A Homogeneous Substitute Material for the Core Layer of Photovoltaic Composite Structures

M. Aßmus*, J. Nordmann, K. Naumenko, H. Altenbach

*Chair of Engineering Mechanics, Institute of Mechanics, Faculty of Mechanical Engineering, Otto von Guericke University Magdeburg,
Universitätsplatz 2, 39106 Magdeburg, Germany*

Abstract

The present contribution is concerned with the computational homogenisation of mechanical properties of the heterogeneous photovoltaic module core layer. This is required since the mechanical approach with a novel layerwise theory used for efficient global structural analysis is based on homogeneous layers. This layerwise theory is build on the direct approach for plates, where all considerations are restricted to the deformable midsurface of the individual layer. Due to the geometric structure of the core layer, an anisotropic material behavior of the effective substitute material results. The resulting orientation dependence of mechanical properties is specified and visualized. Finally, based on aforementioned work, the results of anisotropic elasticity gained for the three dimensional CAUCHY continuum, are transferred to stiffness measures of surface elasticity.

Keywords: photovoltaic module, composite structure, homogenisation, anisotropy, direct approach, surface elasticity

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*Corresponding author

Email address: marcus.assmus@ovgu.de (M. Aßmus)

1. Introduction

1.1. Motivation

Considerations on Structural Mechanics of photovoltaic modules gain increasing importance as, besides aspects of performance yield, the aspect of mechanical durability come to the fore. Not least for the reason that power losses can be accompanied by mechanical damage. In the design process, engineers can specifically influence the mechanical stresses of the components involved, by varying geometrical and material parameters. However, adequate mechanical approaches are required. Nowadays computational structural analyses, such as with the Finite Element Method, have become established also in this field which is underlined by an impressive number of works, e.g. [1, 2, 3, 4]. Since computational analysis of fully detailed (three dimensional) structures with regard to photovoltaic modules, is inefficient in terms of computational effort, calculation time, and accuracy of results, it is reasonable to split the mechanical problem on different scales.

As depicted in Figure 1, a photovoltaic module consists of multiple brittle solar cells, loosely connected with electrical contacts and embedded in a soft encapsulation, representing the core layer. This core layer is enclosed by two skin layers, which consist, by restriction to symmetrical structures, of two identical layers of high stiffness. Glasses are used for the skin layers (front/back cover), silicon for the solar cells, and polymeric materials like ethylene-vinyl acetate as encapsulation. Typical geometrical measures and material parameters also used in the present work are given in Table 1. Material parameters presented therein are merely exemplaric but represent the elastic mismatch between core (index c) and skin layers (index s).

In [5], a novel approach to global and local structural analysis was introduced. For global structural analysis it is reasonable to use thin walled structural theories due to the slenderness of photovoltaic structures. Here, it is comprehensible to work with an abstract structure where all layers are homogeneous. One problem in this procedure is the heterogeneity of the core layer of a photovoltaic module. To avoid the expensive analysis of all the details including solar cells and connectors etc. on structural scale, a homogeneity postulate for the core layer was made in [6] which is based on considerations of overall structural behavior. Descriptions of these considerations can be found in [6] and [5]. There it is stated that the solar cells and further structural ingredients in the core layer do not contribute to the overall stiffness of the composite structure. It is assumed that the core layer of a photovoltaic module is represented by the encapsulation material solely. This postulate has since been applied successfully in numerous works, e.g. [6, 7, 8, 5]. A confirmation of this postulate can be found in [9], illustrated by an experimental comparison. However, the question

Figure 1: Anatomy of a photovoltaic module in exploded view with inhomogeneous structure of the core layer (representation not true to scale)

Table 1: Geometrical and isotropic material parameters of a typical photovoltaic composite structure as the basis for present investigations [5]

K	L_1^K [mm]	L_2^K [mm]	h^K [mm]	E^K [N/mm ²]	ν^K [–]
s	1620	810	3.2	$73.0 \cdot 10^3$	0.30
c			1.0	7.9	0.41

arises how big the influence of the up to now neglected solar cells on the mechanical behavior of the core layer is. This question is to be investigated in this contribution.

1.2. Organisation and Procedure

Photovoltaic modules made of crystalline solar cells traditionally have a heterogeneous core layer concerning the geometrical structure and mechanical properties while the mechanical model to solve boundary value problems in the context of a global structural analysis with eXtended LayerWise Theory (XLWT) [6] is based on a homogeneous core layer. These contrary assumptions of reality and model are depicted in Fig. 2 (top left) indicated by red arrows. The present work is driven by the idea of a proof for the homogeneity assumption from [6]. Otherwise, if this postulate loses validity, the goal is to generate mechanical properties for the core layer. In this case further assumptions must be specified. For this, the core layer of a photovoltaic module is subjected to a numerical homogenisation which results in a spatial average of mechanical properties. As is customary in homogenization theory, a representative portion is used for this purpose only [10, 11], where we follow the ergodic hypothesis [12]. The representative portion is specified by considerations of the periodic topology which can be identified at the core layer structure. At the unit cell defined, anisotropic [13] material behaviour is assumed generally. Through symmetry considerations it is possible to determine a material symmetry. This limits the number of linear independent material parameters and thus restricts the number of virtual material tests required to determine these material parameters. These tests are specified while it turns out, that it is entirely sufficient to carry out four different material tests. Since in the present treatise linear elasticity theory is used, required material parameters can be determined through simple relationships between homogenised stresses and strains. The generated substitute material is valid for the whole core layer with respect to classical (three dimensional CAUCHY) continua. Since the mechanical model to solve boundary value problems is a non-classical, bidimensional continuum, a transformation from the elasticity tensor of CAUCHY continuum to stiffness tensors of linear surface elasticity of a COSSERAT-like plate continuum is required. The formulation of XLWT is based on such a surface continuum (also directed continuum [14]) which is a pragmatic approach of the linear planar COSSERAT surface [15]. While COSSERAT surfaces have three additional rotational degrees of freedom compared to the three dimensional CAUCHY continuum, directed surfaces consider only the out-of-plane rotations, but also three translational degrees of freedom. The restriction to out-of-plane rotations is justifiable since the structural rigidity of a bidimensional surface against in-plane rotations is much higher than resistance against the out-of-plane rotations, at least at the present problem. Furthermore, we restrict ourselves to the planarity of the surface continuum in the reference placement. The transition is based on separate considerations of membrane, shear, and bending states of the unit cell. Resulting stiffness tensors are presented and discussed. The whole procedure is visualised in Figure 2, while the paper is organised as follows: Section 2 focusses on a identification of a representative portion from the periodic field of the photovoltaic module core layer. The symmetry properties of the particular unit cell are also discussed. Based on this, in the following section, the effect of this symmetries on the elastic tensor are derived and tests to determine the required material parameters are defined. In this section, the numerical solution strategy is also presented, where the focus is on the boundary conditions used. In Section 4 the results of the homogenisation are presented, verified and visualised. Therefore, three and two dimensional graphical representations of resulting YOUNG's, bulk, and shear modulus as far as of POISSON's ratio are given. Section 5 concludes with the transfer of the anisotropic elasticity tensor previously determined for three dimensional CAUCHY continua to the stiffness tensors for membrane, bending and shear state of the surface continua. The work concludes with a summary and an outlook on possible further work.

Figure 2: Procedure to generate a homogeneous substitute material for the surface continuum

1.3. Preliminaries and Notation

In the present work, all considerations are restricted to CARTESIAN coordinates with orthonormal bases $\mathbf{e}_i$ or $\mathbf{e}_\alpha$ and $\mathbf{n}$ respectively, in three dimensional EUCLIDEAN space $\mathbb{E}^3$. In what follows, we limit ourselves to geometrical and physical linearity.

Throughout the whole work, a direct notation is preferred while tensor and vector-matrix formulation are used simultaneously. Zero rank tensors (or scalars) are symbolised by italic letters (e.g. a), first rank tensors (or monads) by italic lowercase bold letters (e.g. $\mathbf{a} = a_i \mathbf{e}_i$ or $\mathbf{b} = b_j \mathbf{e}_j$), second rank tensors (or dyads) by italic uppercase bold letters (e.g. $\mathbf{A} = A_{lm} \mathbf{e}_l \otimes \mathbf{e}_m$ or $\mathbf{B} = B_{no} \mathbf{e}_n \otimes \mathbf{e}_o$), and fourth rank tensors (or tetrads) by italic uppercase bold calligraphic letters (e.g. $\mathcal{A} = A_{pqrs} \mathbf{e}_p \otimes \mathbf{e}_q \otimes \mathbf{e}_r \otimes \mathbf{e}_s$), where EINSTEIN sum convention is applied. Coefficient matrices of tensors are set in square brackets, e.g.

$$[H_{ij}] = \begin{bmatrix} H_{11} & H_{12} & H_{13} \\ H_{21} & H_{22} & H_{23} \\ H_{31} & H_{32} & H_{33} \end{bmatrix} \quad \text{for } \mathbf{H} = H_{ij} \mathbf{e}_i \otimes \mathbf{e}_j. \quad (1)$$

Letters in blackboard bold (e.g. $\mathbb{R}$ or Sym) are reserved for sets, groups, and spaces. Here, Lin is the space of linear mappings, Sym is the space of symmetric second rank tensors, Orth is the general orthogonal group, $\text{Orth}^+ \subset \text{Orth}$ is the set of proper orthogonal tensors, $\mathbb{G}$ is the group of generators, $\mathbb{T}$ is the symmetry group, $\mathbb{N}$ is the set of vector normals, $\mathbb{R}$ is the set of real numbers, $\mathbb{Z}^+$ is the set of positive integers, and $\mathbb{E}^3$ is the three dimensional EUCLIDEAN space.

Basic operations for tensors are the scalar product

$$\mathbf{a} \cdot \mathbf{b} = a_i b_j \mathbf{e}_i \cdot \mathbf{e}_j = a_i b_i = \alpha \quad \alpha \in \mathbb{R}, \quad (2)$$

121 the dyadic product

$$\mathbf{a} \otimes \mathbf{b} = a_i b_j \mathbf{e}_i \otimes \mathbf{e}_j = \mathbf{C}, \quad (3)$$

122 the composition of a second and a first rank tensor

$$\mathbf{A} \cdot \mathbf{a} = A_{lm} a_i \mathbf{e}_l \otimes \mathbf{e}_m \cdot \mathbf{e}_i = A_{li} a_i \mathbf{e}_l = \mathbf{d}, \quad (4)$$

123 the composition of two second rank tensors

$$\mathbf{A} \cdot \mathbf{B} = A_{lm} B_{no} \mathbf{e}_l \otimes \mathbf{e}_m \cdot \mathbf{e}_n \otimes \mathbf{e}_o = A_{lm} B_{mo} \mathbf{e}_l \otimes \mathbf{e}_o = \mathbf{D}, \quad (5)$$

124 the double scalar product between two second rank tensors

$$\mathbf{A} : \mathbf{B} = A_{lm} B_{no} \mathbf{e}_l \otimes \mathbf{e}_m : \mathbf{e}_n \otimes \mathbf{e}_o = A_{lm} B_{ml} = \beta \quad \beta \in \mathbb{R}, \quad (6)$$

125 and the double scalar product between a fourth and a second rank tensor

$$\mathcal{A} : \mathbf{B} = A_{pqrs} B_{no} \mathbf{e}_p \otimes \mathbf{e}_q \otimes \mathbf{e}_r \otimes \mathbf{e}_s : \mathbf{e}_n \otimes \mathbf{e}_o = A_{pqrs} B_{sr} \mathbf{e}_p \otimes \mathbf{e}_q = \mathbf{F}. \quad (7)$$

126 The transpose of the tensor $\mathbf{A}$ is defined as

$$\mathbf{A}^\top = A_{ml} \mathbf{e}_l \otimes \mathbf{e}_m \quad (8)$$

127 while the transpose of $\mathcal{A}$ as

$$\mathcal{A}^\top = A_{rspq} \mathbf{e}_p \otimes \mathbf{e}_q \otimes \mathbf{e}_r \otimes \mathbf{e}_s. \quad (9)$$

128 A tensor $\mathbf{B}$ is invertible if

$$\mathbf{B}^{-1} \cdot \mathbf{B} \cdot \mathbf{a} = \mathbf{a}, \quad (10)$$

129 so that

$$\mathbf{B}^{-1} \cdot \mathbf{B} = \mathbf{B} \cdot \mathbf{B}^{-1} = \mathbf{1} \quad \text{and} \quad (\mathbf{B}^{-1})^{-1} = \mathbf{B} \quad (11)$$

130 holds true. An orthogonal tensor $\mathbf{B}$ is defined as

$$\mathbf{B}^\top \cdot \mathbf{B} = \mathbf{1} \quad \mathbf{B}^\top = \mathbf{B}^{-1}, \quad (12)$$

131 where $\mathbf{1} = \mathbf{e}_i \otimes \mathbf{e}_i$ is the second rank unit tensor. The RAYLEIGH product ($\star$) maps all basis vectors of a tensor simultaneously
132 without changing its components. Applying a second on a fourth rank tensor results in

$$\mathbf{B} \star \mathcal{A} = A_{pqrs} (\mathbf{B} \cdot \mathbf{e}_p) \otimes (\mathbf{B} \cdot \mathbf{e}_q) \otimes (\mathbf{B} \cdot \mathbf{e}_r) \otimes (\mathbf{B} \cdot \mathbf{e}_s). \quad (13)$$

133 If $\mathbf{B}$ is orthogonal ($\mathbf{B} \in \text{Orth}$) then the RAYLEIGH product is a rotation (or reflection) of $\mathcal{A}$.

