New strategy for anchorage reliability assessment of GFRP bars to concrete using hybrid artificial neural network with genetic algorithm

Fei Yan, and Zhibin Lin¹

Department of Civil and Environmental Engineering, North Dakota State University, Fargo, ND58018-6050, USA.

Abstract

Anchorage is of critical importance in the glass fiber-reinforced polymer (GFRP) bar reinforced concrete structures to allow reinforcing GFRP bars to provide sufficient bond to concrete. This study presents a new strategy for anchorage reliability assessment of GFRP bars to concrete by integrating superiorities of artificial neural network (ANN) and genetic algorithm (GA). The new methodology harnesses not only the strong nonlinear mapping ability in the ANN to approximate the performance function (PF) and solve its partial derivatives in terms of the design variables, but also global searching ability in the GA to explore the optimal initial weights and biases of the ANN to avoid falling into local minima during the network training. The ANN-based first order second moment (FOSM) method and Monte Carlo simulation (MCS) method were first derived. Implementation of the proposed hybrid ANN-GA procedures for GFRP bar anchorage reliability analysis were then achieved by the targeted reliability index and development length. Both the ANN-based FOSM and MCS methods were utilized for determining the reliability index and probability of failure of GFRP bar anchorage. The further implementation of the proposed strategy was achieved by a graphical user interface toolbox in Matlab environment for practical use.

Keywords: Glass fibers, Polymer-matrix composites (PMCs), Debonding, Statistical properties/methods, Numerical analysis.

1. Introduction

GFRP reinforcing bars, due to their superior corrosion resistance, as well as the high strength to weight ratio and cost efficiency [1-4], have been taken as an alternative substitute for steel bars for addressing the every-increasing deteriorated reinforced concrete structures. Sufficient development length of reinforcing bars plays an important role in preventing bond premature failure and ultimately ensures the safety of the structures [5]. Anchorage reliability of GFRP bars to concrete therein is one of the most critical indices for implementation of such engineered material to the concrete structures. A reasonable reliability index of the development length must be designated to allow the GFRP bar to yield desirable flexural failure prior to anchorage failure.

Anchorage reliability assessment requires a performance function (PF) with respect to a set of design variables, while the PFs are usually implicit in most cases. Although data generated from either numerical simulation or

¹ Corresponding author: Tel: +1 701-231-7204; Fax: +1 701-231-6185. E-mail addresses: zhibin.lin@ndsu.edu

experimental tests are commonly used for determining the PFs [6, 7], there still remain high challenges: a) Effectiveness of numerical simulation. The GFRP bar bond-slip behavior exhibits a highly nonlinear contact feature between GFRP bars and concrete [8], resulting in high variation in modeling (parameter selection and optimization); and b) Limitation of experimental tests. Most laboratory tests, due to limited facilities, time consuming and cost, may be conducted under certain particular conditions, which in turn do not accommodate all critical design variables (e.g., bar position, bar diameter and concrete cover) commonly experienced in construction. As a result, both numerical simulation and experimental tests neither consider the different characteristics of GFRP materials nor distinguish issues inherent to particular applications to construct the PFs for anchorage reliability analysis.

Alternatively, use of the ANN algorithm enables to approximate the implicit PF through a set of inputs and a desirable output without need of solving explicit function of the PFs [9-11]. This technique has been widely accepted in structural reliability analysis, and has been validated to be more comparable over conventional approaches [12]. Deng et al. [12] presented their work on structural reliability analyses through the ANN-based first-order second-moment (FOSM) method and Monte Carlo simulation (MCS) method. The ANN technique in their study was utilized to predict the implicit PFs and determine their partial derivatives with respect to design variables for determining failure probability and reliability index. Their analysis revealed that the results predicted by the ANN-based FOSM and the MCS methods have higher accuracy over conventional reliability analysis methods. Papadrakakis et al. reported the reliability analysis of complex structural system using the ANN-based MCS method. The critical load factor and failure probability considering plastic-hinge collapse were accurately captured [13].

Multi-layer feed-forward neural network with back-propagation (BP) algorithm due to its strong ability of data mining is usually taken as a preferred choice for complex problems with highly nonlinear correlations [14-17]. The BP algorithm adopts a local searching technique through the gradient decent method to adjust the weights and biases back from the output layer to the preceding layers iteratively, thereby minimizing the mean square error between the actual and predicted outputs [18, 19]. However, this algorithm may experience inherent drawback where the training phase is too low to avoid local minima. To overcome this, the genetic algorithm (GA) is embedded into the ANN to post a global searching ability, referred to the hybrid ANN-GA model for the network training. By taking advantage of the capability of identifying the global optimal solutions, the initial weights and biases in the ANN are evolved firstly, and then assigned to the ANN as the initial values for the subsequent iterations. Cheng and Li [20] used the ANN-based GA model for structural reliability analysis, in which the ANN model was used to approximate the limit state function, while the GA to estimate the failure probability. The developed method confirmed that the hybrid ANN with GA is more effective, particularly when the failure probability tends to be extremely small. This hybrid modeling strategy has also been used in several engineering fields, such as the slump of ready mix concrete [18], the permeability of the reservoir [21], and yet has not been applied in reliability analysis of GFRP bars in concrete.

This study is to develop a systematic strategy using the hybrid ANN-GA model for the anchorage reliability assessment of GFRP bars to concrete. The developed procedures of the strategy cover the detailed data selection and processing, PF modeling and validation, and the ANN-based reliability assessment in terms of the FOSM and MCS

methods. Implementation of the ANN-GA model for GFRP bar anchorage reliability assessment is then exemplified in a step-by-step manner. A graphical user interface (GUI) system in Matlab environment is designed for practical use.

