Similarity analysis of the streamer zone of Blue Jets

N.A. Popov¹, M.N. Shneider² and G. M. Milikh³

¹Moscow State University, Moscow, Russia

²Department of mechanical and aerospace engineering Princeton University, Princeton, NJ, USA

³Astronomy Department University of Maryland, College Park, MD, USA

Abstract

Multiple observations of Blue Jets (BJ) and Gigantic Blue Jets (GBJ) show that BJ and GBJ emits a fan of streamers similar to a laboratory leader. Moreover, in the exponential atmosphere those long streamers grow preferentially upward, producing a narrow cone confined by the aperture angle. It was also noticed that BJ are similar to the streamer zone of a leader (streamer corona) and the modeling studies based on the streamers fractal structure were conducted. Objective of this paper is to study the fractal dimension of the bunch of streamer channels emitted by BJ and GBJ, at different altitude and under the varying electric field. This similarity analysis has been done in three steps: First we described the dendritic structure of streamers in corona discharge applying the fractal theory. Then using this model and the data from existing laboratory experiments we obtained the fractal dimension of the branching streamer channels. Finally the model was validated by the observations of BJ available from the literature.

1. Introduction

About two decades ago, researchers discovered upward-propagating collimated flashes of light originating above thunderstorms [Wescott *et al.*, 1995]. Due to their distinctive, principally, blue color, they were termed “blue jets” (BJ). They resemble tall trees with quasi-vertical trunk and filamentary branches. [Pasko *et al.*, 2002] discovered the so-called gigantic blue jets (GBJ), propagating into the lower ionosphere. Number of GBJ observations were made since [Su *et al.*, 2003; Kuo *et al.*, 2009; Cummer *et al.*, 2009; Chou *et al.*, 2011; Neubert *et al.*, 2011; Soula *et al.*, 2011; Liu *et al.*, 2015].

Multiple observations of BJ show that they emits a streamers similar to the streamers of corona discharge of a laboratory leader. Streamers in corona discharge are self-similar plasma structures termed fractals [Wiesmann *et al.*, 1986; Pasko *et al.*, 2000; Pasko *et al.*, 2002; Popov, 2002]. According to the current understanding the streamer propagates due to the formation of electron avalanches in the strong electric field E of the space charge near the streamer tip. The propagation direction of those avalanches is determined by the shape of the streamer’s electric field. The probability of streamer propagation in a given direction is determined by the spatial distribution of the electric field near the streamer tip.

In the case of BJs the leader trunk barely exits the cloud and we only see the streamer zone [Raizer *et al.*, 2006] and [da Silva and Pasko, 2013]. Contrary to BJs GBJs lightning leaders escape the thundercloud upwards and propagate through the stratosphere. An interesting hypothesis [da Silva and Pasko, 2013] stated that when GBJ reach *the jump altitude when the streamer zone length is longer than the scale height of the atmospheric air density profile*, the streamer zone reaches the ionosphere.

Note that we focus on study of the streamer corona, which is the basic of the blue jets (BJs), as well as gigantic blue jets (GBJs). Furthermore BJs and GBJs are of the similar structure, and the both are eventually evolve into the streamer region. Notice that in this paper we limit ourselves to the analysis of the structure of the streamer region of GBJ since it is much better recognizable on the existing images.

The dendritic structure of streamers in corona discharge was treated by means of the fractal theory [Satpathy, 1986], which main parameter is the fractal dimension D . Note that the fractal dimension D of the plasma structure is determined by the power index γ , which depends on the reduced electric field value near the streamer tip [Popov, 2002]. [Pasko and George, 2002] numerically simulated the streamer corona of a positive leader as a fractal process. They suggested that the power index γ , that links the probabilities a random choice with electric field value, is $\gamma = 1$ and obtained the fractal dimension of the dendritic structure $D = 1.37 \pm 0.13$. The numerical simulations of the streamer corona of blue jets and blue starters as a fractal structures were first presented in [Pasko and George, 2002] although computed for $\gamma = 1$.

The objective of this paper is to apply the **theory of fractal dimension** of streamers in the corona discharge to study formation and propagation of blue jets and gigantic blue jets. The proposed model concerns GJs, BJs and streamer corona. However, since the existing images of the streamer corona and of BJs cannot resolve the object's spatial structure, in the present paper we focus entirely on the GBJs. Thus the model is validated by comparison with the existing optical observations of gigantic blue jets.