134
135 Using the tensor notation, latin indices (e.g. i, j, k, l) run through the values 1, 2, and 3, while greek indices (e.g. α, β)
136 run through the values 1 and 2. The vector valued HAMILTONIAN (Nabla operator) is defined as $\nabla = \mathbf{e}_i \partial / \partial x_i$ for the three
137 dimensional theory. $\nabla \square$ is the gradient of a tensor, where $\square$ is a tensor of any rank. Detailed insight into tensor calculus
138 is given, for example, in ALTENBACH [16], LAI, RUBIN & KREML [17], or BERTRAM [18].

139
140 In vector-matrix notation, vectors are denoted as upright lowercase sans serif bold letters (e.g. displacement vector
141 $\mathbf{u} = [u_1 \ u_2 \ u_3]^\top$) and matrices as upright uppercase sans serif bold letters (e.g. stiffness matrix $\mathbf{K}$ or matrix of shape
142 functions $\mathbf{N}$).

2. Photovoltaic Structures and Periodic Fields

Since we are interested in the effective material properties of the components between front and back cover (see definition of core layer in Figure 1), we consider the meso-structure of the photovoltaic module core layer, where the solar cells are embedded in an encapsulant. This structure is visualised in Figure 3 (top left). For the sake of simplicity, we neglect the metallic contact between and at the solar cells. This is legitimate, since the connection of the solar cells by the metallic connectors is relatively loose. It is assumed that they do not contribute to the stiffness of the core layer. Instead of considering the whole layer, it is possible to consider just a representative part. By doing so, one can recognize a plane periodicity of this mesoscopic structure depicted in Figure 3 (bottom left). The unit cell identified has the shape of a cuboid and thus shows the following geometrical properties:

- lengths $L_1 = L_2 \neq L_3$
- all interior angles are 90°

Here, all $L_i \forall i = \{1, 2, 3\}$ are the edge lengths of the unit cell in the corresponding directions. This unit cell also possesses five symmetry planes shown in Figure 3 (right). They can be defined by a set of five vector normals $\mathbb{N}$.

$$\mathbb{N} = \{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3, \mathbf{n}_4, \mathbf{n}_5\} \quad \text{with } \mathbf{n}_4 = \frac{\mathbf{e}_1 + \mathbf{e}_2}{\sqrt{2}} \quad \text{and } \mathbf{n}_5 = \frac{\mathbf{e}_2 - \mathbf{e}_1}{\sqrt{2}} \quad (14)$$

Hereout, four normals are coplanar and at angles of $\pi/4$ to each other. The fifth normal vector is perpendicular to this plane. This type of material symmetry is known as tetragonal symmetry. The number of independent material parameters exhibited by this symmetry group and the necessary material tests to determinate these parameters entirely will be clarified in the sequel. It is worth to mention, that the symmetry identified here is purely geometrically induced.

Figure 3: Core layer structure, corresponding periodic field with representative portion, and resulting unit cell with detected symmetry planes

3. Determination of Material Parameters in Three Dimensional Elasticity

3.1. Linear Elasticity

In three dimensional linear elasticity of classical continua, the stress tensor $\mathbf{T} = T_{ij} \mathbf{e}_i \otimes \mathbf{e}_j$ is given by a linear mapping of the strain tensor $\mathbf{E} = E_{ij} \mathbf{e}_i \otimes \mathbf{e}_j$.

$$\mathbf{T} = \mathbf{C} : \mathbf{E} \quad (15)$$

This is known as generalized Hooke's law, which is valid if displacements and rotations remain small. Therein, $\mathbf{E}$ is the infinitesimal strain tensor determined as follows.

$$\mathbf{E} = \frac{1}{2} [\mathbf{H} + \mathbf{H}^\top] \quad (16)$$

Here, $\mathbf{H} = \nabla \mathbf{u}$ is the displacement gradient while $\mathbf{u} = u_i \mathbf{e}_i$ is the displacement vector. Both, stress and strain tensor are symmetric, such that each tensor comprises six independent coefficients. The symmetry of $\mathbf{T}$ follows from the assumptions of vanishing torque stresses and body couples, known as BOLTZMANN's axiom [19], and $\mathbf{E}$ is symmetric since the strain tensor is the difference between two metric tensors (current and reference placement). The fourth rank elasticity tensor $\mathbf{C}$ used in (15) has 3^4 coefficients C_{ijkl} .

$$\mathbf{C} = C_{ijkl} \mathbf{e}_i \otimes \mathbf{e}_j \otimes \mathbf{e}_k \otimes \mathbf{e}_l \quad (17)$$

It possesses the following properties [20, 21].

$$\begin{aligned} \mathbf{A} : \mathbf{C} : \mathbf{B} &= \mathbf{B} : \mathbf{C}^\top : \mathbf{A} & C_{ijkl} &= C_{klij} & \text{major symmetry} \\ \mathbf{A} : \mathbf{C} &= \mathbf{A}^\top : \mathbf{C} & C_{ijkl} &= C_{jikl} & \text{left subsymmetry} \\ \mathbf{C} : \mathbf{A} &= \mathbf{C} : \mathbf{A}^\top & C_{ijkl} &= C_{ijlk} & \text{right subsymmetry} \end{aligned}$$

This is valid for all second rank tensors $\mathbf{A}, \mathbf{B} \in \mathbb{L}(\text{lin})$. A reduction to 36 linear-independent coefficients results from left and right subsymmetry. The coefficient matrix of the elasticity tensor can now be written as (6×6) -matrix. Based on the major symmetry of $\mathbf{C}$, there are only 21 independent coefficients.

$$\mathbf{C} = \begin{bmatrix} C_{1111} & C_{1122} & C_{1133} & C_{1123} & C_{1113} & C_{1112} \\ & C_{2222} & C_{2233} & C_{2223} & C_{2213} & C_{2212} \\ & & C_{3333} & C_{3323} & C_{3313} & C_{3312} \\ & & & C_{2323} & C_{2313} & C_{2312} \\ & & & & C_{1313} & C_{1312} \\ \text{sym} & & & & & C_{1212} \end{bmatrix} \quad (18)$$

Matrix (18) represents the coefficients of the general anisotropic case (triclinic) where VOIGT's original notation [22] is applied. Super indices (contracted notation) and normalisations are avoided to preserve the structure of representation. Corresponding stresses $\mathbf{T}$ and strains $\mathbf{E}$ are now written as vectors due to their symmetry properties where a simple, half circulating mnemonic rule is applied to the coefficient matrix of the symmetric second rank stress or strain tensor respectively.

$$\mathbf{t} = [T_{11} \quad T_{22} \quad T_{33} \quad T_{23} \quad T_{13} \quad T_{12}]^\top \quad \mathbf{e} = [E_{11} \quad E_{22} \quad E_{33} \quad 2E_{23} \quad 2E_{13} \quad 2E_{12}]^\top \quad (19)$$

The factor 2 in the shear components of the strain vector $\mathbf{e}$ is introduced to ensure the strain energy density W to be $2W = \mathbf{t}^\top \mathbf{e} = \mathbf{T} : \mathbf{E}$. Hooke's law can now also be written in the following vector-matrix form.

$$\mathbf{t} = \mathbf{C} \mathbf{e} \quad (20)$$

3.2. Anisotropy

Further simplifications concerning the elasticity matrix need material symmetries. Those are represented by symmetry groups which are determined by symmetry classes. As shown for example in [18], they are classified by 7 major symmetry groups valid in linear elasticity theory. These are triclinic, monoclinic, rhombic, tetragonal, trigonal, cubic, and hexagonal systems. They can be generated from a minimal set of versors $\mathbf{Q}^{G_q}$ where $q = \{1, \dots, \mathbb{Z}^+\}$ is the number of different elements in the associated symmetry group. These elements can be summarized in a resulting versor $\mathbf{Q} \in \mathbb{T}$ in order to derive a constitutive tensor of the corresponding symmetry group. This is done via the RAYLEIGH product $\star$. The operator $\star$ maps $\mathbf{Q}$ on every basis vector of $\mathcal{C}$.

$$\mathbf{Q} \star \mathcal{C} = C_{ijkl} (\mathbf{Q} \cdot \mathbf{e}_i) \otimes (\mathbf{Q} \cdot \mathbf{e}_j) \otimes (\mathbf{Q} \cdot \mathbf{e}_k) \otimes (\mathbf{Q} \cdot \mathbf{e}_l) \quad \forall \mathbf{Q} \in \text{Orth}^+ \subset \text{Orth} \quad (21)$$

Equation (21) also holds true for all $\mathbf{Q}^{G_q} \in \mathbb{T}$. Orth^+ is a subgroup of the group of all orthogonal transformations, representing proper orthogonal tensors with positive determinant, used to describe pure rotations (no reflections). Considering tetragonal symmetry identified in Section 2, two generators $\mathbf{R}^{(o)} \in \mathbb{G}$ are existing from which $\mathbf{Q}$ is formed. The coefficient matrices of $\mathbf{R}^{(o)} = R_{ij}^{(o)} \mathbf{e}_i \otimes \mathbf{e}_j \quad \forall o = \{1, 2\}$ are defined as follows [23].

$$[R_{ij}]^{(1)} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad [R_{ij}]^{(2)} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \quad (22)$$

Here, $\mathbf{R}^{(1)}$ causes a rotation of $\pi/2$ around $\mathbf{e}_3$ and $\mathbf{R}^{(2)}$ a rotation of π around $\mathbf{e}_1$. The concatenation of these generators in the form

$$\mathbf{R}^{(1)} \cdot \mathbf{R}^{(2)} = \mathbf{Q}^{G_1}, \quad \mathbf{Q}^{G_1} \cdot \mathbf{R}^{(1)} = \mathbf{Q}^{G_2}, \quad \mathbf{Q}^{G_1} \cdot \mathbf{R}^{(2)} = \mathbf{Q}^{G_3}, \quad \text{et cetera}, \quad (23)$$

results in the group of tetragonal systems $\mathbb{T}$. This group contains eight different elements $\mathbf{Q}^{G_q} \in \mathbb{T} \quad \forall q = \{1, \dots, 8\}$ obtained by the generators shown in (22). The composition of these eight elements results in $\mathbf{Q}$ used in equation (21).

$$\mathbf{Q}^{G_8} \cdot \mathbf{Q}^{G_7} \cdot \mathbf{Q}^{G_6} \cdot \mathbf{Q}^{G_5} \cdot \mathbf{Q}^{G_4} \cdot \mathbf{Q}^{G_3} \cdot \mathbf{Q}^{G_2} \cdot \mathbf{Q}^{G_1} = \mathbf{Q}^{\text{tetra}} \quad (24)$$

Applying this tensor to the constitutive tensor

$$\mathcal{C}^{\text{tetra}} \stackrel{!}{=} \mathbf{Q}^{\text{tetra}} \star \mathcal{C} \quad (25)$$

leads to required simplifications in the coefficients of the constitutive tensor corresponding to the tetragonal system. The following coefficients become identically zero when we are restricted to the representation given in Equation (18).

$$C_{1123} = C_{1113} = C_{1112} = C_{2223} = C_{2213} = C_{2212} = C_{3323} = C_{3313} = C_{3312} = C_{2313} = C_{2312} = C_{1312} = 0 \quad (26)$$

Furthermore, following identities are existing now.

$$C_{1111} = C_{2222} \quad C_{1133} = C_{2233} \quad C_{2323} = C_{1313} \quad (27)$$

Thus, tetragonal symmetry is specified by 6 independent coefficients. The elasticity matrix reduces to the following expression.

$$\mathbf{C}^{\text{tetra}} = \begin{bmatrix} C_{1111} & C_{1122} & C_{1133} & 0 & 0 & 0 \\ & C_{1111} & C_{1133} & 0 & 0 & 0 \\ & & C_{3333} & 0 & 0 & 0 \\ & & & C_{2323} & 0 & 0 \\ & & & & C_{2323} & 0 \\ \text{sym} & & & & & C_{1212} \end{bmatrix}. \quad (28)$$

In the sequel, we drop the superscript *tetra* for reasons of clarity and keep in mind that from now on the representation is limited to the tetragonal system.

