“EQUIVALENT” PERMEABILITY AND FLOW IN COMPLIANT POROUS MEDIA

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ABSTRACT

Resin flow modeling for liquid composite molding processes is generally based on assumptions of rigid porous media. This is invalid for process variations utilizing compliant mold. Yet the models built on rigid porous media assumption are used with some success in analyzing such infusions.

Previous work showed that for certain porous media the one dimensional flow patterns are similar to those in rigid porous media and the deformation effects can be included in a scaling factor for permeability.

This note analyzes the one-dimensional linear and radial flows in porous media with generic constitutive relations between resin pressure, thickness and permeability. It shows that as long as the deformation remains moderate, the effect of deforming porous medium may be incorporated in a single scaling factor for material permeability. This scaling factor depends on material and applied injection pressure, but does not change with time, flow-front position or type of infusion (linear or radial).

INTRODUCTION

Resin flow modeling for liquid composite molding processes is usually based on assumptions of rigid porous media. The state of fiber bed is not supposed to change with resin pressure. This assumption provides a simple linear description [2, 3] and allows for robust and fast solution methods [4]. These methods in turn allow one to optimize process design and even introduce on line control [5] that would be impractical with slower, more complex models.

Unfortunately, this assumption of rigid porous media strictly holds for processes that use rigid stationary mold such as Resin Transfer Molding (RTM). The level of pre-stressing applied to the fiber bed in this process, necessary to prevent fiber wash-out, makes deformation changes due to resin pressure negligible. Once a compliant mold or vacuum bag is used for processes such as in Vacuum Assisted RTM (VARTM), the developing resin pressure field leads to thickness changes of porous media rendering the rigid porous media assumption invalid as fiber content changes [6]. But while there have been attempts to address the compliance of the fiber bed in the numerical flow model [7-10], none of these have come close to the versatility and performance of RTM filling simulations.

As a result, process designers have been using the RTM model with a great deal of success – to simulate the flow in VARTM infusions. The practical justification to this approach comes from several sources:

1. Experimental permeability measurements in VARTM settings correlate well with constant permeability and thickness flow patterns, although the permeability value differs from the one measured in RTM settings
2. The approach has been used for some time without challenge. When compared, the predicted flow patterns and fill times reasonably match experimental values if the permeability was properly established, for example measured in a VARTM experimental setting [11-14].

3. Lopatnikov et. al showed analytically that flow patterns in one-dimensional flow of linearly elastic material to behave as rigid porous media [1].

Lopatnikov's work provides the solution for linear one-dimensional flow, including pressure distribution, but it is limited by the Kozeny-Carman permeability relation and, somewhat impractical applied deformation model (linearly elastic). Extension is called for to verify that this behavior extends to other deformation models.

However, to establish the apparent permeability for one-dimensional experiment – linear or radial – one does not need to have a complete solution. Assuming that the flow is single-scaled and no unsaturated regions remain behind the flow-front, we know that the flow through any location remains constant (first integral). Utilizing this fact, we can solve for advancing flow under general constitutive relations that describe the thickness and permeability change.

Although one will not be able to recover the complete pressure field from this solution, but will be able to find the equivalent permeability that one can use in the RTM model to predict the flow patterns. The equivalent permeability will be determined from the relation between flow-front position and time.

ASSUMPTIONS

We will assume that the process is adequately described by Darcy's law and that quasi-steady solution may be applied. Multi-scale effects (delayed micro-pore saturation) are negligible. The former is generally true, the latter may or may not be true, but its accuracy can be generally verified for individual cases. For radial infusion model we will assume that the material is isotropic. We assume no significant flow in the through-the-thickness direction, meaning that the solution is not quite applicable for scenarios in which flow enhancement layer is used if flow-front lags significantly on the tool side. We will assume that the infusion pressure p_{in} remains constant throughout the process and so does the fluid viscosity η . In almost all cases this is true. As far as preform compaction description goes, we will assume that both the material

thickness h , porosity ϕ and in-plane permeability K can be described as dependent on resin pressure p :

$$\begin{aligned} K &= K(p) \\ \phi &= \phi(p) \\ h &= h(p) \end{aligned} \quad (1)$$

This rules out viscous component of compaction behavior but it would still allow plastic behavior and even hysteresis behavior due to “debulking” as long as the loading (fluid pressure increase) is monotonous, increasing from zero to whatever final value it attains (usually p_{in}).

Practically, the application of “debulking”, repetitive loading and unloading cycles by vacuum cycling, would mean that the relations (1) will change with each debulking cycle. Thus the dependency (1) will be different after successive loading cycles. This would make it necessary to characterize material in each such case of interest under monotonously increasing fluid pressure.

FLOW ADVANCEMENT ANALYSIS FOR LINEAR FLOW

Geometry of 1-D resin linear flow experiment is simple (Figure 1). Resin is infused from one end of a rectangle under pressure p_{in} and its progress $L(t)$ is monitored in time.