2. Fractal dimension of a bunch of multiple streamers

Widely used model [Niemeyer *et al.*, 1984; Wiesmann *et al.*, 1986] describes the spatial structure of branching channels. In this model, the plasma channels propagate toward the highest electric field, provided that the field exceeds the threshold value E_{cr} . The spatial distribution of the electric field is determined from solution of the Laplas equation for the potential having the boundary conditions which consider the conducting channels. All these channels are taking as equal, while the field inside the channels is constant. The probability of the streamer propagation in z-direction P_z is determined by the field distribution near the streamer head.

$$P_z \propto E_z^\gamma - E_{cr}^\gamma \quad (1)$$

where E_z is the field in the direction z , γ is the power index.

According to [Popov, 2002], the power index γ can be obtained from the equation for the effective ionization frequency:

$$\gamma = \frac{\partial \ln(\nu_{ion} - \nu_{att})}{\partial \ln E} \quad (2)$$

where $\nu_{ion} - \nu_{att}$ is the difference between the ionization frequency and electron attachment frequency for a given sort of the gas. It depends only on the reduced electric field E/N value.

The major results of the model [Niemeyer et al., 1984; Wiesmann et al., 1986] is that the plasma structures allow spatial scaling, thus they are self-similar structures or fractals. As shown in [Satpathy, 1986] the fractal dimension D depends entirely on the dimensionality of space d and the power index γ in Eq. (1), $D = D(d, \gamma)$.

Figure 1 reveals the power index as function of the reduced electric field $\gamma(E/N)$ for the air, here it is shown by the dashed line. It is computed for the E/N range 200 - 1200 Td (1 Townsend is equal to 10^{-17} V/cm²). The respective ionization and electron attachment frequencies $\nu_{ion}(E/N)$ and $\nu_{att}(E/N)$ were obtained by the Bolsig+ code [Hagelaar and Pitchford, 2005].

In addition, Fig. 1 reveals the fractal dimension D of the plasma structures formed in BJ, here it is shown by the solid line. The data $D(\gamma) = D(\gamma, d = 3)$ obtained in [Satpathy, 1986] for the 3D Laplace field was used. These data were determined by the model [Niemeyer et al., 1984; Wiesmann et al., 1986] from the solution of 3-D Laplas equation for the potential having the boundary conditions which consider the conducting channels. [Satpathy, 1986] computed the number of conducting channels $N(R)$ which cross the sphere of radius R . Then the fractal dimension $D(\gamma)$ was found using Eq. (5). The fractal dimension of the plasma structures vs. E/N (Fig. 1) was found under the assumption that the maximum electric field in the streamer head is constant and equal to E .

The data [Kuo et al., 2009] shows the reduced electric field in the streamer head inside the GBJ in the altitude range $h = 40 - 90$ km. It was obtained using the ratio of the observed intensities of the optical bands $N_2(2P)$ and $N_2(1N)$. Found in the work [Kuo et al., 2009] maximum reduced electric field was $E/N = 400$ Td. Notice that due to the shift between the maximum electric field and maximum intensities of the $N_2(2P)$ and $N_2(1N)$ bands in the streamer head [Naidis, 2009; Bonaventura et al., 2011] the real value $(E/N)_{max}$ in the streamers becomes about 1.4 times greater than the optical data gives. According to [Kuo et al., 2009; Bonaventura et al., 2011] we assume that in the streamer head where the branching probability is highest, the value $(E/N)_{max}$ reaches 500 - 600 Td (it is shown by the dotted area in Fig. 1).

As it follows from Fig. 1 for $E/N = 500 - 600$ Td the fractal dimation of the branching plasma channels yields:

$$D = 2.05 \pm 0.04 \quad (3)$$

Note that it significantly differs from $D = 1.37 \pm 0.13$ used by [Pasko *et al.*, 2000].

3. Average number of streamers in blue jets and gigantic jets

According to the fractal theory the full length L of the streamers inside the surface having the radius R can be presented by [Niemeyer *et al.*, 1984; Wiesmann *et al.*, 1986]:

$$L \propto R^D \quad (4)$$

While the number of streamers crossing this surface yields

$$N(R) = \frac{dL}{dR} = \left(\frac{R}{R_0} \right)^{D-1} \quad (5)$$

where R_0 is the characteristic scale within which the streamer branches. The value R_0 cannot be obtained from the fractal theory and thus it should be provided as an input to the model.

Another important result from the fractal theory describes the average inter-streamer distance Λ as the function of R :

$$\Lambda(R) \propto \frac{d^2 L}{dR^2} \propto \left(\frac{R}{R_0} \right)^{D-2} \quad (6)$$

Equation (6) has the proportionality sign which assumes the existence of a dimensional factor, the latter is missing from Equation (6). The factor will determine dimensionality of the average inter-streamer distance.