3.3. Test Definitions

As one can clearly see, it is possible to link between the material symmetry and the mesostructure of the photovoltaic module core layer components. In present case, the unit cell is subject to tetragonal symmetry, resulting from the symmetry planes depicted in Figure 3. As shown in Equation (28), this kind of symmetry has six linear independent material parameters. Material tests at the unit cell are required to determine these parameters. Since the implementation of tests have to be performed to determine the required material parameters is nearly impossible in the context of experimental investigations, virtual material tests with the aid of a computational method are used. As we will show in the sequel, four different tests are sufficient to determine all parameters. In a first step these tests are to be defined.

We consider a unit cell setup with linear displacement boundary conditions what corresponds to the homogeneous strain boundary condition [24]. Further typical and possible representatives are periodic boundary conditions. Compared to these periodic boundary conditions, the use of homogeneous strain boundary conditions results in a too rigid structure. Different studies show that in case of low volume fractions of the inclusions, only slightly different material parameters result by comparing results from applying different boundary conditions, e.g. [25]. Since the volume fraction of the solar cell at the unit cell is quite low, it is also legitimate to choose homogeneous strain boundary conditions. This boundary condition fulfills the HILL-MANDEL macro homogeneity condition [26, 27], where a proof is given in [24]. The application of homogeneous strain boundary conditions is described by

$$\mathbf{u} = \bar{\mathbf{H}} \cdot \mathbf{x} \quad \forall \mathbf{x} \in \partial\mathfrak{B} \quad (29)$$

where $\bar{\mathbf{H}} = \bar{H}_{ij} \mathbf{e}_i \otimes \mathbf{e}_j$ is the averaged displacement gradient which is prescribed on all material points $\mathbf{x} = X_i \mathbf{e}_i$ of the boundaries in the reference placement.

To differentiate between the tension and shear tests we use superscript indices. Here, ‘uut’ stands for **u**ni**ax**ial **t**ension tests, and ‘ssh’ for **s**imple **s**hear **t**ests. We define deformations by prescribed strains as follows, where we divide between uniaxial tension tests

$$[\bar{H}_{ij}]_1^{\text{uut}} = \begin{bmatrix} E_{11} & 0 & 0 \\ 0 & - & 0 \\ 0 & 0 & - \end{bmatrix} \quad [\bar{H}_{ij}]_2^{\text{uut}} = \begin{bmatrix} - & 0 & 0 \\ 0 & E_{22} & 0 \\ 0 & 0 & - \end{bmatrix} \quad [\bar{H}_{ij}]_3^{\text{uut}} = \begin{bmatrix} - & 0 & 0 \\ 0 & - & 0 \\ 0 & 0 & E_{33} \end{bmatrix}, \quad (30)$$

and simple shear tests

$$[\bar{H}_{ij}]_1^{\text{ssh}} = \begin{bmatrix} 0 & E_{12} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad [\bar{H}_{ij}]_2^{\text{ssh}} = \begin{bmatrix} 0 & 0 & E_{13} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad [\bar{H}_{ij}]_3^{\text{ssh}} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & E_{23} \\ 0 & 0 & 0 \end{bmatrix}. \quad (31)$$

As can clearly be seen, due to the symmetries of the unit cell, the in-plane tension tests ($\bar{\mathbf{H}}_1^{\text{uut}}, \bar{\mathbf{H}}_2^{\text{uut}}$) and in-plane shear tests ($\bar{\mathbf{H}}_1^{\text{ssh}}, \bar{\mathbf{H}}_2^{\text{ssh}}$) each result in identical parameters. Therefore, it is entirely sufficient to neglect $\bar{\mathbf{H}}_2^{\text{uut}}$ and $\bar{\mathbf{H}}_2^{\text{ssh}}$

3.4. Homogenisation and Parameter Determination

Since the homogeneous strains are determined entirely by prescribing the displacement gradient on the entire boundary of the unit cell, resulting stresses are required. The stresses in $\mathfrak{B}$ induced by the different tests can be decomposed into an homogeneous part $\bar{\mathbf{T}}$ and an fluctuating part $\tilde{\mathbf{T}}$.

$$\mathbf{T}(\mathbf{x}) = \bar{\mathbf{T}} + \tilde{\mathbf{T}}(\mathbf{x}) \quad (32)$$

Considering sufficiently large integration areas, the fluctuations disappear.

$$\int_{\mathfrak{B}^{\text{UC}}} \tilde{\mathbf{T}}(\mathbf{x}) \, dV = \mathbf{0} \quad (33)$$

The averaged stresses can now be described by the following equations.

$$\bar{\mathbf{T}} = \frac{1}{V} \int_{\mathfrak{B}^{\text{UC}}} \mathbf{T} \, dV \quad \forall \mathbf{x} \in \mathfrak{B} \quad (34)$$

236 In order to determine coefficients in $\mathbf{C}$, one has to use the inverse Hooke's law.

$$\mathbf{e} = \mathbf{S} \mathbf{t} \quad (35)$$

237 Here, $\mathbf{S} = \mathbf{C}^{-1}$ is the compliance matrix where the components can be determined in the following way.

$$\mathbf{S} = \frac{1}{\det \mathbf{C}} \text{adj } \mathbf{C} \quad (36)$$

238 Herein, 'det' denotes the determinant and 'adj' the adjugate of a nonsingular matrix. Detailed procedures for these
239 computations can be found in, e.g. [28]. Applying this procedure to the elasticity matrix given in Equation (28), the
240 compliance matrix can be written as follows.

$$\mathbf{S} = \begin{bmatrix} S_{1111} & S_{1122} & S_{1133} & 0 & 0 & 0 \\ & S_{1111} & S_{1133} & 0 & 0 & 0 \\ & & S_{3333} & 0 & 0 & 0 \\ & & & S_{2323} & 0 & 0 \\ & & & & S_{2323} & 0 \\ \text{sym} & & & & & S_{1212} \end{bmatrix} \quad (37)$$

241 The coefficients of $\mathbf{S}$ are correlated to those of $\mathbf{C}$.

$$S_{1111} = \frac{C_{1111}C_{3333} - C_{1133}^2}{[-2C_{1133}^2 + (C_{1111} + C_{1122})C_{3333}][C_{1111} - C_{1122}]} \quad (38)$$

$$S_{1122} = \frac{-C_{1122}C_{3333} + C_{1133}^2}{[-2C_{1133}^2 + (C_{1111} + C_{1122})C_{3333}][C_{1111} - C_{1122}]} \quad (39)$$

$$S_{1133} = -\frac{C_{1133}}{-2C_{1133}^2 + (C_{1111} + C_{1122})C_{3333}} \quad (40)$$

$$S_{3333} = \frac{C_{1111} + C_{1122}}{-2C_{1133}^2 + (C_{1111} + C_{1122})C_{3333}} \quad (41)$$

$$S_{2323} = \frac{1}{C_{2323}} \quad (42)$$

$$S_{1212} = \frac{1}{C_{1212}} \quad (43)$$

242 By means of the inverse Hooke's law, the compliance coefficients can be uniquely determined according to the different
243 test definitions. A tension test just yields a uniaxial stress state with normal strains and no shear strains. In contrast
244 thereto, a shear test yields a single shear stress and the corresponding shear strain, whereby no normal stresses and
245 normal strains occur. Hence it is possible to formulate the occupancy of the vectors $\mathbf{e}$ and $\mathbf{t}$ for all four tests in the
246 following way.

$$\text{induced by } \overline{\mathbf{H}}_3^{\text{utt}} : \quad \mathbf{t}_3^{\text{utt}} = [0 \quad 0 \quad \overline{T}_{33} \quad 0 \quad 0 \quad 0]^T \quad \mathbf{e}_3^{\text{utt}} = [E_{11} \quad E_{22} \quad E_{33} \quad 0 \quad 0 \quad 0]^T \quad (44)$$

$$\text{induced by } \overline{\mathbf{H}}_1^{\text{utt}} : \quad \mathbf{t}_1^{\text{utt}} = [\overline{T}_{11} \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]^T \quad \mathbf{e}_1^{\text{utt}} = [E_{11} \quad E_{22} \quad E_{33} \quad 0 \quad 0 \quad 0]^T \quad (45)$$

$$\text{induced by } \overline{\mathbf{H}}_3^{\text{ssh}} : \quad \mathbf{t}_3^{\text{ssh}} = [0 \quad 0 \quad 0 \quad \overline{T}_{23} \quad 0 \quad 0]^T \quad \mathbf{e}_3^{\text{ssh}} = [0 \quad 0 \quad 0 \quad 2E_{23} \quad 0 \quad 0]^T \quad (46)$$

$$\text{induced by } \overline{\mathbf{H}}_1^{\text{ssh}} : \quad \mathbf{t}_1^{\text{ssh}} = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad \overline{T}_{12}]^T \quad \mathbf{e}_1^{\text{ssh}} = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 2E_{12}]^T \quad (47)$$

Stress measures overlined indicate averaged values according to Equation (34). With these vectors and the inverse Hooke's law (35), equations for the elements of the compliance matrix S_{ijkl} respectively the elasticity matrix C_{ijkl} or the material parameters can be formulated. We will only formulate these equations for the S_{ijkl} . A tension test leads to three equations and every shear test to a single one. Thus, overall eight equations exist. With the knowledge about the symmetry group of the compliance and elasticity matrix out of Section 3.2 just six of the eight equations are needed to calculate the effective compliances S_{ijkl} .

$$\text{resulting from } \overline{\mathbf{H}}_1^{\text{utt}} : \quad S_{1111} = \frac{E_{11}}{\overline{T}_{11}} \quad (48)$$

$$S_{1122} = \frac{E_{22}}{\overline{T}_{11}} \quad (49)$$

$$S_{1133} = \frac{E_{33}}{\overline{T}_{11}} \quad (50)$$

$$\text{resulting from } \overline{\mathbf{H}}_3^{\text{utt}} : \quad S_{3333} = \frac{E_{33}}{\overline{T}_{33}} \quad (51)$$

$$\text{resulting from } \overline{\mathbf{H}}_1^{\text{ssh}} : \quad S_{2323} = \frac{2E_{23}}{\overline{T}_{23}} \quad (52)$$

$$\text{resulting from } \overline{\mathbf{H}}_3^{\text{ssh}} : \quad S_{1212} = \frac{2E_{12}}{\overline{T}_{12}} \quad (53)$$

Considering the relations between S_{ijkl} and C_{ijkl} given in Equations (38)–(43) it is possible to uniquely determine the elastic parameters for tetragonal symmetry.

$$C_{1111} = \frac{S_{1111}S_{3333} - S_{1133}^2}{\left[S_{3333}(S_{1111} + S_{1122}) - 2S_{1133}^2\right]\left[S_{1111} - S_{1122}\right]} \quad (54)$$

$$C_{1122} = \frac{-S_{1122}S_{3333} + S_{1133}^2}{\left[S_{3333}(S_{1111} + S_{1122}) - 2S_{1133}^2\right]\left[S_{1111} - S_{1122}\right]} \quad (55)$$

$$C_{1133} = -\frac{S_{1133}}{S_{3333}(S_{1111} + S_{1122}) - 2S_{1133}^2} \quad (56)$$

$$C_{3333} = \frac{S_{1111} + S_{1122}}{S_{3333}(S_{1111} + S_{1122}) - 2S_{1133}^2} \quad (57)$$

$$C_{2323} = \frac{1}{S_{2323}} \quad (58)$$

$$C_{1212} = \frac{1}{S_{1212}} \quad (59)$$

3.5. Computational Solution

3.5.1. Environment

The computational simulations have been carried out with an eight core Intel i7-5960X CPU and 64 GB RAM on a 64 bit Windows 10 operating system using the finite element program system ABAQUS version 6.14. For numerical parameters like tolerances and precisions, ABAQUS default values have been used.

