Overview article

Failure of Metals I – Brittle and Ductile Fracture

A. Pineau^{1*}, A.A. Benzerga^{2,3,4*}, T. Pardoen⁵

¹ Centre des Matériaux, Mines ParisTech, UMR CNRS 7633, B.P. 87, 91003 Evry, France

² Department of Aerospace Engineering, Texas A&M University, College Station, TX 77843-3141, USA

³ Department of Materials Science and Engineering, Texas A&M University, College Station, TX 77843

⁴ Center for intelligent Multifunctional Materials and Structures (CiMMS)

College Station, TX 77843, USA

⁵ Institute of Mechanics, Materials and Civil Engineering, Université catholique de Louvain

1348 Louvain-la-Neuve, Belgium

December 18, 2015

Abstract

This is the first of three overviews on failure of metals. Here, brittle and ductile failure under monotonic loadings are addressed within the context of the *local approach to fracture*. In this approach, focus is on linking microstructure, physical mechanisms and overall fracture properties. The part on brittle fracture focuses on cleavage and also covers intergranular fracture of ferritic steels. The analysis of cleavage concerns both BCC metals and HCP metals with emphasis laid on the former. After a recollection of the Beremin model, particular attention is given to multiple barrier extensions and the crossing of grain boundaries. The part on ductile fracture encompasses the two modes of failure by void coalescence or plastic instability. Although a universal theory of ductile fracture is still lacking, this part contains a comprehensive coverage of the topic balancing phenomenology and mechanisms on one hand and microstructure-based modeling and simulation on the other hand, with application examples provided.

* Corresponding authors

A. A. Benzerga: benzerga@tamu.edu;

A. Pineau: andre.pineau@mines-paristech.fr

Contents

Preface

4

1 Introduction

5

2 Cleavage Fracture

6

2.1	Preliminary remarks	6
2.2	Mechanisms of stress intensification	8
2.3	Multiple barriers models and crossing of grain boundaries	9
2.3.1	Basic model	9
2.3.2	Dynamic behavior	10
2.3.3	Grain boundary crossing	11
2.4	Statistical modeling – Application to fracture toughness	12
2.4.1	Beremin model and Weibull stress	12
2.4.2	Beremin model – Effect of plastic strain	15
2.4.3	Application of Beremin model to the prediction of fracture toughness	16
2.5	Cleavage in other BCC metals	17
2.6	Cleavage in HCP metals	18
2.6.1	Cleavage plane	18
2.6.2	Cleavage fracture of zinc	18

3 Introduction to intergranular fracture in ferritic steels

19

Summary

20

4 Ductile Fracture

21

4.1	Phenomenology	21
4.1.1	Fracture path	21
4.1.2	Measures of fracture	22
4.1.3	Measures of stress state	24
4.1.4	Fracture locus	26
4.1.5	Crack growth resistance curves	27
4.1.6	Shear failure versus failure in shear	28
4.1.7	Dynamic behavior	28
4.1.8	Effects of temperature and strain rate	29
4.1.9	Size effect	30
4.2	Mechanisms	31
4.2.1	Void nucleation	31
4.2.2	Void growth	32
4.2.3	Void distortion	33
4.2.4	Void coalescence	34
4.2.5	From damage to cracking	35
4.2.6	Mechanisms of shear failure	35
4.2.7	Mechanisms at the crack tip	36
4.2.8	Dynamic aspects	38

Version 8	Overview Article	3
4.2.9	Effects of temperature and strain rate	39
4.3	Failure by Microvoid Coalescence	39
4.3.1	Ordered void distributions	40
4.3.2	Disordered void distributions	41
4.4	Failure by Instability	43
4.4.1	Crack-free specimens	43
4.4.2	Precracked specimens	44
4.5	Predictive Modeling	44
4.5.1	Damage nucleation	44
4.5.2	Structure of constitutive relations	47
4.5.3	Growth and distortion	50
4.5.4	Coalescence	57
4.5.5	Dynamic aspects	62
4.6	Applications	63
4.6.1	Uncoupled models	63
4.6.2	Material parameter identification	64
4.6.3	Intrinsic fracture loci	65
4.6.4	Fracture in notched bars	68
4.6.5	Size effect	69
4.6.6	Fracture toughness	70
5	Concluding Remarks	73

Preface

It is a great pleasure for André Pineau to have this opportunity to thank the Editors of *Acta Materialia* for awarding him the 2013 *Acta Materialia* Gold Medal. Professor Pineau is particularly grateful to Professors Thaddeus Massalski of Carnegie Mellon University and Carolyn Hansson of the University of Waterloo, former and current Executive Secretary of the Board of Governors of *Acta Materialia*, respectively. Professor Hansson presented him with the award at a special symposium on “Materials Fundamentals of Fatigue and Fracture”, organized as part of the MRS Fall Meeting of the Materials Research Society (MRS) in December 2013. This symposium was organized by Professor A.A. Benzerga from Texas A&M University, Professor E.P. Busso, at that time at Ecole des Mines de Paris and now at ONERA, Professor D.L. McDowell from the Georgia Institute of Technology and Professor T. Pardoen from the Université catholique de Louvain in order to honor André Pineau. Many of Professor Pineau’s former and current students and colleagues attended this symposium and the award ceremony, which was greatly appreciated by the organisers and the laureate. As Professor Pineau stated in his acceptance speech, an award of this type is above all a recognition of the work and contributions made by many of his former students and collaborators throughout his career.

After this MRS Symposium and discussions with *Acta Materialia*’s Editorial Board, it was decided that three *Acta Materialia* papers would be submitted to cover the main symposium topics and offer to the community an overview of the current state of the art in the field of the “local approach to fracture”, as applied to metals. This approach promoted by Professor Pineau throughout his career aims at looking at fracture from the viewpoint of the physical mechanisms of failure in order to link metallurgy aspects, microstructure, fracture properties and structural integrity assessment procedures. A. Pineau would like to thank also Professor Christopher Schuh who has been the *Acta Materialia* coordinating editor for these papers.

Overview I deals with ductile and brittle (cleavage and intergranular) fracture. These two modes of failure are treated separately. An attempt is made to deal with a wide range of materials, including ferritic steels and BCC metals (Mo, Nb), HCP metals (Zn, Mg), aluminum alloys and titanium alloys. In brittle fracture, emphasis is placed not only on micro-mechanisms but also on the sources of scatter and the size effect. A broad overview of the recent developments in the area of ductile fracture is presented. The emphasis is also laid on the micro-mechanisms of ductile failure including the growth and coalescence of cavities, often initiated from second phase particles. Recent advances in the development of constitutive equations coupling the details of damage evolution with the mechanical behavior of materials are presented. The development of damage tolerant microstructures is essential for engineering applications. For a long time this research activity has remained essentially empirical. A better knowledge of the micro-mechanisms of failure has largely contributed to the birth of new methodologies for evaluating the ductility and the fracture toughness of metals, in particular for brittle cleavage fracture.

Overview II focuses on the very important mode of failure that is fatigue. The overview is structured in two parts: Part I considers the development and early propagation of cracks in terms of the basic damage mechanisms and the relative size of the crack with respect to microstructural feature(s). In Part II, emphasis is laid on Ni-based superalloys and the interaction between oxidation, creep and microstructure (including coatings) in the formation

and propagation of cracks. This part of the overview addresses both experimental and modeling aspects.

In Overview III, an attempt is made to describe the damage micro-mechanisms in new nano-structured materials and ultrafine grained metals. After a brief presentation of the various ways used to produce these materials and of the specific deformation mechanisms controlling their high strength, emphasis is laid on the inherent low ductility or/and low cracking resistance in both bulk ultra-fine grained and nanocrystalline metals, some nano-twinned metals, thin metallic coatings on substrates and free-standing thin metallic films and nanowires. The main goal of this overview is to reveal the commonalities among these materials systems in the origins of the lack of ductility and fracture resistance, in factors governing fatigue resistance, and in ways to improve these properties

Other topics related to the micro-mechanisms of failure in metals, such as fracture at elevated temperature and environmentally assisted cracking, are not covered in these overviews. Finally, while an attempt has been made to give credit through extensive referencing to established as well as emerging research, it is likely that some important work has been overlooked.

1 Introduction

Among the various damage mechanisms introduced in the preface, we begin by studying those associated with brittle and ductile fracture in metallic alloys. Brittle fracture includes both cleavage and intergranular fracture. Ductile fracture encompasses failure by cavitation or by plastic instability. The main objective of this article is to overview the methodologies that are based on the study of the mechanisms operating at a local, i.e. microscopic scale and, through a multiscale approach, on the transfer of this local information to the macroscale, that over which the performance of structural components as well as materials characteristics or “properties” are usually defined. Several reviews and books have been published on this subject (see e.g. [1–3]) but very few of them provide a comprehensive synthesis of the state of the art. In particular, a special effort is made here to incorporate the most recent developments in the theoretical and numerical modeling of both brittle and ductile fracture.

The methodologies referred to above fall under what is now called “the local approach to fracture”, which has been largely developed for brittle fracture with the original Beremin model introduced in the late 70’s and early 80’s [4, 5]. Brittle fracture has been reviewed recently by the authors [6]. In the present paper emphasis is laid on the latest developments, in particular those dealing with the multiple barrier models and the crossing of grain boundaries by cleavage cracks. This aspect of brittle fracture has a special importance when the materials are tested in the rising part of the ductile-to-brittle transition (DBT) curve. The topic of ductile fracture has also been independently reviewed by the authors in two separate monographs [6, 7]. Another review by Besson [8] focused on modeling. While we defer to these reviews for many details, the main mechanisms and concepts are overviewed for completeness. In doing so, we lay emphasis on the latest developments adopting a narrative that seamlessly combines ductile fractures in structural components and metalworking. In addition, significant advances have recently been made in developing more robust models, which ultimately will reduce the many uncertainties associated with currently used models.

The influence of crack tip constraint and stress triaxiality on ductile and brittle fracture is of major importance for the assessment of structural integrity of many industrial components. This assessment is usually made by using linear and nonlinear fracture mechanics concepts. Compared with these concepts, micromechanical models developed in the frame of the local approach to fracture have the advantage that the corresponding material parameters for fracture can be transferred in a more general way between various specimen geometries. In the early version of the Gurson model for ductile fracture [9–11], crack initiation and propagation are a natural outcome of the local softening of the material due to void coalescence, which starts when a critical void volume fraction, f_c , is reached over a characteristic distance, l_c . In principle, the parameters f_c and l_c can be determined from rather simple tests such as tensile tests using smooth and notched round bars in combination with numerical analyses of these tests, or from micromechanical models. Similarly, the Weibull stress model originally proposed by Beremin [5] provides a framework to quantify the complex interactions among specimen size and geometry deformation level and material flow properties when dealing with brittle (cleavage or intergranular) fracture. The Beremin model in its simplest form also uses two parameters only.

The identification and determination of the damage parameters in the Gurson or in the Beremin model require a hybrid methodology of combined testing and numerical simulation. The full description of this methodology is out of the scope of the present paper. More details can be found elsewhere [6]. Here it is enough to say that, contrary to the classical fracture mechanics methodology, the local approach to fracture is not subject to any size requirement for the specimens as long as the same fracture phenomena occur.

This article is organized according to failure modes: cleavage, intergranular, and ductile fracture. In the part devoted to cleavage the early theories for this mode of failure are briefly presented first. Then more recent theoretical developments are presented and applied to ferritic steels and other metals with either a BCC or HCP structure. Intergranular fracture in ferritic steels is also briefly reviewed. Then, ductile fracture is presented in some detail.

2 Cleavage Fracture

2.1 Preliminary remarks

Cleavage fracture preferentially occurs over dense atomic planes (See Table 1). Three fracture surfaces observed on ferritic steels are shown in Fig. 1a, b, c. These micrographs reveal that the orientation of cleavage facets, change when they cross sub-boundaries, twin boundaries or grain boundaries. Steps or ridges appear on the fracture surface to compensate for the local misorientation, in particular at grain boundaries. The crossing of grain boundaries by cleavage cracks is analyzed in more detail in the following. For BCC metals and in the case of mechanical twins, these steps look like indentation marks which are named “tongues” (Fig. 1c). In order to maintain the equilibrium of the crack front, the nearest steps gather to form a single step of higher height leading to the formation of “rivers” as observed in Fig. 1a and b. These rivers align with the direction of the local propagation of the cleavage cracks. On a macroscopic scale the surfaces of the cleavage facets tend to be normal to the maximum principal stress (mode I fracture).

Intergranular fracture corresponds to another brittle mode of failure observed in polycrystalline metals. This mode of failure is often observed when the segregation of impurities such as P, As, S, etc. . . at grain boundaries takes place (See e.g. [12]). The transition between cleavage and intergranular fracture takes place when the ratio R_{CI} is lower than one [3]. This ratio is defined as

$$R_{CI} = 1.20 - \frac{\gamma_b}{2\gamma_S} \quad (1)$$

where γ_b is the free energy (per unit area) of the boundary and γ_S the free energy of a surface exposed by cleavage. Cottrell [13–15] has shown that, in pure metals, γ_b depends mainly on the [macroscopic](#) shear modulus, μ , whilst γ_S depends on the [macroscopic](#) bulk modulus, K . This means that the ratio R_{CI} can be written as

$$R_{CI} = 1.20 - \alpha \frac{\mu}{K} \quad (2)$$

where α is a numerical constant close to one. Table 2 gives the values of R_{CI} for a number of metals. In a large number of pure metals, including pure Fe, intergranular fracture should be the preferential mode of failure. However, in many cases, the segregation of beneficial impurities, such as carbon, along the grain boundaries in iron tends to suppress intergranular brittleness.

Cleavage cracks can be blunted by the emission of dislocations. Rice and Thomson [16] have investigated the conditions under which this blunting mechanism operates. These authors compared the energy needed for the propagation of a cleavage crack over a unit area, that is $2\gamma_S$, to the energy U_L needed to nucleate a dislocation loop of Burgers vector b and of radius r in a slip plane intersecting the cleavage plane. They derived a criterion for “intrinsic” brittleness corresponding to pure cleavage without crack blunting. Intrinsic brittleness occurs when the ratio between the shear modulus, μ , and the bulk modulus, K , is lower than a critical value given by

$$(\mu/K)_{CD} = 10\gamma_S/bK. \quad (3)$$

The propensity for blunted cleavage (called ductile fracture in [16]) increases with the ratio μ/K . A number of values for this ratio are reported in Table 2 which indicates that, in almost all metallic materials, blunted cleavage should occur.

The critical radius r^* of the dislocation loop is only a few b , which is of the same order of magnitude as the dislocation core. This casts doubt on the validity of the preceding calculation. This explains why subsequently, the above theoretical transition from blunted cleavage to pure cleavage was reanalyzed by Rice [17] who used the Peierls concept to analyze dislocation nucleation from a crack tip. The shear stress on the slip plane is now a periodic function of period b . The work done to displace the two half bodies on both sides of the slip plane by $b/2$ is the *unstable stacking fault energy* denoted γ_{us} . It was found that adding shear modes (II and III) has a strong influence on the value of the strain energy release rate for the nucleation of a dislocation loop. With those calculations and considering that blunting requires only the nucleation of partial dislocations, it is finally found that in mode I and 10% shear modes, the critical condition for blunting is γ_S/γ_{us} larger than 4.4 for FCC and 2.4 for BCC metals.

The values of the ratio γ_S/γ_{us} reported in Table 3 were obtained from Frenkel-Peierls-Nabarro potential or from embedded-atom-models (EAM). These values are very approximate. This table shows that in most FCC metals, except iridium, blunting is expected to always take place. For BCC metals, lithium is expected not to cleave. Conversely, pure cleavage should be observed before dislocation nucleation for Cr, Mo and W and also Fe, Nb, V and Ta which are closer to the border line.

It must be pointed out that these calculations apply for a temperature of 0 K and that at higher temperatures thermal activation will favour crack blunting. Moreover these calculations apply to pure metals and not to engineering materials in which cleavage fracture is initiated from second-phase particles, as shown below.

The normal stress or cleavage stress, σ_c , theoretically needed to fracture a crystal by cleavage can be determined provided that the bonding energy, U , between the atoms located across the cleavage plane is known. It can be shown that σ_c can be expressed as [3]

$$\sigma_c = (E\gamma_S/b)^{1/2}. \quad (4)$$

With the typical values valid for iron, $E = 200$ GPa, $b = 0.3$ nm, $\gamma_S \approx 0.1 \mu b \approx 1$ J/m², Eq. 4 leads to $\sigma_c \approx E/10$. Mechanisms of stress amplification must therefore be invoked since the measured cleavage stresses are much lower than this theoretical value.

2.2 Mechanisms of stress intensification

The first mechanism implies the formation of slip bands and, under given circumstances, of mechanical twins as sources of stress concentration [18,19]. It is assumed that cleavage initiates when the local normal stress due to dislocation pile-up reaches a critical value σ_{lc} over a critical distance. This mechanism which is based on a criterion of cleavage initiation is not fully satisfactory because it predicts that the calculated cleavage stress is temperature dependent which is not the general rule. The second mechanism due to Cottrell [20] assumes that cleavage is growth controlled. In this model the condition for propagating a microcrack initiated at the intersection of two slip planes is calculated. This model predicts that the cleavage stress can be written as

$$\sigma_f = k'_y d^{-1/2} \text{ with } k'_y = \frac{2\mu\gamma_S}{\pi(1-\nu)k_y} \quad (5)$$

where d is the grain size and k_y is the coefficient of the Hall-Petch equation predicting the variation of yield strength σ_y with grain size:

$$\sigma_y = \sigma_i + k_y d^{-1/2} \quad (6)$$

where σ_i is the lattice friction stress which is a function of temperature. The cleavage stress predicted from Eq. 5 is independent of temperature.

Smith [21] modified the Cottrell theory to account for the fact that in mild steels cleavage fracture initiates from very brittle platelets of cementite particles located along the grain boundaries.

Initiation and propagation of cleavage cracks from particles have been reanalyzed using the theory of the deformation of heterogeneous solids containing inclusions [22, 23]. The

inclusions were assumed to have an oblate or prolate spheroidal shape (Fig. 2). One example of such inclusions in steel is shown in Fig. 3. The (local) maximum principal stress σ_I applied to the particle is given by [23]:

$$\sigma_I = \Sigma_I + k(\Sigma_{eq} - \sigma_0) \quad (7)$$

where Σ_I is the remote maximum principal stress, Σ_{eq} is the remote von Mises effective stress, σ_0 is the initial yield strength of the matrix, and k is a factor that depends on particle shape and loading orientation. In this equation, stress intensification corresponds to the second term on the right hand side. The criterion for brittle fracture of the particle thus writes: $\sigma_I = \sigma_{Ic}$ with σ_{Ic} the critical stress considered as a material parameter. Note that this criterion does not distinguish between particle cracking and particle-matrix decohesion so that σ_{Ic} can also be viewed as the smallest of the two.

At failure of the particle, the energy stored in the matrix delineated by the representative elementary volume (RVE) shown in Fig. 2 is given by

$$E_V = \Sigma_{eq} \varepsilon_{eq} = \frac{\sigma_0^2}{E_p} \left[\frac{\sigma_{Ic}/\sigma_0 + \lambda}{T + 2/3 + \lambda} - 1 \right] \left[\frac{\sigma_{Ic}/\sigma_0 + \lambda}{T + 2/3 + \lambda} \right] \quad (8)$$

where ε_{eq} is the equivalent strain, E_p is the tangent modulus of the material, $T \equiv \Sigma_m/\Sigma_{eq}$ is the stress triaxiality ratio with $\Sigma_m = \Sigma_{kk}/3$ the mean normal stress, $\lambda = w^2/2\alpha - 2/3 + 2/3\alpha$ with $w = c/a$, the aspect ratio of the particle, $\alpha = 3/2 w^2(2L-1)$ and $L = \ln((2-w)/w)$ [12]. The surface energy of the crack in the particle can be written as $E_S = 3/4\gamma_S (f/h)$, where $f = 2/3c^2h/R^3$ is the volume fraction of inclusions.