Solving for pressure field under usual [2] quasy-steady assumptions, the continuity equation for 1D linear flow has a very simple form

$$\frac{d}{dx} (h(p) \cdot \langle v \rangle) = 0 \quad (2)$$

We assumed $\partial p / \partial t$ and consequently the accumulation part $\partial h / \partial t$ to be negligible. In our case the justification of this assumption has some limits as scaling analysis shows that the ratio of accumulation vs. convective terms will scale with $2 \cdot \Delta h / h_0$ where Δh is the thickness change and h_0 is the original thickness. Thus (2) can be considered accurate for small thickness changes

and bearable for usual values in the analyzed process (10%). It would be inadmissible for compliant materials.

And the volume averaged velocity $\langle v \rangle$ can be evaluated from resin pressure using Darcy's law:

$$\langle v \rangle = -\frac{K(p)}{\eta} \frac{dp}{dx} \quad (3)$$

Here η , is the resin viscosity. Integrating Eq. (2) and combining it with Eq.(3) will result in

$$C = \frac{K(p).h(p)}{\eta} \frac{dp}{dx} \quad (4)$$

The value of constant C (flow per unit width) can be obtained by separating the equation and integrating the position x from 0 to L (flow-front position) and pressure p from p_{in} to 0:

$$C = \frac{1}{L.\eta} \int_{p_{in}}^0 K(p).h(p)dp \quad (5)$$

Now that C is known, we can write the (quasi-steady) equation for flow front advancement. The pressure at flow-front is 0 and consequently permeability and porosity will be evaluated for this pressure

$$\frac{dL}{dt} = -\frac{K(0)}{\eta.\phi(0)} \frac{dp}{dx} \Big|_{x=L} \quad (6)$$

And we can evaluate the derivative on right side of (6) using Eqs. (4) and (5) to obtain

$$\frac{dL}{dt} = \frac{K(0)}{\eta.\phi(0)} \frac{\eta}{K(0).h(0)} \frac{1}{L.\eta} \int_0^{p_{in}} K(p).h(p)dp \quad (7)$$

Or, after algebraic simplification it can be recast as

$$L. dL = \frac{1}{\phi(0).\eta} \frac{\int_0^{p_{in}} K(p).h(p)dp}{h(0).p_{in}} . p_{in} dt \quad (8)$$

Note that as long as the inlet pressure does not change, the value of integral depends neither on time nor on flow-front position and is *constant*. This result can be compared with the well-known similar equation for rigid porous media. Let us write it for fully compacted state ($p=0$):

$$L. dL = \frac{1}{\phi(0).\eta} K(0).p_{in} dt \quad (9)$$

Quite obviously, the deformable porous media flow advance exhibits the same behavior, i.e., flow with constant effective permeability K_{eff} which by comparing Eq.(8) to Eq.(9) is

$$K_{eff} = \frac{\int_0^{p_{in}} K(p).h(p) dp}{h(0).p_{in}} \quad (10)$$

From this equation, for any particular material with known constitutive properties the value of K_{eff} can be evaluated.

FLOW ADVANCEMENT FOR RADIAL FLOW

Again, the geometry of radial flow experiment – assuming isotropic material - is familiar (Figure 2)

In this case we will track the radius of flow-front $R(t)$. The continuity equation in radial coordinates is

$$\frac{d}{dr} (r.h(p). \langle v \rangle) = 0 \quad (11)$$

Darcy's law only changes the independent coordinate to r

$$\langle v \rangle = - \frac{K(p)}{\eta} \frac{dp}{dr} \quad (12)$$

The constant C describes the flow rate through any radius up to $R(t)$ and it will be

$$C = r. \frac{K(p).h(p)}{\eta} \frac{dp}{dr} \quad (13)$$

And the spatial integration to determine C will go from injection radius R_{in} to R :

$$C = \frac{1}{\log(R/R_{in}).\eta} \int_{p_{in}}^0 K(p).h(p) dp \quad (14)$$

Again, the integral itself does not depend on time or flow-front position, only on the constitutive models and injection pressure. The flow-front motion will be described by a similar equation

$$\frac{dR}{dt} = - \frac{K(0)}{\eta.\phi(0)} \frac{dp}{dr} \Big|_{r=R} \quad (15)$$

And equations (14) and (15) will provide us with separable governing equation

$$\frac{dR}{dt} = \frac{K(0)}{\eta \cdot \phi(0)} \frac{\eta}{K(0) \cdot h(0)} \frac{1}{R \cdot \log(R/R_{in}) \cdot \eta} \int_0^{p_{in}} K(p) \cdot h(p) dp \quad (16)$$

Or, after algebraic simplification it will result in

$$R \cdot \log(R/R_{in}) dR = \frac{1}{\phi(0) \cdot \eta} \frac{\int_0^{p_{in}} K(p) \cdot h(p) dp}{h(0) \cdot p_{in}} \cdot p_{in} dt \quad (17)$$

As before, comparing this equation with the one for rigid porous media in initial state (i.e., $p=0$):

$$R \cdot \log(R/R_{in}) dR = \frac{1}{\phi(0) \cdot \eta} K(0) \cdot p_{in} dt \quad (18)$$

We arrive at equation for effective permeability (10).