3.5.2. Geometry

The choice of geometrical dimensions is based on a photovoltaic module with 72 quadratic solar cells which have a planar edge length of 125 mm and a thickness of 0.2 mm which is a typical solar cell size. The planar distance between the solar cells is 4 mm. The solar cells are arranged coplanar while they are positioned centrally and plane parallel to the core layer interfaces. These solar cells are completely and uniformly surrounded by the encapsulation material. This encapsulation extends over the entire thickness of the core layer which should be 1 mm in the present case. Considering

Figure 4: Geometry and dimensions of the unit cell and including components (representation not to scale)

the representative portion shown in Section 2, the half of the planar distance between the solar cells have to be chosen for the encapsulation surrounding the solar cell in-plane. Like mentioned, metallic connectors are neglected entirely. Geometrical details of the unit cell examined are shown in Figure 4 while precise data can be found in Table 2. It turns out, that the solar cell has only a very small volume fraction of less than 20% of the total volume of the unit cell which is $16,641 \text{ mm}^3$.

3.5.3. Materials

The unit cell consists of a solar cell and the encapsulation only. Materials considered are briefly presented here. The solar cell, which presents the inclusion, is made of polycrystalline silicon (PS). Polycrystalline silicon is assumed to be isotropic while it is a brittle and stiff material. Theoretical limits of YOUNG's modulus for multicrystalline silicon can be specified with intermediate values of $130 \cdot 10^3$ and $188 \cdot 10^3 \text{ N/mm}^2$ [29]. Applying an analytical homogenisation, namely the VOIGT-REUSS-HILL [30, 31, 32] average, by using values of monocrystalline silicon (cubic symmetry) given in [33], yields macroscopically isotropic, effective medium values presented in Table 3. According to [34] this approach corresponds to rough approximations of effective characteristics of polycrystalline aggregates but however, find a broad application when determining effective properties of polycrystalline silicon and which results in sufficient accuracy for present investigations. The matrix material is made of Ethylene vinyl acetate (EVA) which is an elastomeric polymer. This rubber-like material is very soft and flexible. It's mechanical behaviour has a pronounced temperature dependency [35]. In present study we focus on mechanical behaviour at room temperature only. Mechanical parameters for this temperature range can be found in [36], which are given in Table 3 and used here. Considering the strongly differing

Table 2: External dimensions of the two components used at the unit cell

material	matrix (EVA)	inclusion (PS)
L_1 [mm]	129	125
L_2 [mm]	129	125
L_3 [mm]	1	0.2
$\mathcal{V}$ [-]	0.812	0.188

Table 3: Elastic material parameters of the two main core layer components in a photovoltaic module

material	EVA [36]	PS (based on [33] & [30, 31, 32])
Y [N/mm ²]	7.9	$163.1 \cdot 10^3$
ν [-]	0.41	0.22
G [N/mm ²]	2.8	$66.8 \cdot 10^3$
K [N/mm ²]	14.6	$97.1 \cdot 10^3$

elastic properties given there, following relations can be identified.

$$\begin{aligned}
 YR &= \frac{Y^{\text{EVA}}}{Y^{\text{PS}}} = 4.84 \cdot 10^{-5} & GR &= \frac{G^{\text{EVA}}}{G^{\text{PS}}} = 4.19 \cdot 10^{-5} \\
 PR &= \frac{\nu^{\text{EVA}}}{\nu^{\text{PS}}} = 1.86 & KR &= \frac{K^{\text{EVA}}}{K^{\text{PS}}} = 1.50 \cdot 10^{-4}
 \end{aligned}$$

3.5.4. Boundary Conditions

To determine the six independent coefficients in the stiffness matrix, as shown in Section 3.4, four different and independent tests defined in Section 3.3 are required. Those tests are visualised in Figure 5 where a unit cube is used to clarify the deformations induced for reasons of better visibility. For reasons of clarity and comprehensibility the six surfaces $\Gamma = \partial\mathcal{B}$ of the unit cell are named. The four surfaces orthogonally to the $\mathbf{e}_1$ - $\mathbf{e}_2$ -plane are named according to the cardinal points and the two surfaces parallel to this plane are named by bottom and front boundary. Thus, this leads to the following notations for the surfaces in the different directions $\mathbf{e}_i$. Γ_N for positive $\mathbf{e}_2$, Γ_W for negative $\mathbf{e}_1$, Γ_S for negative $\mathbf{e}_2$, Γ_O for positive $\mathbf{e}_1$, Γ_b for negative $\mathbf{e}_3$ and Γ_f for positive $\mathbf{e}_3$. With these definitions it is easily and precisely possible to state which boundary conditions are applied on which surface test-dependent. Since our unit cell is cut out of a photovoltaic module core layer, every outer surface of the unit cell have to remain plane at the deformation process. The resulting deformations are shown in Fig. 5, too. To obtain these deformations in the tensile tests, constraints must be formulated to guarantee that the surfaces keep plane. This is implemented by, in the FE-model on every free surface a master node (superscript index M) is defined and the remaining nodes of the surface are the slave nodes (superscript index j). Then the equation

$$u_i^{\Gamma_p, M} - u_i^{\Gamma_p, j} = 0 \quad \forall p = \{N, S, O, W, f, b\} \quad (60)$$

ensures, that every slave node has the same displacement as the master node. On the remaining two surfaces the displacements are specified. These two surfaces are placed on opposite sides. On one side the displacement in the tensile test direction is nonzero and on the opposite side it is zero. To prohibit rigid body movements, one point of the surface with no displacement is choosen (superscript index P) and the remaining degrees of freedom are disabled. In the shear tests, the procedure is slightly different. The correct deformations are induced by displacement boundary conditions only. They are defined on every surface. The boundary conditions applied are specified in Tables 4, 5, 6 and 7.

3.5.5. Discretisation and Convergence

The doamin of the unit cell $\mathcal{B}$ have to be spatially discretized in order to solve the boundary value problems defined in preceding section computationally. Due to the materials used, problems with volume locking are expected. This is underlined by the bulk to shear modulus ratio deviating significantly from each other [37].

$$\begin{aligned}
 \frac{K^{\text{EVA}}}{G^{\text{EVA}}} &= 5.22 & \frac{K^{\text{PS}}}{G^{\text{PS}}} &= 1.45
 \end{aligned}$$

To counter volume locking, two different kinds of elements were considered. One is called C3D20R. It is a 20-node quadratic brick element with reduced integration. The other one is called C3D8I. It is a incompatible mode 8-node linear brick element. Both elements are tested since they compensate locking effects. Furthermore, five different discretisation

Table 4: Boundary conditions for the in-plane tension test $[\bar{H}_{ij}]_1^{\text{uat}}$

boundary	boundary conditions	constraints
Γ_S	—	$u_2^{\Gamma_S, M} = u_2^{\Gamma_S, j}$
Γ_N	—	$u_2^{\Gamma_N, M} = u_2^{\Gamma_N, j}$
Γ_W	$u_1 = u_{2,3}^P = 0$	—
Γ_O	$u_1 = 0.05 \text{ mm}$	—
Γ_f	—	$u_3^{\Gamma_f, M} = u_3^{\Gamma_f, j}$
Γ_b	—	$u_3^{\Gamma_b, M} = u_3^{\Gamma_b, j}$

Table 5: Boundary conditions for the out-of-plane tension test $[\bar{H}_{ij}]_3^{\text{uat}}$

boundary	boundary conditions	constraints
Γ_S	—	$u_2^{\Gamma_S, M} = u_2^{\Gamma_S, j}$
Γ_N	—	$u_2^{\Gamma_N, M} = u_2^{\Gamma_N, j}$
Γ_W	—	$u_1^{\Gamma_W, M} = u_1^{\Gamma_W, j}$
Γ_O	—	$u_1^{\Gamma_O, M} = u_1^{\Gamma_O, j}$
Γ_f	$u_3 = 0.05 \text{ mm}$	—
Γ_b	$u_3 = u_{1,2}^P = 0$	—

Table 6: Boundary conditions for the in-plane shear test $[\bar{H}_{ij}]_1^{\text{ssh}}$

boundary	boundary conditions	constraints
Γ_S	$u_2(X_1) = u_2/L_1(X_1 + L_1/2)$ with $u_2 = 0.05 \text{ mm}$, $L_1 = 129 \text{ mm}$	—
Γ_N	$u_2(X_1) = u_2/L_1(X_1 + L_1/2)$ with $u_2 = 0.05 \text{ mm}$, $L_1 = 129 \text{ mm}$	—
Γ_W	$u_1 = u_2 = 0$	—
Γ_O	$u_1 = 0$, $u_2 = 0.05 \text{ mm}$	—
Γ_f	$u_3 = 0$	—
Γ_b	$u_3 = 0$	—

Table 7: Boundary conditions for the out-of-plane shear test $[\bar{H}_{ij}]_3^{\text{ssh}}$

boundary	boundary conditions	constraints
Γ_S	$u_2 = u_3 = 0$	—
Γ_N	$u_2 = 0$, $u_3 = 0.05 \text{ mm}$	—
Γ_W	$u_1 = 0$	—
Γ_O	$u_1 = 0$	—
Γ_f	$u_3(X_2) = u_3/L_2(X_2 + L_2/2)$ with $u_3 = 0.05 \text{ mm}$, $L_2 = 129 \text{ mm}$	—
Γ_b	$u_3(X_2) = u_3/L_2(X_2 + L_2/2)$ with $u_3 = 0.05 \text{ mm}$, $L_2 = 129 \text{ mm}$	—

variations are investigated to consider the convergence behaviour of the resulting stresses and to determine the number of finite elements required. For this purpose, all four tests were performed on five discretized domains Ω respectively. As it turns out, $2.1 \cdot 10^6$ elements are needed while calculation times are in the range of approximately 11 to 13 hours dependent on the type of test. Since the quadratic element with reduced integration gives more accurate results, all further calculations refer to this.

Figure 5: Visualisation of the four different deformation modes induced for virtual material tests on exemplary cube

3.6. Homogenisation in discretised domain

The following equation is describing the coefficient-wise determination of volume averaged stresses of the finite element solution equivalent with Equation (34).

$$\bar{T}_{ij} = \frac{1}{\sum_{GP=1}^{NG} V^{GP}} \sum_{GP=1}^{NG} T_{ij}^{GP} V^{GP} \quad \forall GP \in \Omega \quad (61)$$

Stress values of the computational solution are examined at the integration points GP . These values have to be weighted with the corresponding volume of the integration point. Only after that, averaging can be performed for the whole unit cell Ω . The averaged coefficients of the stress tensor are used for the description of the stress vectors in Equations (44), (45), (46), and (44).