Consider $D = 2$ (see Eq. (3)) we obtain that $\Lambda(R) \approx const$ i.e. the average inter-streamer distance changes only slightly as the distance R increases. Moreover, using Eq. (5) one can find the total number of the streamers in the Blue Jet, $N(R)$, at the height h_{max} . The latter is the maximum height where the optical steamers are the most distict, therefore they will be used in our analysis. Here $R = h_{max} - h_0$ and h_0 is the height at which the BJ starts, while R_0 is the characteristic scale of the leader head at the height h_0 . The latter height controls the value of the peak current (see Discussion section after Table 1.)

Knowing $N(R)$ one can find the average distance between the streamers

$$R_{av} = 2 \cdot R_{str} / \sqrt{N(R)} \quad (7)$$

Here R_{str} is the observed radius of the streamer corona at the height h_{max} . Furthermore the obtained value of R_{av} was checked against the streamer radius observed at the height h_{max} .

4. Relation between the streamer speed and reduced diameter

In order to complete and test the fractal model we use the semi-empirical relations derived in [Raizer *et al.*, 1998; Raizer *et al.*, 2007]. The latter are based on studies of the laboratory streamers. The main input parameter for those relations is the voltage $\Delta U_t = U_t - U_0$ where U_t and U_0 are the potential of the streamer tip and of the external electric field respectively. The semi-empirical relations provides us with:

Field at the streamer tip $E_m \approx 1.5 \cdot 10^4 (N / N_0) \text{ kV/m}$;

Radius of the tip $r_m \approx 3.3 \cdot 10^{-5} \Delta U_t (N_0 / N) \text{ m}$; $\Delta U_t [\text{kV}]$;

Speed of the positive streamer $v_s \approx 5.3 \cdot 10^4 \Delta U_t \text{ m/s}$; $\Delta U_t \geq 5 \text{ kV}$;

If radius of the streamer tip is known, the voltage ΔU_t and the streamer speed could be obtained. On the other hand, knowing the streamer speed V_{str} one can obtain ΔU_t and r_m . Note also that ΔU_t is unequivocally coupled to E_m and r_m . For instance, for semi-spherical streamer tip $\Delta U_t \approx E_m r_m$

Let us present the equations in terms of the reduced streamer diameter $d_{rd} = 2 r_m \cdot (N/N_0)$ and reduced electric field $E_m \cdot (N_0/N)$. We exclude ΔU_t from the equations for r_m and V_{str} and then obtain the following relation between the streamer speed and its reduced diameter

$$V_{str}(\text{m/s}) = 0.8 \cdot 10^6 \cdot d_{rd}(\text{mm}) \quad (8)$$

The solid line in Figure 2 shows the streamer speed vs. the reduced diameter computed with Eq. (8). The two dashed lines are adapted from [Kanmae *et al.*, 2012]. They were computed for the E_s values 120 and 150 kV/cm, and validated by comparison with the observations of the Red Sprite streamers. Figure 2 shows that our semi-empirical model is in agreement with that of [Kanmae *et al.*, 2012].

5. Discussion

We begin with analysis of the GJ images [Pasko *et al.*, 2002; Su *et al.*, 2003]. The image from [Su *et al.*, 2003] is revealed in Fig. 3 while two other images are not shown, just described here. Listed in the top three rows of Table 1 are the hight h_{st} where the BJ starts, and the maximum

height h_{max} where the optical streamers are most distinct, along with the radii of the streamer coronas R_{str} measured at h_{max} .

Listed in Table 1 is also the relative air density at h_{st} and h_{max} altitude. It was obtained from the model of the earth atmosphere, and then used to compute the diameter of the leader head R_0 . We assumed then that the diameter of the leader head R_0 is 1 cm at the ground, and it varies with the height inversely proportional to the air density.

Such height dependence is based on the following considerations. According to the modern understanding [Bazelyan and Raizer, 1998; da Silva and Pasko, 2013] the value R_0 is determined by the characteristic rate of the electron attachment to the molecular oxygen. The latter process involves either two particles ($e + O_2 \rightarrow O^- + O$), or three particles ($e + O_2 + O_2 \rightarrow O_2^- + O_2$). At high altitude where BJs occur the dominant is the dissociative electron attachment, which is two particle reaction (see [da Silva and Pasko, 2013]). Therefore we assume the linear dependence $R_0 \propto (N_0/N)$.

Theoretical estimates, based on the fractal model, show the total amount of the streamers N_{tot} at h_{max} and the average inter-streamer distance (see Eq. (7)).

As it shown in [Milikh et al., 2016], in the streamer zone of a leader the strong inter-streamer interaction occurs when the streamer diameter is about the same as the distance between the streamers. Therefore we assume that the reduced streamer diameter is equal to the reduced inter-streamer distance.

