphic processes are defined by a range of optimal values.

Keywords:

optimization, erosion, hydrology, landscape modeling

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1 **2. Software availability**

2 Name of software: *GullyErosionProfiler1D*
3 Developer: Francis Rengers
4 Contact address: U.S. Geological Survey, 1711 Illinois St., Golden, CO 80401.
5 frengers@usgs.gov
6 Year first available: 2012
7 Required software: Matlab
8 Source: <https://github.com/csdms-contrib/gullyerosionprofiler1d>
9 Availability and cost: GNU licensed freeware
10
11 Name of software: *Param_{scan}*
12 Developer: Monte Lunacek
13 Contact address: National Renewable Energy Laboratory, 15013 Denver W
14 Pkwy, Golden, CO 80401. Monte.Lunacek@nrel.gov
15 Year first available: 2014
16 Required software: Python
17 Program Language: Python
18 Source: <https://github.com/frengers/JanusOpt/tree/master/src>
19 Availability and cost: GNU licensed freeware
20

21 **3. Introduction**

22 Landscape change results from erosion and sedimentation processes that
23 govern mass redistribution over long time periods; consequently, direct ob-
24 servation of landscape evolution is rarely possible. In order to properly un-
25 derstand how different geomorphic processes and landscape properties influ-
26 ence long-term erosion it is necessary to use models as proxies for large-scale
27 landscape evolution. In most scenarios it is infeasible to construct physical
28 models (Tucker, 2009) because important properties (such as rain-drop size)
29 do not scale with model landscapes. As a substitute for direct observation,
30 computer models serve as a useful alternative (Tucker and Hancock, 2010).

Computational landform evolution models are composed of geomorphic transport laws (GTLs) (Dietrich et al., 2003), and a key challenge for computer modeling is choosing model parameter values for the GTLs. Parameters such as soil/rock erodibility are difficult to measure (Elliot et al., 1989; Prosser and Dietrich, 1995), therefore there is large uncertainty in the values assigned to these parameters. It is common to use steady and uniform values for some fluxes such as soil infiltration, which can presumably be estimated from field measurements; however, point infiltration rates can vary by orders of magnitude within the same watershed (Sharma et al., 1980). Consequently, process parameter values are often estimated by calibration (e.g. Barkwith et al. (2015)), which in turn requires comparison between observed and predicted landscape properties. Moreover, there is uncertainty in the initial and boundary conditions. Here we explore the use of optimization techniques to search for input values for these uncertain model parameters.

In this study, we have focused on a model of gully erosion. Gullies are deeply incised channels that migrate upstream via erosion from near-vertical headcuts, which are analogous to waterfalls in rivers (Figure 1). Water erosion in the model works to incise the gully vertically, and headcuts migrate upstream as they erode. The model was developed to explore how the processes of headcut erosion and water incision compete to form a concave-up longitudinal profile while maintaining a near-vertical headcut over hundreds of years (Rengers and Tucker, 2014). Our model, and the range of parameter values used in the model, are based on observations from the West Bijou Creek study site in eastern Colorado (Rengers and Tucker, 2014, 2015).

We applied the Covariance Matrix Adaptation Evolution Strategy (CMA-ES) (Hansen and Ostermeier, 2001), a parameter optimization scheme, to determine the most appropriate model parameter values. This approach is an inverse modeling method in which a model is run with different input parameters, and model output is subsequently compared to a known observed value. The best model parameters are determined as those that provided the closest fit between the model output and the observations.

4. Background

The overall longitudinal profile of a gully results from the competition between fluvial erosion and sedimentation processes (Rengers and Tucker, 2014). These processes are conceptualized in Figure 2. Wall failure as shown in the conceptual model can result from a variety of processes including:

Figure 1: View of a headcut migrating upstream and extending gully erosion.