At failure, the ratio E_V/E_S is usually much greater than one, as illustrated in Fig. 4. This indicates that most of the energy is spent in plastic deformation of the matrix. This ratio is plotted in Fig. 4 as a function of T and for various values of σ_0 and w , with $\sigma_{Ic} = 2000$ MPa, $\gamma_S = 2$ J/m², $h = 10^{-6}$ m which are typical values for a ferritic steel. It is thus found that most of the energy is spent in the plastic deformation of the matrix. This is largely verified at low T , low yield strength and for spherical particles where it is observed that $E_V \approx 10^3 E_S$. A similar conclusion would have been reached if one had assumed that fracture took place from the surface separating the particle from the matrix.

2.3 Multiple barriers models and crossing of grain boundaries

2.3.1 Basic model

Schematically, cleavage of ferritic steels most frequently results from the succession of three elementary steps illustrated in Fig. 5 (See also [24–29]):

- Slip induced cracking of brittle particle (most often carbide) in ferritic steels;
- Propagation of the microcrack across the particle or the particle/matrix interface and then along a (100) cleavage plane of the neighboring matrix;
- Propagation of the grain-sized or, in bainitic steels, packet-sized crack to neighboring grains across the grain boundary.

The first event is governed by a critical stress σ_{Ic} through Eq. (7). This expression applies when the particle size is larger than 0.1–1 μm [24]. Below this size a dislocation-based theory must be used [30]. The values of σ_{Ic} can be statistically distributed (see e.g. [31,32]). Eq. (7) shows that for a given stress state the strain necessary to initiate particle cracking increases with temperature because of the variation of yield strength with temperature.

The various mechanisms in cleavage fracture can be described in terms of local values of the fracture toughness, $K_{Ia}^{c/f}$ (carbide/ferrite) and $K_{Ia}^{f/f}$ (ferrite/ferrite), or $\gamma^{c/f}$ and $\gamma^{f/f}$ in terms of free surface energy, that must be reached by the crack to cross the first barrier (particle/matrix) and the second barrier (grain boundary), as schematically shown in Fig. 6. The crossing of the first barrier has been analyzed previously. The grain boundary crossing will be analyzed in detail below. A number of studies have shown that in ferritic steels, the crack arresting boundaries were those which are largely misorientated [33–35] and in particular those with a large twist angle [12,36–40]. The particle and “grain” size distribution functions (f_c , f_g) have thus to be considered, as schematically shown in Fig. 7 [35]. In this figure the critical values of the particle and grain size, C^* and D^* are simply related to the local value of the maximum principal stress, σ_I , by a Griffith-like expression [41]

$$C^* = \left(\frac{\delta K_{Ia}^{c/f}}{\sigma_I} \right)^2 \quad \text{and} \quad D^* = \left(\frac{\delta K_{Ia}^{f/f}}{\sigma_I} \right)^2. \quad (9)$$

Very few experimental results have been published in the literature to test the validity of this basic model. However a number of results are gathered in Table 4 where the details concerning a study on a bainitic steel [35] are given. It is worth noting that the local values of the calculated fracture toughness $K_{Ia}^{c/f}$ and $K_{Ia}^{f/f}$ are much lower than the macroscopic fracture toughness, K_{Ic} . Several reasons can be invoked to explain this difference. The first one lies in the calculations. The local values for the maximum principal stress due to stress concentration related to crystallographic aspects can be much larger than the macroscopic stress used in the calculations. The second reason is related to possible dynamic effects, not accounted for in these calculations, as discussed in the next section. In Table 4, it is also worth noting that $K_{Ia}^{f/f}$ is larger than $K_{Ia}^{c/f}$. This conclusion combined with other observations obtained from acoustic emission measurements [35] suggest that the micromechanisms operating during fracture toughness determinations at increasing temperature are not necessarily the same. At very low temperature, cleavage can be controlled by the initiation of microcracks from carbides, while at increasing temperature cleavage is controlled by the propagation of microcracks arrested at grain boundaries [42]. In such conditions the cleavage stress is not necessarily constant with temperature. A number of authors have shown that the energy $\gamma^{c/f}$ is independent of temperature and is of the order of 7 J/m² [26,28] while $\gamma^{f/f}$ is much larger and is strongly temperature dependent (See e.g. [25–29]). The ratio $\gamma^{f/f}/\gamma^{c/f}$ can reach values as large as 10–50.

2.3.2 Dynamic behavior

The dynamic behavior of a microcrack nucleated from a carbide particle and propagating within the ferrite matrix was studied by Kroon and Faleskog [43]. These authors carried out unit cell-type dynamic FEM calculations (Fig. 8). The initiation of cleavage was modelled

explicitly by introducing a small pre-existing crack within the carbide. This microcrack propagated through the carbide and eventually into the surrounding ferrite. The carbide was assumed to be purely elastic and the ferrite to be elastic-viscoplastic with a yield strength at vanishing zero strain rate equal to R_e . Macroscopic constitutive equations allowing for different strain rate sensitivity were adopted. Crack growth resistance was simulated using a cohesive surface, where the traction was governed by an exponential cohesive law. Crack growth rates as large as the Rayleigh wave velocity and strain rates as large as 10^4 – 10^6 s⁻¹ were simulated.

Results showing the variation of the macroscopic stress, Σ_z , as a function of carbide size, c , at crack arrest are reported in Fig. 9. These results were obtained for various values of the stress triaxiality, T . In this figure the values obtained from the static and elastic Griffith theory [41] are also shown as a reference. The Griffith curve is located above all the curves corresponding to the elastic viscoplastic matrix. This situation might appear as being rather counter-intuitive since plastic flow is expected to increase the resistance to crack growth. However, as stated by Kroon and Faleskog [43], the Griffith curve applies to a stationary crack whereas in their calculations the crack has a significant speed when it reaches the carbide/ferrite interface. Fig. 9 suggests also that small particles may play a more prominent role in cleavage fracture than what might be expected from the straightforward application of the Griffith theory. The strain rate sensitivity and the dynamic aspect of crack growth thus come into play in the initiation and in the continued growth of a cleavage crack.

2.3.3 Grain boundary crossing

Grain boundaries in ferritic steels offer an important resistance to the propagation of cleavage cracks. This is frequently observed in the ductile-brittle transition (DBT) regime where secondary cracks are often observed below the fracture surface to be arrested at grain boundaries. See the pioneering work by Hahn et al. [44] and more recently the work by Lambert et al. [35] who showed using acoustic emission that cracks could be arrested at grain (or packet) boundaries. Based on observations in hydrogen charged Fe - 3% Si alloy, Gell and Smith [36] demonstrated that the resistance of grain boundaries to the transmission of cleavage cracks was more important for twist than for tilt misorientation. The specific role of grain boundaries in cleavage crack growth resistance has not yet been completely clarified. However recent studies [37–40, 45] have contributed to a better understanding of this critical step in cleavage fracture. In a recent study devoted to the effect of thermal ageing (450 °C – 5,000 h) on the fracture toughness of a pressure vessel steel (A 508), the mechanisms of grain boundary crossing by cleavage cracks have been investigated in more detail [40, 46, 47]. These recent results are presented below.

Fracture surfaces of A508 steel tested at -60 °C are shown in Fig. 10. It is seen that these surfaces are not only covered by cleavage facets but also contain some intergranular facets when a cleavage micro-crack located in a given grain crosses a grain boundary with a significant twist misorientation component. The tilt, ϕ , and the twist angle, ψ , angles between adjacent grains were measured using quantitative stereography [12]. A close examination to Fig. 10 shows that the number of inclined cleavage facets at grain boundaries per unit length increases with the twist angle, while the value of the tilt angle has no effect, as expected.

These observations have suggested that the crossing of a twist grain boundary by a cleav-

age crack can be described in four steps (See also Qiao and Argon [37,38]) as schematically shown in Fig. 11.

- Step 1: A cleavage crack propagates in the first grain and is arrested by the grain boundary.
- Step 2: As the applied stress is increased, cleavage microcracks are initiated in grain 2 along (100) facets. The initiation sites may be carbides or “tongues” (intersection of the cleavage plane with mechanical twins) as depicted in Fig. 10b. These micro-cracks propagate along the cleavage facets of the second grain.
- Step 3: These microcracks join the cleavage crack in grain 1 and break the grain boundary. One obtains a perturbed crack front pinned by the grain boundary and the risers between adjacent cleavage facets in grain 2.
- Step 4: The crack continues to propagate in grain 2 with its perturbed crack front.

The energy balance has been calculated. It was assumed that the critical steps were those corresponding to steps 3 and 4. The energy dissipated along the risers was neglected. The details of this analysis are given elsewhere [12,40]. It was shown that this model was able to reproduce the increasing number of segmentations in a given grain with the twist angle. It was also established that the number of segmentations for a given twist angle decreases with the grain boundary energy. This is in good accordance with observations showing that in aged material the number of segmentations for a given twist angle was lower than in as-received material. The decrease in grain boundary energy of aged material is associated with the segregation of phosphorus along the grain boundaries which was shown to take place during aging at 450 °C.

Altogether, the crossing of grain boundaries with a large twist component corresponds to a significant increase of the calculated cleavage stress, as indicated in Fig. 12 [40]. Grain boundaries play thus a central role on cleavage fracture in ferritic steels and, in particular, in the DBT regime where the critical step for triggering overall cleavage is the propagation of grain-sized microcracks.

It can be added that there exist relatively few metallurgical methods to control the tilt/twist character of the grain boundaries in structural materials. However it has been observed that the misorientation between the ferritic packets was much more important in martensitic steels than in bainitic steels. These modifications in misorientation distribution between the effective units for cleavage fracture are accompanied by significant improvements in fracture toughness [48].

2.4 Statistical modeling – Application to fracture toughness

2.4.1 Beremin model and Weibull stress

The variety of microstructural elements implied in the micromechanisms for cleavage cracking in a polycrystalline (multiphase) material generates a statistical distribution of the fracture stress. Rather surprisingly, it was only in the 1980's that models have been proposed to account for this statistical effect (for a review, see e.g. [49–51]). Nowadays one of the most widely used models is derived from the original work of Beremin [5]. For a review of this

model based on the Weibull weakest link theory [52, 53] see Mudry [54], Pineau [24, 55, 56] and Evans [57].

Assuming that the material contains a population of microdefects (particles or grain-sized microcracks) distributed according to a simple (power or exponential) law, $p(a)$, the weakest link theory states that the probability to failure $P(\sigma)$ of a representative volume element, V_o , is given by

$$P(\sigma) = \int_{a_c(\sigma)}^{\infty} p(a) da \quad (10)$$

where a_c is simply given by the Griffith expression, similar to Eq. 4, i.e.

$$a_c = \frac{2E\gamma_s}{\alpha\sigma^2} \quad (11)$$

where α is a numerical constant.

In a volume V , which is uniformly loaded and which contains V/V_o statistically independent volumes the probability to failure can thus be expressed as

$$P_R = 1 - \exp \left[-\frac{V}{V_o} P(\sigma) \right]. \quad (12)$$

The function $p(a)$ is not very well known. Theoretically this function can be established from a perfect knowledge of the three steps involved in cleavage fracture of polycrystals. However when the critical step for cleavage fracture is associated with the propagation of microcracks initiated from particles whose size a is known, the distribution $p(a)$ can be determined experimentally assuming that all the particles are involved in the formation of cleavage cracks. Two types of laws have been largely used:

$$\text{- A power law } p(a) = \gamma a^{-\beta} \quad (13)$$

which contains two parameters γ and β

$$\text{- An exponential law } p(\text{size} > a) = \exp \left[-\left(\frac{a - a_u}{a_o} \right)^N \right] \quad (14)$$

where a_o , a_u and N are parameters of the distribution.

The simple power law applied to Eqs. 10 and 12 leads to

$$P_R = 1 - \exp \left[-\frac{V}{V_o} \left(\frac{\sigma}{\sigma_u} \right)^m \right] \quad (15)$$

with the Weibull shape factor m , such as

$$m = 2\beta - 2. \quad (16)$$

This shape factor characterizes the dispersion, which is the higher the lower m ; σ_u is a measure of the “fracture resistance” of the elementary volume, V_o .

It should be noted that m and σ_u are theoretically temperature independent. Similarly the exponential law (Eq. 14) leads to [58]

$$P_R = 1 - \exp \left\{ -\frac{V}{V_o} \exp \left[-\left(\frac{1/\sigma^2 - 1/\sigma_u^2}{1/\sigma_{co}^2} \right)^N \right] \right\}, \quad (17)$$

$$\text{with } \sigma_{co} = (2E\gamma_S/\alpha a_o)^{1/2} \text{ and } \sigma_u = (2E\gamma_S/\alpha a_u)^{1/2}. \quad (18)$$

Eq. 15 is a simplified expression which contains no explicit threshold. In three dimensions and in the presence of smooth stress gradients, this equation can be written as

$$P_R = 1 - \exp \left(-\frac{\int_{PZ} \sigma_1^m dV}{V_o \sigma_u^m} \right) \quad (19)$$

where the volume integral is extended over the plastic zone (PZ). This equation can be rewritten as

$$P_R = 1 - \exp \left[-\left(\frac{\sigma_W}{\sigma_u} \right)^m \right] \quad (20)$$

where σ_W is referred to as the Weibull stress given by

$$\sigma_W = \left[\frac{1}{V_u} \int_{PZ} \sigma_1^m dV \right]^{1/m} \quad (21)$$

in which V_u simply replaces V_o .

A number of authors have introduced a threshold stress, σ_{th} , directly into Eq. 19 (See e.g. Bakker and Koers [59]; Xia and Cheng [60]; Gao et al. [61] proposed a slightly modified form for Eq. 19 and Eq. 20, writing:

$$P_R = 1 - \exp \left[-\left(\frac{\sigma_W - \sigma_{W \min}}{\sigma_u - \sigma_{u \min}} \right)^m \right] \quad (22)$$

where $\sigma_{W \min}$ represents the minimum value of σ_W at which cleavage fracture becomes possible (for a full discussion, see also Gao et al. [62, 63]; Gao and Dodds [64]).

In the literature, it is often underlined that Eq. 20 does not include a threshold. This is only partly true since in the original Beremin model, an implicit Weibull threshold stress was also included. In the Beremin theory it is assumed that cleavage fracture cannot occur in the absence of plastic deformation, i.e. below the yield strength, σ_y . This means that cleavage cannot occur when the calculated plastic zone size ahead of the crack tip is lower than a critical value, X_c . This assumption is similar to the RKR model [65]. This threshold in terms of K is simply given by

$$K_{I \min} \approx R_p \sqrt{3\pi X_c}. \quad (23)$$

The Beremin model contains only two independent parameters, m and the product $\sigma_u^m V_u$. A number of studies on structural steels have shown that m is close to 20 when no threshold is introduced (See e.g. Beremin [5]; Minami et al. [66]). Lower values for m are found when a

threshold is introduced (See e.g. Gao and Dodds [64]; Petti and Dodds [67, 68]. Low values for m have also been observed for intergranular fracture, as shown below.

It is noted that the Beremin theory does not assign a completely clear meaning to the defects distributions given by Eqs. 13 & 14. It is tempting to develop a more “precise” theory including the three steps indicated previously for cleavage fracture. This would lead to very sophisticated expressions for the probability of failure which should always be calculated from the weakest link theory. Several attempts to develop such expressions have been made in the literature. One of the most advanced attempts based on a physical description of cleavage fracture was made by Chen [25, 26]. However this author has not been able to derive an explicit expression as simple as Eqs. 15 & 17 to determine the probability of failure. It should be realized that the nature of the defects introduced in Eqs. 13 & 14 may change with test temperature. At low temperature, in the lower part of the DBT curve the critical defects are likely the coarsest carbides. On the other hand, at high temperature and in the DBT regime the critical defects are grain-sized microcracks. The distribution laws for these two types of defects are expected to be different. This variation in the nature of the defects with temperature or with loading and constraint may induce variations in the values of the parameter σ_u (and eventually m) in Eq. (15).

2.4.2 Beremin model – Effect of plastic strain

Detailed observations in various steels showed that the number of microcracks nucleated from carbides increased with plastic strain (See e.g. Kaechele and Tetelman [69]; Chen and Cao [26]). This is the reason why a number of authors, following the work by Bordet et al. [70–72] have modified the original Beremin model in order to include the effect of plastic strain on the nucleation of microcracks and the deactivation of these microcracks if they do not propagate immediately. The work by Bernauer et al. [73] should also be mentioned. At a material point located within the plastic zone the probability of cleavage fracture is expressed as

$$P_{\text{cleav}}(t) = \int_0^t P_{\text{prop}} dP_{\text{nucl}} \quad (24)$$

where t represents the “loading time,” whilst P_{prop} and dP_{nucl} are the probability of crack propagation and the increment of probability to nucleate a microcrack. In Bordet’s model, dP_{nucl} is written as

$$dP_{\text{nucl}} = N_r \sigma_y d\varepsilon_{\text{eq}}^p = N_o (1 - P_{\text{nucl}}) \sigma_y d\varepsilon_{\text{eq}}^p \quad (25)$$

where $\varepsilon_{\text{eq}}^p$ is the equivalent plastic strain, N_r and N_o are respectively the remaining and the initial number of cleavage initiation sites. Normalizing the yield strength, σ_y , by a given stress σ_{y0} and the plastic strain by $\varepsilon_{\text{eq}0}^p$, integration of Eq. 25 leads to

$$P_{\text{nucl}} = 1 - \exp \left(- \frac{\sigma_y}{\sigma_{y0}} \frac{\varepsilon_{\text{eq}}^p}{\varepsilon_{\text{eq}0}^p} \right). \quad (26)$$

As for the original Beremin model, assuming that the nucleated microcracks can be treated as Griffith flaws distributed according to a power law, and invoking the principle of weakest-link over the plastic zone (PZ) volume, the overall fracture probability can then be

expressed as

$$P_R = 1 - \exp \left(- \int_{PZ} \int_0^t P_{\text{prop}} dN_{\text{nuc}} dV \right) = 1 - \exp \left[- \frac{\sigma_W^*}{\sigma_u^*} \right] \quad (27)$$

where σ_u^* is a scaling parameter and σ_W^* is a modified Weibull stress defined as

$$(\sigma_W^*)^m = \int_{PZ(\sigma_I > \sigma_{th})} \left\{ \int_0^{\varepsilon_{eq}} \frac{\sigma_y}{\sigma_{y0}} (\sigma_I^m - \sigma_{th}^m) \left[1 - P_{\text{nuc}}(t) \frac{d\varepsilon_{eq}}{\varepsilon_{eq0}} \right] \right\} \frac{dV}{V_o}. \quad (28)$$

2.4.3 Application of Beremin model to the prediction of fracture toughness

Two extreme conditions must be considered corresponding to small scale and large scale yielding.

Small scale yielding solution

The application of the Beremin model (Eq. 20) to the crack tip situation is straightforward if it is assumed that the stress-strain field ahead of the crack tip under small scale yielding is simply scaled by the ratio $r/(K/\sigma_y)^2$ or $r/(J/\sigma_y)$ where σ_y is the yield strength and r is the distance ahead of the crack tip [5, 24]. The probability distribution for the fracture toughness, K_{Ic} of a specimen with a thickness, B and containing a 2D crack can simply be expressed as

$$P_R = 1 - \exp \left[- \frac{K_{Ic}^4 B \sigma_y^{m-4} C_{mn}}{V_u \sigma_u^m} \right] \quad (29)$$

where C_{mn} is a numerical factor dependent on the work-hardening exponent (n of a power-law hardening relation) and on the Weibull shape factor m . Tables giving the values of C_{mn} for various values of m and n have been published recently [74]. It can be noted that the size effect in Eq. 29 is measured by the product $K_{Ic}^4 B$. This dependence derives simply from the fact that the volume of material sampled by the plastic zone of size R is simply given by $R^2 B$, i.e. $K^4 B$ since R is proportional to K^2 . The above expression can be extended to a 3-D crack of length l in which the stress intensity factor, K , is a function of the curvilinear abscissa, s [3].