Finally using Eq. (8) we compute the streamer speed, which is in the range of $(0.3-2.0) \cdot 10^7$ m/s. Note that the experiments [Wescott et al., 2001; Pasko et al., 2002; Su et al., 2003] lack fast speed optical detectors, thus the temporal resolution of the observed images was insufficient for accurate detection of the streamer speed. As mentioned by [Pasko et al., 2002] “the estimated speed in the range $(1.9 - 2.2) \cdot 10^6$ m/s is therefore a lower bound on the actual speed, which was probably higher”.

Observables	[Pasko et al., 2002]; [da Silva and Pasko, 2012]	[Su et al., 2003, fig. 2]
Height of streamers start h_{st}	35 km	37.5 km
Maximum height streamers reach h_{max}	67 km	73 km
Radius of the streamer corona R_{str} at h_{max}	5 km	5.9 km

Estimates	[Pasko et al., 2002]; [da Silva and Pasko, 2012]	[Su et al., 2003, fig. 2]
Relative air density $N(h=0)/N(h_{st})$	145	204

$N(h=0)/N(h_{max})$	9,000	24,500
Total branches N_{tot} at h_{max}	22,000	17,400
Average inter-streamer distance, R_{av} at h_{max}	67.3 m	89.4 m
Reduced streamer diameter	7.5 mm	3.7 mm
Streamer speed (from Eq. (8))	$6 \cdot 10^6$ m/s	$3 \cdot 10^6$ m/s

Table 1. Results of observations of Gigantic Blue Jets adopted from [Su et al., 2003; Pasko et al., 2002] (the three top rows). Estimates based on the fractal model (rows 4-6 from the top). Reduced streamer diameters are shown in the row second from the bottom. The streamer speed is presented in the bottom row.

Moreover the GJ current evaluations were carried out for the experimental conditions [Neubert et al, 2011]. The full GJ current has been estimated based on the model of a single streamer which reaches the altitude h_{max} [Raizer et al., 2007]. The current amplitude of the streamer was multiplied then by the number of streamer channels $N(h_{max})$ at this altitude. The latter value was determined with the help of the fractal theory, Eq. (5). The full GJ current I_{GJ} was estimated for the leader head height $h_0 = 35$ km [Neubert et al, 2011] and for the h_{max} between 60 and 70 km. It was found that I_{GJ} ranges between 3 and 5 kA which consistent with that detected by [Neubert et al, 2011].

6. Conclusions.

Multiple observations of Blue Jets (BJ) show that BJ emit a fan of streamers similar to a laboratory leader (streamer corona). In the exponential atmosphere those long streamers grow preferentially upward, producing a narrow cone confined by the aperture angle. In the presented paper the dendritic structure of streamers in corona discharge was treated by means of the fractal theory. The fractal dimension of the branching streamer channels was found.

Some BJ images available from the literature were analyzed and the average inter-streamer distance was estimated with the help of the fractal model. Based on our previous work we assumed that in the streamer zone of the leader the inter-streamer distance is about the same as the streamer diameter. Using this assumption and the semi-empirical model [Raizer et al., 1998] we obtained the average streamer speed and compared it with the observations.

According to [Popov, 2002] the fractal dimension of the streamer channels is determined only by the reduced electric field $(E/N)_h$ in the streamers head. Therefore, applied to the problem of high-altitude lightning, the presented theory predicts that the fractal dimension should stay almost constant when the streamers propagate at different altitudes and different values of air density. In

addition, [Popov, 2002] showed that the fractal of the group of the streamer channels is not affected by the applied voltage thus it does not depend on the leader potential difference with respect to the ionosphere.

Full GJ current I_{GJ} was estimated for the observational conditions [Neubert *et al.*, 2011]. It was shown that I_{GJ} ranges between 3 and 5 kA which consistent with the detected values.

Acknowledgments

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Figure Captions

Figure 1. The fractal dimension D (solid line) and the power index γ (the dashed line) versus reduced electric field E/N computed for the streamer tips. The rectangular box contains the variations of the coefficient γ and of the fractal dimension in the E/N reduced field range 500-600 Td.

Figure 2. Speed versus the reduced diameter. The solid line is computed using relation (8) of our model. The two dashed lines are adapted from [Kanmae *et al.*, 2012] where they were computed for $E_s = 120$ and 150 kV/cm, and validated by comparison with the observations of the Red Sprite streamers.

Figure 3. Image of the gigantic blue jet [Su *et al.*, 2003]. The leader starts at $h_{st} = 37.5$ km; the maximum altitude considered $h_{max} = 73$ km; radius of the streamer corona $R_{str} = 5.9$ km measured at h_{max} .

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323

324 **Figure 2.** Streamer speed versus the reduced diameter. The solid line is computed using relation
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