2. ANN-based anchorage reliability assessment

2.1. Framework of ANN-based anchorage reliability assessment

A systematic framework of the ANN-based reliability analysis is proposed, as demonstrated in a flow chart shown in Figs. 1(a) and 1(b). It mainly consists of three phases as shown in Fig. 1(a). Firstly, the database for the network training needs to be created, where the design variables $X = (x_1, x_2, \dots, x_n)^T$ and its corresponding response $g(X)$ (i.e., PFs) can be obtained from experimental results. Statistical characteristics in terms of means and standard deviations are then computed and prepared for the reliability analysis. Secondly, with the predetermined database, the hybrid ANN-GA modeling method is used for determining the PFs of anchorage reliability of GFRP bars in concrete. The computational strategy is to integrate the ANN with the GA to predict the PF and calibrate its effectiveness, thereby resulting in the prerequisite for the subsequent reliability assessment at the third step. Specifically, the ANN is used to map the relationship between the design variables and the PFs, while the GA is used to optimize the initial weights and biases of the ANN. Toward the end, the reliability assessment using the ANN-based FOSM and MCS methods are used to derive reliability index and the failure probability. Note that the development length collected in the database is too short for adequate anchorage, leading to the reliability index to be negative. Thus, the mean of development length is corrected based on a targeted reliability index.

(a) Reliability analysis procedures

(b) ANN-GA model

Fig. 1 Concept of ANN-based anchorage reliability analysis

Among the detailed procedures above, effective prediction of the PFs with respect to the design variables at the second step plays important role in the accuracy of the final reliability assessment. For the hybrid ANN-GA modeling of the PFs, the ANN needs to be constructed in advance due to the fact that the chromosome length in the GA is dependent on the network architecture. As illustrated in Fig. 1(b), the basic information, such as the population size and optimized target, is firstly initialized in the GA. The weights and biases of the ANN are encoded to constitute a set of chromosomes, forming an initial population to evolve. The norm of the errors between the predicted output vector and expected output vector is defined as the fitness. The chromosomes with the smaller fitness (i.e., the smaller errors) are selected for crossover and mutation at a certain probability, generating offspring inheriting excellent genes from their parents. With that, the worse chromosomes in the parent population are replaced with these new superior ones, creating a new population. Meanwhile, the best chromosome is decoded and transmitted into the ANN to determine whether the optimal target is achieved. Otherwise, the evolution subjected to the same rules will continue refining the network prediction. As long as the optimized target meets the requirement, the best chromosome containing the best solutions for the ANN prediction are decoded and assigned to the network as the initial weights and biases for training. The weights and biases are then adjusted to gain the relationship between the design variables and the PF. Finally, the predicted PF will be further calibrated through data from either experimental tests or numerical simulation.

2.2. Data selection and processing

The selected data for the network training must cover the most critical factors for the PF, while eliminating the secondary factors that may cause unexpected disturbance during the network prediction. As one of the most critical factors for the bar anchorage PF, the bond strength and its influence factors need to be investigated. A preliminary database was collected from existing 179 beam-test specimens in the literature [22-36], covering the variables of interest, such as bar diameter, concrete strength and cover, bar position and surface, development length, and transverse confinement. The statistical characteristics are summarized in Table 1, where *Surface* denotes the surface treatment of GFRP bars; *Position* denotes the bar position; ρ denotes the transverse reinforcement ratio; d_b denotes the bar diameter; c denotes the concrete cover; l_d denotes the development length; f'_c denotes the concrete compressive strength; f_u denotes the ultimate strength of GFRP bars. In addition, the surface treatments were quantified by 1, 2 and 3 for the helical lugged, spiral wrapped and sand coated, respectively. Bar positions were quantified by 1 and 2 for top and bottom, respectively.

Table 1 Statistical characteristics of variables affecting bond strength

Factors	Minimum	Maximum	Mean	Standard deviations
Surface	1	3	1.79	0.69
Position	1	2	1.17	0.38

ρ	0	0.08	0.02	0.02
d_b (mm)	9.53	28.70	18.98	5.48
c (mm)	9.53	406.00	63.43	47.65
l_d (mm)	38.10	799.91	271.02	220.55
f'_c (MPa)	27.56	65.29	34.00	8.41
f_u (MPa)	469.00	931.00	648.14	110.98

Fig. 2 displays the influences of those variables on bond strength, τ_b . Generally, it was observed that the bar surface treatment, bar position and transverse reinforcement ratio have no significant effects on the bond strength. Differently, the concrete compressive strength, the ratio of concrete cover to bar diameter, and the ratio of development length to bar diameter have high impacts on bond strength: as $\sqrt{f'_c}$ and c/d_b increased, τ_b increased linearly; while τ_b decreased nonlinearly as l_d/d_b increased. According to the data diversity principle, *Surface*, *Position*, and ρ will not be included for the network training, while still remaining for the purpose of the PF prediction in design codes for a comparison.