This size dependence has also been introduced in the master curve (MC) concept [51, 75]. In this curve, the probability to failure is written as

$$P_R = 1 - \exp \left[- \frac{B}{B_o} \left(\frac{K_{Ic} - K_{\min}}{K_o - K_{\min}} \right)^4 \right] \quad (30)$$

where B_o is an arbitrary normalized thickness, $K_{\min} \approx 20 \text{ MPa}\sqrt{\text{m}}$ and K_o follow a “universal” temperature dependence:

$$K_o = 30 + 70 \exp[0.019(\theta - \theta_o)] \quad (K \text{ in MPa}\sqrt{\text{m}}, \theta \text{ in } ^\circ\text{C}) \quad (31)$$

where θ_o is the temperature at which the mean (median) fracture toughness for a 25 mm thick specimen is equal to $100 \text{ MPa}\sqrt{\text{m}}$.

It should be added that Chen [25, 26] assumed that the scatter in fracture toughness measurements mainly results from the random distribution of the location of cleavage sites

and not from the distribution of carbide size. Again the development of a complete model including the three steps for cleavage fracture is almost impossible and would require many parameters.

Large scale yielding solution

In many situations the analytical expression given by Eq. 29 cannot be applied because the conditions for small scale yielding are not satisfied. This is especially true when increasing the temperature and reaching the DBT temperature. The approximation of the crack tip field with the Hutchinson-Rice-Rosengren (HRR) solution [76, 77] is no longer valid. The results for the Weibull stress can only be obtained from FE calculations. Fig. 13 shows schematically the variation of the fracture toughness calculated in terms of the Weibull stress when large scale yielding (LSY) conditions are reached at a given temperature and when the test temperature is increased.

Semi analytical expressions for the Weibull stress have been given for 2D cracks by Lei et al. [78] and O'Dowd et al. [79]. Considering an elastic-plastic material with a Ramberg-Osgood material law expressed as

$$\frac{\varepsilon}{\varepsilon_o} = \frac{\sigma}{\sigma_o} + \alpha \left(\frac{\sigma}{\sigma_o} \right)^n \quad (32)$$

where n is the strain-hardening exponent and σ_o and ε_o are reference stress and strain, respectively, the solution for the Weibull stress can be expressed as

$$\frac{\sigma_W}{\lambda \sigma_o} = \left[\frac{\hat{\phi}(n)}{1 - m/2(n+1)} \left(\frac{J}{\alpha \varepsilon_o \sigma_o L \lambda^{n+1}} \right)^2 \right]^{1/m} \quad (33)$$

where $\lambda > 1$ is a parameter which scales the yield stress σ_o ; $L = (V_u/B)^{1/2}$; m as above and $\hat{\phi}(n)$ is a function of the strain-hardening exponent n . Incorporating this expression for the Weibull stress into Eq. (20) leads to an expression similar to Eq. (29).

Applications of the Beremin model

The Beremin model and the MC approach have now been largely used in the literature to model the variation of fracture toughness with (i) specimen size [12, 80], (ii) specimen geometry [81], (iii) loading rate [82], (iv) inhomogeneities of the materials as in welds [35, 83], (v) irradiation induced embrittlement [6]. For more detailed review, see Pineau [55] and François et al. [3].

2.5 Cleavage in other BCC metals

BCC metals other than Fe have been investigated in much less detail. This is partly due to the smaller use in engineering applications of the high melting BCC refractory metals (V, Nb, Ta, Cr, Mo, W). Commercially available group V-A refractory metals (V, Nb, Ta) are considerably more ductile than commercially available group VI-A refractory metals (Cr, Mo, W). This mainly results from the much higher solubility of impurities in the V-A metals than in the VI-A metals.

Cleavage in molybdenum has been investigated in some detail, see [84, 85]. Niobium has also been investigated with more attention [84, 86–88]. These authors have studied pure Nb

and Nb-Zr solid solutions. The cleavage stress was measured as a function of grain size using blunt notched specimens. This cleavage stress was found to be higher for the fine-grain sized materials (63 to 66 μm) than for the coarse-grain sized materials (135 to 165 μm). It has been suggested that the cleavage stress is controlled by a Cottrell [20] type critical flaw size, which in turn is controlled by the grain size.

The well-known RKR (Ritchie, Knott, Rice) model [65] was used by Padhi and Lewandowski [88] to predict the variation of fracture toughness with temperature and loading rate. The values of the cleavage stress determined from these fracture toughness tests were found to be larger than those measured on blunt notched specimens (typically 1700 MPa compared to 1400-1500 MPa in 60 μm grain size Nb). This suggests that the sampling volumes which were quite different in these two specimen geometries, could also affect the value of the cleavage stress. Additional tests and analysis are necessary to show if this size effect can be interpreted using the Beremin approach.

2.6 Cleavage in HCP metals

2.6.1 Cleavage plane

HCP metals (Zn, Ti, Mg) exhibit a wider variety of deformation modes, compared to BCC metals. These modes include slip and twinning on various systems. In all HCP metals the dominant slip mode occurs with the Burgers vector $a = 1/3 \langle 11\bar{2}0 \rangle$ with the primary (0002) basal slip plane (e.g. Be, Ti, Zr, Mg) or the prismatic planes (e.g. Ti, Zr). All HCP metals have the basal plane {0002} as the preferential cleavage plane.

HCP metals, such as Mg and Zr, have a low ductility when tested at low temperature. Recent atomistic simulations have shown that Mg is intrinsically brittle at 0K [89]. In Ti and Zr, pure cleavage along 0002 plane is not observed, except under stress corrosion conditions, see e.g. for Zr alloys Kubo et al. [90]; Schuster and Lemaignan [91, 91]; Cox [92]. This is why in this section only cleavage fracture of Zn is briefly considered.

2.6.2 Cleavage fracture of zinc

Zinc exhibits cleavage fracture along the basal plane and also along the prismatic planes, as underlined by Hughes et al. [93]. Cleavage on basal plane has been studied more thoroughly, in particular on zinc single crystals [94, 95]. Cleavage fracture on basal plane has been studied in some detail on hot dip galvanized steel sheets [96, 97]. It was confirmed that the critical stress normal to {0002} cleavage plane is largely reduced when the amount of plastic strain in the basal plane, γ^{bas} , is increased. Similar observations have been made on crack propagation behavior in strongly textured zinc sheets (Lemant and Pineau [98]). The study on hot dip galvanized sheets showed that the cleavage stress can be written as

$$\sigma_c = \sigma_{\text{th}} + \frac{k'}{\gamma_o + \gamma_{\text{ep}}^{\text{bas}}} \quad (34)$$

where σ_{th} , k' and γ_o are material parameters. Eq. (34) bears a similitude with the criterion $\sigma_c \propto 1/\gamma_{\text{ep}}^{\text{bas}}$, which was proposed by Gilman [94]. To the authors' knowledge, no studies have been devoted to a statistical analysis of the cleavage behavior of this material.

3 Introduction to intergranular fracture in ferritic steels

As stated previously, theoretical calculations (See section 2.1 and Table 2) indicate that intergranular fracture would be expected in many polycrystalline metals, instead of transgranular cleavage, which is the main mode of failure observed at low temperature when the material is tested below the DBT temperature. With this in mind, the non-occurrence of intergranular fracture in ferritic steels is usually attributed to the reinforcement of the grain boundaries by the segregation of a number of impurities, in particular carbon. Conversely, the segregation of other impurities, for example phosphorus, can change the brittle mode of failure from cleavage to intergranular. This corresponds to what is named temper embrittlement in steels. This phenomenon is still a subject of debate in spite of numerous studies over the past 50 years.

Many authors assumed grain boundary impurity segregation (in particular phosphorus) to be responsible for temper embrittlement [99]. It is also often assumed that co-segregation of carbon and phosphorus takes place [46, 100, 101]. This interaction is repulsive such as the content of phosphorus segregated along the grain boundaries is increased whilst that of carbon is decreased. This interaction effect might also contribute to the deleterious effect of impurity segregation on intergranular strength. A number of authors, e.g. [102], have rejected this assumption and have attempted to explain the enhanced impurity segregation by a decreased solubility limit of impurities or a diminished free carbon concentration. Next neighbor interaction between segregating species leading to an interfacial miscibility gap or another 2D phase transition was also discussed by others, in particular Militzer and Wieting [103].

Temper embrittlement is usually observed in large components like rotors for steam turbines or pressure vessels. This is due to the fact that long times are necessary to cool down these heavy components from the normalizing or tempering temperatures. Impurities have enough time to diffuse to the grain boundaries when the steel is the temperature range close to 500 °C and above.

Very few detailed studies have been made to determine quantitatively the variation of the critical intergranular fracture stress, σ_{CI} , with test parameters, using the procedures similar to those used for the measurement of the cleavage stress, in particular tests on notched bars with various notch radii. However the pioneering work by Kameda and Mc Mahon [104] should be mentioned. These authors showed that the critical intergranular fracture stress was directly related to the amount of impurity (Sb) segregated on the grain boundaries. A similar study by Naudin et al. [105] showed that the intergranular fracture stress was decreasing linearly with the amount of phosphorus segregated on the grain boundaries. However a minimum amount of phosphorus had to be segregated (5-10%) before intergranular fracture occurred (See Fig. 14) [105]. In another recent study devoted to the statistical aspect of intergranular fracture, Wu and Knott [106] observed that σ_{CI} was independent of test temperature and was distributed according to either a normal or a Weibull law.

More detailed studies on the effect of intergranular fracture on the fracture toughness of pressure vessel steel A508 have been made [107–109]. These studies showed that, in the presence of intergranular fracture, the Beremin theory should be slightly modified to account for the effect of temperature on fracture toughness, since σ_{CI} was found to be an increasing function of test temperature. Kantidis et al. [107] showed that the variation of

fracture toughness with temperature could be represented using the original Beremin model (Eq. 20), provided that the Weibull stress includes a temperature dependence, i.e.

$$\sigma_{\text{WW}}^m = \int_{\text{PZ}} \sigma_{\text{I}}^m [1 + \lambda(T - T_o')]^m \frac{dV}{V_o} \quad (35)$$

where σ_{WW} is the modified Weibull stress, $\lambda > 0$ is a material parameter and T_o' is a reference temperature. Eq. 35 indicates that when the temperature is lower than T_o' , intergranular fracture occurs at lower stresses as compared to the original Weibull stress. In this study it was also shown that the shape factor m for intergranular fracture was much lower (~ 10) than the shape factor for cleavage (~ 20). This means that the scatter in test results is larger for intergranular fracture than for cleavage.

Another aspect of segregation induced embrittlement has been indicated previously when it was mentioned that the upper shelf energy was reduced by temper embrittlement. Detailed studies on this effect were made by Hipsley and Druce [110]. These authors investigated the effect of isothermal ageing at 500 °C of quenched-and-tempered low alloy steel on the fracture resistance of the material. They showed that this thermal ageing produced reductions in the resistance to ductile fracture, as measured by impact fracture upper-shelf energy, ductile fracture toughness (J_{Ic}) and tensile ductility. The segregation of phosphorus to carbide/matrix interfaces during ageing was observed by Auger spectroscopy and was shown to be mainly responsible for this reduction in ductile fracture resistance.

Summary

A unifying theory of fracture must be physically based and not just phenomenological. In the previous part devoted to cleavage and intergranular fracture, an attempt has been made to illustrate the benefits of the micromechanical modeling of fracture. In particular, it has been shown that the local approach to brittle fracture allows to quantify failure of not only volume elements but also cracked bodies. This approach captures the influence of a large number of physical parameters describing the relevant, yet sufficiently detailed microstructure of materials under complex loading conditions. For a long time, it was believed that fracture toughness in the transition regime is an intrinsic material property. Micromechanical approaches, in particular those introducing statistical effects, have clearly shown that the cleavage fracture toughness is actually specimen-size dependent. This property is now introduced in ASTM standards [111].

4 Ductile Fracture

When polycrystalline metal alloys do not cleave, they may fail in a ductile, most often transgranular manner. Failure may occur by void coalescence or by mechanical instability of the specimen itself, Fig. 15. In the first, voids nucleate at inclusions then grow, aided by plasticity. When they eventually coalesce, a crack forms and the material fractures. Voids may also promote failure by instability, but here they are not necessary. Failure by instability usually involves one or more shear bands. Material separation across the shear band eventually involves void nucleation to coalescence, albeit at lower length scales. In this section, we overview both processes of fracture. Emphasis is laid on quasi-static response at low homologous temperatures, [but dynamic aspects of ductile failure are also covered](#). We begin by succinctly recalling the basic phenomenology and appropriate measures of fracture in various metallic systems (Section 4.1). Then we describe the fundamental operating mechanisms across multiple length scales (Section 4.2). In Sections 4.3 and 4.4, we separately review the analyses of failure by microvoid coalescence and by shear localization. We conclude with a long section on predictive modeling (Section 4.5) within a local approach to fracture [56]. For each class of models, we address material parameter identification and point to current challenges and open issues.

4.1 Phenomenology

Ductile fracture involves extensive plasticity and rough fracture surfaces. It manifests in different ways depending on the material system, level of constraint, and boundary conditions.

4.1.1 Fracture path

In notched round tensile bars, the fracture path is usually nominally flat or zigzagged, depending on the plane of view and heterogeneity of the material [112]. In plane strain bars, however, the fracture path is usually slanted indicating failure by instability [113]. A mix of flat and slant fracture is observed in round tensile bars leading to the well-known cup and cone. In thin sheets where plane stress prevails, slant fracture may occur through the thickness or in the plane of the sheet. In thin-walled tubes under torsion, when failure is reported, it occurs subsequent to shear localization, which is dependent on strain rate [114, 115]. In compression, upset cylinders fail by longitudinal or oblique cracks, both confined to the barrel [116]; in less-ductile materials, such as magnesium alloys or brass, catastrophic fracture splits the specimen in two halves [117]. In actual structural components or more elaborate specimen geometries, the fracture path will depend on the multiaxiality of loading and the occurrence of plastic instabilities (necking and shear bands).

The above listed examples are for initially crack-free specimens, which are typically used to investigate the influence of states of stress and strain on fracture. They are also used to study damage initiation and accumulation, as well as the transition from diffuse damage to localized cracking, especially when instabilities are avoided. Pre-cracked specimens are typically used in experimental fracture mechanics to evaluate fracture toughness. Here too, various geometries are used. Crack tunneling is a typical phenomenon observed in sufficiently thick geometries, such as compact-tension (CT) specimens: the crack advances faster in the

mid-section than near the free surfaces. Another phenomenon in precracked specimens is the often observed transition from flat to slant fracture, also observed in wide plates and in burst-open pressure vessels. Particularly complex crack paths are involved in ductile tearing of thin sheets which, in addition to the competition between flat and slant fracture, sometimes exhibits a crack tip necking process [118,119] or the so called “flip-flap” mechanism in which the orientation of the slant plane regularly changes as the crack propagates [120,121]. The occurrence of these crack paths, described in more detail in Section 4.2.7 from the viewpoint of the physical underlying mechanisms, depends on sheet thickness, strain hardening capacity and strain rate [118,122], as well as on boundary conditions in the case of the “flip-flap” mode.

4.1.2 Measures of fracture

No measure of ductile fracture resistance is intrinsic to the material. Yet, in practice, nominal measures are used, if only to compare different materials. These measures are in terms of strain, stress or energy per unit area.

In notched round bars of ductile materials, the force versus diameter reduction curve usually exhibits an abrupt drop after reaching a maximum at limit load. The knee in this curve is associated with macrocrack initiation, as will be discussed under “Mechanisms” below (Section 4.2). It is thus tempting to define a strain to fracture initiation, $\bar{\epsilon}_i$, based on an effective strain

$$\bar{\epsilon} = 2 \ln \frac{D_0}{D}, \quad (36)$$

evaluated for $D = D_i$ at the knee. In (36) D and D_0 are the current and initial diameters, respectively. Similarly, a fracture strain, $\bar{\epsilon}_f$, may be defined based on D_f at complete loss of load-bearing capacity. If the material is anisotropic a variant of (36) is used. The effective strain $\bar{\epsilon}$ represents an average strain in the minimal section at the notch root. The fracture strain $\bar{\epsilon}_f$ is conveniently measured post-mortem, after complete material separation. Since the mechanical fields are perturbed after crack initiation, $\bar{\epsilon}_i$ is thought to be a better measure of ductility for a given stress state. In principle, however, $\bar{\epsilon}_i$ must be size-dependent as the knee in the load-deflection response would be barely detectable in an increasingly large (geometrically similar) bar.

In uniaxial tension of a cylindrical rod or flat sheet, either the total elongation to fracture or the reduction in area at fracture are commonly used as measures of ductility. Neither are fundamental measures. The elongation to fracture combines a size-independent uniform elongation and a size- and gauge-length dependent post-necking elongation. However, the reduction in area is more representative of some true fracture strain since it is roughly gauge-length independent. On that basis, one can adopt definition (36), or a variant in sheet specimens, to estimate $\bar{\epsilon}_f$ at fracture in a tensile bar. Access to $\bar{\epsilon}_i$ in uniaxial round bars is also possible using non standard metrology. In plane strain tension and sheet specimens, which fail by mechanical instability, $\bar{\epsilon}_i$ is not accessible unless post-necking deformation is negligible, in which case $\bar{\epsilon}_i \approx \bar{\epsilon}_f$.

In torsion, the shear strain and shear stress in the tube are based on the angle of twist, θ , and torque, M^t , and given by $\gamma = R\theta/L$ and $\tau = M^t/AR$, where R and A are the mean radius and cross-sectional area, respectively, and L is the gauge length. Dimensions R , A

and L remain constant during the test so that problems related to necking-induced geometric softening or dilation are in principle avoided. Under these circumstances, γ is a measure of true strain. In ductile metals subjected to low to moderate strain rates, the τ - γ curve does not exhibit a maximum. When fracture is reported, the shear strain to fracture initiation, γ_i , is an adequate measure of ductility. At higher shear strain rates (typically above 50/s), a maximum in shear stress is observed, most likely due to thermal softening [123].

In compression or upsetting, if deformation is frictionless then fracture is not expected in cylinders. In practice, it does occur, however, subsequent to barrelling in ductile metals and to shear instability in less-ductile metals. The better the lubrication the less severe the barrelling. Incremental loading may be used to ensure more uniform expansion. The magnitude of true axial strain is based on current height $\ln(h_0/h)$ which, to the neglect of elastic strains, is given by generalizing (36) to

$$\bar{\epsilon} = \left| 2 \ln \frac{D_0}{D} \right|, \quad (37)$$

assuming isotropic and uniform expansion. When barrelling sets in, the height-based strain measure becomes inadequate and use of (37) at the apex is preferred. The diameter D may be obtained using a radial extensometer, curvature measurement using digital image correlation (DIC) or approximate formulae. Surface markers may be used to follow the evolution of biaxial strains after barrelling. In ductile metals, cracking is visible since it initiates on the surface for some value D_i of the diameter. The initiation strain is thus taken from (37) at $D = D_i$. However, the fracture strain is ill-defined because separation is not complete. In less-ductile metals, such as magnesium alloys and brass, $\bar{\epsilon}_f \approx \bar{\epsilon}_i$ and may be inferred from current height measurements. If markers are used then additional information on surface strains to fracture may also be obtained [116, 124].

Pre-cracked specimens are used to determine crack growth resistance curves in terms of a measure J_R of the J integral versus crack extension, Δa . Ideally, these curves are intrinsic to the material under conditions of confined plasticity (so-called small-scale yielding), which are favored in thick components. There are, however, theoretical and practical issues hampering the use of J - Δa as universal curves. The J integral is rigorously path-independent for a nonlinear elastic material and infinitesimal strains. Limitations associated with unloading and finite deformations (more generally nonproportional loadings) are well documented [125]. In practice, the resistance curve is found to depend on specimen geometry and size. Yet, nominal fracture properties are extracted from J - Δa curves, such as the crack initiation toughness, J_{Ic} , and the tearing modulus, dJ/da . They may be useful for comparing materials and for designing against fracture. Beyond J_{Ic} , crack growth resistance increases and eventually reaches a steady-state value J_{ss} . Steady state toughness measurement would require crack growth over distances that are much longer than can be afforded in standard specimens. Experiments indicate that the geometry-dependence of initiation toughness is less than that of steady-state toughness, as suggested by the evolution of J_R (see Section 4.1.5). Another measure of crack initiation toughness is based on the critical value of the crack tip opening displacement, $\delta_{Ic} = \alpha_n J_{Ic} / \sigma_0$, where α_n is the so-called Shih factor [126].