67 ground water seepage (Howard and McLane, 1988), piping (Verachtert et al.,
 68 2010), direct wash erosion over headcut walls (DeLong et al., 2014), mass
 69 failure (Bradford and Piest, 1980; Dietrich and Dunne, 1993; Istanbuluoglu
 70 et al., 2005; Montgomery, 1999), and plunge pool erosion through undercut-
 71 ting (Gardner, 1983; Gilbert and Hall, 1907; Stein and LaTray, 2002; Wohl
 72 et al., 1994). At our specific study site headcut wall failure was primarily
 73 associated with mass failure via soil saturation resulting from overland flow
 74 (Rengers and Tucker, 2015). Regardless of the specific process that governs
 75 headcut wall failure, erosion at the headcut has the ability to dramatically
 76 alter the overall profile of a gully over time depending on the rate of wall
 77 failure, and the influence of fluvial erosion (Gardner, 1983; Stein and Julien,
 78 1993; Rengers and Tucker, 2014).

79 Three scenarios demonstrate the end-member possibilities required to dy-
 80 namically maintain a gully headcut (Figure 3). First, if headcut wall failure

occurs at a high rate, but fluvial processes are unable to transport the material away from the toe of the headcut; the headcut will become buried and a section of the gully channel will become locally convex (Figure 3a). Alternatively, if fluvial erosion operates at a high rate, eroding both the headcut lip and sediment deposited by headcut wall failure, then the gully channel will become locally concave (Figure 3b). Neither of the prior scenarios would preserve a discrete headcut step. A headcut is only preserved in the unique situation where fluvial erosion upstream of the headcut is limited and fluvial erosion downstream of the headcut is sufficient to remove material deposited from headcut wall failure (Figure 3c). This conceptual view of gully longitudinal profile evolution has been implemented in a numerical model in order to quantitatively test the hypothesis proposed in this conceptual framework (Rengers and Tucker, 2014).

4.1. Study Site

The gully erosion model is based on observations from a gully system that drains to West Bijou Creek on the high plains of eastern Colorado, USA (Rengers and Tucker, 2014, 2015). The underlying geology at the study site is a sequence of poorly lithified sandstone and shale units of Cretaceous to early Paleogene age (Barclay et al., 2003). The bedrock is shallow and weathers into cohesive clay-loam and loam soils (Entisols and Mollisols primarily). Site vegetation is primarily grassland, with sparse shrubs on steeper hillslopes. Both the soil and vegetation characteristics are believed to have an important control on gully formation and evolution (Rengers and Tucker, 2014).

Airborne lidar of the study site was flown on 23 April 2007 (National Center for Airborne Laser Mapping, 2007). The point data had a vertical accuracy of 10 cm and the point density was sufficient to construct a DEM with 1 m resolution (National Center for Airborne Laser Mapping, 2007).

4.2. Governing equations

Several models have been developed previously for gully erosion, but they do not explicitly account for headcut erosion (Woodward, 1999; Sidorchuk, 1999). Consequently, we use a model developed by Rengers and Tucker (2014) for gully erosion that explicitly accounts for headcut migration and focuses on the competition between fluvial erosion, headcut wall failure and sedimentation, and the resistive forces of vegetation and soil. This model does not specifically represent the processes that cause the initial gully incision and headcut formation, but rather starts with a headcut that can migrate

117 over time. The major components of this model are displayed in Figure 2,
 118 and based on the following set of equations.

Figure 2: Conceptual diagram showing the model components.

Figure 3: Three scenarios that show the possible evolution of a headcut in a gully environment. The arrow length indicates relative magnitude of each process. The top cartoon shows the starting condition, while the bottom cartoon shows the final condition. (a) Headcut burial from more deposition than erosion. (b) Headcut destruction due to more fluvial erosion than sedimentation. (c) Headcut propagation in the upstream direction, and preservation of the headcut step.

For overland flow, we assume steady-state discharge per unit width specified by:

$$q = \frac{(P - I)}{B} A \quad (1)$$

119 where P [dimensions of length per time, LT^{-1}] is precipitation rate, I [LT^{-1}]
 120 is infiltration rate, B [L] is the bottom width of the channel, and A [L^2] is
 121 drainage basin contributing area. Basin area is calculated from downstream
 122 distance along the valley profile, x , using Hack's law (Hack, 1957):

$$A = fx^H, \quad (2)$$

123 with the values $f = 0.19$ and $H = 2.3$, which were derived from the DEM
 124 of the study catchment (Rengers and Tucker, 2014). The flow depth h is
 125 calculated using a form of the Manning roughness equation,

$$h = \left(\frac{qn_m}{S^{1/2}}\right)^{3/5}, \quad (3)$$

126 where n_m is the Manning roughness coefficient, and S is the channel bed
 127 slope. We used different Manning's n_m parameters depending on the land-
 128 cover type. The unique parameter name for n_m representing grass is *mann*,
 129 and for bare soil it is *nb* (Table 1).