Fig. 2 Influences of design variables on bond strength

In addition, since the data collected are generally in a big numerical range, it is necessary to normalize them into a regular range to enhance the training efficiency of the network. The algorithm of the data normalization is shown in Eqn. (1):

$$x_n = \frac{y_{max}-y_{min}}{x_{max}-x_{min}}(x - x_{min}) + y_{min}, \quad (1)$$

where x_n is the normalized value of variable x ; x_{max} and x_{min} are the maximum and minimum of x , respectively; y_{max} and y_{min} are the maximum and minimum of normalized target, respectively. When the normalized target with $y_{max} = 1$ and $y_{min} = -1$ are used, then we have

$$x_n = 2 \frac{(x-x_{min})}{x_{max}-x_{min}} - 1. \quad (2)$$

2.3. Performance function modeling for anchorage reliability

2.3.1. Performance function definition

The first step of a reliability analysis is to construct the PF in terms of a set of design variables $X=(X_1, X_2, \dots, X_n)^T$, where $X_i (i = 1, 2, \dots, n)$ is the i^{th} design variable. In general, the basic items of the PF can be classified into the structural resistance, R , which is dependent on the properties of the structure itself, and the load effect, S , which is resulted from external loads. As such, the PF of the anchorage reliability are demonstrated in Eqn. (3):

$$Z = g(X) = R - S = \tau_b \pi d_b l_d - f_u \pi d_b^2 / 4. \quad (3)$$

Note that the bond strength has implicit form in terms of the some design variables in existing Canadian and Japanese FRP design codes, which posts a high challenge in conventional reliability analysis as we stated previously. By using the ANN technique, the procedures of solving the implicit bond strength are replace with directly constructing a relationship between the design variables and the PF, while the relationship will be generated, as long as the inputs and its corresponding output are given.

2.3.2. Modeling performance function based on design standards

The bond strength of FRP bars to concrete, illustrated in Table 2, are given by national and international design codes for constructing the PF. Introducing τ_b in Eqn. (3) yields the PF for its anchorage reliability. However, note that some design variables have implicit form in bond strength equations, which is difficult to deal with for the conventional FOSM method. For example, the contribution of the bar diameter to the bond strength is associated with the coefficient K_3 in Eqn. (5), which is equal to 0.8 when $A_b \leq 300 \text{ mm}^2$ and 1.0 for other cases, where A_b denotes the cross sectional area of the bar. It is difficult to solve the partial derivative with respect to d_b due to its implicit form. Similar difficulty are also observed for the concrete cover associated with d_{cs} in Eqns. (5) and (6), and the bar diameter and concrete cover associated with α_1 in Eqn. (7). Note that the bond strength prediction in ACI 440.1R-06 code has an explicit form with respect to design variables, and thus demands less efforts for a reliability analysis.

Table 2 Bond strength predicted by national and international design standards

Design standards	Design equations	Notes
ACI 440.1R-06 [37]	$\tau_b = \sqrt{f'_c} (0.332 + 0.025 \frac{c}{d_b} + 8.3 \frac{d_b}{l_d})$ (4)	d_{cs} : the smaller of the distance from concrete surface to the center of the bar or two-thirds the spacing of the bars being developed (mm);
CSA S806-02 [38]	$\tau_b = \frac{d_{cs} \sqrt{f'_c}}{1.15(K_1 K_2 K_3 K_4 K_5) \pi d_b}$ (5)	K_1 : bar location factor; K_2 : concrete density factor; K_3 : bar size factor; K_4 : bar fiber factor; K_5 : bar surface profile factor.
CSA S6-06 [39]	$\tau_b = \frac{f_{cr}(d_{cs} + K_{tr} E_{FRP} / E_s)}{0.45 \pi d_b K_1 K_5}$ (6)	f_{cr} : the cracking strength of concrete (MPa); K_{tr} : transverse reinforcement index (mm); E_{FRP} : elastic modulus of FRP bar (MPa); E_s : elastic modulus of steel (MPa);
JSCE [40]	$\tau_b = f_{bod} / \alpha_1$ (7)	f_{bod} : designed bond strength of concrete; α_1 : a confinement modification factor.

2.3.3. Modeling performance function based on the ANN and GA

The hybrid ANN-GA strategy is to use the ANN to predict the PF according to the given design variables, and use the GA to evolve the initial weights and biases of the ANN. There is no need to derive the bond strength compared to the PF modeling based on the design standards.

(1) Key issues of the ANN modeling

The architecture of a network has great influence on the prediction of PF. It has been proved that the performance improvements by adding additional hidden layers (second, third or even more) are very small or even worse. A network containing one hidden layer with adequate neurons is capable of approximating any continuous function with satisfactory precision [41-43]. While for the number of neurons in hidden layer, it is usually determined through a number of trials [44, 45]. In addition, according to the discussion in Section 2.2, the design variables do not involve the bar surface, bar position and transverse reinforcement ratio in the input layer, whereas take the bar diameter, concrete cover, development length, concrete compressive strength and bar ultimate strength into account, as shown in Fig. 3.

Fig. 3 Preliminary architecture of the ANN

The data imported from the database to the network is classified into training set, validating set and testing set, respectively. The training data is used for network training by paring a set of inputs with the corresponding expected output. The validating data is used to avoid over-fitting. If the accuracy over the training data yields an increase, but the accuracy over the validating data stays the same or decreases, then over-fitting occurs and training needs to be stopped. The testing data is used to test the final solution that guarantees the predictive capability of the network [46]. In addition, the Levenberg–Marquardt algorithm is taken as a preferred choice for network training [17, 47]. The nonlinear hyperbolic tangent sigmoid transfer function is usually used in the hidden layer while the linear transfer function in the output layer. However, these transfer functions can switch over their positions between the hidden and output layers to achieve best training results.