In thin pre-cracked specimens, large-scale yielding is unavoidable. In such cases, fracture resistance is evaluated by some measure of work of fracture. A particularly elegant and

simple approach to quantify the tearing resistance of thin metallic sheets is the Essential Work of Fracture (EWF) method [127]. The method separates out the energy expended in the fracture process zone, called the essential work of fracture W_e (J/m²), from the plastic dissipation in the remote plastic zone W_p (J/m³). The underlying idea is to use self-similar pre-cracked specimens with various ligament lengths l , typically double edge notch tension specimens, and determine, for each specimen, the total work of fracture per unit ligament area. The self-similarity argument enables to mathematically separate W_e and W_p , which scale with l and l^2 , respectively. The method is adapted mainly to near plane-stress states and provides a physically meaningful material cracking resistance indicator. The latter is useful in a materials research perspective towards guiding better microstructure design but is of limited use in structural analysis, except if W_e can be related to J_{Ic} for instance. Such correlation has indeed been proposed [128] but is not always valid [129].

For the sake of generalization, the total work of fracture Γ (J/m²) to be expended after the crack has advanced by an amount Δa for continued propagation consists of three contributions [130]:

$$\Gamma(\Delta a) = \Gamma_0 + \Gamma_n + \Gamma_p \quad (38)$$

where Γ_0 is the work in the fracture process zone (see Section 4.2.7 for corresponding damage mechanisms) and represents, in principle, the intrinsic resistance to cracking; Γ_n is the work per unit crack advance required for crack tip necking; and Γ_p is the extrinsic contribution resulting from gross plastic dissipation. The term Γ_n may become prominent in thin sheets, see Section 4.1.1. If the fracture toughness is defined as $J_{Ic} = \Gamma_0 + \Gamma_n$, either at crack initiation or after a small pre-defined amount of crack advance, then the EWF W_e may be viewed as a mean value of $\Gamma_0 + \Gamma_n$ over the entire propagation.

To sum up, appropriate strain-to-failure measures are typical in initially crack-free specimens whereas heuristic extensions of Griffith's energy release rate to elastoplastic materials are used to provide measures of fracture toughness. Other useful energy-based measures include the essential work of fracture, which is typically used in thin structures, or simply the total work of fracture, which may be used in crack-free specimens as well to alleviate some size-dependence. In the literature critical stresses are often reported as measures of fracture. In initially crack-free specimens, the effective stress at incipient cracking is about the yield stress to 50% above (or below) it, depending on strain hardening. This creates inherent difficulties with stress measures, which are generally inadequate.

4.1.3 Measures of stress state

To describe fracture under multiaxial loadings, scalar descriptors of stress and strain states are used. An important descriptor is the stress triaxiality ratio, or simply *triaxiality*, defined by:

$$T = \frac{\sigma_m}{\sigma_{eq}} \equiv \frac{I_1}{3\sqrt{3}J_2} \quad (39)$$

with σ_m the mean normal stress and σ_{eq} the von Mises effective stress. Also, $I_1 = \sigma_{kk}$ is the first invariant of the stress tensor and $J_2 = \sigma'_{ij}\sigma'_{ij}/2$ the second invariant of its deviator. In

terms of the principal stresses, taken in no particular order,

$$\sigma_m = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3) \quad (40)$$

$$\sigma_{eq}^2 = \frac{1}{2} [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]. \quad (41)$$

Introducing the stress ratios of any two principal stresses to a non-zero principal stress, say $\rho_1 = \sigma_1/\sigma_3$ and $\rho_2 = \sigma_2/\sigma_3$ ($\sigma_3 \neq 0$), the triaxiality is generally expressed as

$$T = \frac{\sqrt{2}}{3} \text{sign}(\sigma_3) \frac{\rho_1 + \rho_2 + 1}{\sqrt{(\rho_1 - \rho_2)^2 + (\rho_2 - 1)^2 + (\rho_1 - 1)^2}}. \quad (42)$$

Typical values and bounds on T are reported in Table 5. Plots of T versus an appropriate stress ratio are depicted in Fig. 16 for $\sigma_3 > 0$. Under axisymmetric loading with a tensile axial stress, the triaxiality can be varied between $-2/3$ (biaxial compression) and infinity (hydrostatic tension). In (plastic) plane strain, it can be varied between 0 (pure shear) and infinity (hydrostatic tension). In plane stress, however, the triaxiality can only be varied between $-1/3$ (compression) and $2/3$ (biaxial tension); but in practice it is limited to the narrower range $1/3$ to $2/3$, except when one stress component becomes negative, as in deep drawing for instance. In notched round bars, $T \sim 0.6$ – 2 whereas higher values are attained ahead of the crack tip in thick components.

To complete the description of stress state, a second dimensionless ratio is introduced:

$$L = \sqrt{3} \tan \theta; \quad \cos(3\theta) = \frac{1}{2} \det \frac{3}{\sigma_{eq}} \boldsymbol{\sigma}'. \quad (43)$$

This parameter enables to discriminate among deviatoric loadings that share the same value of the effective stress (or J_2) via the third invariant of the stress deviator $J_3 = \det \boldsymbol{\sigma}'$. In terms of principal stresses, this amounts to examining the influence of the intermediate stress, σ_{II} , since

$$L = \frac{2\sigma_{II} - \sigma_I - \sigma_{III}}{\sigma_I - \sigma_{III}} \quad (44)$$

where $\sigma_I \geq \sigma_{II} \geq \sigma_{III}$ are ordered principal stresses. Parameters L and θ are known as the Lode parameter and Lode angle, respectively, after Lode [131] who examined the effect of σ_{II} on yielding by subjecting tubes of various metals to combined tension and internal pressure. Typical values and expressions of L are reported in Table 6. For axisymmetric and plane strain loadings, L takes a constant value, irrespective of the stress ratios. On the other hand, in plane stress L can be varied between -1 and $+1$, depending on the stress ratio (Fig. 17).

Strain or strain-rate triaxiality measures may also be defined. A strain triaxiality ratio as in (39) is typically not used in ductile fracture. However, strain ratios are widely used in sheet metal forming. For example, in upsetting $\bar{K} = -\dot{\epsilon}_z/\dot{\epsilon}_\theta$ defines the ratio of axial to hoop strain rates [124]. Another useful measure is the Lode variable ν for the strain rate, defined as

$$\nu = -\frac{3\dot{\epsilon}_{II}}{\dot{\epsilon}_I - \dot{\epsilon}_{III}} \quad (45)$$

where $\dot{\epsilon}_I \geq \dot{\epsilon}_{II} \geq \dot{\epsilon}_{III}$ are ordered principal strain rates. Just like L , this variable lies between -1 and $+1$, with $\nu = -1$ for a simple contraction (or biaxial extension), $\nu = 0$ for simple shear,

and $\nu = +1$ for a simple extension (or biaxial contraction). The difference in definitions (44) for L and (45) for ν is for convenience.

Stress and strain ratios, as well as T and L , are evolving fields just like the stress and strain tensors. One advantage of working with T and L (or $\bar{K}$ and ν) is that they remain constant under proportional stressing (or straining). This enables correlations of stress state with ductility measures to be established. However, when strong spatial gradients are present, such correlations may become meaningless.

The stress state in front of a crack tip depends on crack tip and specimen geometry, notably its thickness t , as well as material nonlinearity. If the plastic zone size r_p is much smaller than t the near-tip region undergoes plane strain conditions. If not, the stress state is more complex to describe and evolves towards near plane stress when $r_p \gg t$. Fig. 18 illustrates the gradients of T ahead of a stationary blunted crack in both limits. In plane strain, the highest stress triaxiality is about 3–4, except close to the crack surface, with higher values realized for a large strain hardening exponent n . In thin sheets, the near-tip variation of triaxiality depends on location, mid-plane versus surface (Fig. 18b) with the far-field converging to the so-called HRR solution [76,77]. In particular, values of T as high as 2 may be reached inside the fracture process zone. This shows that the plane stress approximation generally breaks down in thin sheets with cracks, especially with the development of crack-tip necking. For a propagating crack, the crack-tip geometry evolves along with the strain singularity and this leads to a concurrent evolution in normal stresses, hence triaxiality. First established for elastic ideally plastic materials under small scale yielding [132], these facts were verified using FE simulations, e.g. [68,133] and have implications in ductile to brittle transitions, e.g. [134,135].

4.1.4 Fracture locus

There is ample evidence that ductile fracture depends on states of stress and strain. Crack tunneling, for instance, is a qualitative manifestation of triaxiality effects. To quantify this dependence, loci of some measure of fracture versus a measure of stress state, say the triaxiality, are commonly obtained from experiments. More generally, a fracture locus may be defined in terms of a strain-to-failure, $\bar{\epsilon}_i$ or $\bar{\epsilon}_f$, viewed as a function of T and L , both taken as weakly varying upon straining. Cross-sections of the surface $\bar{\epsilon}_f(T, L)$ are illustrated in Fig. 19. In general, the higher the triaxiality the lower the fracture strain. This effect of stress triaxiality on ductile fracture is established for sufficient levels of superimposed hydrostatic stress [23,136–138]. On the other hand, the effect of the Lode parameter, if at all significant, is not well characterized. Experimentally, it is difficult to infer plots of $\bar{\epsilon}_f$ versus L at fixed T . Mostly under plane stress loading can L be varied over its full range, but not independently of T , as inferred from Table 6 and Fig. 17. In general, the same value of the pair (T, L) can be realized for distinct stress states.

For positive triaxialities lower than about 2/3, lower-than-expected fracture strains have been reported for certain materials [139–141]. It is unclear whether such trends are simply the consequence of shear fractures, i.e., fracture by mechanical instability of the test piece. Nevertheless, understanding such effects is key in a variety of processes where combined tension and shear loadings prevail, such as in high-speed machining, metalworking and ballistic fractures. For negative triaxialities, strong variations in T and L occur at fracture locations,

as described next.

For nominally negative triaxialities, which are relevant to metalworking, surface states of biaxial compression–tension or even biaxial tension typically arise. A wide range of states can be generated considering various geometry and friction parameters. These include the height-to-diameter ratio in compression, the plate thickness in rolling and bending, and the die or roll friction in compression or rolling. In such cases, a useful concept is that of upsettability limit (Fig. 20), which represents the relation between extensional and contraction strains at fracture. The fact that the loci of fracture strains lie parallel to the strain path for homogeneous (frictionless) compression has first been noted by Kudo and Aoi [116] and confirmed by others [124,142]. Invariably, the intercept of the upsettability limit curve with the ordinate axis corresponds to plane strain. Fig. 21 shows the nonlinear strain paths to fracture obtained by Kudo and Aoi using surface markers. Remarkably, fracture occurs when the nonlinear strain paths cross the fracture line. Having established such seemingly path-independent fracture loci in upsetting has immense practical interest. Whether such behavior generalizes to arbitrary nonproportional stressing paths is far from trivial, as will be discussed in Section 4.6.3. It also bears emphasis here that what is meant by “fracture” in upsetting is surface crack initiation. As noted above in Sections 4.1.1 and 4.1.2 complete fracture does not occur in ductile metals in compression. In less ductile metals, however, shear-splitting fracture does occur and the (global) strain paths essentially follow the homogeneous compression path to fracture. This is the case for example in high-strength aluminum and Mg alloys.

4.1.5 Crack growth resistance curves

In principle, there is a direct connection between the fracture locus and J_R curves. The initiation toughness J_{Ic} must be related to some local critical strain corresponding to the stress state prevailing ahead of the crack tip. However, not only the loading is nonproportional it also evolves for a propagating crack. In addition, any critical strain must be defined over a critical distance from simple dimensional considerations. In practice, therefore, it is customary to characterize materials using crack growth resistance curves, as defined in Section 4.1.2. What is emphasized here is the constraint effect on J_R curves measured on nominally plane strain specimens (Fig. 22). Research conducted in the 80’s and 90’s has documented ample evidence for the strong dependence of J_R curves upon specimen size or loading configuration (e.g. bending- versus tension-dominated) [11,81,143–145]. The validity of the J_R curve to assess the integrity of structures for long crack extensions was thus not guaranteed anymore [145].

In the framework of fracture mechanics, the constraint effect can be quantified through either the so-called T-stress (for small scale yielding only) or the so-called Q-stress, which measures the departure from the HRR solution [146,147]. These additional terms essentially mean that the stress triaxiality profile deviates from the small scale yielding expected profile, thus affecting the evolution of damage. Implementing T-stress or Q-stress based methods requires to experimentally construct a two-parameter failure criterion instead of only one (J_{Ic}). Such extensions have found limited use in engineering practice due to the complexity in the determination of the two-parameter criterion. Alternatively, these constraint effects have provided a strong motivation for the development of the local approach to fracture.

The initiation toughness J_{Ic} is affected by constraint as well. An obvious manifestation of this is when the specimen thickness t is decreased enough so that plane strain conditions cease to prevail. In some materials the fracture toughness monotonically increases with decreasing t (curve a in Fig. 23); in others, a peak value of J_{Ic} is reported at some intermediate thickness (curve b). Possible mechanisms responsible for such differences will be addressed in Section 4.2.7. The key point here is that fracture toughness is not well defined outside the high constraint plane strain regime and that the total work of fracture Γ decomposed as in equation (38) above provides a rationale for a thickness-dependent fracture toughness.

4.1.6 Shear failure versus failure in shear

When ductile failure occurs by mechanical instability of the test piece, the fracture surface exhibits total or partial slanting, which is often referred to as *shear failure*. This typically occurs in plane strain bars and thin sheets, in the shear lips of a cup-cone fracture, etc. Shear failure is associated with the formation of shear bands [113]. In particular, it may occur even in the absence of a shear component in the remote loading. On the other hand, failure in shear refers to the phenomena of fracture under remote shear (or shear-dominated) loading. Because shear testing is typically cumbersome fracture data in shear is rather limited. The most comprehensive data base on failure in shear was generated by Johnson and co-workers for a dozen materials using torsion of thin-walled tubes [123, 148]. In recent years, other specimen designs known as the butterfly specimen [139] and the double-notched torsion specimen [140] have been used, not without ambiguity in the actual stress states that are generated or the fracture strains that are reported, as noted by Haltom et al. [149] who employed tubular tension/torsion specimens. In general, understanding shear failure, as a potentially adverse limit to ductility and formability, is of utmost importance. The understanding of material failure in shear is also important in some applications.

4.1.7 Dynamic behavior

Dynamic aspects in ductile failure manifest in different ways and inevitably bring about the effects of inertia, temperature and strain rate. Inertia enters at macroscopic, and possibly microscopic, scales. At the macroscale, wave effects may enhance local stresses and affect plastic flow localization in ways not encountered under quasi-static or low-velocity impact loadings. Strain rate and temperature effects arise naturally in dynamic loadings, whether in the regime of thermally activated or that of phonon-drag controlled plastic flow [150]. To access the basic phenomenology of dynamic ductile failure, it is not possible to use the specimens typically employed in quasi-static experiments, whether pre-cracked or not. Partly for this reason, some measures of dynamic fracture are rather peculiar. Important measures include the spall strength and ballistic limit. Ductile spall refers to the process of internal failure of condensed matter due to tensile stresses exceeding the tensile strength [150, 151]. It is the result of the interaction of two or more rarefaction waves, in the wake of a shock. Typical experiments probing into dynamic fracture include: (i) plate impact [152]; (ii) Taylor impact [153]; (iii) dynamic ring expansion [154]; (iv) laser shock experiments [155]; and (v) ballistic impact [156, 157]. In low-velocity impact experiments, which typically access intermediate strain rates, such as the Charpy impact test, inertial effects are small but those

of temperature and strain rate are important, particularly for rate-sensitive materials such as BCC metals.

In a typical spall experiment, a compressive shock is produced at the front surface of a target material. The first rarefaction wave forms on the trailing side of the shock front and propagates in the same direction. A second rarefaction wave develops subsequent to the shock interacting with the back (free) surface of the target; it goes opposite the shock direction. It is the interference of these two rarefaction waves that produces a spall plane where dynamic fracture eventually occurs. In dynamic ring expansion, inertial effects manifest in the formation of multiple necks [154], an information that can be used along with careful post-mortem measurements to estimate the fracture strain in dynamic loadings [158]. The latter is generally found to be greater than under quasi-static loading conditions. In particular, when applied to materials with a low ambient rate-sensitivity, such as Al alloys, the increase in fracture strain can presumably be related to dynamic effects [158]. In ballistic impact, three categories of failure mode are observed: adiabatic shear plugging, ductile failure and discing [159]. The ballistic limit velocity depends on the failure mode [160]. Adiabatic shear is favored if the rate of thermal softening exceeds the rate of strain hardening of the target material. The intersection of shear bands with a free surface leads to asymmetric deformation and this does not favor further propagation of adiabatic shear bands. Failure is then completed by ductile tearing or by discing, particularly if the target plate has a low through thickness toughness. Initially, the penetration process is shear-dominated, but at later stages, combined tensile and shear contributions become equally important [156].

In all of the above, an important aspect that distinguishes dynamic from quasistatic fracture is fragmentation. Hence, much effort has been devoted to the characterization and prediction of the number and fragment size distribution. Systematic studies of material microstructure effects are still lacking in dynamic ductile failure, although there is some evidence that material purity and plastic anisotropy affect the fracture process; see e.g. [159] in ballistics and more recently [155] for ductile spall. There is also a need to quantify the effects of stress triaxiality in dynamic fracture of ductile metals. However, such studies are difficult to conduct on the sole basis of experiments.

4.1.8 Effects of temperature and strain rate

The fracture loci of ductile materials as well as their crack growth resistance curves exhibit temperature and strain-rate dependence to various extents. Figure 24 depicts some typical trends. In general, the fracture strain in simple tension, $\bar{\epsilon}_f$, increases with increasing temperature, as illustrated in Fig. 24a for AZ31B Mg alloy [161] and in Fig. 24b for Al alloy 6005A [162]. The same trend is observed in various steels, e.g. [138, 163], as well as iron and copper alloys [138]. However, in other steels the fracture strain decreases with increasing temperature, exhibits a “ductility-minimum point” then increases again [164, 165]. Eventually, at higher temperatures closer to T_m the ductility decreases [164]. The ductility-minimum phenomenon is typical of austenitic stainless steel types 304 and 316 [164] as well as some C-Mn-Cr-Mo and pressure vessel steels [165, 166].

The effect of strain rate on ductility (elongation to fracture, reduction of area or tensile fracture strain) depends on the material and strain rate range. In Mg alloys the fracture strain decreases with increasing strain rate, Fig. 24a. This is also the case of some Al alloys

and steels, except at dynamic rates of loading for which $\bar{\epsilon}_f$ increases, e.g. [163]. In other Al alloys, however, the fracture strain is found to increase with increasing strain rate, as illustrated in Fig. 24b after [162]. Similar behavior is observed in some steels [164].

The effect of loading rate on fracture toughness has been mostly investigated in steels. Increasing the loading rate generally increases the fracture toughness¹. Here too, there are exceptions particularly in ferritic pipe steels as reported in [168] or HY80 in [169].

Regarding the effect of temperature on fracture toughness, the trends may be inferred from the upper shelf of ductile to brittle transition curves. Those obtained by Charpy impact generally exhibit an absorbed energy that increases with increasing temperature. However, on the upper shelf J_{Ic} is generally found to decrease with increasing temperature [168,169]. For materials exhibiting the ductility-minimum point, Amar and Pineau [165] have characterized the temperature dependence of notch ductility as well as fracture toughness. They found that J_{Ic} exhibits a minimum in about the same temperature range, Fig. 24c.

The mechanisms associated with the above phenomena will be addressed in Section 4.2.9 with some analysis deferred to the end of Sections 4.6.4 and 4.6.6.