130

131 Continuity of mass for bed sediment can be expressed as:

$$\frac{\partial h_T}{\partial t} = \frac{d - e}{(1 - \phi)} \quad (4)$$

132 where h_T is bed elevation, d is the deposition flux [volume per unit time
 133 per unit bed area: LT^{-1}], e is the erosion flux [LT^{-1}], and ϕ [L^3L^{-3}] is the
 134 porosity of sediment on the bed.

135 The erosion flux is modeled as a function of excess boundary shear stress,

$$e = k(\tau - \tau_c) \quad (5)$$

136 where k [L^2TM^{-1}] is the erodibility constant, τ [$ML^{-1}T^{-2}$] is the bed shear
 137 stress, τ_c [$ML^{-1}T^{-2}$] is a critical shear stress below which erosion is negli-
 138 gible. Bed shear stress for steady, uniform, wide-channel flow is:

$$\tau = \rho ghS \quad (6)$$

139 where ρ [ML^{-3}] is the density of water and g [LT^{-2}] is gravitational accel-
 140 eration. The deposition rate depends on the product of settling velocity and
 141 near-bed sediment concentration. Following Davy and Lague (2009), this
 142 can be expressed as:

$$d = d^* c_s \nu_s + \frac{q_{sLat}}{B} \quad (7)$$

$$c_s = \frac{q_s}{q} \quad (8)$$

143 where d^* is a dimensionless parameter equal to 1 if the sediment flux is
 144 uniformly distributed through the water column, c_s is sediment concentration
 145 in the water column (dimensionless), ν_s [LT^{-1}] is the settling velocity, q_{sLat}
 146 [L^2T^{-1}] represents sediment coming into the channel from lateral sources,
 147 and q_s [L^2T^{-1}] is the sediment discharge per unit width.

148 In addition to fluvial sediment, we also route the sediment derived from
 149 headcut wall retreat. The headcut is treated as a local source of sediment
 150 with a volume flux per unit valley width of:

$$V_r = RH_c \quad (9)$$

151 where V_r [L^2T^{-1}] is the rate of sediment eroded per unit block length, R
 152 [LT^{-1}] is the retreat rate, and H_c is the height of the headcut. Motivated by
 153 the observation that slump deposits tend to accumulate within a few meters
 154 of the main headcut face, we describe the depositional pattern of headcut-
 155 derived sediment using an exponential pattern that decays downstream from
 156 the headcut face:

$$d_{hc}(l) = \frac{V_r}{l_*} e^{\frac{-l}{l_*}} \quad (10)$$

157 where d_{hc} is the deposition rate due to wall collapse downstream of the head-
 158 cut, l is the distance downstream of the headcut, and l_* is a characteristic
 159 length scale for wall-collapse deposits. This rule mimics the observation that
 160 the thickness of slump deposits tends to taper downstream over a distance
 161 of one to several meters.

162 Equations 5-10 can be inserted into Equation 4 to obtain an equation for
 163 bed evolution:

$$\frac{\partial h_T}{\partial t} = \begin{cases} \frac{\frac{q_s}{q} \nu_s - k(\tau - \tau_c)}{1 - \phi} + RH_c \frac{1}{l_*} e^{\frac{-l}{l_*}} & \text{if } \tau > \tau_c; \\ \frac{\frac{q_s}{q} \nu_s}{1 - \phi} + RH_c \frac{1}{l_*} e^{\frac{-l}{l_*}} & \text{if } \tau \leq \tau_c. \end{cases} \quad (11)$$

164 Equation 11 was solved numerically. Figure 4 shows an example of the
 165 model performance, which can be compared with an observed profile at each

166 time-step. The observed profile was derived from the airborne lidar-derived
 167 DEM of the study site, with 1 m grid spacing.