II) Hybrid ANN-GA modeling

The information exchange between the ANN and GA is implemented through encoding and decoding the chromosome. The synthesis workflow of the ANN-GA model is demonstrated in Fig. 4. Basically, the actual values of the weights and biases are first randomly generated in the ANN program, and each data set corresponds to a solution to the neural network, as shown in Fig. 4 (a). These data sets are then encoded into a group of chromosomes, comprising an initial population in the GA program, as shown in Fig. 4 (b). To evolve the weights and biases, the chromosomes are decoded to calculate their fitness, which is defined as the norm of the PF differences between the predicted vector and expected vector for convenience. Thus, the best chromosome demonstrates the smaller fitness and vice versa. The superior chromosomes have the larger probabilities to participate in the evolutionary operations in terms of selection, crossover and mutations, creating new chromosomes inheriting the excellent genes from their parent ones, as demonstrated in Fig. 4 (c). These new inborn chromosomes are compared with the old ones, and substitute those worse chromosomes in the former population, thereby comprising a new superior population, as shown in Fig. 4 (d). After that, the new population follows the same rules to start a new evolutionary loop until the fitness meet requirement of the allowable error of the prediction.

Fig. 4 Synthesis of workflow of the ANN-GA model

2.3.4. Performance function prediction and validation

Generally, the PF was predicted and validated through two major steps: first, by comparing with the ANN model to demonstrate its optimized effect on the predicted ability; next, by comparing with the model based on design standards to demonstrate its superiority among different modeling methods.

The database created in section 2.2 was used for PF modeling. Out of the total 179 samples, 109 records were randomly selected for the training data, 35 records for the validating data, and 35 records for the testing data. For the ANN-based modeling, the architecture of the network adopting 5-11-1 was proved to yield good results. On the other hand, the parameters of the GA were initialized as shown in Table 3, where the length of chromosome was calculated to 78 based on the architecture of the network. During the evolutions, those excellent chromosomes were selected with a probability of 0.9, and then crossed and mutated with a probability of 0.7 and 0.1, respectively.

Table 3 Parameter initializations for the GA

Parameters	Values
Population size	100
Length of chromosome	78
Maximum number of generations	250
Selection probability	0.9
Crossover probability	0.7
Mutation probability	0.1

Fig. 5 displays the variations of the fitness over the number of evolutions. It is clear that the best fitness decreased as the evolution increased, and leveled off after a threshold near to 200. Thus, the appropriate maximum number of generations of the GA was determined to be 200.

Fig. 5 Variations of the best fitness

The PF modeled with the ANN-GA was first compared to that modeled with the ANN to demonstrate the optimization effect, as shown in Fig. 6. The relative error was defined as the ratio of the difference between the predicted result and experimental result to the experimental result. It can be seen that the errors of the ANN-GA model was limited within the interval of $[-0.05, 0.05]$ for all data sets, while those of the ANN model distributed along $[-0.10, 0.10]$. Thus, the ANN-GA model significantly improved the accuracy of the predicted PF compared with the ANN model.

Fig. 6 Comparisons between the ANN-GA and ANN models

On the other hand, the linear regression analysis of the predicted and target values is illustrated in Fig. 7, comprising of training data, validating data, testing data and all data. It can be seen that the Pearson's correlation coefficient (R-value) was 0.97, 0.95, 0.95 and 0.96 for training data, validating data, testing data and all data, respectively. This indicates that the prediction model of the ANN-GA fitted the experimental results very well. Specifically, the R-value of the training data showed good learning ability for the network to approximate the actual values. And the R-value of the testing data represented that the trained network was competent for generalizing data between the design variables and the PF.

Fig. 7 Regression analyses of data predicted by the ANN-GA model as compared to experimental results

In the following, the PFs modeled with different methods were compared in detail, as shown in Fig. 8. It was observed that the PFs modeled with the ANN-GA exhibited the closest predictions to the experimental results for all

data sets, whereas those modeled based on CSA S6-06 displayed the largest deviations. Meanwhile, the model of ACI 440.1R-06 showed good agreement with the experimental results, performing best among other design standards. The models of CSA S806-02 and JSCE manifested the most conservative results compared to the experimental results. Thus, it is reasonable to use the ANN-GA to model the PF for reliability assessment due to its accuracy as well as its convenience.

Fig. 8 PF calculations

Therefore, these comparisons among different modeling approaches and validations by experimental results further confirm that the PFs generated by the proposed ANN-GA algorithm are viable with applications to the subsequent reliability analysis.

2.4. ANN-based anchorage reliability assessment

2.4.1. ANN-based performance function derivation

As the simplest type of the ANN, the feed-forward neural network with BP algorithm was employed in this study. It consists of the input layer, one or more hidden layers and the output layer, in which each layer has a number of interconnected neurons that send message to each other. Design variables are regarded as the preliminary information to be assigned to the input layer, and then pass through the hidden layer to the output layer. The weights therein are used to measure the contribution that the preceding neuron set to the current one. Biases are added to the sums calculated at each neuron (except input neuron) during the feed-forward process [48, 49]. The working principle of a single neuron unit processor is depicted as shown in Fig. 9.