4.1.9 Size effect

The influence of specimen size on the fracture stress of brittle materials is well documented, as discussed previously in Section 2. Accurate determination of the plane strain fracture toughness of ductile materials requires careful attention to the relationship between crack length and specimen size. However much less is known about the effect of specimen size on the failure of ductile materials in initially crack-free tensile specimens. In these cases, failure measures are obtained from tests on laboratory specimens that are much smaller than structural components. Implicit to the use of these laboratory specimens is the absence of a specimen-size-effect. A number of studies, e.g. on a low alloy Mn steel [170], on a duplex stainless steel [171] and on a high toughness HY100 steel [172], conducted on geometrically similar notched specimens have shown that the tensile strain to failure and the scatter band of ductility values increased in small specimen sizes. Possible explanations of this phenomenon will be discussed in Section 4.6.5 and become more clear once the basic mechanisms are elucidated in Section 4.2. In other studies [173,174], a decrease in the Charpy specimen size was shown to decrease the normalized upper-shelf energy. Since neither the loading rate nor the notch geometry were scaled, such results demonstrate a definite effect of specimen geometry, not necessarily of size, on impact energy. The size effect observed in the ductile fracture of the above studies should be investigated in more detail because of its practical importance.

A universal theory of ductile fracture is expected to rationalize the above basic phenomenological aspects of fracture, including fracture paths and available measures. No such theory exists at present.

¹This holds for the ductile range. The brittle fracture toughness decreases with increasing loading rate [167,168].

4.2 Mechanisms

Plateau et al. [175] showed the first fractographs of dimpled fracture surfaces using scanning electron microscopy (SEM); also see [176,177]. A typical fracture surface is shown in Fig. 25 after [178]. The presence of dimples is commonly used to rationalize fracture by void growth to coalescence, although it does not inform on the void growth kinetics nor does it tell where all the dimples originate from. The main dimples are traces of voids formed at second-phase particles or inclusions. The other dimples fill a spectrum of length scales and there is uncertainty on where they come from and at what stage in the deformation and damage process. In ductile fracture, relevant mechanisms operate at various length scales. In what follows we discuss mechanisms, first at the scale of individual heterogeneities (particles), then aspects of their collective manifestation in shear bands, in crack initiation and at the crack tip.

4.2.1 Void nucleation

That metals fracture prematurely due to void formation at second-phase particles was first established by Tipper [179]. An early review of the phenomenon by Goods and Brown [30] addressed both homogeneous and heterogeneous nucleation, highlighting in the latter the role of second phase particles and grain boundaries. The nucleation from large inclusions has been studied in model materials [180] and observed by means of high-resolution X-ray tomography [181], as illustrated in Fig. 26. In a soft matrix (here pure Al), nucleation predominately occurs by particle decohesion (Fig. 26a). By way of contrast, in a hard matrix (here a structural Al alloy), particle cracking is typically favored (Fig. 26b). These two mechanisms occur to various extents in engineering materials [23,182,183]. When particle plasticity is limited or inexistent, particle cracking may be viewed as a cleavage process, the fundamentals of which have been reviewed in Section 2. On the other hand, the fine-scale mechanism of particle–matrix decohesion may be thought of as involving the coalescence of prismatic vacancy loops at the interface [184], followed by interface crack propagation.

Parameters other than the matrix hardness play a role in particle induced void nucleation. Table 7 summarizes the key trends. Also, particle fragmentation is often reported for elongated inclusions when loaded along their length. In addition, other damage nucleation mechanisms are reported in multiphase material systems whereby microcracks are formed in a brittle phase, e.g., in duplex stainless steels [185], in dual phase steels or TRIP multiphase steels [186–188], or at weak interfaces and twin boundaries [189,190].

Void nucleation in shear dominated loading has recently been observed *in situ* in SEM [191]. In this particular instance of a hard particle of alumina (Fig. 27), nucleation occurs by decohesion at the south-east and north-west corners, where the interfacial normal tractions are expected to be tensile. It is likely that decohesion occurred earlier than could be visualized but the absence of hydrostatic tension did not provide a driving force for growth. Eventually, significant void distortion takes place which will be discussed in context below.

There likely is a lower cutoff particle size in void nucleation. Although detailed analyses of this are missing, the cutoff can be inferred from mere inspection of a large ensemble of void nucleation sites, as illustrated in [112], or by discrete dislocation and strain gradient plasticity arguments. For the usual dislocation densities encountered in worked metals, the

cutoff size is about $0.1\mu\text{m}$ or thereabout.

4.2.2 Void growth

The growth of voids nucleated at artificially embedded, weakly bonded inclusions was studied by Marini et al. [180] who validated the general form of the void enlargement law by Rice and Tracey [192]. Later, the growth of porosity in sintered iron was studied by Becker et al. [193] motivating some important choices in micromechanical parameters entering the Gurson model [9] modified by Tvergaard and Needleman [10,194]. More recently, the growth of artificially inserted voids has been studied *in situ* by means of tomography by Weck et al. [195]. Fig. 28 shows the void enlargement that occurs in a miniature ($0.215 \times 0.25 \times 2\text{mm}^3$) gage section of copper subjected to simple tension after an effective strain of 0.5 (about twice the necking strain). Because the void concentrates stress, it elongates at first at a rate roughly twice that of the specimen. Later, it extends more slowly as will be elaborated upon in the modeling section. The voids also grow laterally to an extent commensurate with the necking-induced increase in triaxiality. At the strain reached in Fig. 28, the lateral void-diameter to void-spacing ratio is about one third. Beyond this stage, the kinetics of void growth drastically changes due to plastic flow localization in the elongated ligaments, even if the authors in [195] analyzed it within the conventional Rice-Tracey framework.

Tomography studies such as in [181] and [195] are still largely limited to simple tension and to large porosity levels (~ 0.065 in [195]); [with some exceptions, see \[196\] for a recent review](#). Several other studies of void growth in engineering materials were carried out using more tedious destructive methods but enabling triaxiality effects and small porosity regimes to be explored. Notable among these are the experiments by Pardoen et al. [197], Koss and co-workers [198] and Benzerga et al. [112]. One common finding from these investigations is that local porosity levels at the onset of a macroscopic crack in the notched specimens are ~ 0.01 . This figure is relatively low compared with [193], yet quite large if one considers that mean void enlargement ratios in excess of 4–5 may be reached from an initial porosity of about 10^{-4} typical of structural metals.

Another important consideration in engineering alloys is the possible coexistence of void growth and continuous void nucleation. In contrast with materials involving embedded large porosity, structural materials usually contain several populations of second phase particles acting as potential void nucleation sites. In some other materials void formation can be deformation-induced, without involvement of particles, as indicated above. Situations where nucleation occurs at multiple separate scales pose serious challenges for modeling within the conventional continuum thermomechanics framework. In particular, if nucleation occurs at fine precipitates that directly affect the plastic flow characteristics (strength and hardening) then scale separation breaks down, as will be further discussed below. In many cases, there is a clear separation of scales whereby the critical voids enlarge while the fine particles, such as carbides in steels, remain intact (Fig. 29). Cohesion of carbides with the matrix may play a role in this, but it is also likely that the absence of nucleation at carbides is the result of a size effect. Submicron-sized voids are usually located within a grain and dislocation-void interactions may become prominent [184]. Thus, in general, a size effect is concurrent with the emergence of key crystallographic aspects to void growth which may persist even for larger voids [199]. For example, in HCP metals where plasticity is dominated by prismatic

slip, such as Zr, hexagonal cross-sectional void shapes were correlated with the alternate activation of multiple (prismatic) systems [200]. A more recent study [201] based on the mechanics of prismatic dislocation loops has shown that void nucleation and void growth are slowed down for small particle size, i.e. when the size of the void is less than about 400 times the Burgers vector. For larger particles, the volume change, as assumed in the following, was found to be independent of the void radius.

4.2.3 Void distortion

The void enlargement that occurs at sufficiently high triaxiality is usually accompanied with void shape changes. But the latter are more prominent at low stress triaxialities. In this regime, even with slow or inexistent void growth, the void distortion can be damaging. The micromechanics of damage cracking by void distortion at low stress triaxiality is a subject of continuing interest. Several types of void distortion occur dependent upon (i) the presence of a shear component, and (ii) the importance of void-particle interactions. Their relevance to damage accumulation and fracture in engineering materials varies accordingly. Consider first the distortion of voids free of particles in a uniaxial bar, Fig. 30. Void elongation takes place, nearly with no net void growth within the triaxiality range $T = 0.33$ – 0.5 [195]. Theoretical calculations in the dilute limit (porosities typically lower than 0.01) indicate that for strictly uniaxial loading ($T = 1/3$) the voids would elongate indefinitely [202], unless they link up along their length. Furthermore, elongated voids can rotate, as illustrated in Fig. 31a, here simply convected with the material's plastic spin. Such rotations can impact the mode of linkup among voids.

On the other hand, the behavior of a void, free of particle constraints, in a shear field has been simulated (Fig. 31b) and tentatively observed (Fig. 31c). In a shear field, an initially spherical void would elongate then rotate [203] and eventually close into a penny-shape crack [204, 205], unless localization in the ligament takes place before closure [206]. Presumably, the void-rotation mediated closure induces localization and can thus provide a physical mechanism of failure under shear dominated loadings. Note that the situation depicted in Fig. 31c corresponds to remote uniaxial loading whereby a shear band traverses the array of voids. While it differs from that in Fig. 31b, it illustrates yet another case of void distortion, namely void shearing by a shear band whose width is smaller than the void size.

A more realistic situation is one in which void-particle interactions are taken into account. In uniaxial loading, void nucleation is not complete even in intensely strained, highly stressed regions. This is illustrated in Fig. 32a which shows that pieces of the inclusion were not torn free. Analysis shows that unless the triaxiality is larger than $2/3$ interfacial regions about the equator of the particle undergo compressive stresses [207, 208]. The forced contact between void and particle is referred to as *void locking* by the particle. Void locking leads to a net increase in porosity, even under pure uniaxial loading, in contrast with the case without inclusions (Fig. 30). Void locking also plays a major role under shear-dominated loading. With nucleation in shear taking place as described in Fig. 27, the locking prevents the void from closing so that the resulting distortion is essentially similar to one that would be reached after an initially spherical void has rotated, but with closing prevented, as illustrated in Fig. 31b. Analysis shows that such void distortions can be damaging in the sense that

they lead to plastic flow localization, hence failure [206]. The void locking phenomenon is more frequent than reported in the literature. Fig. 32c illustrates a situation in the neck of a notched steel bar just below the macroscopic crack so that the triaxiality is expected to be equal or somewhat greater than $2/3$. It is also likely that the locking here be the manifestation of a size effect given that the locked pieces of the inclusion are as thin as 200nm. More generally, void locking is believed to play an important role under predominantly deviatoric loadings, such as in tension, compression and shear.

4.2.4 Void coalescence

The physics of void coalescence has long been quite obscure, as noted by Ashby among others [209]. While some aspects are still unclear, the phenomenon has been documented through a series of empirical observations. Void coalescence occurs in a variety of modes, each having several variants, depending on microstructural factors, loading conditions and plastic flow characteristics. The phenomenon may be categorized into three modes, Fig. 33. One commonly observed mode is coalescence by internal necking of the intervoid ligament (Fig. 33a). It had been hypothesized by Henry [210] and Cottrell [211], rationalized in plane strain by Thomason [212], and discussed by Brown and Embury [213] and Argon et al. [214], interestingly with limited micrographical evidence (see [30]). The second mode of void coalescence is by internal shearing of the intervoid ligament (Fig. 33b). Localized shear can cause distant cavities to coalesce, giving rise to local failure by void sheeting [215]. The third generic mode is “necklace” coalescence (Fig. 33c). It consists of voids linking up along their length, a situation encountered in steels with elongated inclusions [216] or alloys with aligned particle clusters [161,217]. This mode is thought to be more benign to ductility, although it could drive ductile delaminations if combined with directional internal necking [7].

Real-time *in situ* observations, such as tomography, have been instrumental in providing many useful details, although as noted above they are still limited to model materials and simple loadings [195,196]. For example, Fig. 33d shows two instances of internal necking in the void array shown earlier in Fig. 28. In the first, regular internal necking takes place at the equators of two neighboring voids in the same row. In the second, internal necking occurs away from the equators as voids in different rows can also linkup, due to prior elongation (strong distortion). The same occurs in the copper-based composite with alumina particles known as Glidcop (Fig. 33e) for which the void array distortion was shown in Fig. 30. Given the current configuration of the array, irregular internal necking here takes place at $\pm 45^\circ$ from the loading direction. In all, the knife-edge separation typical of internal necking is clearly observed as in Fig. 33a. Note that the net true strains in Fig. 33d and Fig. 33e are 1.0 and 0.5, respectively.

Some details in Fig. 33 deserve a mention. The internal necking and internal shearing situations in (a) and (b) occur under a triaxial stress state developing inside the neck of a bar with a shallow notch. Here, internal shearing was favored by void rotation from an initial orientation at 45° from the loading axis. That in (c) corresponds to plane strain uniaxial loading after some necking. Thus, in all instances in Fig. 33 void coalescence proceeded while unimpeded by the void locking mechanism, either because of a sufficiently high triaxiality, (a)–(c), or the absence of particles altogether, (d)–(e). To complete the picture and point

to a potentially critical failure mechanism, Fig. 34 illustrates possible linkup processes in the model materials studied by Babout et al. [218]. Corresponding nucleation mechanisms were shown in Fig. 26. In the soft matrix, the particle-locked voids increased their volume while elongating, as expected in uniaxial tension. However, because locking operates at the equators, internal necking is not possible, despite the close packing. Eventually, coalescence by localized shear may occur. In the hard matrix, on the other hand, the penny-shaped cracks that formed subsequent to particle cracking (Fig. 26b) blunted a little before linking up together. While the mode of linkup between the most closely packed particles is by localized shear, the linkup between the distant particles is neither shearing nor necking; in the limit, internal necking of a ligament separating two cracks is meaningless. A particularly important, generally neglected aspect of ductile fracture is failure with limited void growth, even for shear-free remote loading. Yet, it is likely that the failure of technologically important materials, such as high strength steels and aluminum alloys, is governed by the linkup of crack-like voids. This failure mechanism is poorly picked up by state of the art computational micromechanics and thus poses profound challenges for analytical modeling. The advent of ultra-high-resolution tomography begins to document certain details of such processes, for example in aluminum alloys [219] and magnesium alloys [220].

4.2.5 From damage to cracking

When the individual mechanisms of void growth, distortion and coalescence operate at their respective nucleation sites, the material is said to be in a damaged state. This pertains to an initially crack-free specimen as well as to the plastic zone ahead of a crack tip. As the individual mechanisms proceed, damage accumulates until either a crack initiates, a pre-existing one propagates, or plastic [instability](#) occurs. In the latter case, [localization](#) manifests again either as a shear band leading to rupture of a crack-free specimen or a transition from normal to slant cracking. For illustration, Figure 35 depicts the damage to cracking transition in one of those four situations. It shows how the above fundamental mechanisms map onto the macroscopic load-displacement response in an initially crack-free specimen (here a round notched bar). At some stage past the maximum load, damage has gradually accumulated through the processes of void nucleation, growth and distortion, as well as a few isolated and inconsequential void-coalescence events. The knee in the curve at stage (c) corresponds to the onset of a macroscopic crack via the coalescence of the largest voids; see [112] for a detailed analysis. The descending part of the curve corresponds to crack propagation. In later stages, the faster load drop is due to the formation of shear lips. The fracture strains $\bar{\epsilon}_i$ and $\bar{\epsilon}_f$ mentioned in Section 4.1.2 correspond to the stages shown as (c) and (f) in Fig. 35. It is clear from the figure that these two measures are substantially different in a notched bar.

4.2.6 Mechanisms of shear failure

Shear failure may occur subsequent to diffuse necking [113] or localized necking, e.g. [221]. It has been observed in single crystals, polycrystals and amorphous metals. This fact would appear to imply that the material's microstructure has no effect on some general features of localization and favors a continuum mechanics approach to analyze it. However, microstruc-

ture affects the process in two ways. First, microscopic inhomogeneities may play a key role in triggering shear bands. Second, microstructural factors ultimately affect constitutive parameters that are essential in the localization of plastic deformation. On the one hand, one should be cognizant of the interplay between microscopic and macroscopic aspects of localization. On the other hand, it is important not to overly attempt to reconstruct the genesis of shear failure simply by observing fracture surfaces. Within a shear band, it is possible for voids to eventually nucleate, grow and coalesce. However, these processes may or may not be the primary driving mechanism.

Presumably, shear failure may occur without prior necking as well. This is the case, for instance, in torsional loading. In a tensile rod, a load maximum is usually observed and is related to cross-sectional area reduction and subsequent geometric softening. In torsion of thin-walled tubes, geometric softening is not possible. Yet, a maximum load is eventually reached [123, 148] and is attributed to thermal softening [115, 123]. As the plastic strain accumulates rather uniformly in the specimen, its temperature increases with the amount of heat produced depending on strain rate, material properties and specimen size. The temperature increase can be estimated in the adiabatic limit, e.g., [174, 222]. In general, however, it is affected by the fraction of plastic work locked in the underlying defect structure, known as the stored energy of cold work [223]. The phenomenon of thermal softening is thus likely to be a key factor in failures under shear-dominated loadings, including at low to moderate strain rates in less ductile metals [148]. Thermal softening eventually triggers plastic flow localization by shear and fracture would ensue as reported in [123, 148]. For much higher strain rates see Section 4.2.8.

Whether shear failure occurs because of “tensile” shear bands or thermal shear bands, the fracture surface usually exhibits parabolic dimples [140, 224], see Fig. 36. It is interesting to note that the sheared dimples appear in opposite directions on the two fractured surfaces when these are examined so as to reconstruct the pristine specimen. Sometimes, tear dimples are observed when there is a gradient in the axial opening stress, such as in bending-dominated fracture mechanics experiments (typically CT specimens). However, in these the parabolic dimples appear in the same direction on the two faces.

4.2.7 Mechanisms at the crack tip

Since no physical crack is atomically sharp, it has an initial crack-tip opening δ_0 , Fig. 37. Under static load, a plastic zone develops and the crack tip blunts. The morphology of the blunted tip is either regular (close to semi-circular) or irregular (exhibiting angular profiles) depending on details in the operating plasticity mechanisms. Typically, a sharp fatigue precrack has a submicron initial opening δ_0 and the opening δ may reach hundreds of microns in tough metals before crack growth initiation. As explained in Section 4.1.3, large normal stresses develop ahead of the crack tip, which in plane strain are three times the yield stress or higher. This favors the nucleation of voids on second phase particles. The size of the highly stressed zone, hence the crack opening δ itself, must be large enough so that the void nucleation sites, with mean spacing X_0 , are encompassed by the high stress region. Concurrently, a zone of high stress triaxiality develops at a distance roughly equal to δ from the crack tip where the plastic strains become also quite large. The conditions for void growth are thus favorable in this region. At some point, the size of the void gets

large enough and the distance from the crack tip small enough that plastic localization takes place, leading to a coalescence mechanism between the crack tip and the void, similar to the mechanism observed between two voids. In plane strain or near plane strain conditions, the equivalent strain, averaged over the volume element containing this void, is much smaller than the fracture strain measured with tensile or notched specimens due to the larger stress triaxiality, see Section 4.1.4. The linking of the first row of voids with the crack tip along the original precrack front constitutes the true physical cracking initiation, and sets the true physical fracture toughness, whether it is expressed by δ_c , K_{Ic} or J_{Ic} . Depending on the initial volume fraction of voids, as well as other parameters such as the void size and the strain hardening capacity, four scenarios are distinguished based on experimental studies (polished cross-sections of cracked specimens in interrupted tests or 3D tomography images) or numerical studies described in section 4.6.6:

1. At sufficiently high porosity, the void near the tip is influenced by its nearest neighbors, which roughly experience the same rate of growth. The interaction among the voids, including voids farther away from the tip, results in higher rate of growth for all voids. Coalescence between several voids and with the crack starts early, almost simultaneously. This is the *multiple void interaction mechanism* [225].
2. For sufficiently small void volume fraction, a single void process prevails, as envisioned by Rice and Johnson [226]. The void nearest to the tip grows with little influence from its neighbors further from the tip. This is the *void by void coalescence mechanism*. For instance, local fracture strains of about 0.4–0.75 have been measured at the tip of an initiating crack in HSLA steels [227], a value that would not be attained with the multiple void interaction mechanism, which involves typical fracture strains around 0.1 to 0.25 due to the very high stress triaxiality associated with the uniaxial mode of deformation [228]. Most metallic alloys have initial void volume fraction smaller than 10^{-2} and thus would fail by a void by void coalescence mechanism.
3. The two former cracking mechanisms imply a tensile mode of localization/coalescence associated to an internal necking mode. Examples of shear coalescence in a fracture process zone under plane strain conditions are provided by observations of “zig-zag” void by void coalescence in low hardening steels [229–231]. The early shear localisation process between the blunted crack tip and the nearest void is detrimental to the fracture toughness as it involves much less plastic work than a full void growth/coalescence to final impingement.
4. In the literature, there are several examples of critical δ_{Ic} equal to 10, 100 or even 1000 times X_0 , e.g. [177, 232] leading to a scenario that does not fit with the first three mechanisms, as represented in Fig. 37 by mechanism 4. In this case, the fracture process zone at crack initiation contains a very high number of voids and can be assimilated to a damage volume element inside a sharply notched tensile specimen, rather than to a crack tip; in this case the first coalescence events do not necessarily occur with the crack tip. The fundamental reason underlying this capacity to transition from scenario 2 to scenario 4 is still an open issue that has been overlooked in the literature.