Figure 4: Example of a model run with an initial profile (thick black line) described by the polynomial in Equation 12. The black line is the observed profile. The thin gray line shows the model profile at different time steps.

168 4.3. Parameters for Optimization

169 We have selected several parameter values to optimize from the governing
 170 equations (Table 1). For precipitation we use a range from 5–100 mm/hr .
 171 In this model precipitation drives overland flow and thus low values of pre-
 172 cipitation will not lead to flow depths sufficient to generate fluvial erosion of
 173 the channel bed. We chose a default value of 40 mm/hr because site data
 174 show that most rain events larger than this rainfall intensity generate some
 175 overland flow. Our minimum precipitation value is sufficient to cause over-
 176 land flow, but it is possible that larger rainfall intensities may be required
 177 to generate erosion over time. The infiltration default parameter value of 20
 178 mm/hr was determined based on infiltration measurements in the field. At
 179 the study site, the saturated infiltration rate was measured at 15 locations
 180 using a double-ring infiltrometer. The final (steady) infiltration rate at each

181 site varied from 20 mm/hr to $> 300 mm/hr$. During the infiltration tests,
 182 we anecdotally observed that the extremely large infiltration rate measure-
 183 ments were often underlain by soil cracks. Because the model infiltration
 184 rate represents an ‘effective’ value for the watershed as a whole, we used an
 185 optimization range of 5–45 mm/hr . This range represents potential ‘effec-
 186 tive’ infiltration rates, and excludes extremely high measurements that are
 187 isolated to single patches within the watershed.

188 Our erosion parameters were obtained from published datasets. The
 189 range of values for the critical shear stress of grass-mantled soil (τ_c) was
 190 obtained from experimental work done by Prosser and Dietrich (1995). The
 191 critical shear stress for bare soil (τ_{cw}) was obtained from data on US De-
 192 partment of Agriculture field experiments (Elliot et al., 1989). The default
 193 value for τ_{cw} represents an experimentally determined value for clay-loam
 194 soil (Elliot et al., 1989), which is the dominant soil in the gully channels at
 195 the field site. Default Manning’s roughness coefficient values were obtained
 196 from standard roughness values for overland flow on bare soil and grassy
 197 vegetation, respectively (Chow, 1959).

198 We also sought to optimize the initial starting profile for our model run.
 199 We varied the initial headcut height based on the range of headcut heights
 200 observed at the field site. A polynomial equation was used to fit a gently
 201 concave-up curve to the longitudinal profile measured with airborne lidar, in
 202 order to approximate an initial profile. This polynomial equation is written
 203 with three coefficients (z_1 , z_2 , and z_3) as:

$$z = z_1x^2 + z_2x + z_3 \quad (12)$$

204 where z is the elevation of each cell in the model. The coefficients of this
 205 model were varied to determine the optimal starting condition.

206 Finally, the amount of time that the model simulates (t) will influence the
 207 match between the measured and modeled profiles. Prior model simulations
 208 suggest that a time range between 400 and 450 yr is appropriate (Rengers
 209 and Tucker, 2014) (Table 1).

210 4.4. CMA-ES

211 In order to find the optimal value for each parameter in the model, we use
 212 CMA-ES to efficiently analyze the many combinations of possible parameter
 213 values. This is done by running the model with different sets of parameter
 214 values. Each time the model is run with a unique set of parameter values

Table 1: Parameters for Optimization

Parameter default value	Value Range used for Optimization
Precipitation (P) = 40 <i>mm/hr</i>	5 – 100 <i>mm/hr</i>
Infiltration (I) = 20 <i>mm/hr</i>	5 – 45 <i>mm/hr</i>
Critical Shear Stress of Grass (τ_c) = 140 <i>Pa</i>	100 – 200 <i>Pa</i>
Critical Shear Stress of Bare Soil (τ_{cw}) = 6.2 <i>Pa</i>	2 – 200 <i>Pa</i>
Manning’s coefficient for Bare Soil (nb) = 0.02	0.01 – 0.055
Manning’s coefficient for Grass (mann) = 0.05	0.01 – 0.055
Initial Headcut Height (h) = 2 <i>m</i>	1 – 4 <i>m</i>
Initial Profile Coefficient (z1) = 0.00008364	8.0×10^{-5} – 9.5×10^{-5}
Initial Profile Coefficient (z2) = -0.09355	-9.5×10^{-2} – -8.5×10^{-2}
Initial Profile Coefficient (z3) = 1022.42	1022 – 1025
Model simulation time (t) = 418 <i>yr</i>	400 – 450 <i>yr</i>