Fig. 9 Working principle of single neuron

It is assumed that the activation function $A_k(\cdot)$ is applied on all neurons of the k^{th} layer; n_k is the number of neurons of the k^{th} layer; $W_k = [w_{ij}^k]$ is the n_{k-1} by n_k weight matrix between the $(k-1)^{th}$ layer and the k^{th} layer and hence, the weight vector between the neurons of preceding layer and the j^{th} neuron of current layer can be expressed as $W_k^j = [w_{1j}^k, w_{2j}^k, \dots, w_{n_{k-1}j}^k]^T$; and $B_k = (b_1^k, b_2^k, \dots, b_{n_k}^k)^T$ is the bias vector of the k^{th} layer. Then the receive vector of the k^{th} layer is expressed as: $Y_k = (Y_1^k, Y_2^k, \dots, Y_{n_k}^k)^T$, in which the value of the j^{th} neuron Y_j^k is,

$$Y_j^k = \sum_{i=1}^{n_{k-1}} w_{ij}^k x_i + b_j^k. \quad (8)$$

The vector Y_k is transformed to the same dimensional output vector, $Z_k = (Z_1^k, Z_2^k, \dots, Z_{n_k}^k)^T$, as shown below:

$$Z_k = A_k(Y_k) = (A_k(Y_1^k), A_k(Y_2^k), \dots, A_k(Y_{n_k}^k))^T. \quad (9)$$

The partial derivative matrix is by the form:

$$D_{Z_k Y_k} = \frac{\partial Z_k}{\partial Y_k} = \text{diag} \left[\frac{dZ_1^k}{dY_1^k}, \frac{dZ_2^k}{dY_2^k}, \dots, \frac{dZ_{n_k}^k}{dY_{n_k}^k} \right] = \text{diag} [A'_k(Y_1^k), A'_k(Y_2^k), \dots, A'_k(Y_{n_k}^k)]. \quad (10)$$

Specially, for a network with one hidden layer, the receive vector Y_1 and output vector Z_1 between the input layer and hidden layer are illustrated as:

$$Y_1 = W_1^T X + B_1 \quad (11)$$

$$Z_1 = A_1(Y_1) = A_1(W_1^T X + B_1) \quad (12)$$

The receive vector Y_2 and output vector Z_2 between the hidden layer and output layer are derived from:

$$Y_2 = W_2^T Z_1 + B_2 = W_2^T (A_1(W_1^T X + B_1)) + B_2, \quad (13)$$

$$Z_2 = A_2(Y_2) = A_2(W_2^T (A_1(W_1^T X + B_1)) + B_2), \quad (14)$$

where, Z_2 herein is exactly the PF, $g(X)$.

Moreover, the gradient vector of the PF is given as:

$$\nabla g(X) = W_1 D_{Z_1 Y_1} W_2 D_{Z_2 Y_2}. \quad (15)$$

2.4.2. ANN-based FOSM method

The conventional FOSM method is based on a first-order Taylor series approximation of the PF linearized at the point located on the failure surface [50]. The limit state function is:

$$Z = g(X) = 0. \quad (16)$$

If $x^* = (x_1^*, x_2^*, \dots, x_n^*)^T$ is a point located on the limit state surface, which is satisfied with:

$$g(x^*) = 0. \quad (17)$$

Then the PF is approximated by a Taylor series at x^* as expressed:

$$Z = g(x^*) + \sum_{i=1}^n \frac{\partial g(x^*)}{\partial x_i} (X_i - x_i^*) = g(x^*) + (\nabla g(x^*))^T (X - x^*). \quad (18)$$

The mean and standard deviation of the PF herein is:

$$\mu_Z = g(x^*) + \sum_{i=1}^n \frac{\partial g(x^*)}{\partial x_i} (\mu_{X_i} - x_i^*) = g(x^*) + (\nabla g(x^*))^T (\mu_X - x^*), \quad (19)$$

$$\sigma_Z = \sqrt{\sum_{i=1}^n \left[\frac{\partial g(x^*)}{\partial x_i} \right]^2 \sigma_{X_i}^2} = \|\nabla g(x^*) \sigma_X\|. \quad (20)$$

The sensitivity coefficient is defined as:

$$\alpha_X = -\frac{\nabla g(x^*) \sigma_X}{\|\nabla g(x^*) \sigma_X\|}. \quad (21)$$

The reliability index can be gained as follows:

$$\beta = \frac{\mu_Z}{\sigma_Z} = \frac{g(x^*) + (\nabla g(x^*))^T (\mu_X - x^*)}{\|\nabla g(x^*) \sigma_X\|}. \quad (22)$$

For the ANN-based FOSM, the reliability index needs to be solved by iterations, in which the steps are described as follows:

Step 1. Assume that the initial checking point $x^* = \mu_X = (\mu_{X_1}, \mu_{X_2}, \dots, \mu_{X_n})^T$, and $\sigma_X = (\sigma_{X_1}, \sigma_{X_2}, \dots, \sigma_{X_n})^T$;

Step 2. Calculate $g(x^*)$ and $\nabla g(x^*)$, in which $g(x^*)$ can be directly obtained through the network simulation, and $\nabla g(x^*)$ can be calculated according to Eqn. (10) and Eqn. (15);

Step 3. Calculate β according to Eqn. (22);

Step 4. Calculate the new x^* according to the equation below, in which α_X can be derived from Eqn. (21),

$$x^* = \mu_X + \beta \sigma_X \alpha_X; \quad (23)$$

Step 5. Repeat steps 2 through 4 until the difference of $\|x^*\|$ is smaller than a threshold.

2.4.3. ANN-based MCS method

Traditional MCS method is commonly used to solve complex problem involving random variables of known or assumed probability distributions. For the ANN-based MCS, the PF can be easily obtained through network simulation and hence, the failure probabilistic estimated by MCS method is illustrated in the following equation [42, 51],

$$p_f = \int_{g(X) \leq 0} f(X) dx = \int_{-\infty}^{\infty} I[g(X)] f(X) dx = \frac{1}{N} \sum_{i=1}^N I[g(X)], \quad (24)$$

where $f(X)$ is the joint probability density function; $I[g(X)]$ is the indicator function defined as: $I[g(X)] = 1$ when $g(X) < 0$, while $I[g(X)] = 0$ when $g(X) \geq 0$. This direct sampling method of MCS is denoted as MCS-DS.