The tearing of thin metallic sheets reveals additional complexity. As explained in Section 4.1.3, the stress triaxiality in the fracture process zone is much lower in near plane stress condition, hence the local strains that must be attained before coalescence are much larger than under plane strain. In plane stress, out-of-plane necking occurs ahead of the crack tip owing to the absence of through-thickness constraint; see Section 4.1.1. Inside this neck, voids grow following mechanisms 2 or 4 described above. Voids at the center of a sheet grow faster than at the outer surfaces due to the larger stress triaxiality there. Hence the local fracture strain leading to coalescence between the crack tip and the first voids is lower at the center. This explains the “bath tub” profile depicted in Fig. 37 associated with intense crack tip necking, as observed in a variety of metallic alloys all exhibiting moderate to high strain hardening capacity [118]. Necking is a source of additional dissipation that contributes to the elevation of apparent fracture toughness in thin metallic sheets, as indicated in Section 4.1.5. The other reason for the higher fracture toughness is the larger local fracture strain before failure due to the lower stress triaxiality. Hence, even though the stress level in the fracture process zone is lower than in plane strain, more energy is dissipated in thin specimens due to the large local plastic strains. This will be rationalized further in Section 4.6.6. *Note that in some precracked thin metallic sheets, cracking initiation is more related to plastic localization than to damage extension, as discussed in Section 4.4.2 later.*

Unlike for crack initiation, a propagating crack remains nominally sharp with an opening δ that is no larger than the diameter of the voids involved in the coalescence mechanism, see e.g. [233]. This holds for both thick and thin components. Hence, some metals could exhibit a type 4 mechanism at crack initiation which transitions into a type 1, 2 or 3 mechanism during propagation. This is associated with a change in the crack tip stress and strain fields during propagation as indicated in Section 4.1.3. The change of the crack tip profile and of the stress and strain fields has important consequences on the ductile-brittle transition, as well as on the choice of a representative parameter to characterize the resistance to crack growth. For instance, the crack tip opening displacement is a meaningful quantity mainly at crack initiation; whereas a parameter such as the crack tip opening angle has been found to be more representative of the intrinsic resistance to tearing during propagation, e.g. [234].

4.2.8 Dynamic aspects

The mechanisms pertaining to dynamic ductile failure (see Section 4.1.7 for phenomenological aspects) have been reviewed by Curran et al. [235]. They identified the same basic processes of void nucleation, growth and coalescence as under quasi-static loading conditions. In particular, second-phase particles and other inhomogeneities play the same role as preferred damage nucleation sites. Hence, one can project much of what is learned from quasi-static experiments, where microvoid kinetics can be studied, either by “freezing” through interrupted tests, or in real time, possibly in situ, as in tomography experiments. Metallurgical aspects of spalling have been reviewed by Meyers and Aimone [151]. The extensive reviews in [151, 235] contain a collection of micrographs illustrating the operating mechanisms and shall not be repeated here for brevity. To some extent, the parallel drawn by Curran et al. between the dynamic and quasi-static mechanisms is appropriate. However, some differences are worth pointing out. Obviously, the effects of higher strain rates and elevated temperatures, for example due to adiabatic heating, will manifest on void enlargement rates.

A related aspect is the possible formation of shear bands due to thermal softening alone or other microstructural transformations [157, 236], in the absence of any cavitation damage. Also, the stress enhancement due to dynamic effects may trigger nucleation mechanisms that would otherwise not be active. This is especially the case in pure metals under impulsive loading. Finally, the effect of inertia at the microscale, for instance on void growth and coalescence, is difficult to ascertain based on mere physical observations, but can impart rate-dependency in the fracture behavior beyond that which is rooted in the rate-sensitivity of the constituent material. Therefore, while the basic mechanisms of ductile failure under dynamic loadings are seemingly the same, the extent to which temperature, strain rate and inertia affect the various stages of fracture can only be assessed when experimental observations are supported by predictive models. Elements of that will be provided in Section 4.5.5.

4.2.9 Effects of temperature and strain rate

As described in Section 4.1.8, the temperature and strain (or loading) rate dependence of ductility and toughness exhibits a wealth of behaviors with general trends admitting several exceptions that are peculiar to some material systems. The mechanisms underlying such dependencies cannot be separated from the mechanisms of plastic flow. In general, this dependence is affected by the rate- and temperature sensitivity of the flow stress in two ways. At the macroscale, tensile properties tend to increase with increasing strain rate for instance [237]. This ultimately affects the energy dissipated before fracture, hence fracture toughness. At the microscale, the variation of flow stress with temperature or strain rate may ultimately affect the conditions for void nucleation [30] but also the rate of enlargement of voids and possibly their linkage [238]. In addition, the rate- and temperature sensitivity of plastic flow can lead to plastic instabilities in ways not encountered in weakly rate-sensitive materials. Examples include the Portevin-Le-Châtelier (PLC) effect and Lüders bands which are related to the static or dynamic strain aging phenomena [239, 240]. The latter has been clearly associated with the “minimum-ductility point” phenomenon (Fig. 24c).

4.3 Failure by Microvoid Coalescence

Ductile fracture always involves void or microcrack coalescence at some level. However, there are situations where failure is essentially controlled by coalescence (Fig. 15b) as opposed to situations where failure is controlled by macroscopic plastic instability (Fig. 15a). The mode of failure by microvoid coalescence notably occurs in notched or pre-cracked specimens, as well as round smooth bars. With the basic phenomena (Section 4.1) and their underlying mechanisms (Section 4.2) in mind, here we analyze this failure mode by using a mix of empirical evidence, as in the previous two sections, as well as direct numerical simulations with discrete voids. In doing so, we begin by considering the simpler case of ordered void distributions, then we discuss aspects related to the physical reality of disordered distributions.

4.3.1 Ordered void distributions

Finite element simulations of periodic arrays of voids have been carried out with increasing levels of sophistication going back to the early works of Needleman [241] and Tvergaard [194, 242], eventually culminating in the work of Koplik and Needleman [243]. Such simulations have revealed essential aspects of the failure process that are consistent with empirical evidence, but at a level of quantitative detail that would have been difficult, if not impossible, to access experimentally. An extensive and systematic review of these simulations was presented by Benzerga and Leblond [7]. For axisymmetric stress states (fixed Lode parameter $L = \pm 1$) an appropriate model is sketched in Fig. 38. The matrix is often modeled using rate-independent J_2 flow theory, but other representations have been used to account for crystal plasticity [244, 245], plastic anisotropy [246, 247], matrix dilatation through a secondary population of voids [248, 249], or rate-dependency [245].

The salient features of a typical cell model calculation are summarized in Fig. 39. The effect of stress triaxiality on the average response of the cell is shown in Fig. 39a. The abrupt drop in stress carrying capacity, which occurs at all values of $T > 1/3$, is a natural outcome of the calculations. Except at high triaxialities ($\gtrsim 3$) the drop takes place in a narrow window of strain over which there is a steep increase in void volume fraction f (Fig. 39b) and void flattening, as indicated by a decrease in the void aspect ratio w (Fig. 39c). This sudden change in behavior, which is indicated by x in the plots, is due to a bifurcation as revealed by examining the lateral strain E_r in Fig. 39d. During the load drop seen in Fig. 39a, E_r remains constant thus indicating that the lateral strain rate has vanished. In other words, the deformation mode of the cell has gone from triaxial to purely uniaxial extension even if the remote stress triaxiality is fixed throughout.

Microscopically, the bifurcation revealed in Fig. 39d corresponds to plastic flow localization in the intervoid ligament with elastic unloading taking place above and below the cavity. This behavior is reminiscent of the phenomenon of necking in bars. In fact, it does correspond to the necking of the intervoid ligament when the latter has become long enough, as evidenced experimentally in Section 4.2.4. A physical rendering of internal necking is shown in Fig. 40a where two replicas of the simulated cell are superposed for clarity. This situation corresponds to a post-localization state on the descending part of a loading curve such as in Fig. 39a. The decrease in void aspect ratio w post-localization (Fig. 39c) is indicative of an acceleration in the lateral void growth, which ultimately would lead to void impingement. Fig. 40b illustrates the process of internal necking in actual void distributions observed in tomography [195].

In general, the average cell response as in Fig. 39 is determined by the competition between the strain hardening of the matrix material and porosity induced softening. It is noted, however, that macroscopic softening is not necessary for the onset of internal necking, as would be obtained if the voids were initially more closely packed in the lateral direction [243]. The situation of uniaxial tension ($T = 1/3$) is worth pointing out. In this case, after a short transient (barely visible in Fig. 39b) the porosity does not evolve and consequently the macroscopic response is almost identical to the uniaxial response of the matrix (Fig. 39a, also see [250]). Although the voids elongate much in this case (see Fig. 39c) they never get close enough for internal necking to set in. This fact has direct implications of the intrinsic fracture locus of a material with initial pores.

Similar simulations for non-axisymmetric loadings and non-cylindrical cavities require full 3D calculations. Earlier work in this direction has essentially corroborated the 2D calculations of Koplik and Needleman [251–253]. Three-dimensional calculations have enabled to investigate Lode effects beyond stress triaxiality, notably under states of combined tension and shear [119, 203, 254–257].

Quite recently, Tekoglu et al. [258] have investigated the competition between microvoid coalescence and macroscopic plastic localization using a voided band embedded between two non-porous infinite slabs of material. At sufficiently high triaxiality, they found a clear separation between the two modes of localization. On the other hand, at lower stress triaxiality, the two modes occur simultaneously.

4.3.2 Disordered void distributions

Engineering materials rarely contain initial porosity, and if they do neither the voids nor the particles are regularly distributed. The question arises therefore as to whether nonuniform particle distribution and heterogeneous void nucleation affect crack initiation and growth, and if so, to what extent. The analysis of void coalescence for ordered void distributions revealed that the key parameter controlling the onset of internal necking and the transition to rapid void growth is the relative void spacing. The latter can be quite heterogeneous if particles, or more generally void nucleation sites, are clustered. If coalescence proceeds after significant amount of void growth, failure is said to be “growth controlled”. When nucleation is rapidly followed by void coalescence with limited amount of void growth, the fracture mechanism is said to be “nucleation controlled”.

The current state of understanding of the elementary mechanisms of nucleation, growth and coalescence of voids and their mutual interactions described in the earlier sections is relatively advanced, and has allowed the development of predictive models, as reviewed subsequently in Section 4.5. These models would lead to quite accurate predictions of ductile failure if the materials were as ideal as those treated by void cell calculations (Section 4.3.1) or those used as model materials for fundamental experimental studies (Section 4.2.4). However, microstructure and composition are inherently heterogeneous and this may affect the process of ductile fracture in ways that remain to be elucidated, particularly in what concerns statistical aspects. Various types of heterogeneities are illustrated in Fig. 41 involving: (1) distribution of chemical composition of the second phases or distribution of smaller size particles or defects in the second phases or at the interfaces, (2) distribution of second phase size, shape, orientation and position, (3) distribution of strength mismatch between hard and soft phase, (4) crystallographic orientation of the grain; (5) grain boundary and near grain boundary metallurgical heterogeneities, such as precipitate-free zones.

Type 1 heterogeneities lead to a distribution of particle nucleation strengths, which are also affected by particle size (type 2), especially if nucleation is limited by the cleavage strength of particles; e.g. in cast Al alloys with Si rich particles [259, 260]. Not only the intrinsic resistance to void nucleation varies from one second phase to another, but also the loading experienced by each phase depends on type 2 and type 5 heterogeneities. Depending on particle size, shape and orientation, proximity to another particle, hardness, crystallographic orientation of the host grain, and possible spatial variations of the matrix strength (e.g. in near grain-boundary precipitate-free zones [261] or in welded zones [262]), nucle-

ation may occur while the macroscopic response is still elastic or, in the other extreme, at large plastic strains to allow for stress buildup at nucleation sites [263]. Also, some voids can be present in the material from the processing steps, e.g. hydrogen related voids in Al alloys [264]. Furthermore, nucleation can be a combination of particle fracture and particle decohesion inside the same material essentially due to the dispersion of particle shape and orientation [23, 162]. Clustering effects were qualitatively observed in a metal in which particle-rich and particle-free regions were deliberately introduced [265] or simulated, see e.g. recent FE simulations [266]. Void nucleation is thus inherently a discontinuous process made of a succession of discrete nucleation events. This may actually lead not only to scatter but also to a size effect (Section 4.1.9). The duplex stainless steel studied by Devillers et al. [171] and containing about 20% ferrite and 80% austenite was aged at 400°C. This aging treatment produced a strong embrittlement effect due to the α/α' demixtion in the ferrite phase which is richer in Cr than the austenite. This embrittlement is due to the formation of cleavage cracks in the ferrite phase and arrested at the interface between ferrite and austenite. Final fracture occurs by cavity growth and coalescence initiated from these cleavage cracks. The damage appears heterogeneously distributed as millimeter-scale clusters.

Nonuniform void distributions do not significantly affect the void growth rates [267–270] and the interaction between voids during the growth phase is usually weak in the case of realistic initial porosity (i.e. below 1%). However, these interactions may become significant inside void clusters, e.g. [271], involving voids of various dimensions. For instance, a small void in the neighborhood of a large cavity grows much faster than in a single size population [249, 272]. The void growth rate depends on the crystallographic orientation of the host grain [244]. The process of void growth may also be influenced by details in the void nucleation mechanism. Voids nucleated by particle fracture are initially flat while voids nucleated by complete decohesion of the particle are initially rounded. This leads to differences in the void growth rates. In multiphase materials, the stress triaxiality in the softer phase is significantly larger than the overall triaxiality, with a concentration factor that depends on how the soft phase is constrained by the hard phase; see Fig. 41 and [273, 274], leading to major variations in void growth rates. Differences in rate sensitivity between the phases can also impact the void growth rates locally. It is thus not surprising, although generally overlooked in models, that significant differences in void growth rates (factor 5 or more) have recently been documented by tracking *in situ* the growth of individual voids in 3D tomography; see [275] for Ti and [276] for Al.

A population of voids of varying sizes and shapes thus grow when a metal is plastically deformed, also involving a distribution of spacings and orientations of the ligaments between neighboring cavities, Fig. 42. The mechanism of coalescence is affected by all these parameters. Hence, coalescence will be a sequential and statistical process, with the first coalescence events occurring inside void clusters and/or in zones involving high triaxiality and/or large plastic deformation (Fig. 42b). A coalescence event gives rise to a larger flat void which resembles a microcrack that can propagate through further discrete coalescence events with voids located near the crack tip (Fig. 42c), while other coalescence events take place at other locations in the material. At some point, one main crack dominates while plasticity ceases elsewhere, leading to the final loss of stress carrying capacity (Fig. 42d). This kind of percolation process, see e.g. [277], is still not well understood and the details depend on many factors related to the homogeneity of the process and the capacity of the

material to resist fracture when one microcrack is created. One issue is that clustering effects are convoluted with the probability of finding a cluster in a region of high-triaxiality and strain concentration. The analysis of all these heterogeneity effects (and their modeling, see Section 4.5) remains a widely open subject of investigation, at the core of the local approach to fracture.

4.4 Failure by Instability

There is a key difference between failure by void coalescence and by plastic instability. In the first, damage by void nucleation, distortion and coalescence is essential. In the second, damage may or may not be important depending on what drives the plastic instability. Here, we briefly overview the necessary and sufficient conditions for shear band formation from an otherwise macroscopically homogeneous stress and strain state (crack-free specimens) then address transitions to shear failure in initially precracked specimens.

4.4.1 Crack-free specimens

Ductility can also be limited by mechanical instability of the test piece or component. This usually involves the formation of shear bands and failure along the most intense shear band. In practice necking-type instabilities may also be viewed as precursors to failure, as in sheet metal forming. Plastic instabilities, such as shear bands, often involve strain localization at a length scale greater than the mean spacing between material inhomogeneities (voids).

[In materials that can be modeled as rate-independent](#), the onset of shear banding can be understood in terms of Rice’s localization criterion [278], which expresses necessary conditions for the loss of ellipticity of the incremental plasticity problem. This criterion constitutes the basis for a number of analytical studies of plastic flow localization in materials and has motivated a series of computational analyses with more complex boundary conditions. [For rate-dependent plastic flow, an alternative mathematical method for analyzing the onset of shear localization is based on a linear perturbation analysis](#), e.g. [279,280]. Published results to date, whether analytical or computational, may be viewed as a collection of potentially sufficient conditions for shear banding (Table 8). As such, they mainly tackle the forward problem: given some materials characteristics, rationalize the formation of a shear band. The combination of any two factors, say for instance the presence of a vertex in the yield surface along with thermal softening, would increase the propensity for shear band formation. In practice, however, the problem that is often posed is of an inverse type: given that the specimen or component exhibits shear failure, what characteristics of that material might have caused shear banding? Because of the ductility limiting character of shear bands, it is important to understand their origins and, in particular, whether they can be mitigated via microstructural design. Solving the inverse problem is generally difficult because most engineering materials exhibit a collection of “sufficient conditions” for shear banding, i.e., some strength differential, deviation from pure Schmid behavior, initial porosity (no matter how small), plastic anisotropy along with other microstructural evolutions that can induce strong yield surface curvatures (due to kinematic hardening or void coalescence for instance), etc. Although this is a relatively mature topic, physics-based predictions with quantitative assessment towards experiments are still lacking.

4.4.2 Precracked specimens

Often, the failure mechanism in thin sheets of high strength Al, steel or Ti involves a slant mode, e.g. [122, 281–284]. This slant mode develops soon after crack initiation with the occurrence of shear bands near the crack tip. Recent high-resolution in situ synchrotron X-ray laminography observations combined with digital volume correlation revealed, for an Al-Cu-Li alloy 2198, that the shear localization process develops before any damage can be observed [285]. In general, the following conditions have been shown to favour slant fracture: (1) a low strain hardening capacity [284]: Al 6056 alloy exhibits a slant profile in the age hardened heat treated condition T751 ($\sigma_0 = 300\text{MPa}$, $n \sim 0.06$) but a flat fracture mode in the annealed state ($\sigma_0 = 70\text{MPa}$, $n \sim 0.2$); (2) plastic anisotropy; Sutton et al. [286] reported that the fracture of 2024-T3 Al sheet was slant when loaded in the L-T configuration but flat when loaded in the T-L configuration; (3) the possibility to nucleate a second population of voids; Bron et al. [287] have shown for a 2024 T4 Al alloy that the slant regions were covered of both primary and secondary dimples. The presence of secondary voids favours the transition from internal necking coalescence to the void sheet shear type coalescence mechanism; (4) increasing the thickness changes the stress state at the crack tip by increasing the stress triaxiality, leading to a decrease of the propensity towards slant fracture, see Asserin-Lebert et al. [284]; (5) flat surfaces; side grooves generally force the crack to remain flat.