an objective function is produced that describes the fit between the model output and the observed data. The goal of an evolution strategy is to evolve a set of parameter values that minimize the objective function. This best set of parameters will yield model output that best matches the observed data.

An evolution strategy is an iterative process where each iteration evaluates λ randomly chosen points, with a point being a single set of parameter values. Only the best μ points are selected to be part of the next iteration. In our experiments, we used a $\mu = 7$ and $\lambda = 14$ strategy. The μ best points are used to produce a new set of λ random points during the next iteration based on a normal probability distribution whose characteristics evolve over the course of the entire process. This is often referred to as the mutation distribution, and the exact way this distribution is defined and evolves differentiates the various evolution strategies.

Often the quality of the solution is not highly sensitive on the exact choice of μ or λ . Generally, problems that are particularly noisy or have more variables would require larger population values. The disadvantage of a larger population size is that it requires more evaluations per iteration, which decreases the optimization time, and can slow the rate at which the algorithm learns the topology of the problem. For our problem, $\lambda = 14$ provided a good balance between optimization speed and quality of solution, and μ is typically set to half of λ .

An evolution strategy with Covariance Matrix Adaptation (CMA) uses

237 a covariance matrix to explicitly rotate and scale the mutation distribution
 238 (Hansen and Ostermeier, 2001). The orientation and shape of the distribu-
 239 tion are directly calculated based on the history of the best points evaluated
 240 and the shape of the best μ points in the current iteration. Hansen and
 241 Ostermeier (2001) define the reproduction phase from iteration g to iteration
 242 $g + 1$ as:

$$x_k^{(g+1)} = \langle x \rangle_\mu^{(g)} + \sigma^{(g)} \mathbf{B}^{(g)} \mathbf{D}^{(g)} z_k^{(g+1)}$$

243 where $z_k^{(g+1)}$ are randomly generated from an $N(0, I)$ distribution. This cre-
 244 ates a set of base points that are rotated and scaled by the eigenvectors ($\mathbf{B}^{(g)}$)
 245 and the square root of the eigenvalues ($\mathbf{D}^{(g)}$) of the covariance matrix C . The
 246 single global step size, $\sigma^{(g)}$, is calculated based on the progress of the search
 247 process. Finally, the points are translated to center around $\langle x \rangle_\mu^{(g)}$, the mean
 248 of the μ best point of the iteration.

249 To compute covariance, CMA-ES uses a time-dependent portion of the
 250 search history. The evolution path, which is the average direction from the
 251 first iteration to the current iteration, updates after each iteration using a
 252 weighted sum of the current evolution path, $p_c^{(g)}$, and a vector that points
 253 from the mean of the μ best points in the iteration g to the mean of the μ best
 254 points in iteration $g + 1$. A principle-components analysis on the evolution
 255 path is used to update the mutation distribution.

256 When a larger population (λ) is used, the best μ individuals may help de-
 257 scribe the topology around the mean of the current iteration (N. Hansen and
 258 S.D. Müller and P. Koumoutsakos, 2003). This is called the rank- μ update.
 259 Hansen and Kern (2004) have empirically shown that CMA-ES performs well
 260 on noisy problems with many solutions when large populations and rank- μ
 261 updates are employed. Although in this specific study, the problem is not
 262 particularly noisy.

263 The distribution used by CMA-ES is initially isotropic. As a result, the
 264 initial value of σ is important for exploration. However, the step-size (σ)
 265 often increases before CMA-ES exploits a particular region. This makes the
 266 exact choice of the value less critical as long as it's not too small. Therefore,
 267 we chose a step-size of 0.1, which, because the parameters of our problem are
 268 scaled between $[0, 1]$, is sufficient for exploring the search space.