However, the direct sampling points X mostly locate at the neighborhood of the maximum of joint probability density function, which lead to few occurrences for $g(X) < 0$ when the failure probability is extremely small. Thus, the efficiency and precision of MCS by direct sampling is relatively lower. To overcome such weakness, importance sampling (IS) is introduced accordingly, and denoted as MCS-IS. By modifying Eqn. (24), the failure probability is calculated as follows [13, 51]:

$$p_f = \int_{g(V) \leq 0} \frac{f(V)}{p(V)} p(V) dV = \int_{-\infty}^{\infty} \frac{I[g(V)]f(V)}{p(V)} p(V) dV = \frac{1}{N} \sum_{i=1}^N I[g(V_i)] \frac{f(V_i)}{p(V_i)}, \quad (25)$$

where $p(V)$ is the importance sampling function; and $V = (V_1, V_2, \dots, V_n)^T$ is generated samples according to $p(V)$. Note that the most probable failure point is the design checking point x^* . Thus, the new variable V can use x^* as the mean of the generated samples, specifying $\mu_V = x^*$ and $\sigma_V = \sigma_X$. The procedures for the MCS-IS method is detailed in the following steps.

Step 1. Calculate the design checking point x^* by the aforementioned steps in the ANN-based FOSM method;

Step 2. Generate samples of the design variables V , in which $\mu_V = x^*$ and $\sigma_V = \sigma_X$;

Step 3. Calculate the sum of the probability density function (PDF) of all samples, in which the PDF of each design variable is equal to $f(V_i)/p(V_i)$;

Step 4. Calculate the failure probability according to Eqn. (25).

3. Implementation to GFRP bar anchorage reliability assessment

3.1. Target reliability index

The limit state for anchorage of GFRP bars in concrete is defined as the state that the bond stress τ achieves to the maximum bond strength τ_b as the bar stress σ at loaded end reaches the ultimate strength f_u , i.e., $\sigma = f_u$ and $\tau = \tau_b$ occur simultaneously. The corresponding probability of the anchorage limit state is denoted as:

$$p_{fa} = P(\sigma = f_u, \tau = \tau_b) = P(\sigma = f_u) \cdot P(\tau = \tau_b | \sigma = f_u) = p_f \cdot p_{fc}, \quad (26)$$

where, p_{fa} is the failure probability of anchorage; p_f is the failure probability of bar stress reaches the maximum; p_{fc} is the conditional failure probability of the bond stress reaches the maximum given that bar stress has reached the maximum. Note that the bar stress at loaded end is determined based on the concrete flexural capacity of normal section, the reliability index β_f and its corresponding p_f can be determined according to the suggestions by ACI 440.1R-06 and Szerszen et al. [37, 52], as shown below:

$$\beta_f = 3.5 \text{ and } p_f = 2.326 \times 10^{-4}. \quad (27)$$

Moreover, it is necessary to stipulate the reliability index of anchorage relatively higher than that of both strength limit state and serviceability limit state for higher reliability. Thus, β_{fa} is raised to an upper level, and its corresponding p_{fa} are shown below:

$$\beta_{fa} = 4.0, \text{ and } p_{fa} = 3.167 \times 10^{-5}. \quad (28)$$

Introducing Eqns. (27) and (28) back into equation (26) yields

$$\beta_{fc} = 1.098, \text{ and } p_{fc} = 1.362 \times 10^{-1}, \quad (29)$$

which means that in order to make $\beta_{fa} = 4.0$, it is necessary to attach $\beta_{fc} = 1.098$ on the basis of $\beta_f = 3.5$. β_{fc} is the target reliability index for determining the development length of GFRP bars to concrete.

3.2. Development length estimation

The development lengths in the literature are relatively smaller than those should be for sufficient anchorage to concrete, leading to most points calculated by Eqn. (3) fall into the negative domain. This further results in the reliability index to be negative. Therefore, statistical parameters of the development length cannot be directly used for anchorage reliability analysis. In order to apply the proposed ANN-based methods to reliability analysis, it is necessary to recalculate the development length. For another, some design variables contributing to bond strength have no explicit expressions in both Canadian and Japanese design standards, and also had relatively larger difference between the test results compared with ACI 440.1R-06. Thus, it is reasonable to employ ACI 440.1R-06 for the development length estimation. The target reliability index $\beta_{fc} = 1.098$ is used as the terminating condition for iterations.

Fig. 10 Development length calculation based on the targeted reliability index

The flow chart of the calculation is illustrated in Fig. 10. Firstly, the means of the design variables of d_b , c , l_d , f'_c and f_u adopted the values listed in Table 1. Meanwhile, in order to reduce the discrete range of the design variables, standard deviation is assumed to be $\sigma = 0.1 \cdot \mu$. Next, an array was created for the storage of development length with an increment with 0.01 mm. Meanwhile, the target reliability index and error tolerance were initialized for subsequent iterations. Thereafter, reliability analysis was carried out based on the steps specified in the ANN-based FOSM method. The development length kept increasing until the reliability index was larger than

the target reliability index. Finally, the estimated development length was calculated to be 1133.05 mm with the corresponding reliability index of 1.098. The mean and standard deviation of the development length are $\mu_{l_d} = 1133.05$ and $\sigma_{l_d} = 0.1 \cdot \mu_{l_d} = 113.31$ respectively.