4.5 Predictive Modeling

Computational micromechanics has contributed its fair share to the development of the field of ductile fracture, e.g., [243, 288–295]. Numerous lessons can be drawn from cell model studies, which have become a major tool in understanding material behavior at intermediate scales. Classically based on finite elements, these analyses will continue to guide the development of improved ductile fracture models and provide a database for model validation at the mesoscale. Three groups of parameters can be varied and their effects investigated: (i) microstructural, as related to void attributes (void volume fraction, shape and spacing), e.g., [202, 243, 293, 295]; (ii) matrix parameters, such as plastic anisotropy [246, 247] and the plastic flow model [238, 244]; and (iii) loading parameters such as T [202, 243] and L , e.g., [119, 206, 245, 254, 296, 297]. While the cell model can be employed in large-scale simulations of fracture [11, 290, 293, 298] the ultimate objective is to develop micromechanics-based constitutive models in closed form, whenever scale separation is possible.

4.5.1 Damage nucleation

Attention is here focused on the size-independent regime with particle size effects mentioned where appropriate. A necessary condition for void nucleation is usually derived based on an energy argument similar to Griffith's [41] for brittle fracture. Thus, the energy release must exceed or equal the surface energy required to create new surfaces, the expression of the latter depending on the mechanism: particle cracking versus matrix–particle decohesion. A condition similar to equation (11) shows that the required stress scales as $1/\sqrt{a}$ with a the particle size [299]. In general, however, nucleation is observed at stresses an order of magnitude lower. As indicated in Section 2, this hints at stress enhancement mechanisms,

generally involving plastic flow in the matrix and dislocation accumulation near the particle interface. Various sufficient conditions have thus been developed in the literature. At the scale of an individual particle, the fundamental condition is stress based, but often results in a criterion of the critical strain type [30, 184, 288]. Ashby for instance analyzed the process of void nucleation at a particle inside a shear band [184]. The model is also qualitatively relevant to predominantly tensile stress states. At sufficient stress levels, comparable to the macroscopic yield stress, prismatic vacancy loops are punched out in the tensile stress regions while interstitial dislocation loops are nucleated in the compressive regions. Microvoids eventually form at the north-west or south-east interface areas (case of decohesion) when the vacancy loops coalesce therein.

Assuming that a continuum approach to void nucleation is applicable, the following criterion may be arrived at [214, 300]:

$$\sigma_I + \Sigma_m = \min(\sigma^d, \sigma^c) \quad (46)$$

where σ_I is the (local) maximum principal stress, Σ_m is the (remote) mean normal stress, and σ^d and σ^c are the interface and particle strengths, respectively. These do not correspond to the interface cohesive strength or ideal particle strength, rather to some effective strength measures. Such material parameters can be inferred from experimental measurements [182], eventually by inverse identification. Considering various particle geometries, Argon [300] developed approximate formulae for σ_I under a remote shear stress, then assumed that the effect of Σ_m can be modeled through superposition. Beremin [23] developed an improved model accounting for plastic strain incompatibility between matrix and inclusion and for particle shape effects. Their criterion reads:

$$\sigma_I = \Sigma_I + \hat{\sigma}_I = \min(\sigma^d, \sigma^c)$$

where Σ_I is the remote maximum principal stress and $\hat{\sigma}_I$ is a polarization stress arising in the inclusion due to strain incompatibility [22]. Their approximate criterion writes:

$$\Sigma_I + k(\Sigma_{eq} - \sigma_0) = \min(\sigma^d, \sigma^c) \quad (47)$$

where the quantities on the left hand side were introduced after (7).

In both (46) and (47) the stress fields are considered as uniform in the particle. By way of consequence, these criteria distinguish between particle cracking and decohesion only through the material-dependent critical stresses σ^c and σ^d . However, for a matrix undergoing plastic deformation the fields inside the elastic particle are not uniform. Later, accurate numerical solutions were derived for spherical particles [301] and for prolate spheroidal particles [207] embedded in an infinite elasto-plastic strain-hardening matrix subjected to axisymmetric proportional loadings. Here, results from Lee and Mear [207] are summarized in terms of stress concentration factors at the interface and in the particle, defined by:

$$\kappa^I = \frac{\max(\sigma_{\eta\eta}|_{\eta=\eta_0})}{\Sigma_{33}}; \quad \kappa^P = \frac{\max(\sigma_I^P|_{\eta \leq \eta_0})}{\Sigma_{33}}, \quad (48)$$

respectively, where Σ_{33} denotes the remote axial stress, $\sigma_{\eta\eta}$ the normal stress in spheroidal coordinates, σ_I^P is the major principal stress in the particle and $\eta = \eta_0$ defines the matrix–particle interface (η degenerates to r in spherical coordinates). Equation (48) indicates that

κ^I measures the maximum tensile normal traction at the interface while κ^P measures the maximum normal stress in the particle. Dimensional analysis shows that κ^I and κ^P generally depend on the remote stress triaxiality T , the modulus mismatch E^P/E , the matrix strain-hardening exponent N , Poisson's ratios, the matrix yield strength through the ratio σ_0/E , and the particle aspect ratio w^P . Note that $\kappa^I = \kappa^P$ for a linearly elastic matrix since the stress field is then uniform in the particle [302]. Salient features include the following:

- At low plastic strains, stress concentration at the interface is strongly affected by the modulus mismatch E^P/E (Fig. 43). At larger strains, however, κ^I asymptotes to a constant which depends on N , w^P and T , but not on E^P/E .
- The asymptotic value of κ^I increases with increasing either w^P or N , and with decreasing T , as illustrated in Fig. 44a. Also, the effect of particle aspect ratio is inferred by comparing Fig. 43a with Fig. 43b.
- In general, stress concentration in the particle does not exhibit an asymptotic behavior. The concentration factor κ^P increases with increasing either w^P or E^P/E , and with decreasing T . It is weakly sensitive to strain hardening.
- Stress concentration in the particle is always greater than that at the interface, as soon as plastic flow sets in (Fig. 44b). The difference between κ^P and κ^I is exacerbated for a large aspect ratio w^P (as shown in figure) or larger N .
- Both κ^P and κ^I decrease with increasing triaxiality, but not at the same rate. The ratio κ^P/κ^I is a decreasing function of T . Therefore, as T is increased void nucleation has a greater propensity to occur by interfacial decohesion instead of particle fracture.

The prediction of an asymptotic behavior for the interfacial factor κ^I (Fig. 44) indicates that void nucleation by particle–matrix debonding may not be a continuous process. The fact that the asymptotic value of κ^I is reached at strains smaller than 0.05 suggests that if decohesion does not take place in the early stages of deformation, it most likely will not occur subsequently. This prediction does not take into account possible effects at dislocation scales [303].

It is tempting to make use of Lee and Mear's stress concentration factors to improve upon the Argon–Beremin criteria. *Concurrent* void nucleation conditions may be written as

$$\kappa^I \Sigma_I = \sigma^d \quad \text{versus} \quad \kappa^P \Sigma_I = \sigma^c \quad (49)$$

for interfacial debonding and particle cracking, respectively. Predicting the specific mode of void nucleation, whichever occurs first according to (49), presents the advantage of identifying the initial void shape and volume. As shown in the tomographs of Fig. 26, particle cracking leads to the formation of a microcrack whereas interfacial debonding, when complete, leads to the formation of a void having the same volume as that of the inclusion. While criterion (49) is superior in principle to criteria (46) and (47), the expressions of the stress concentration factors are not known at present in closed form. For practical purposes, the Beremin model provides an adequate micromechanical description of the void nucleation condition. It matches the trends of the Lee–Mear analysis regarding the effect of particle

aspect ratio and stress triaxiality. In addition, it accounts for cases of transverse loading and oblate particles. Other closed-form homogenization schemes can be used to estimate the particle stress; see e.g. [304].

When void nucleation is by particle fracture, criteria (47) or (49)₂ are adequate since the normal stress is roughly uniform over the meridian plane [207]. However, interfacial debonding is a process with an initiation and a growth stage, as evidenced by cell model studies using cohesive interfaces [288]. Depending on stress triaxiality, void locking by the particle may prevent complete debonding, typically for $T < 2/3$ [208] (see Fig. 32). Even at higher triaxiality, the strain range over which debonding takes place can be significant [288]. Under such circumstances, criterion (49)₁ may be adequate for initiation but not for complete void formation. To illustrate this, Fig. 45 shows the distribution of the interfacial normal traction as a function of the angle ϕ measured from the x_3 axis, as calculated in [207]. For ease of visualization, Fig. 46 qualitatively depicts the corresponding traction distributions. Under uniaxial loading (Fig. 45a), if local debonding initiates it is more likely to proceed for spherical particles than for elongated ones (see the sketch of Fig. 46a). For higher remote triaxialities (Fig. 45b), the interfacial normal traction is tensile everywhere and more uniformly distributed, irrespective of the particle aspect ratio. In this case, once debonding initiates it likely proceeds with complete separation.

The effect of particle size, which was long debated, is now understood as the consequence of local particle volume fraction fluctuations leading to increased interactions among particles. Above a certain particle size, there is no absolute particle size effect on the plastic strain required for void nucleation by debonding. This does not mean that particle size does not matter at all. There is a minimum size below which interfacial cracks are not energetically favorable [214, 305]. This critical size was initially estimated at about 25 nm but is likely above 100 nm [306]. Also, for particle cracking, a size effect is associated with a Weibull-like distribution of defects in the particles affecting their brittle strength (Section 2). In addition, for small particles, say below 1 μm , a size effect in debonding by gradual decohesion may emerge because of the cohesive length [288].

While void nucleation models based on critical stress conditions and stress concentration factors are more basic, some practical difficulties are associated with them. One example is the incomplete decohesion due to stress state or loading orientation. In practice, continuum void nucleation models have been developed and gained wide attention [307]. Other models have attempted to account for more complex stress state effects, linking in particular the void nucleation condition with the material toughness on phenomenological grounds [308]. Finally, for large particle volume fractions, as in multiphase steels, the nucleation of voids leads to a loss of stress carrying capacity in the reinforcing phase. Such situations are challenging to model as they lead to a breakdown in scale separation. In particular, void nucleation inevitably affects the hardening law of the material. A composite model has recently been developed to address this issue [304].

4.5.2 Structure of constitutive relations

A formulation coupling plasticity and damage is possible and useful. In the absence of damage, plasticity and creep essentially depend on the stress deviator, except for extreme pressures (typically $p/K > 10^{-2}$ with K the bulk modulus). With the equivalent stress σ_{eq}

defined in (41), a work-conjugate equivalent strain rate is defined as

$$\dot{\epsilon}_{\text{eq}} = \left\{ \frac{2}{3} [(\dot{\epsilon}_1 - \dot{\epsilon}_2)^2 + (\dot{\epsilon}_2 - \dot{\epsilon}_3)^2 + (\dot{\epsilon}_3 - \dot{\epsilon}_1)^2] \right\}^{1/2} \quad (50)$$

where $\dot{\epsilon}_1$, $\dot{\epsilon}_2$, and $\dot{\epsilon}_3$ are the principal strain rates. Given a mechanism of deformation (low-temperature plasticity, twinning, power law creep, etc.), the strain rate for that mechanism is governed by

$$\dot{\epsilon}_{\text{eq}} = F(\bar{\sigma}, \theta, S_i, P_i). \quad (51)$$

Here, $\bar{\sigma} = \sigma_{\text{eq}}$ in the absence of damage, θ is the temperature, S_i a collection of state variables (dislocation density, grain size, etc.), and P_i a set of material properties (atomic volume, elastic moduli, diffusion constants, etc.) If multiple mechanisms are active, their rates are summed up. The rate of change of structure is described by equations of the type:

$$\dot{S}_i = G_i(\bar{\sigma}, \theta, S_j, P_j). \quad (52)$$

In the presence of damage, the set of state variables S_i is augmented with a subset of variables D_i that describe the microstructural state of damage: volume, shape, orientation, and eventually spatial distribution of voids or microcracks. Each mechanism or stage of damage accumulation (void nucleation, growth, distortion, internal necking, internal shearing, etc.) is described by a rate equation:

$$\dot{D}_i = H_i(\sigma_{\text{eq}}, \sigma_{\text{m}}, \sigma_{\text{I}}, \bar{\sigma}, \theta, S_j, P_j) \quad (53)$$

where σ_{I} is the maximum principal stress, the significance of which is clear in the context of nucleation. If multiple damage mechanisms are active in the elementary volume their rates are summed up, not without difficulty in some cases, to be noted where appropriate.

The coupled set of constitutive equations (51)–(53) can be integrated over any loading history to give a complete description of plasticity and damage accumulation, and provide the time or strain to fracture. While models that define $\dot{\epsilon}_{\text{eq}}$ through (51) are satisfactory those that describe the evolution of structure, including damage, with time or strain are more difficult to formulate. Although progress has been made in formulating and applying evolution equations for state variables in plasticity [309], their extensions to damage, based on physical models, are still lacking. It is therefore convenient to make simplifying assumptions about the structure, as relevant to plasticity, equations (52), while retaining some level of complexity related to damage, equations (53). Two alternative assumptions are typically used and were, for example, made for the construction of deformation and fracture mechanism maps [209, 310]. The simplest is the assumption of constant structure, i.e.,

$$S_i = S_i^0 \quad (54)$$

which is often made at low temperatures. The second is that of steady state:

$$\dot{S}_i = 0 \quad (55)$$

such as in high-temperature deformation. Neither is a good assumption for ductile damage. For plasticity, however, either assumption leads to eliminating (52) and to the simplified equation:

$$\dot{\epsilon}_{\text{eq}} = F(\bar{\sigma}, \theta) \quad (56)$$

since the S_i 's and P_i 's are either constant or determined by the external variables of stress and temperature. In what follows, the only state variables kept in the formulation are those describing the microstructural state of damage, i.e., the subset of D_i 's. Also, temperature effects are assumed to affect damage only through plasticity. Thus, the governing constitutive relations simplify into (56) and

$$\dot{D}_i = H_i(\sigma_{\text{eq}}, \sigma_m, \sigma_I, \bar{\sigma}, D_j, P_j). \quad (57)$$

Note that damage enters (56) if only implicitly, depending on how $\dot{\epsilon}_{\text{eq}}$ is interpreted in the presence of damage.

Generally, the damage state variables D_i include the void volume fraction, or porosity f , two void aspect ratios, w_j , vector directors describing the void orientation with respect to a fixed laboratory frame, and three ligament parameters, χ_k . Void growth, rotation and coalescence result from the evolution of these variables. The evolution of f beyond nucleation is governed by the equation of continuity which, to the neglect of elasticity, expresses the plastic incompressibility of the matrix material according to:

$$\dot{f} = (1 - f) \dot{\epsilon}_{kk} \quad (58)$$

so that $\dot{f}$ derives directly from a flow potential $\Phi(\boldsymbol{\sigma}; D_i, \bar{\sigma})$ since $\dot{\epsilon}_{kk} \propto \partial\Phi/\partial\sigma_m$ by normality. The derivation of flow potentials, mainly in the form of yield functions in the rate-independent limit, follows itself a general variational principle aimed at obtaining porous metal plasticity models by means of homogenization. The reader is referred to [7] for background and details. The same variational structure is however not available for the evolution of the aspect ratios and orientations of voids. For these, Eshelby concentration tensors for linear-comparison materials are employed as basis [311] and heuristically augmented in the nonlinear case using computational limit analysis on unit cells [312]. The evolution of the ligament parameters χ_k is relevant to void coalescence. It is governed by matrix incompressibility and specific evolution of the w_k 's post-localization due to the onset of elastic unloading above and below the voids [202, 243, 313].

In continuum damage mechanics, specialized to a single scalar damage variable D for illustration, D is structureless but is often thought of as the area fraction of voids or cracks; as such, it varies between 0 at the start of life and 1 at the end. In the above formulation, the only variables that truly vary between 0 and 1 (end of life) are the ligament parameters. In fact, fracture in the sense of abrupt loss of load bearing capacity of the elementary volume ensues if any of the χ_k 's nominally reaches 1. It is clear that the volume fraction of voids need not be 1 at complete fracture. In what follows, advances in void growth theory are used to report evolution equations for the D_i 's while progress in void coalescence theory is mainly used to deliver critical values D_i^c of the damage variables. When these are available, intrinsic as well as apparent fracture loci can be predicted. In the fully coupled formulation, crack growth resistance curves can also be obtained, using various simulation techniques, for any type of specimen. In a given model, not all damage variables are allowed to evolve. A model is obviously simpler if several D_i 's are subjected to the like of the ‘‘constant structure’’ assumption (54).

4.5.3 Growth and distortion

Isolated-Void Models McClintock [314] and Rice and Tracey [192] developed models for the growth of cylindrical and spherical voids, respectively, embedded in an infinite medium made of a rigid-perfectly plastic material. For spherical voids, the net void growth ratio is an appropriate damage variable, $D = D_1 \equiv \ln(R/R_0)$. For high positive stress triaxiality, the void growth rate is given by

$$\dot{D} \equiv \frac{\dot{R}}{R} = \alpha(\nu) \exp\left(\frac{3}{2} \frac{\sigma_m}{\bar{\sigma}}\right) \dot{\epsilon}_{\text{eq}} \quad (59)$$

where ν is the strain rate Lode variable defined in (45) and $\alpha(\nu) = \alpha' + \beta \nu$, α' and β being constants. In the original model [192] $\alpha' = 0.279$ and $\beta = 0.004$ so that $\alpha = 0.283$ under axisymmetric tensile loading ($\nu = +1$). Because the Lode dependence is small, the first term dominates and α is usually taken as independent of ν . Later, Huang [315] computed more accurate, supposedly converged, dilatation rates in the axisymmetric case giving $\alpha = 0.427$, that is about a 50% increase. For more general loadings, Rice and Tracey rationalized the following heuristic extension:

$$\frac{\dot{R}}{R} = 2\alpha' \sinh\left(\frac{3}{2} \frac{\sigma_m}{\bar{\sigma}}\right) \dot{\epsilon}_{\text{eq}} + 2\beta\nu \cosh\left(\frac{3}{2} \frac{\sigma_m}{\bar{\sigma}}\right) \dot{\epsilon}_{\text{eq}} \quad (60)$$

which is believed to be quite accurate for arbitrary values of T and ν , but only for spherical voids and using $2\alpha' = 0.846$ instead of 0.558 to account for Huang's correction. At low triaxialities, spherical voids do not retain their shape. Because of this, equation (60) becomes increasingly inaccurate upon straining. This has implications for predicted fracture strains; see Section 4.6.3. Rice and Tracey have also discussed void shape changes on approximate grounds.

The Gurson Model The only damage variable is f and void shape changes are neglected. This is a good approximation for the high-triaxiality behavior of plastically isotropic materials with equiaxed inclusions and voids. The rate of growth of damage is given by

$$\frac{\dot{f}}{f(1-f)} = \frac{3}{2} q \frac{\bar{\sigma}}{\sigma_{\text{eq}}} \sinh\left(\frac{3}{2} \frac{\sigma_m}{\bar{\sigma}}\right) \dot{\epsilon}_{\text{eq}} \quad (61)$$

where σ_{eq} , σ_m and $\bar{\sigma}$ are related through the Gurson flow potential:

$$\Phi^{\text{GT}}(\boldsymbol{\sigma}; f) \equiv \frac{\sigma_{\text{eq}}^2}{\bar{\sigma}^2} + 2qf \cosh\left(\frac{3}{2} \frac{\sigma_m}{\bar{\sigma}}\right) - 1 - q^2 f^2 = 0 \quad (62)$$

and $q = 1$ in the original model [9], otherwise a heuristic factor introduced by Tvergaard [194]. Several studies indicate that the correction factor q is not universal. To illustrate this, recall that for hydrostatic loading, Gurson's estimate of the yield stress is exact so that $q = 1$ in that case, irrespective of porosity. For more general loadings, q must be viewed as a function of f so long as void shape changes and hardening effects are disregarded. In the limit of a

dense material ($f \rightarrow 0$) equation (62) reduces to the von Mises potential and a void growth rate is inferred from (61) as

$$\frac{\dot{R}}{R} = \frac{q}{2} \sinh \left(\frac{3}{2} \frac{\sigma_m}{\bar{\sigma}} \right) \dot{\varepsilon}_{\text{eq}} \quad (63)$$

For this estimate to match Huang's correction one must have $q \approx 1.7$. The maximum correction is thus needed for $f \rightarrow 0$ since lower values were found to match finite-element cell model results for finite porosities, for example by Tvergaard [194] ($q = 1.5$) or Koplik and Needleman [243] ($q = 1.25$). The physical meaning of heuristic factor q has been somewhat debated for some time. Thus, Perrin and Leblond [316] developed an estimate for $q = 4/e \approx 1.47$ based on a self-consistent calculation that suggests that $q > 1$ is due to weak interactions among voids. However, the fact that q is maximum in the case of dilute porosities and that $q = 1$ is exact for infinite triaxiality suggests that multiple factors affect the magnitude of q , the void interaction effect being one aspect.