4. Homogeneous Material Characteristics

4.1. Coefficients

Considering the inverse Hooke's law in form of Equation (35) under applying stress and strain measures given in Equations (44), (45), (46), and (44) resulting from the four different material tests defined in Section 3.3 separately, it is possible to determine the compliance coefficients according to Equations (48), (49), (50), (52), and (53). Considering the relations between elasticity and compliance coefficients given in Equations (54), (55), (56), (58), and (59), the following six coefficients are determined.

$$\begin{aligned} C_{1111} &= 495.29 \text{ N/mm}^2 & C_{1133} &= 13.20 \text{ N/mm}^2 \\ C_{3333} &= 22.68 \text{ N/mm}^2 & C_{2323} &= 3.46 \text{ N/mm}^2 \\ C_{1122} &= 18.20 \text{ N/mm}^2 & C_{1212} &= 40,000.00 \text{ N/mm}^2 \end{aligned}$$

4.2. Verification

For the verification of the coefficients obtained it is necessary to the admissible range of the material parameters of the corresponding symmetry. To determine the limits of this range we start with the strain energy density function in vector matrix formulation [17].

$$W = \frac{1}{2} \mathbf{t} \mathbf{e} = \frac{1}{2} \mathbf{e}^T \mathbf{C} \mathbf{e} > 0 \quad \forall \mathbf{e} \neq \mathbf{0} \quad (62)$$

$$W = \frac{1}{2} \begin{bmatrix} E_{11} \\ E_{22} \\ E_{33} \\ 2E_{23} \\ 2E_{13} \\ 2E_{12} \end{bmatrix}^T \begin{bmatrix} C_{1111} & C_{1122} & C_{1133} & 0 & 0 & 0 \\ & C_{1111} & C_{1133} & 0 & 0 & 0 \\ & & C_{3333} & 0 & 0 & 0 \\ & & & C_{2323} & 0 & 0 \\ & & & & C_{2323} & 0 \\ & & & & & C_{1212} \end{bmatrix} \begin{bmatrix} E_{11} \\ E_{22} \\ E_{33} \\ 2E_{23} \\ 2E_{13} \\ 2E_{12} \end{bmatrix} \quad (63)$$

The strain energy density function W is positive definite. It becomes zero if and only if all strain components are zero. Setting strain coefficients to a nonzero value successively, yields that this must also apply to the elasticity matrix. Therefore, the following restrictions are required:

- all diagonal elements of $\mathbf{C}$ must be positive: $C_{1111} \wedge C_{3333} \wedge C_{2323} \wedge C_{1212} > 0$
- the determinant of the elasticity matrix must be positive: $\det(\mathbf{C}) > 0$
- there is an inverse of $\mathbf{S} = \mathbf{C}^{-1}$, which is also symmetric and positive definite:
 $\exists \mathbf{S} : \mathbf{S} = \mathbf{S}^T, S_{1111} \wedge S_{3333} \wedge S_{2323} \wedge S_{1212} > 0$

The properties of the compliance matrix $\mathbf{S}$ are the same as the properties of the elasticity matrix. Up next we set two ($W(E_{11}, E_{22})$, $W(E_{11}, E_{33})$, $W(E_{22}, E_{33})$) respectively three ($W(E_{11}, E_{22}, E_{33})$) strain values to nonzero values to investigate the properties of the upper submatrix. These four conditions lead to the following submatrices

$$\begin{bmatrix} C_{1111} & C_{1122} \\ \text{sym} & C_{1111} \end{bmatrix}, \quad \begin{bmatrix} C_{1111} & C_{1133} \\ \text{sym} & C_{3333} \end{bmatrix}, \quad \begin{bmatrix} C_{1111} & C_{1133} \\ \text{sym} & C_{3333} \end{bmatrix}, \quad \begin{bmatrix} C_{1111} & C_{1122} & C_{1133} \\ & C_{1111} & C_{1133} \\ \text{sym} & & C_{3333} \end{bmatrix}$$

which are all positive definite and nonsingular. The mathematic proof of these properties is given, when the determinant is positive and nonzero. Four equations can be extracted out of these examinations, defining the limiting values of the material parameters.

$$C_{1111}^2 - C_{1122}^2 > 0 \quad (64)$$

$$C_{1111}C_{3333} - C_{1133}^2 > 0 \quad (65)$$

$$C_{1111}(C_{1111}C_{3333} - C_{1133}^2) - C_{1122}(C_{1122}C_{3333} - C_{1133}^2) + C_{1133}(C_{1122}C_{1133} - C_{1111}C_{1133}) > 0 \quad (66)$$

$$C_{1111} \wedge C_{3333} \wedge C_{1122} \wedge C_{1133} \wedge C_{2323} \wedge C_{1212} > 0 \quad (67)$$

These equations must be satisfied, otherwise the calculated elasticity coefficients are not stable and it is not possible to do further calculations with them. Comparing the restrictions in Equations (64) to (67) with resulting values in Section 4.1, gives the proof of the stability conditions.

$$\begin{aligned}
\text{condition (64)} : C_{1111}^2 C_{1122}^2 &= 244980.94 \text{ N}^2/\text{mm}^4 && \Rightarrow \text{fulfilled} \\
\text{condition (65)} : C_{1111} C_{3333} - C_{1133}^2 &= 11058.94 \text{ N}^2/\text{mm}^4 && \Rightarrow \text{fulfilled} \\
\text{condition (66)} : C_{1111}(C_{1111} C_{3333} - C_{1133}^2) - C_{1122}(C_{1122} C_{3333} - C_{1133}^2) \\
&\quad + C_{1133}(C_{1122} C_{1133} - C_{1111} C_{1133}) = 5389911.49 \text{ N}^3/\text{mm}^6 && \Rightarrow \text{fulfilled} \\
\text{condition (67)} : C_{1111} \wedge C_{3333} \wedge C_{1122} \wedge C_{1133} \wedge C_{2323} \wedge C_{1212} &> 0 && \Rightarrow \text{fulfilled}
\end{aligned}$$

4.3. Result visualisation

4.3.1. Basic idea

The graphical representation of the anisotropic material parameters is based on the decomposition of the elasticity tensor into orthogonal subspaces. Here, we present the final set of equations in a vector-matrix notation. The interested reader is referred to BÖHLKE & BRÜGGEMANN [38] or RYCHLEWSKI [39] where the following equations are formulated in tensor notation. The vector-matrix notation used here is preferred for calculations with computer algebra systems. For this purpose we choose the vector of spatial direction $\mathbf{d}(r, \phi, \theta)$ and three vector normals $\mathbf{n}_i \forall i = \{1, 2, 3\}$.

$$\mathbf{d} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} = \begin{bmatrix} \sin(\phi) \cos(\theta) \\ \sin(\phi) \sin(\theta) \\ \cos(\phi) \end{bmatrix} \quad \mathbf{n}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad \mathbf{n}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad \mathbf{n}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

These vectors are visualised in Figure 6. The vector normals have following properties.

$$\mathbf{n}_i \mathbf{n}_j = 0 \quad \forall i \neq j \quad |\mathbf{n}_i| = \sqrt{n_{i1} + n_{i2} + n_{i3}} = 1 \quad \forall i, j = \{1, 2, 3\}$$

To extract scalar parameters of the spatial system, the directions $\mathbf{d}_V$ and $\mathbf{m}_V$ have to be applied to stresses and strains numerous times. The index ‘V’ stands for vector. The vector $\mathbf{d}_V$ needed here is built based on the the matrix $\mathbf{D}$ which is result of the application of $\mathbf{d}$ to itself.

$$\mathbf{D} = \mathbf{d} \mathbf{d}^T = \begin{bmatrix} d_1 d_1 & d_1 d_2 & d_1 d_3 \\ & d_2 d_2 & d_2 d_3 \\ \text{sym} & & d_3 d_3 \end{bmatrix} \quad \Leftrightarrow \quad \mathbf{d}_V = \begin{bmatrix} d_1 d_1 \\ d_2 d_2 \\ d_3 d_3 \\ d_2 d_3 \\ d_1 d_3 \\ d_1 d_2 \end{bmatrix}$$

The same procedure also applies to the vector $\mathbf{m}_V$, while the mathematically equivalent matrix $\mathbf{M}$ is based on a combination of $\mathbf{d}$ and $\mathbf{n}$.

$$\mathbf{M} = \frac{\sqrt{2}}{2} (\mathbf{d} \mathbf{n}_i^T + \mathbf{n}_i \mathbf{d}^T) = \frac{\sqrt{2}}{2} \begin{bmatrix} d_1 n_{i1} + n_{i1} d_1 & d_1 n_{i2} + n_{i1} d_2 & d_1 n_{i3} + n_{i1} d_3 \\ & d_2 n_{i2} + n_{i2} d_2 & d_2 n_{i3} + n_{i2} d_3 \\ \text{sym} & & d_3 n_{i3} + n_{i3} d_3 \end{bmatrix} \quad \Leftrightarrow \quad \mathbf{m}_V = \frac{\sqrt{2}}{2} \begin{bmatrix} 2n_{i1} d_1 \\ 2n_{i2} d_2 \\ 2n_{i3} d_3 \\ d_3 n_{i2} + n_{i3} d_2 \\ d_3 n_{i1} + n_{i3} d_1 \\ d_1 n_{i2} + n_{i1} d_2 \end{bmatrix}$$

Furthermore, the sixdimensional vector $\mathbf{1}_V$ is needed which represents the identity matrix $\mathbf{1}$ in the Voigt notation.

$$\mathbf{1} = \begin{bmatrix} 1 & 0 & 0 \\ & 1 & 0 \\ \text{sym} & & 1 \end{bmatrix} \quad \Leftrightarrow \quad \mathbf{1}_V = [1 \quad 1 \quad 1 \quad 0 \quad 0 \quad 0]^T$$

Figure 6: Spherical coordinates and normal vectors in the Euclidean space $\mathbb{E}^3$ and resulting sphere (globular with isotropy)

A representative vector for all corresponding normals is introduced.

$$\mathbf{N}_i = \mathbf{n}_i \mathbf{n}_i^\top = \begin{bmatrix} n_{i1}n_{i1} & n_{i1}n_{i2} & n_{i1}n_{i3} \\ \text{sym} & n_{i2}n_{i2} & n_{i2}n_{i3} \\ & & n_{i3}n_{i3} \end{bmatrix} \quad \Leftrightarrow \quad \mathbf{n}_V = \begin{bmatrix} n_{i1}n_{i1} \\ n_{i2}n_{i2} \\ n_{i3}n_{i3} \\ n_{i3}n_{i2} \\ n_{i3}n_{i1} \\ n_{i1}n_{i2} \end{bmatrix}$$

Now, that the system is parametrized, it is possible to specify the final set of equations like follows [38].

$$\frac{1}{Y(\mathbf{d})} = \mathbf{d}_V^\top \mathbf{S} \mathbf{d}_V \quad (68)$$

$$\frac{1}{3 K(\mathbf{d})} = \mathbf{1}_V^\top \mathbf{S} \mathbf{d}_V \quad (69)$$

$$\frac{1}{2 G(\mathbf{d}, \mathbf{n}_i)} = \mathbf{m}_V^\top \mathbf{S} \mathbf{m}_V \quad (70)$$

$$-\frac{\nu(\mathbf{d}, \mathbf{n}_i)}{Y(\mathbf{d})} = \mathbf{d}_V^\top \mathbf{S} \mathbf{n}_V \quad (71)$$

For visualisation, an adequate coordinate system have to be chosen now. Here, the best choice is to use spherical coordinates. Fig. 6 shows these coordinate system and the normal vectors introduced. When Equations (68) or (69) are plotted and the result yields to a sphere, the material behaviour is isotropic. The discrepancy between the resulting figure and a sphere describes the amount of anisotropy of the material parameter. In contrast, plots of equation (70) or (71) yields curve in a plane with the normal vector $\mathbf{n}_i$. Now the ideal form of the curve is a cycle because it represents isotropic material behaviour in a plane.

4.3.2. Presentation of results

In the sequel we present the orientation dependence of mechanical properties by means of YOUNG's, bulk and, shear modulus as far as Poisson's ratio resulting from homogenisation results.

Figure 7: Orientation distribution of Young's modulus Y in different views

YOUNG's modulus. The YOUNG's modulus body is visualised in Figure 7. As depicted, there is a strong variation of elasticity in the e_1 - e_2 -plane. On the other hand, the elasticity orthogonal to this plane is substantially lower and is subject to much lower variations. Following orientation dependant parameters can be identified.

$$\begin{aligned} Y_1 &= 487.6 \text{ N/mm}^2 \\ Y_2 &= 487.6 \text{ N/mm}^2 \\ Y_3 &= 22.7 \text{ N/mm}^2 \\ Y_{\max} &= 734.6 \text{ N/mm}^2 \end{aligned}$$

The maximum YOUNG's modulus $Y_{\max}$ can be found in the direction bisecting the angle between e_1 and e_2 and then by every offset of $\pi/2$.

Bulk modulus. A graphical representation is given in Figure 8. Considering the e_1 - e_2 -plane, there are no variations of bulk modulus. The bulk modulus orthogonal to this plane is substantially lower also. The stretched view shows that there are slight variations orthogonal to e_1 - e_2 -plane. Specific values can be given as follows.

$$\begin{aligned} K_1 &= 397.0 \text{ N/mm}^2 = K_{\max} \\ K_2 &= 397.0 \text{ N/mm}^2 \\ K_3 &= 7.7 \text{ N/mm}^2 \end{aligned}$$

It gets clear that the bulk modulus is much larger in the plane.