3.3. Reliability index estimation

3.3.1. Performance function modeling with the ANN-GA model

The ANN-based methods are used to estimate the anchorage reliability index of GFRP bars to concrete. It is assumed that all the design variables follow the normal distribution. Both mean and St. of the development length adopted the new calculated values $\mu_{l_d} = 1133.05$ and $\sigma_{l_d} = 113.31$. Meanwhile, the means and St. of the other design variables followed the same rule as that of section 3.2, in which $\sigma = 0.1 \cdot \mu$. All information is summarized in Table 4. In addition, for the randomly generated variables, there are no test results that can be used as the corresponding targets. Considering that ACI 440.1R-06 display better performance in the aforementioned discussions, it was reasonable to use it to calculate the PF as the target output of the network. Also, the nonlinear transfer function was used in the hidden layer, and the linear transfer function in the output layer.

Table 4 Corrected statistical characteristics of design variables

Design variables	Distribution type	Mean	Standard deviation
d_b	Normal	18.98	1.90
c	Normal	63.43	6.34
l_d	Normal	1133.05	113.31
f'_c	Normal	34.00	3.40
f_u	Normal	648.14	64.81

One hundred samples were generated for networking learning, in which the training, validating and testing sets account for 60%, 20% and 20% respectively. As demonstrated in Fig. 11, the training data and validating data displayed the PF predicted by the ANN-GA in solid lines and the target PF in dash lines. Clearly, the results of the ANN-GA model matched well with those calculated by ACI 440.1R-06, with less than 0.01% difference. Meanwhile, for the testing data, the predicted and actual output differ little, with the maximum difference of 0.03% and hence, the network is capable of predicting output accurately according to the design variables that conforms to the respective probabilistic distributions.

Fig. 11 Training and validating of the ANN-GA model

3.3.2. ANN-based FOSM method

For the ANN-based FOSM method, the key step is to derive both $g(X)$ and $\nabla g(X)$ from the network. The calculation procedures were detailed in the following steps. First, assigning the means of design variables to the initial checking point x^* ,

$$x^* = \mu_X = (\mu_{X_1}, \mu_{X_2}, \dots, \mu_{X_n})^T = (18.98, 63.43, 1133.05, 34.00, 648.14)^T \quad (30)$$

Also, the vector of the standard deviations of design variables was denoted as:

$$\sigma_X = (1.90, 6.34, 113.31, 3.40, 64.81)^T \quad (31)$$

Next, the weights of the input layer (W_1) and biases from the input layer to the hidden layer (B_1), and the weights of the hidden layer (W_2) and biases from the hidden layer to the output layer (B_2) were derived from the network, as shown in the following:

$$W_1 = \begin{pmatrix} -0.8537 & 0.2486 & -0.8818 & -0.3274 & 1.4514 \\ 0.0778 & 0.9255 & -0.4445 & -0.2479 & 0.5890 \\ 0.5801 & -0.3983 & 0.3394 & -0.3280 & -0.0382 \\ -0.5762 & -0.1051 & 0.5454 & -0.0512 & -0.5662 \\ -0.2205 & 0.2168 & -0.4198 & -0.3398 & 0.7984 \\ -0.2195 & 1.1514 & -0.5824 & 0.0426 & 0.5410 \\ 0.1619 & 0.8293 & 0.7387 & -1.8839 & 1.7730 \\ 0.4595 & 0.0687 & 0.7245 & -0.4512 & 1.1714 \\ -0.2387 & 0.1005 & 0.7975 & 0.2413 & 0.1357 \\ -0.8262 & -0.3569 & 1.5100 & -0.0667 & 1.2506 \\ -1.3065 & 1.2610 & -0.5053 & -0.0474 & 0.4526 \end{pmatrix}, B_1 = \begin{pmatrix} 1.5615 \\ -0.6552 \\ -0.7526 \\ 0.5366 \\ 0.1107 \\ -0.5510 \\ 0.3937 \\ 1.5450 \\ -0.2673 \\ -1.4895 \\ 1.0291 \end{pmatrix}. \quad (32)$$

$$W_2 = \begin{pmatrix} -0.3038 \\ -0.4138 \\ -0.6789 \\ 0.7506 \\ -0.6196 \\ 0.4053 \\ 0.0205 \\ 0.0776 \\ 0.4045 \\ -0.0041 \\ 0.0396 \end{pmatrix}^T, B_2 = (-0.6572). \quad (33)$$

By substituting Eqn. (32) into Eqn. (11) and Eqn. (12), Y_I and Z_I were calculated as follows:

$$Y_1 = \begin{pmatrix} 1.8412 \\ -0.5242 \\ -1.1639 \\ 0.2784 \\ 0.1563 \\ -0.2057 \\ -1.4637 \\ 0.1131 \\ -1.1532 \\ -3.6431 \\ 1.4214 \end{pmatrix}, Z_1 = \begin{pmatrix} 0.9509 \\ -0.4809 \\ -0.8223 \\ 0.2715 \\ 0.155 \\ -0.2029 \\ -0.8984 \\ 0.1126 \\ -0.8188 \\ -0.9986 \\ 0.8899 \end{pmatrix}. \quad (34)$$

By substituting Eqn. (34) into Eqn. (13), Y_2 was calculated to be (-0.4649). Meanwhile, the partial derivatives of the nonlinear transfer function and linear transfer function were deduced according to Eqn. (10), as shown below:

$$D_{Z_1 Y_1} = \begin{pmatrix} 0.0958 \\ 0.7687 \\ 0.3238 \\ 0.9263 \\ 0.9760 \\ 0.9588 \\ 0.1929 \\ 0.9873 \\ 0.3295 \\ 0.0027 \\ 0.2081 \end{pmatrix}, D_{Z_2 Y_2} = 1 \quad (35)$$