269 CMA-ES is not able to handle boundary constraints because it uses a nor-
 270 mal distribution to sample the search space. Therefore, we used a constraint-
 271 penalty method similar to Hansen et al. (2009), to ensures that our parameter

272 values are evaluated within the feasible region of our model. When a point
273 falls outside of the model boundaries, we evaluate the point at the closest
274 boundary and then apply a penalty to the solution that is proportional to
275 the distance where we evaluated the point, the closest boundary.

276 5. Methods

277 We used the CMA-ES algorithm on the gully headcut retreat model.
278 This algorithm was run on a high-performance cluster at the University of
279 Colorado, Boulder, using 12 nodes, each of which had 4 cores. The specified
280 range of parameter values was normalized so that parameter values ranged
281 from 0 to 1. However, the constraint-penalty approach that we employed
282 allowed parameter values to slightly exceed those bounds. An initial value for
283 each parameter was randomly chosen from this range. In order to determine
284 the fit between the model and the measured profile, we compared the modeled
285 profile to the measured profile using the root mean squared error (RMSE) as
286 the objective function.

287 Thirty independent trials of CMA-ES were run so that the parameter
288 search would begin with thirty randomly selected initial parameter sets. This
289 was done because Hansen and Ostermeier (2001) suggest that the quality of
290 solutions found by CMA-ES using a small initial step-size are often deter-
291 mined by the location of the starting point (Hansen and Ostermeier, 2001).
292 Within a single trial there were multiple iterations, and each iteration con-
293 sisted of a model run with parameters specified by the CMA-ES algorithm.
294 CMA-ES used the objective function output from each iteration to inform
295 parameter value choices for the next iteration by mutating, that is, it varied
296 the search steps stochastically and then selected the parameters that best
297 minimized the objective function. A given trial run terminated once the pa-
298 rameters converged toward unique values, or the search-step size was equal
299 to or less than 1×10^{-6} .

300 6. Results and Discussion

301 The CMA-ES approach resulted in far fewer iterations than would be re-
302 quired if a brute-force approach was used. If we explored 10 potential values
303 for each of the 11 parameters, this would require more than 25 billion model
304 runs to search every combination. Each model simulation takes approxi-
305 mately 20 minutes to completion, therefore a brute force approach would

306 result in 5.2×10^{11} minutes of model simulation time. By contrast, we found
 307 that in CMA-ES trials, the parameters often converged on an optimal value
 308 within 60-80 iterations (Figure 5a), and the resulting model profile obtained
 309 from parameters that minimized the objective function fit the target (i.e.
 310 observed) profile well (Figure 5b).

Figure 5: (a) Convergence of each variable toward an optimal value. (b) Modeled longitudinal profile of a gully with a headcut using the best-fit parameters (RMSE=17.5 m), and compared to the measured longitudinal profile.

311 The optimal solution for each trial does not result in the same set of
 312 parameter values. This is due to the random initial parameter values chosen
 313 at the beginning of each trial. However, most of the solutions obtained from
 314 the CMA-ES parameter search resulted in a close fit between the modeled
 315 and measured longitudinal profile (Figure 6). This shows that the search
 316 algorithm can robustly discover potential optimal parameter values in a large

317 solution space. Some optimal solutions from CMA-ES resulted in profiles
318 that failed to match the measured headcut location (Figure 6), but these
319 represented only four solutions within the top 18 model solutions.

320 We explored how each iteration of the 30 CMA-ES trial runs converged
321 toward an optimal set of parameters, and found a duality within the final pa-
322 rameter values (Figure 7). Some parameters are highly sensitive and converge
323 on a unique value. These are parameters that represent boundary conditions
324 including: the starting profile shape (z_1 , z_2 , and z_3), the starting headcut
325 height (h), and the model simulation time (t). By contrast, parameters con-
326 trolling competing geomorphic processes work in pairs such as precipitation
327 and infiltration, the critical shear stress for grass versus bare soil, and the
328 Manning’s roughness coefficient for grass versus bare soil.