FLOW RATE SCALING

For linear flow, the flow rate entering the preform (per unit width) will be

$$Q = - \left. \frac{K(p_{in}) h(p_{in})}{\eta} \frac{dp}{dx} \right|_{x=0} \quad (19)$$

Combining (4) and (5) to eliminate C we obtain the value of pressure derivative at location x as

$$\frac{dp}{dx} = \frac{1}{L} \frac{\int_0^{p_{in}} K(p) \cdot h(p) dp}{K(p(x)) \cdot h(p(x))} \quad (20)$$

using $x=0$ and $p(0)=p_{in}$ we obtain

$$Q = \frac{h(p_{in})}{\eta} \frac{\int_0^{p_{in}} K(p) \cdot h(p) dp}{h(p_{in}) \cdot p_{in}} \cdot \frac{p_{in}}{L} \quad (21)$$

For rigid material with permeability K_{effQ} and thickness $h(p_{in})$ pressure gradient will be constant and (19) becomes

$$Q = \frac{h(p_{in})}{\eta} K_{effQ} \cdot \frac{p_{in}}{L} dt \quad (22)$$

Comparing these two results we will obtain:

$$K_{effQ} = \frac{\int_0^{p_{in}} K(p) \cdot h(p) dp}{h(p_{in}) \cdot p_{in}} \quad (23)$$

Similarly, doing the same exercise for radial flow, we will have

$$Q = -2\pi R_{in} \frac{K(p_{in})h(p_{in})}{\eta} \frac{dp}{dr} \Big|_{r=R_{in}} \quad (24)$$

Pressure derivative will be obtained from (13) and (14) by eliminating C to obtain

$$Q = 2\pi \frac{h(p_{in})}{\eta} \frac{\int_0^{p_{in}} K(p) \cdot h(p) dp}{h(p_{in}) \cdot p_{in}} \cdot \frac{p_{in}}{\log(R/R_{in})} dt \quad (25)$$

Comparing this to the solution for rigid material leads to the same “equivalent” permeability for flow rate K_{effQ} (Eq. (23)). The relation between two “equivalent permeability” values is simply a ratio of inlet and flow-front thicknesses and reflects the different reference thickness at inlet:

$$\frac{K_{eff}}{K_{effQ}} = \frac{h(p_{in})}{h(0)} \quad (26)$$

DISCUSSION AND CONCLUSIONS

The results above justify to a significant degree the heuristic approach of using RTM model and permeability measured in VARTM experiment to predict resin flow advancement into compliant porous media. The restriction introduced by the omitted term in Eq. (2) is usually insignificant for materials used in composite processing. Thus, under rather generic assumptions, the flow advancement behavior in compliant porous media corresponds to that in rigid media with effective permeability scaled by Equation (10). Moreover, the scaling factor is demonstrated to be *the same* for both radial and tangential flow. Rigorously, it shows only that the method is correct for the two simplest possible flow cases, but it significantly expands the results of Lopatnikov [1]. Further analysis would require numerical work but the extrapolation of these results to generic infusion pattern seems logical. No generic proof is attempted here and numerical approach may be needed to verify such conclusion.

The scaling factor above differs from scaling factor needed to obtain proper incoming flow rate Eq. (23). That is direct result of changing thickness and the shift between the

two is directly related to fabric compliance. This “flow-based” scaling factor is same for linear and radial flow again.

The physics behind the integral within the scaling factor Eq. (10) suggests that any change in infusion pressure under VARTM conditions will lead to non-proportional result change in flow-front positions. This is because while the effective permeability does not change in flow-front positions. This is because while the effective permeability does not depend on location or time, it depends on the inlet pressure, which is related to the maximal change in thickness as in [1]. If inlet pressure changes, the integration bounds in Eq. (10) or (23) will change, changing the equivalent permeability. Eq. (10) or (23) allows one to relate that change to material compliance in quantitative manner.

For permeability measurements, the results above explain the dual “RTM” and “VARTM” permeability differences. They may also help to quantify this discrepancy. Furthermore, the scaling factor Eq. (10) can be used to relate the variability in VARTM measured permeability to the variability of deformation behavior as described in constitutive Eq. (1).

Full analysis of these constitutive equations (1) is beyond the scope of this note, but one should bear in mind that fabric compaction curves are not really elastic. Even for monotonous loading when “elastic” curve can be used, the curve will reflect the loading history, such as “debulking” applied to fabric preform. Thus, change in manufacturing practices or experimental procedure can, somewhat contra-intuitively lead to change of equivalent permeability.

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Figure(s)

Figure 1: 1-D linear flow experiment

Figure 2: 1-D radial flow experiment for isotropic material