Figure 8: Orientation distribution of bulk modulus K in different views

Shear modulus. Results for the shear modulus are visualised in Figure 9. It represents a circle in e_1 - e_2 -plane, and is significantly smaller in e_1 - e_3 - and e_2 - e_3 -plane while the latter planes do not show circle-like pictures. The following characteristic values can be quantified for the shear module.

$$\begin{aligned} G_{13} &= G_{23} = 3.46 \text{ N/mm}^2 \\ G_{12} &= 40,000 \text{ N/mm}^2 \end{aligned}$$

The values of G_{13} and G_{23} confirm the postulate of [6] approximately. Effective transverse shear behaviour is nearly identical to the shear modulus of the encapsulant EVA (see Table 3). The in-plane shear modulus is significantly higher. Observing Figure 9 (top left), it becomes clear that the shear modulus in e_1 - e_2 is isotropic. For the other two planes indicated there, this is not the case.

Poisson's ratio. Poisson's ratios for the homogenised substitute material are depicted in Figure 9 also. They show a pronounced direction-dependent behavior in all planes presented there while e_1 - e_3 - and e_2 - e_3 -plane show identical results. Orientation dependent values can be given as follows.

$$\begin{aligned} \nu_{12} &= \nu_{21} = 0.022 \\ \nu_{13} &= \nu_{23} = 0.57 \\ \nu_{31} &= \nu_{32} = 0.026 \end{aligned}$$

It turns out that at in-plane deformations, longitudinal strains are vanishingly small while transverse strains are exorbitantly high, which is related to the rubber-like behavior of the encapsulation. In contrast, at out-of-plane deformations, longitudinal strains are vanishing what is related to the stiffening effect of the solar cell.

Figure 9: Orientation dependence of shear modulus G and Poisson's ratio ν in e_1 - e_2 , e_1 - e_3 , and e_2 - e_3 plane

5. Transition to Composite Stiffnesses

5.1. Linear Elasticity of Surface Continua

In [6], an approach to global structural analysis has been introduced with an eXtended LayerWise Theory (XLWT). Each layer is considered individually as a homogeneous structure, while constraints are introduced to couple the layers. As usual in the theories of thin-walled structures, the considerations are reduced to the midplane $\mathfrak{B}_\ominus$ of the layer, where $\mathbf{n}$ is the normal to this surface continuum. For this, the three dimensional position vector $\mathbf{x}$ is projected into the plane. Thus, all material points $\mathbf{x}_\ominus$ are coplanar. Since our problem concerns the core layer of a photovoltaic module, we limit the following considerations to one layer only with uniform thickness h .

$$\mathbf{x}_\ominus = \mathbf{P} \cdot \mathbf{x} = X_\alpha \mathbf{e}_\alpha \quad \mathbf{x} = \mathbf{x}_\ominus + X_3 \mathbf{n} = X_i \mathbf{e}_i \quad (72)$$

Here, $\mathbf{P} = \mathbf{1} - \mathbf{n} \otimes \mathbf{n} = \mathbf{e}_\alpha \otimes \mathbf{e}_\alpha$ is the first metric tensor. In contrast to deduction approaches to derive theories for thin-walled structures, the direct approach is applied here. Hence, the starting point is a surface continuum in three dimensional EUCLIDIAN space. This surface continuum takes an in-plane and an out-of-plane stress state into account, while both states are uncoupled. For this reason, the surface is equipped with five independent degrees of freedom (DOF's) related to those (partially dependent DOF's) of the three dimensional CAUCHY continuum in the following way.

$$\left. \begin{aligned} \mathbf{u} &= \mathbf{u}_\ominus + w \mathbf{n} & \mathbf{u}_\ominus &= \mathbf{P} \cdot \mathbf{u} = u_\alpha \mathbf{e}_\alpha \\ \tilde{\boldsymbol{\varphi}} &= \varphi_\alpha \mathbf{e}_\alpha + \varphi_3 \mathbf{n} & \boldsymbol{\varphi} &= -\tilde{\boldsymbol{\varphi}} \times \mathbf{n} = -\varphi_2 \mathbf{e}_1 + \varphi_1 \mathbf{e}_2 \end{aligned} \right\} \forall \mathbf{x}_\ominus \in \mathfrak{B}_\ominus \quad (73)$$

Here, $\mathbf{u}$ and $\tilde{\boldsymbol{\varphi}}$ are displacements and rotations of the three dimensional continuum (both with three components), while $\mathbf{u}_\ominus$, w , and $\boldsymbol{\varphi}$ are the ones for the surface continuum, where following degrees of freedom result in detail: $u_1, u_2, w, \varphi_1, \varphi_2$. In the sequel, we dispense with a derivation and present the governing equations for a single layer, while all equations are restricted to the layer midplane. In a first step, it is reasonable to determine the kinematics of a plane, while we distinguish the membrane strain tensor $\mathbf{E}_\ominus \in \text{Sym}$, the curvature tensor $\mathbf{X} \in \text{Sym}$, and the transverse shear strain vector $\boldsymbol{\gamma}$.

$$\mathbf{E}_\ominus = \nabla^{\text{sym}} \mathbf{u}_\ominus = E_{\alpha\beta} \mathbf{e}_\alpha \otimes \mathbf{e}_\beta \quad (74)$$

$$\mathbf{X} = \nabla^{\text{sym}} \boldsymbol{\varphi} = X_{\alpha\beta} \mathbf{e}_\alpha \otimes \mathbf{e}_\beta \quad (75)$$

$$\boldsymbol{\gamma} = \nabla w + \boldsymbol{\varphi} = \gamma_\alpha \mathbf{e}_\alpha \quad (76)$$

All three tensors represent infinitesimal measures. The kinetics of the surface continuum are described by the membrane force tensor $\mathbf{N} \in \text{Sym}$, the polar tensor of moments $\mathbf{L} = \mathbf{M} \times \mathbf{n}$ with $\mathbf{L} \in \text{Sym}$, and the shear force vector $\mathbf{q}_Q$. $\mathbf{M}$ is the axial tensor of moments, see [5].

$$\mathbf{N} = N_{\alpha\beta} \mathbf{e}_\alpha \otimes \mathbf{e}_\beta \quad (77)$$

$$\mathbf{L} = M_{\alpha\beta} \mathbf{e}_\alpha \otimes \mathbf{e}_\beta \quad (78)$$

$$\mathbf{q}_Q = Q_\alpha \mathbf{e}_\alpha \quad (79)$$

The relations between the defined kinetic and kinematic quantities are given by the following constitutive equations.

$$\mathbf{N} = \mathcal{C}_\ominus : \mathbf{E}_\ominus \quad \mathcal{C}_\ominus = C_{\alpha\beta\gamma\delta} \mathbf{e}_\alpha \otimes \mathbf{e}_\beta \otimes \mathbf{e}_\gamma \otimes \mathbf{e}_\delta \quad (80)$$

$$\mathbf{L} = \mathcal{D} : \mathbf{X} \quad \mathcal{D} = D_{\alpha\beta\gamma\delta} \mathbf{e}_\alpha \otimes \mathbf{e}_\beta \otimes \mathbf{e}_\gamma \otimes \mathbf{e}_\delta \quad (81)$$

$$\mathbf{q}_Q = \mathbf{Z} \cdot \boldsymbol{\gamma} \quad \mathbf{Z} = Z_{\alpha\beta} \mathbf{e}_\alpha \otimes \mathbf{e}_\beta \quad (82)$$

Here, $\mathcal{C}_\ominus$, $\mathcal{D}$, and $\mathbf{Z}$ are the stiffness tensors. $\mathcal{C}_\ominus$ is the fourth rank membrane stiffness tensor, $\mathcal{D}$ the fourth rank bending stiffness tensor, and $\mathbf{Z}$ is the second rank transverse shear stiffness tensor. $\mathcal{C}_\ominus$ and $\mathcal{D}$ have 2^4 coefficients. $\mathbf{Z}$ has only 2^2 coefficients. The fourth rank stiffness tensors also have a left and right subsymmetry, while all stiffness tensors have a major symmetry ($\in \text{Sym}$). While in [6] they are presented for isotropic materials, they have to be determined for a material with tetragonal symmetry in the present contribution.

5.2. Stiffness Tetrads at Tetragonal Symmetry

Although the derivation of stiffness tensors in context of the theory of directed continua is done via a specially postulated potential in form of the strain energy function (see e.g. [40, 41]), there is also a correlation with the three dimensional CAUCHY continuum. Based on this relation, the required stiffnesses for tetragonal symmetry are derived in the sequel. For this purpose, membrane and plate state are decoupled, while the main assumption is the plane stress state, where stress resultants are constant through thickness. This can be concluded in the following form.

$$\frac{\partial T_{ij}}{\partial X_3} = 0 \quad \text{or} \quad \frac{\partial T_{\alpha\beta}}{\partial X_3} = \frac{\partial T_{\alpha 3}}{\partial X_3} = 0 \quad (83)$$

For present photovoltaic composite structure, it is assumed that the core layer has a uniform thickness.

$$\frac{\partial h}{\partial X_1} = \frac{\partial h}{\partial X_2} = 0 \quad (84)$$

It is also assumed that all material parameters are independent of the transverse coordinate X_3 .

$$\frac{\partial C_{\alpha\beta\gamma\delta}}{\partial X_3} = 0 \quad (85)$$

Since in [6] is assumed that thickness changes are not considered at any arbitrary deformation process, $E_{33} = 0$ holds true. Using Hooke's law in form of Equation (20) including tetragonal symmetry from Matrix (28) and taking the above mentioned assumptions into account, one can identify the following set of equations.

$$T_{11} = C_{1111}E_{11} + C_{1122}E_{22} + C_{1133}E_{33} \quad (86)$$

$$T_{22} = C_{1122}E_{11} + C_{1111}E_{22} + C_{1133}E_{33} \quad (87)$$

$$T_{33} = C_{1133}(E_{11} + E_{22}) + C_{3333}E_{33} \quad (88)$$

$$T_{23} = C_{2323}2E_{23} \quad (89)$$

$$T_{13} = C_{2323}2E_{13} \quad (90)$$

$$T_{12} = C_{1212}2E_{12} \quad (91)$$

Considering that $T_{33} = 0$, Equation (88) results in

$$E_{33} = -\frac{C_{1133}}{C_{3333}}(E_{11} + E_{22}),$$

so that Equations (86) and (87) turn to the following expressions.

$$T_{11} = \left(C_{1111} - \frac{C_{1133}^2}{C_{3333}}\right)E_{11} + \left(C_{1122} - \frac{C_{1133}^2}{C_{3333}}\right)E_{22} \quad (92)$$

$$T_{22} = \left(C_{1122} - \frac{C_{1133}^2}{C_{3333}}\right)E_{11} + \left(C_{1111} - \frac{C_{1133}^2}{C_{3333}}\right)E_{22} \quad (93)$$

The correlation between the three dimensional CAUCHY continuum and the surface continuum is given by the over thickness integration of stresses which results in the coefficients of the membrane force tensor N .

$$N_{\alpha\beta} = \int_{-h/2}^{h/2} T_{\alpha\beta} dX_3 \quad \Rightarrow \quad \text{since (83) holds:} \quad N_{\alpha\beta} = T_{\alpha\beta} \int_{-h/2}^{h/2} dX_3 = h T_{\alpha\beta} \quad (94)$$

Now, $T_{\alpha\beta}$ can be substituted with the expressions of Equations (92), (93), and (91). The following expressions result where symmetry properties of N are taken into account.