329 Our observation that the optimal parameter values for boundary condi-
330 tions converge toward singular values supports the fact that landform evo-
331 lution is dependent on the unique initial conditions of the landform. By
332 contrast, the natural competition between geomorphic processes means that
333 it is possible to change the value of parameters representing driving and
334 resisting forces, and retain the same result. Therefore, it is not surprising
335 that parameters representing competing processes can obtain different opti-
336 mal parameter values and produce a similar result. For example, increasing
337 precipitation and infiltration by the same magnitude will result in the same
338 runoff depth. Likewise, a lower critical shear stress for bare soil will increase
339 erosion in the bare soil areas, but a higher critical shear stress for grass will
340 decrease erosion of grassy areas. The combination of these two parameter
341 changes can result in a similar end-result for sedimentation. Changing Man-
342 ning’s roughness coefficient for bare and grassy areas in opposite directions
343 can also result in a similar average flow velocity. Therefore, because these
344 process-controlling parameters can take on multiple values but still produce
345 a similar average value, a unique solution does not emerge.

346 This research suggests that a search algorithm for parameter values bridges
347 the gap between picking parameters based on a range of observed estimates
348 and inefficient, often intractable, brute-force parameter searches. Moreover,
349 even with an efficient search algorithm, the inherent functionality of some pa-
350 rameter values will result in the potential for multiple optimized parameter
351 estimates. Therefore, the ability of a search algorithm to identify parameters
352 that converge toward singular values as well as those that oscillate between
353 multiple values is a critical requirement for any potential strategy. This
354 has implications for most geomorphic process models, which require many

Figure 6: Simulated profiles compared to the measured headcut (dark gray). Fourteen of the model runs (the majority) provide a good fit to the observed headcut, but four solutions result in poor fits despite being the optimal solution suggested by CMA-ES. Note for clarity only the top 60% of the optimal parameter sets are displayed.

355 unconstrained parameters for competing processes.

356 While the overall objective of the optimization scheme was to simply

Figure 7: The parameter values for each iteration of the 30 CMA-ES trial runs. The objective function (f) is highlighted in blue for the parameter values that all led to the lowest differences between the model and the measured profile. Less optimal value of f are displayed in gray. The parameter names are defined in Table 1.

357 obtain parameter values, the technique revealed important insight into the
 358 nature of the parameters in the underlying model. The duality of the param-
 359 eters (unique versus multi-value convergent) showed that parameters repre-
 360 senting competing processes do not result in a single optimal parameter value.
 361 These also tend to be the parameters that govern geomorphic processes. How-
 362 ever, the boundary conditions can be fit uniquely, and this allowed us to find
 363 the initial headcut height and slope geometry that best corresponded to the
 364 conditions leading to a final gully shape.

365 7. Conclusions

366 By using an optimization algorithm and leveraging high-performance clus-
 367 ter resources, it is possible to identify parameter values that lead to best-fit
 368 model solutions. In the context of a one-dimensional model of gully head-
 369 cut propagation the optimized results using CMA-ES show that boundary
 370 condition parameters converge toward a singular value. These boundary con-
 371 dition values provide estimates of the initial longitudinal profile shape and
 372 gully headcut height, which were not known a priori. While it is unlikely that
 373 our optimal results are the exact initial boundary conditions experienced in

the gullies at our field site, this approach provides an objective methodology for specifying the unknown boundary conditions. At a minimum, parameter optimization is a way to exclude possible initial boundary condition scenarios in landform evolution models, where initial conditions are rarely constrained.

This study further showed that parameters controlling geomorphic processes can occupy several different optimal values. The average runoff depth, velocity, and erosion rate in the gully model result from competing processes. We found that it was possible to change competing parameter values governing these processes and obtain a similar optimal result. It is therefore likely that there will always be a range of parameter values for competing processes that converge toward equally optimal solutions. The CMA-ES approach is nevertheless helpful in identifying the range wherein it is possible to vary competing parameters and maintain an optimal solution. Furthermore, this study shows that CMA-ES is an efficient approach for exploring multiple combinations of parameter value choices, thus allowing a tractable alternative to brute-force searching of the parameter space.

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