Thus, upon substitution of Eqn. (35) into Eqn. (15), the solution of the gradient of the PF can be deduced, as shown below:

$$\nabla g(x^*) = (-0.4868 \quad 0.0616 \quad 0.6598 \quad 0.3367 \quad -0.7688)^T \quad (36)$$

Since $\nabla g(X)$ is the normalized result, the actual value can be derived from inversing normalization, and α_X was obtained according to Eqn. (21) as shown below:

$$\alpha_X = (-0.0002 \quad -0.0024 \quad -0.9780 \quad -0.0007 \quad -0.2087)^T \quad (37)$$

Then the reliability index β is deduced according to Eqn. (22),

$$\beta = 0.2662 \quad (38)$$

After that, the first new x^* was calculated according to Eqn. (23),

$$x^* = (19.0000 \quad 0.0634 \quad 1.1589 \quad 0.0340 \quad 0.6442)^T \quad (39)$$

It needs to take a number of iterations until the norm of the difference between the current and last x^* is smaller than the allowable error. The final reliability index was calculated to be $\beta = 1.098$, and the final checking point was:

$$x^* = (20.07 \quad 62.81 \quad 1071.78 \quad 32.76 \quad 691.02)^T. \quad (40)$$

It is clear that β exactly coincides with that calculated by ACI 440.1R-06.

3.3.3. ANN-based MCS method

One hundred thousand samples of the each design variable were generated according to the statistical characteristics listed in Table 4, and were used to form a matrix that would be feed into the trained network as the input vectors. By using the MCS-DS method, the failure probability was easily obtained according to Eqn. (23), in which the samples of the PF less than zero were counted. The final solution was: $p_f = 0.134$, and the corresponding reliability index $\beta = 1.106$. While for the MCS-IS method, by using the x^* in Eqn. (24), the PDF of all samples was calculated according to $f(V_i)/p(V_i)$. Then the final solutions were deduced as: $p_f = 0.135$, and the corresponding reliability index $\beta = 1.105$. Thus, it can be observed that the relative errors of reliability index between the ANN-based MCS-DS and MCS-IS and ACI 440.1R-06 were 0.7% and 0.6%, respectively.

4. Designed graphical user interface (GUI) system for FRP bar anchorage reliability assessment

A GUI toolbox in Matlab environment was developed for both development length estimation and the ANN-based reliability analysis, as shown in Figs. 12 and 13. The development length is predicted based on the target reliability index as long as the means and standard deviations of design variables are known. The computational kernel follows the principles demonstrated in Fig. 10. Fig. 13 displays the ANN-based reliability analysis, including the ANN-based FOSM, MCS-DS and MCS-IS methods. It mainly consists of five toolbars. The upper toolbar was used for statistical characteristics inputs, referred to preprocessor. The two toolbars located in the middle window were used for parameter settings with regard to the ANN and GA respectively. After running of the program, results are directly plotted from the buttons located at lower left side. Reliability index using the ANN-based FOSM, MCS-DS and MCS-IS methods will be generated for users.

The screenshot shows a MATLAB GUI titled "Cal Development Length". It is organized into several sections:

- Statistical Parameters:**
 - Bar Diameter (db):** Distribution Type: Normal, mean: 18.98 mm, st: 1.90 mm.
 - Concrete Cover (c):** Distribution Type: Normal, mean: 63.43 mm, st: 6.34 mm.
 - Compressive Strength of Concrete (fc'):** Distribution Type: Normal, mean: 34.00 MPa, st: 3.40 MPa.
 - Tensile Strength of Bar (fu):** Distribution Type: Normal, mean: 648.14 MPa, st: 64.81 MPa.
- Solution:**
 - Control:** Method: FOSM, DT of Ld: Normal, Target Beta: 1.098.
 - Solve:** Run, Clear buttons.
- Results:**
 - Development length:** Ld = 1133.05 mm.
 - Reliability index:** Beta = 1.09802.

Fig. 12 GUI for Development length estimation

Fig. 13 GUI for the ANN-based reliability analysis

5. Conclusions and future work

This paper introduced a new strategy for the ANN-based anchorage reliability assessment of GFRP bars to concrete. Some conclusions can be drawn as follows:

1) The proposed hybrid modeling methodology integrates the respective superiorities of the nonlinear mapping ability of the ANN and global searching ability of GA. It provides an effective way to approximate the PF and solve its partial derivatives in terms of the design variables, yielding higher accuracy over conventional methods. The relative errors between the predicted and actual values of the ANN-GA model reduced within $\pm 5\%$. Moreover, the PFs calculated based on ACI 440.1R-06 were observed to be closer to the test results than those calculated based on other codes, where the Canadian design code CSA S6-06 exhibited the largest deviations.

2) Both analytical formulations and numerical implementations of the ANN-based GFRP bar anchorage reliability analysis were presented in detail. A reasonable targeted reliability index for determining the development length of GFRP bars to concrete was demonstrated to be 1.098, which ensures that the anchorage failure would not occur before the flexural failure during structural service life. Note that the reliability index predicted by the ANN-based FOSM method is 1.098, 1.106 by the MCS-DS method, and 1.105 by the MCS-IS method, respectively. The proposed strategy in this study can also be used to assess both reliability index and required development length

for five given design variables of the PF. In addition, the designed GUI system was developed under a Matlab environment based on the proposed modeling strategy, which can be directly applied in practical use.

3) Design variables considered in the ANN inputs did not cover surface treatment, bar position and transverse reinforcement ratio due to the limited experimental data. With sufficient training data, such variables would be included as the inputs of the neural network for anchorage reliability prediction.

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