$$N_{11} = h \left(C_{1111} - \frac{C_{1133}^2}{C_{3333}}\right)E_{11} + h \left(C_{1122} - \frac{C_{1133}^2}{C_{3333}}\right)E_{22} \quad (95)$$

$$N_{22} = h \left(C_{1122} - \frac{C_{1133}^2}{C_{3333}}\right)E_{11} + h \left(C_{1111} - \frac{C_{1133}^2}{C_{3333}}\right)E_{22} \quad (96)$$

$$N_{12} = 2h C_{1212} E_{12} \quad (97)$$

445 Comparing coefficients of 3D elasticity and membrane stiffness results in $\mathcal{C}_\ominus$.

$$\begin{aligned}\mathcal{C}_\ominus = & h \left(C_{1111} - \frac{C_{1133}^2}{C_{3333}} \right) (\mathbf{e}_1 \otimes \mathbf{e}_1 \otimes \mathbf{e}_1 \otimes \mathbf{e}_1 + \mathbf{e}_2 \otimes \mathbf{e}_2 \otimes \mathbf{e}_2 \otimes \mathbf{e}_2) \\ & + h \left(C_{1122} - \frac{C_{1133}^2}{C_{3333}} \right) (\mathbf{e}_1 \otimes \mathbf{e}_1 \otimes \mathbf{e}_2 \otimes \mathbf{e}_2 + \mathbf{e}_2 \otimes \mathbf{e}_2 \otimes \mathbf{e}_1 \otimes \mathbf{e}_1) \\ & + 2h C_{1212} (\mathbf{e}_1 \otimes \mathbf{e}_2 \otimes \mathbf{e}_1 \otimes \mathbf{e}_2 + \mathbf{e}_2 \otimes \mathbf{e}_1 \otimes \mathbf{e}_1 \otimes \mathbf{e}_2 + \mathbf{e}_1 \otimes \mathbf{e}_2 \otimes \mathbf{e}_2 \otimes \mathbf{e}_1 + \mathbf{e}_2 \otimes \mathbf{e}_1 \otimes \mathbf{e}_2 \otimes \mathbf{e}_1)\end{aligned}\quad (98)$$

446 The next interesting tensor is the bending stiffness $\mathcal{D}$. Taking only bending state into account, the correlations of the
447 strain coefficients of the membrane strain $\mathbf{E}_\ominus$ with the rotation and curvature coefficients are given by the following
448 relation.

$$\mathbf{E}_\ominus = X_3 \nabla^{\text{sym}} \boldsymbol{\varphi} = X_3 \mathbf{X} \quad (99)$$

449 In index notation this results in the following expressions.

$$E_{11} = \frac{\partial \varphi_1}{\partial X_1} X_3 = \chi_{11} X_3 \quad E_{22} = \frac{\partial \varphi_2}{\partial X_2} X_3 = \chi_{22} X_3 \quad 2E_{12} = \left(\frac{\partial \varphi_1}{\partial X_2} + \frac{\partial \varphi_2}{\partial X_1} \right) X_3 = \chi_{12} X_3 \quad (100)$$

450 The 3D-2D correlations given by the over thickness integration of stresses and correspondings levers, which results in
451 the coefficients of the moment tensor $\mathbf{L}$ (see definition in Equation (78)).

$$M_{\alpha\beta} = \int_{-h/2}^{h/2} X_3 T_{\alpha\beta} dX_3 \quad (101)$$

452 To derive the constitutive equations with respect to the bending moments, the strains $E_{\alpha\beta}$ in Equation (92), (93), and
453 (91) are replaced by Equations (100). Substituting the stresses $T_{\alpha\beta}$ in (101) with modified Equations (92), (93), and (91)
454 results in the following expressions.

$$M_{11} = \frac{h^3}{12} \left(C_{1111} - \frac{C_{1133}^2}{C_{3333}} \right) \chi_{11} + \frac{h^3}{12} \left(C_{1122} - \frac{C_{1133}^2}{C_{3333}} \right) \chi_{22} \quad (102)$$

$$M_{22} = \frac{h^3}{12} \left(C_{1122} - \frac{C_{1133}^2}{C_{3333}} \right) \chi_{11} + \frac{h^3}{12} \left(C_{1111} - \frac{C_{1133}^2}{C_{3333}} \right) \chi_{22} \quad (103)$$

$$M_{12} = \frac{2h^3}{12} C_{1212} \chi_{12} \quad (104)$$

455 Comparing coefficients of 3D elasticity and bending stiffness results in $\mathcal{D}$.

$$\begin{aligned}\mathcal{D} = & \frac{h^3}{12} \left(C_{1111} - \frac{C_{1133}^2}{C_{3333}} \right) (\mathbf{e}_1 \otimes \mathbf{e}_1 \otimes \mathbf{e}_1 \otimes \mathbf{e}_1 + \mathbf{e}_2 \otimes \mathbf{e}_2 \otimes \mathbf{e}_2 \otimes \mathbf{e}_2) \\ & + \frac{h^3}{12} \left(C_{1122} - \frac{C_{1133}^2}{C_{3333}} \right) (\mathbf{e}_1 \otimes \mathbf{e}_1 \otimes \mathbf{e}_2 \otimes \mathbf{e}_2 + \mathbf{e}_2 \otimes \mathbf{e}_2 \otimes \mathbf{e}_1 \otimes \mathbf{e}_1) \\ & + \frac{2h^3}{12} C_{1212} (\mathbf{e}_1 \otimes \mathbf{e}_2 \otimes \mathbf{e}_1 \otimes \mathbf{e}_2 + \mathbf{e}_2 \otimes \mathbf{e}_1 \otimes \mathbf{e}_1 \otimes \mathbf{e}_2 + \mathbf{e}_1 \otimes \mathbf{e}_2 \otimes \mathbf{e}_2 \otimes \mathbf{e}_1 + \mathbf{e}_2 \otimes \mathbf{e}_1 \otimes \mathbf{e}_2 \otimes \mathbf{e}_1)\end{aligned}\quad (105)$$

456 The last missing tensor is the second rank transverse shear stiffness tensor $\mathbf{Z}$ while for determination shear state is
457 taken into account. The coefficients of the transverse shear force vector $\mathbf{q}_Q$ are determined as follows.

$$Q_\alpha = \int_{-h/2}^{h/2} T_{\alpha 3} dX_3 \quad (106)$$

458 Transverse shear strains are assumed to be constant along X_3 .

$$\gamma_1 = \frac{\partial w}{\partial X_1} + \varphi_1 = 2E_{13} \quad \gamma_2 = \frac{\partial w}{\partial X_2} + \varphi_2 = 2E_{23}$$

459 This results in both following relations for the transverse shear strain coefficients

$$Q_1 = h C_{2323} \gamma_1 \quad (107)$$

$$Q_2 = h C_{2323} \gamma_2 \quad (108)$$

460 Comparing coefficients of 3D elasticity and shear stiffness results in $\mathbf{Z}$.

$$\mathbf{Z} = h C_{2323} (\mathbf{e}_1 \otimes \mathbf{e}_1 + \mathbf{e}_2 \otimes \mathbf{e}_2) \quad (109)$$

461 Equations (98), (105), and (109) can be generalized in terms of their bases. This yields compact expressions for the
462 stiffness tensors where the bases are grouped in the spirit of ALTENBACH [42].

$$\mathcal{C}_\ominus = \frac{1}{2} h B_1 (\mathbf{P}_1 \otimes \mathbf{P}_1 + \mathbf{P}_2 \otimes \mathbf{P}_2) + \frac{1}{2} h B_2 (\mathbf{P}_1 \otimes \mathbf{P}_1 - \mathbf{P}_2 \otimes \mathbf{P}_2) + h B_3 (\mathbf{P}_3 \otimes \mathbf{P}_3) \quad (110)$$

$$\mathcal{D} = \frac{h^3}{24} B_1 (\mathbf{P}_1 \otimes \mathbf{P}_1 + \mathbf{P}_2 \otimes \mathbf{P}_2) + \frac{h^3}{24} B_2 (\mathbf{P}_1 \otimes \mathbf{P}_1 - \mathbf{P}_2 \otimes \mathbf{P}_2) + \frac{h^3}{12} B_3 (\mathbf{P}_3 \otimes \mathbf{P}_3) \quad (111)$$

$$\mathbf{Z} = h B_4 \mathbf{P}_1 \quad (112)$$

463 The recalculated material parameters $B_i \forall i = \{1, 2, 3, 4\}$ are given as follows.

$$B_1 = C_{1111} - \frac{C_{1133}^2}{C_{3333}}$$

$$B_2 = C_{1122} - \frac{C_{1133}^2}{C_{3333}}$$

$$B_3 = C_{1212}$$

$$B_4 = C_{2323}$$

464 Here, the second rank tensors $\mathbf{P}_m \forall m = \{1, 2, 3\}$ are defined as follows.

$$\mathbf{P}_1 = \mathbf{e}_1 \otimes \mathbf{e}_1 + \mathbf{e}_2 \otimes \mathbf{e}_2 \quad (\text{first metric tensor of surface})$$

$$\mathbf{P}_2 = \mathbf{e}_1 \otimes \mathbf{e}_1 - \mathbf{e}_2 \otimes \mathbf{e}_2 \quad (\text{unnamed})$$

$$\mathbf{P}_3 = \mathbf{e}_1 \otimes \mathbf{e}_2 - \mathbf{e}_2 \otimes \mathbf{e}_1 \quad (\text{discriminant tensor})$$

465 Since the unit cell examined is a representative portion of the core layer structure of a photovoltaic module, the
466 abovementioned stiffness tensors hold true for every material point $\mathbf{x}_\ominus$ of the midplane $\tilde{\mathfrak{B}}_\ominus$ of the core layer when using
467 the model of a surface continuum for structural analysis in linear elasticity.

468 5.3. Discussion

469 The material parameters of the surface continuum B_i are related to those of the surface continuum. Following identities
470 hold true.

$$B_1 = Y_1 = Y_2 = C_{1111} - \frac{C_{1133}^2}{C_{3333}}$$

$$B_2 = C_{1122} - \frac{C_{1133}^2}{C_{3333}}$$

$$B_3 = G_{12} = C_{1212}$$

$$B_4 = G_{13} = G_{23} = C_{2323}$$

Due to that relations, it is trivial to generate stiffness parameters for surface elasticity out of homogenisation results of the three dimensional material test. Furthermore, it is easy to see, that by comparing membrane and bending stiffness tensor, following relation holds true.

$$\mathcal{D} = \frac{h^2}{12} \mathcal{C}_\ominus$$

To get a clearer view of the stiffness tensors determined for tetragonal symmetry in context of the orthonormal bases used in Equations (77), (78), and (79), their coefficient matrices are given in the following.

$$\mathcal{C}_\ominus \Rightarrow [C_{\alpha\beta\gamma\delta}]_\ominus = h \begin{bmatrix} B_1 & 0 & 0 & 2B_3 \\ & B_2 & 2B_3 & 0 \\ & & B_2 & 0 \\ \text{sym} & & & B_1 \end{bmatrix} \quad (113)$$

$$\mathcal{D} \Rightarrow [D_{\alpha\beta\gamma\delta}]_\ominus = \frac{1}{12} h^3 \begin{bmatrix} B_1 & 0 & 0 & 2B_3 \\ & B_2 & 2B_3 & 0 \\ & & B_2 & 0 \\ \text{sym} & & & B_1 \end{bmatrix} \quad (114)$$

$$\mathcal{Z} \Rightarrow [Z_{\alpha\beta}] = h \begin{bmatrix} B_4 & 0 \\ \text{sym} & B_4 \end{bmatrix} \quad (115)$$

In [6], a shear correction factor κ is introduced in analogy to what's known as MINDLIN theory [43], accounting shear deformations through the thickness of the layer. Within the scope of the presented treatise, it is also possible to introduce such measure within the shear stiffness tensor. Equation (112) would then be presented as follows.

$$\mathcal{Z} = \kappa \ h \ B_4 \ P_1 \quad (116)$$

The shear correction factor $0 < \kappa \leq 1$ is a tuning parameter for the shear energy contribution. The determination of this parameter is part of special research efforts, cf. [44, 43, 45, 8].

6. Conclusion and Outlook

In the present work, an anisotropic constitutive law for the core layer of a photovoltaic composite structure is generated via computational homogenisation of elastic properties of the constituents. Additionally, the results are transferred to stiffness measures of a surface continuum, which forms the mathematical basis for structural mechanics at slender structures like photovoltaic modules.

Due to symmetry considerations at the core layer of photovoltaic modules, a periodic structure could be identified. This allowed the selection of a representative portion of this layer, whose periodicity is mesoscopic. More detailed observations on symmetry properties of this unit cell led to a geometrically induced anisotropy. This anisotropy identified is known as tetragonal symmetry. Building on this, the constitutive relations for this symmetry class based on linear theory of elasticity are derived, bringing to light the number of missing material parameters. In order to determine them, material tests are defined and a computational homogenisation is used in consequence.

First, the determined parameters were subjected to an examination based on the postulate of positive definiteness of strain energy density whereby all values could be verified. In order to facilitate access to the material behaviour determined, graphical representations of resulting YOUNG's, bulk, and shear moduli as far as of POISSON's ratio plotted on a sphere or a plane are given.

Subsequently, the results for the till now considered CAUCHY continuum are transferred to measures of the surface continuum. First of all, this showed kinetic and kinematic quantities of a surface, where the constitutive relations for the symmetry class had to be defined. The comparison of coefficients of constitutive laws for CAUCHY and surface continuum led to defined stiffness measures, whereby the correlation of parameters from both worlds is unique.

The parameters presented here are specific concerning the geometry and material parameters of the unit cell. When changing

- edge lengths or thickness of solar cell,
- thickness of the encapsulation,
- spacing of solar cells in lateral directions,
- to an uncentered position of solar cell in transverse direction,
- to a deviation of plane-parallelism of solar cell to encapsulant front or back surface,
- material properties of the solar cell,
- material properties of encapsulant,

results presented here lose validity, while the general framework of the methodology as far as the identified symmetry properties of elasticity and stiffness tensors remain valid without loss of generality. The procedures accomplished here have then to be applied once more with the new parameters of the constituents. Again, it can be shown that resulting anisotropy is a universal statement. The symmetry properties of the material class determined here are correlated to the geometric structure of the core layer.

The influence of the anisotropic material behaviour on the global structural behaviour of photovoltaic modules, determined with XLWT in comparison to results where the homogeneity postulate made in [6] is used will be investigated in a following publication. To this end, stiffness tensors for the surface continuum have to be implemented in the subroutine for the user element presented in [7].

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References

- [1] M. Sander, S. Dietrich, M. Pander, M. Ebert, J. Bagdahn, Systematic investigation of cracks in encapsulated solar cells after mechanical loading, *Solar Energy Materials and Solar Cells* 111 (2013) 82–89. doi:10.1016/j.solmat.2012.12.031.
- [2] O. Hasan, A. Arif, Performance and life prediction model for photovoltaic modules: Effect of encapsulant constitutive behavior, *Solar Energy Materials and Solar Cells* 122 (2014) 75–87. doi:10.1016/j.solmat.2013.11.016.
- [3] U. Eitner, S. Kajari-Schröder, M. Köntges, H. Altenbach, Thermal stress and strain of solar cells in photovoltaic modules, in: H. Altenbach, V. A. Eremeyev (Eds.), *Shell-like Structures: Non-classical Theories and Applications*, Springer Berlin Heidelberg, Berlin, Heidelberg, 2011, pp. 453–468. doi:10.1007/978-3-642-21855-2_29.
- [4] F. Kraemer, S. Wiese, E. Peter, J. Seib, Mechanical problems of novel back contact solar modules, *Microelectronics Reliability* 53 (8) (2013) 1095–1100. doi:10.1016/j.microrel.2013.02.019.
- [5] M. Abmus, K. Naumenko, H. Altenbach, A multiscale projection approach for the coupled global-local structural analysis of photovoltaic modules, *Composite Structures* 158 (2016) 340–358. doi:10.1016/j.compstruct.2016.09.036.
- [6] K. Naumenko, V. A. Eremeyev, A layer-wise theory for laminated glass and photovoltaic panels, *Composite Structures* 112 (2014) 283–291. doi:10.1016/j.ijmecsci.2015.03.012.
- [7] J. Eisenträger, K. Naumenko, H. Altenbach, J. Meenen, A user-defined finite element for glass and photovoltaic panels based on a layer-wise theory, *Composite Structures* 133 (2015) 265–277. doi:10.1016/j.compstruct.2015.07.049.
- [8] H. Altenbach, V. A. Eremeyev, K. Naumenko, On the use of the first order shear deformation plate theory for the analysis of three-layer plates with thin soft core layer, *Zeitschrift für Angewandte Mathematik und Mechanik* 95 (10) (2015) 1004–1011. doi:10.1002/zamm.201500069.
- [9] M. Weps, K. Naumenko, H. Altenbach, Unsymmetric three-layer laminate with soft core for photovoltaic modules, *Composite Structures* 105 (2013) 332–339. doi:10.1016/j.compstruct.2013.05.029.
- [10] H.-J. Böhm, A short introduction to continuum micromechanics, in: H.-J. Böhm (Ed.), *Mechanics of Microstructured Materials*, Vol. 464 of *International Centre for Mechanical Sciences (CISM)*, Springer, Vienna, 2004, pp. 1–40. doi:10.1007/978-3-7091-2776-6_1.
- [11] G. Dvorak, *Micromechanics of Composite Materials, Solid Mechanics and Its Applications*, Springer, Dordrecht · Heidelberg · New York · London, 2012. doi:10.1007/978-94-007-4101-0.
- [12] M. J. Beran, *Statistical continuum theories*, Vol. 9 of *Monographs in statistical physics and thermodynamics*, Interscience Publishers, 1968.
- [13] W. Voigt, *Lehrbuch der Kristallphysik (mit Ausschluss der Kristalloptik)*, Springer, Wiesbaden, 1966, Reproduktion des 1928 erschienenen Nachdrucks der ersten Auflage von 1910. doi:10.1007/978-3-663-15884-4.

- [14] P. M. Naghdi, The theory of shells and plates, in: C. Truesdell (Ed.), Linear Theories of Elasticity and Thermoelasticity: Linear and Nonlinear Theories of Rods, Plates, and Shells, Springer, Berlin · Heidelberg, 1973, pp. 425–640, (originally appeared as Vol. 3/6A-2 in Handbuch der Physik in 1972, edited by Siegfried Flügge). doi:10.1007/978-3-662-39776-3_5.
- [15] F. Cosserat, E. Cosserat, *Théorie des corps déformables*, A. Hermann et fils, Paris, 1909.
URL <http://jhirlibrary.jhu.edu/handle/1774.2/34209>
- [16] H. Altenbach, *Kontinuumsmechanik: Einführung in die materialunabhängigen und materialabhängigen Gleichungen*, 2nd Edition, Springer Vieweg, Berlin · Heidelberg, 2012. doi:10.1007/978-3-642-24119-2.
- [17] W. Lai, D. Rubin, E. Kreipl, *Introduction to Continuum Mechanics*, 4th Edition, Butterworth-Heinemann, Oxford, 2010.
- [18] A. Bertram, *Elasticity and Plasticity of Large Deformations: An Introduction*, 3rd Edition, Springer, Berlin · Heidelberg, 2012. doi:10.1007/978-3-642-24615-9.
- [19] R. Tiemeyer, *Axiome der Klassischen Mechanik. Hilberts Problem und Hamels Lösungsversuch in wissenschaftstheoretischer Perspektive*, Logos, Berlin, 2016.
- [20] A. Bertram, Symmetry properties of fourth-order tensors with applications in continuum mechanics, *Investigaciones ciencias e ingenierias* 15 (1) (1985) 41–55.
- [21] S. C. Cowin, Properties of the anisotropic elasticity tensor, *Quarterly Jnl of Mechanics & Applied Math* 42 (2) (1989) 249–266. doi:10.1093/qjmam/42.2.249.
- [22] W. Voigt, *Lehrbuch der Kristallphysik (mit Ausschluss der Kristalloptik)*, Springer, Wiesbaden, 1966, Reproduktion des 1928 erschienenen Nachdrucks der ersten Auflage von 1910. doi:10.1007/978-3-663-15884-4.
- [23] B. D. Coleman, W. Noll, Material symmetry and thermodynamic inequalities in finite elastic deformations, *Archive for Rational Mechanics and Analysis* 15 (2) (1964) 87–111. doi:10.1007/BF00249520.
- [24] R. Glüge, Generalized boundary conditions on representative volume elements and their use in determining the effective material properties, *Computational Materials Science* 79 (2013) 408–416. doi:10.1016/j.commatsci.2013.06.038.
- [25] K. Terada, M. Hori, T. Kyoya, N. Kikuchi, Simulation of the multi-scale convergence in computational homogenization approaches, *International Journal of Solids and Structures* 37 (16) (2000) 2285–2311. doi:10.1016/S0020-7683(98)00341-2.
- [26] R. Hill, Elastic properties of reinforced solids: some theoretical principles, *Journal of the Mechanics and Physics of Solids* 11 (5) (1963) 357–372. doi:10.1016/0022-5096(63)90036-X.
- [27] J. Mandel, Définition d’un repère privilégié pour l’étude des transformations anélastiques du polycristal, *Journal de mécanique théorique et appliquée* 1 (1) (1982) 7–23.
- [28] G. H. Golub, C. F. Van Loan, *Matrix Computations*, 3rd Edition, Johns Hopkins University Press, Baltimore, MD, USA, 1996.
- [29] L. Zhao, A. Maynadier, D. Nelias, Stiffness and fracture analysis of photovoltaic grade silicon plates, *International Journal of Solids and Structures* 97–98 (2016) 355–369. doi:10.1016/j.ijsolstr.2016.07.013.
- [30] W. Voigt, Ueber die Beziehung zwischen den beiden Elastizitätskonstanten isotroper Körper, *Wiedemanns Annalen* 38 (1889) 573–587. doi:10.1002/andp.18892741206.
- [31] A. Reuss, Berechnung der Fließgrenze von Mischkristallen auf Grund der Plastizitätsbedingung für Einkristalle, *Zeitschrift für Angewandte Mathematik und Mechanik* 9 (1) (1929) 49–58. doi:10.1002/zamm.19290090104.
- [32] R. Hill, The elastic behaviour of a crystalline aggregate, *Proceedings of the Physical Society. Section A* 65 (5) (1952) 349–354. doi:10.1088/0370-1298/65/5/307.
- [33] W. F. Hosford, *The Mechanics of Crystals and Textured Polycrystals*, Oxford University Press, New York, 1993.
- [34] E. Kröner, Berechnung der elastischen konstanten des vielkristalls aus den konstanten des einkristalls, *Zeitschrift für Physik* 151 (4) (1958) 504–518. doi:10.1007/BF01337948.
- [35] A. Czanderna, F. Pern, Encapsulation of PV modules using ethylene vinyl acetate copolymer as a pottant: A critical review, *Solar Energy Materials and Solar Cells* 43 (2) (1996) 101–181. doi:10.1016/0927-0248(95)00150-6.
- [36] U. Eitner, *Thermomechanics of photovoltaic modules*, PhD thesis, Martin Luther University Halle-Wittenberg (2011).
URL <http://nbn-resolving.de/urn:nbn:de:gbv:3:4-5812>
- [37] F. Koschnick, *Geometrische locking-effekte bei finiten elementen und ein allgemeines konzept zu ihrer vermeidung*, PhD thesis, Technical University of Munich (2004).
URL <http://nbn-resolving.de/urn:nbn:de:bvb:91-diss2004100700624>
- [38] T. Böhlke, C. Brüggemann, Graphical representation of the generalized hooke’s law, *Technische Mechanik* 21 (2) (2001) 145–158.
URL http://www.uni-magdeburg.de/ifme/zeitschrift_tm/2001_Heft2/Boehlke_Brueggeman.pdf
- [39] J. Rychlewski, Unconventional approach to linear elasticity, *Archives of Mechanics - Archiwum Mechaniki Stosowanej* 47 (2) (1995) 149–171.
- [40] H. Altenbach, A directly formulated linear theory of viscoelastic plates and shells, *Ingenieur-Archiv* 58 (3) (1988) 215–228. doi:10.1007/BF00534332.
- [41] H. Altenbach, P. A. Zhilin, The theory of simple elastic shells, in: R. Kienzler, I. Ott, H. Altenbach (Eds.), *Theories of Plates and Shells: Critical Review and New Applications*, Vol. 16 of Lecture Notes in Applied and Computational Mechanics, Springer, Berlin · Heidelberg, 2004, pp. 1–12. doi:10.1007/978-3-540-39905-6_1.
- [42] H. Altenbach, The direct approach in the theory of viscoelastic shells (in russian), *Habilitation thesis*, Leningrad Polytechnic Institute (1987).
- [43] R. D. Mindlin, Influence of rotatory inertia and shear on flexural motions of isotropic, elastic plates, *Journal of Applied Mechanics* 18 (1951) 31–38.
- [44] E. Reissner, The effect of transverse shear deformation on the bending of elastic plates, *Journal of Applied Mechanics* 12 (1945) 69–77.
- [45] H. Altenbach, An alternative determination of transverse shear stiffnesses for sandwich and laminated plates, *International Journal of Solids and Structures* 37 (25) (2000) 3503–3520. doi:10.1016/S0020-7683(99)00057-8.