Integrating the damage law (61) for dilute porosities and proportional stressing (constant triaxiality T) leads to:

$$f = f_0 \exp \left[\frac{3}{2} q \sinh \left(\frac{3}{2} T \right) \varepsilon_{\text{eq}} \right] \quad (64)$$

which shows the exponential effect of stress triaxiality on void growth, a well known trend since McClintock [314] and Rice and Tracey [192]. The integrated form (64) is valid assuming $\sigma_{\text{eq}}/\bar{\sigma} \sim 1$, which is a better assumption than would seem on the basis of Fig. 39 for example, although it invariably overestimates void growth at high triaxiality and underestimates it at moderate triaxiality.

Incorporating Plastic Anisotropy Engineering materials in wrought form are often plastically anisotropic, even when elastically isotropic. In addition, large plastic strains lead to the development of texture, hence anisotropy. Also, in some materials intragranular cavities form which, at least in the early stages, evolve in a crystal matrix that is inherently anisotropic. Benzerga and Besson [246] have shown that, to first order, the rate of growth of porosity in an orthotropic Hill-type matrix is dictated by

$$\frac{\dot{f}}{f(1-f)} = \frac{3}{h} \frac{\bar{\sigma} \sigma_{\text{eq}}}{\sigma_H^2} \sinh \left(\frac{3}{h} \frac{\sigma_m}{\bar{\sigma}} \right) \dot{\varepsilon}_{\text{eq}} \quad (65)$$

where h is given by

$$h = 2 \left[\frac{2}{5} \frac{h_1 + h_2 + h_3}{h_1 h_2 + h_2 h_3 + h_3 h_1} + \frac{1}{5} \left(\frac{1}{h_4} + \frac{1}{h_5} + \frac{1}{h_6} \right) \right]^{\frac{1}{2}} \quad (66)$$

in terms of anisotropy coefficients h_i entering Hill's equivalent stress σ_H . The latter is expressed in axes pointing onto the principal directions of orthotropy as

$$\sigma_H \equiv \left\{ \frac{3}{2} [h_1 s_{11}^2 + h_2 s_{22}^2 + h_3 s_{33}^2 + 2h_4 s_{23}^2 + 2h_5 s_{31}^2 + 2h_6 s_{12}^2] \right\}^{1/2} \quad (67)$$

where s_{ij} are the components of the stress deviator. Also, $\bar{\sigma}$ is here the flow stress of the undamaged material in some reference direction. The stress quantities σ_H , σ_m and $\bar{\sigma}$ entering (65) are related through the Benzerga–Besson flow potential:

$$\Phi^{BB}(\boldsymbol{\sigma}; f, h) \equiv \frac{\sigma_H^2}{\bar{\sigma}^2} + 2f \cosh\left(\frac{3}{h} \frac{\sigma_m}{\bar{\sigma}}\right) - (1 + f^2). \quad (68)$$

For an isotropic matrix, all h_i 's are equal to 1 so that the h factor in (66) reduces to $h = 2$, $\sigma_H = \sigma_{eq}$ and Gurson's flow potential is recovered. Also, in the limit of a dense material ($f \rightarrow 0$) equation (68) reduces to Hill's potential. Tvergaard's parameter q should be introduced in (65) and (68) as for the Gurson model. Based on the cell model calculations of Keralavarma and Benzerga [247], as reported in [7], the value $q = 1.3$ applies over a wide range of triaxialities for a moderate initial porosity of 0.001.

Integrating the damage law (65) for dilute porosities and proportional loading yields:

$$f = f_0 \exp\left[\frac{3}{h} \sinh\left(\frac{3}{h} T\right) \varepsilon_{eq}\right] \quad (69)$$

where it was assumed that the loading is such that $\sigma_{eq}/\bar{\sigma} \sim 1$, which in the anisotropic case is not granted. Alternatively, T in (69) could be interpreted as the local triaxiality, as opposed to that which is remotely imposed. What is of particular importance is that plastic anisotropy has a net effect, through h , on the rate of void growth as quantified by (65) or (69). The effect can be quite intricate because of the prefactor $\bar{\sigma}\sigma_{eq}/\sigma_H^2$ and directionality effects inherent to anisotropy. However, a general trend is that materials with $h > 2$ are predicted to be more resistant to void enlargement. The effect is all the more important that factor h appears inside the exponential term, just like the stress triaxiality. This effect has recently been evidenced in Mg alloys [317].

Incorporating Void Shape Effects Along with plastic anisotropy, the incorporation of void shape and rotation effects into void growth models constitutes the most important development of void growth theory over the past two decades. Initiated by Rice and Tracey [192] and further developed by Ponte Castaneda and co-workers [311, 318–320] and Gologanu et al. [321–323], this endeavor seems to have settled in the recent work of Madou and Leblond [312, 324–326] who developed most accurate models for general ellipsoids under arbitrary loadings. Salient features of the models, which are mathematically complex, are worth describing before presenting the key equations:

1. Nonspherical voids are not equally sensitive to all three normal stresses. For example, an infinitely extended cylindrical void is not sensitive to the axial stress, only to the radial one [314, 327]. Likewise, a crack-like void is mostly sensitive to the axial stress, not as much to the in-plane stresses. Therefore, the growth of nonspherical voids is governed by some effective hydrostatic stress, σ_h , which is a weighted average of the normal stresses (along the principal axes of the ellipsoidal void):

$$\sigma_h \equiv \alpha_{11}\sigma_{11} + \alpha_{22}\sigma_{22} + \alpha_{33}\sigma_{33} \quad (70)$$

where $0 \leq \alpha_{ij} \leq 1$ are weighting factors that depend primarily on void shape satisfying $\alpha_{kk} = 1$ (slightly negative values are possible due to extreme nonlinearities [7]). The

effect of σ_h on void enlargement is exponential. Only for spherical voids does σ_h reduce to the mean normal stress σ_m entering isotropic models, for example (59) and (61).

2. Nonspherical voids unavoidably rotate, if only convected with the material (Fig. 31a). Models that describe void shape and orientation changes provide key insight into the kind of void distortion that occurs at low stress triaxiality, particularly under combined shear and tension (Fig. 31b). In the model, the rotation of the void axes is governed by Eshelby-like concentration tensors whose coefficients are adjusted using a large database of finite element cell model calculations for various frozen microstructures.
3. In the limit of crack-like voids ($f \rightarrow 0$) the behavior of the material does not reduce to that of the matrix, as would be the case for the Gurson or Benzerga–Besson models in the limit $f \rightarrow 0$. A relevant parameter that emerges from the analyses is some “secondary porosity” denoted g . For example, $g = 4\pi c^3/(3V)$ for penny-shape cracks with c the crack radius and V the elementary volume. Physically, a material with microcracks bears a connection to a material with spherical voids of radius c , except that the damage law would be different. For spherical voids, $g = 0$.
4. The quadratic term of the plastic potential of a material with nonspherical cavities is not governed only by the deviatoric stress, e.g. the like of σ_{eq}^2 in (62), but generally includes the contribution of normal stresses. This has direct implication on damage growth, which has an exponential as well as linear dependence upon normal stresses via σ_h .

In principle, the growth and distortion of ellipsoidal voids is described by rate equations of the type (57). The evolution of porosity is given by the general equation (58) using the Madou–Leblond potential:

$$\Phi^{ML} = \frac{\sigma_q^2}{\bar{\sigma}^2} + 2q(g+1)(g+f) \cosh\left(\kappa \frac{\sigma_h}{\bar{\sigma}}\right) - (g+1)^2 - q^2(g+f)^2 \quad (71)$$

where most terms have been defined above, σ_q^2 being a quadratic form in the stress components and κ a factor that depends on void shape and porosity, just like the α_{ij} coefficients entering σ_h in (70). For spherical voids, the equivalent stress σ_q reduces to the von Mises stress σ_{eq} and $\kappa = 3/2$ thus recovering the Gurson model. If a_1 , a_2 and a_3 denote the current semi-axes of the void along unit directors $\mathbf{e}_1$, $\mathbf{e}_2$ and $\mathbf{e}_3$ then one needs to specify rate equations of the type (57) for any two aspect ratios, say $w_1 \equiv a_3/a_1$ and $w_2 \equiv a_3/a_2$, as well as for the $\mathbf{e}_k$ ’s. Madou and Leblond did not specify such evolution laws explicitly. They offered instead a computationally efficient way to update the variables, and this involved solving an eigenvalue problem at each time step. Details may be found in [312] with comparisons to the finite-deformation cell model calculations of Tvergaard [296] and Nielsen et al. [119] presented by Morin [328].

In practice, problems where void shape effects become prominent include the following: (i) low (and negative) triaxiality behavior [221, 260, 329, 330], including void reorientations under shear-dominated loadings [119, 191]; this behavior concerns situations of *induced anisotropy*; (ii) description of *initial anisotropy* associated with nonspherical inclusions [112, 331], aligned particle clusters, or directional nucleation as is generally the case for particle cracking or

grain-boundary cavitation with the formation of crack-like voids; (iii) accurate modeling of void coalescence [202,313].

Combined Plastic Anisotropy and Void Shape Effects As will be elaborated upon in Section 4.6.4, situations arise where fundamental understanding of coupled void shape and plastic anisotropy effects is needed for robust predictions. Consider the simpler case of spheroidal voids, say $a_1 = a_2 \neq a_3$ so that $w_1 = w_2 \equiv w$ is retained as a single void aspect ratio with $\mathbf{e}_3$ the void axis. Voids are said to be prolate if elongated ($w > 1$) and oblate if flat ($w < 1$). In an early attempt to address this coupling for modeling anisotropic fracture, Benzerga et al. [331] conjectured that the combined effect of void shape and plastic anisotropy is a simple superposition of separate effects. They used the GLD model (specialization of (71) to spheroidal voids) and the Benzerga–Besson model to this end. Their heuristic approach consisted of replacing the deviatoric term of the quadratic stress σ_q in (71) with Hill’s equivalent stress σ_H and injecting the anisotropy factor h given by (66) inside the exponential term. Finite element cell model calculations [247] provided support to the above conjecture in the case of moderate plastic anisotropy. For some cases of strong plastic anisotropy, however, the micromechanical analyses show nontrivial coupling with void shape effects [247, 332]. Keralavarma and Benzerga thus developed a coupled model from first principles, initially for axisymmetric loadings aligned with the microstructure [333] as in [334], and later for more general loadings [335]. The rate of growth of porosity, as inferred from their model specialized to axially symmetric loading about the void’s axis, is governed to first order by

$$\frac{\dot{f}}{(1-f)} = \frac{\sigma_{eq}}{\sigma_H} \left\{ C\eta + q(g+f) \frac{\kappa \bar{\sigma}}{\sigma_H} \sinh \left(\kappa \frac{\sigma_h}{\bar{\sigma}} \right) \right\} \dot{\epsilon}_{eq} \quad (72)$$

where σ_H , σ_h and $\bar{\sigma}$ are related through a potential $\Phi^{KB}(\boldsymbol{\sigma}; f, w, \mathbf{e}_3, h_i)$ of the general form (71) with the quadratic stress σ_q specified through (to first order, L being the Lode parameter)

$$\sigma_q^2 = C(\sigma_H^2 - 2\eta L \sigma_h \sigma_{eq}) \quad (73)$$

and the effective hydrostatic stress σ_h of equation (70) simplified into

$$\sigma_h = \alpha(\sigma_{11} + \sigma_{22}) + (1 - 2\alpha)\sigma_{33}. \quad (74)$$

Here, C , η as well as κ are functions of the plastic anisotropy coefficients h_i ’s, as parameters, and of damage state variables f and w . On the other hand, α and g are only functions of f and w . Mathematical expressions of these functions are complex and so omitted here for brevity. They are however available in closed form [335, 336] enabling straightforward computer calculations. The only adjustable parameter in (72) is Tvergaard’s parameter q which can be calibrated based on micromechanical analyses [336]. Note that (72) reduces to Gurson’s damage law (61) if all h_i ’s are equal to 1 and voids are spherical (since $\alpha = 1/3$, $g = 0$, $\eta = 0$ and $C = 1$). Also, for a Hill matrix containing spherical voids, κ reduces to $\kappa^{BB} \equiv 3/h$ and the Benzerga–Besson damage law (65) is recovered.

It bears emphasis that the KB potential Φ^{KB} is applicable to *non-axisymmetric* loadings and that equations (72) and (73) are very special cases, which yet illustrate key aspects of the model. Another key feature of the model is that it enables void shape changes and

rotations to be simulated under general loadings [337]. Thus, damage evolution is specified not only through the rate of porosity, for example in (72), but also through the rates of w , $\mathbf{e}_3$ and the axes of orthotropy where the h_i 's are defined. Void shape evolution is governed by

$$\frac{\dot{w}}{w} = \frac{3}{2} \left[1 + \left(\frac{9}{2} - \frac{T^2 + T^4}{2} \right) (1 - \sqrt{f})^2 \frac{\gamma - \gamma^G}{1 - 3\gamma} \right] \dot{\varepsilon}'_{33} + \left(\frac{1 - 3\gamma}{f} + 3\alpha - 1 \right) \dot{\varepsilon}_{kk} \quad (75)$$

where $\dot{\varepsilon}'_{33}$ is the deviatoric strain-rate in the direction of the void axis, and $\gamma(f, w)$ and $\gamma^G(f, w)$ may be found in [323]. The evolution of the void axis $\mathbf{e}_3$ is given by

$$\dot{\mathbf{e}}_3 = \mathbf{W} \mathbf{e}_3 \quad (76)$$

which assumes that the voids rotate with the material, $\mathbf{W}$ being the total spin. This is clearly an approximation. An improved representation may be found in [335] on the basis of earlier work by [319] which employs an Eshelby concentration tensor for the spin. Also, the orthotropy axes were taken to rotate with the material given the scarcity of data, if only computational. Other choices are possible but more complex; see e.g., [337–339] and references therein. This in itself constitutes a broad topic of research with connections to finite deformation crystal plasticity. Finally, it is worth noting that Morin et al. [340] have recently proposed an extension of the KB flow potential to ellipsoidal voids thereby extending the Madou–Leblond potential to anisotropic matrices; evolution equations for the damage state variables, other than the porosity, are yet to be developed.

In closing this section on void growth and distortion modeling, some remarks are in order.

Remark 1: Prior to void coalescence, an equation such as (71) seems to provide a universal format for the flow potential of a porous material from which the rate of porosity is derived via equation (58). What defines the quadratic and linear forms, respectively σ_q^2 and σ_h , depends on void shape and plastic anisotropy. Quantities σ_q and σ_h , both homogeneous to a stress, generalize the von Mises effective stress and hydrostatic stress, respectively.

Remark 2: As written, equation (53) giving the rate of damage-related state variables assumes isotropy. Provision for anisotropy entails dependence of the H_i functions in (53) upon all components of the stress tensor. Ductile fracture is inherently anisotropic, even prior to localization, as captured by the latest void growth models. Guided by these, (53) should be amended to include σ_q and σ_h in lieu of σ_{eq} and σ_m , respectively. For specific types of loadings discriminated by either the strain-rate Lode parameter ν or stress Lode parameter L , the two stresses σ_q and σ_h may be expressed in terms of σ_{eq} and σ_m with the internal state variables entering such relations. An indication of this, already evident in (73) and (74), will be exploited in an example below.

Remark 3: The theory of void growth based on damage law (72) predicts a cutoff triaxiality at $T = 1/3$ (uniaxial loading) for which the net void growth is nil. This prediction is consistent with exact results based on finite element analyses [202]. To illustrate this, consider the simple case of plastic isotropy ($\sigma_H \sim \sigma_{eq}$), dilute porosities ($\sigma_{eq} \sim \bar{\sigma}$) and

axisymmetric loading ($\sigma_h/\bar{\sigma} = T + 2/3(1 - 3\alpha)$). The damage law (72) becomes:

$$\frac{\dot{f}}{\dot{\varepsilon}_{\text{eq}}} = C\eta + q(g + f)\kappa \sinh[\kappa(T - \beta)] \quad (77)$$

where β is a shorthand notation for

$$\beta = \frac{2}{3}(3\alpha - 1).$$

Equation (77) reveals that if stationary microstructures exist such that $T - \beta = 0$ and $\eta = 0$ then $\dot{f} = 0$. Recall that the damage law parameters, notably β and η , depend on the internal variables and hence are evolving quantities. The case of uniaxial tension is at least one such instance (there does not seem to be other “simple” loading cases that realize the stationarity of the microstructure). Indeed, for $T = 1/3$ voids elongate irrespective of initial shape. As they develop into cylindrical shapes, $\eta \rightarrow 0$, $\alpha \rightarrow 1/2$ and $\beta \rightarrow 1/3$, which is also the value of imposed triaxiality. It will be shown below why such a stationary microstructure does not lead, in general, to void coalescence by internal necking so that the theoretical fracture strain is infinite. Implications for the intrinsic fracture locus will be discussed further in Section 4.6.3 with due consideration of other aspects, such as inclusion locking effects.

Damage law (72) also predicts possible void closure at $T = 0$ (pure shear). This prediction is in keeping with exact finite element results [119, 203, 296]. In the simplest case of an initially spherical void, the latter evolves into an ellipsoid, which can be approximated by a rotating prolate void. In this case, $\alpha > 1/3$ implying $\beta > 0$ so that the hyperbolic term in (77) is negative. Since η is slightly negative for prolate voids, $\dot{f} < 0$ and this eventually leads to void closure, even with the simple void rotation law (76). Whether void closure actually occurs depends on the possibility of void coalescence, a subject that deserves more attention than it has received in the literature.

By way of contrast, the Rice–Tracey equation (60) predicts that the void growth rate $\dot{R}/R$ is zero in pure shear and finite in tension. Both predictions are correct for a sphere. However, under such low triaxiality conditions the spherical void assumption must be relaxed to obtain the correct behavior (i.e., $\dot{R}/R < 0$ in shear and $\dot{R}/R \rightarrow 0$ in tension). The above two fundamental solutions for simple tension and pure shear underlie much of the debate about fracture at low stress triaxiality.

Remark 4: In the damage law (72), the exponential effect of the effective hydrostatic stress σ_h is modulated by parameter κ . Since κ appears inside the exponential term, a small variation $\Delta\kappa \sim 0.2$ can have large effects on the strain to failure. For spheroidal voids embedded in an isotropic matrix, $0.75 \lesssim \kappa \leq \sqrt{3}$ depending on void shape and porosity with the range being narrowed to $3/2 \leq \kappa \leq \sqrt{3}$ for prolate voids [321, 323]. At sufficiently high triaxiality, κ quickly evolves to $\sim 3/2$. However, with plastic anisotropy accounted for, the range can be much broader, irrespective of triaxiality, void volume fraction and shape. Values as low as 1 and as large as 2 can be realized

[317]. To appreciate this, consider the simplified expression of κ for prolate voids:

$$\kappa = \frac{3}{h} \left\{ 1 + \frac{2h_t \ln(w/W)}{h^2 \ln f} \right\}^{-1/2} \quad (78)$$

where W is the aspect ratio of the elementary cell, h is given by (66) assuming that the void axis is aligned with a principal direction of orthotropy, and h_t is a function of the h_i 's; see [335, 336]. The dominant term in (78) is $3/h$ which shows the prevalence of plastic anisotropy over void shape in setting the rate of void growth. Incidentally, equation (78) also shows that plastic anisotropy via the factor h_t/h^2 can either exacerbate the effect of void shape or eliminate it, even at moderate and low triaxialities. With a similar effect on parameter η , which also depends on the h_i 's, this provides a rationale for the nontrivial coupling between void shape and plastic anisotropy evidenced by cell calculations [247].

Remark 5: The above models of void growth capture some key microstructural effects but are based on continuum micromechanics. As such, their representation of the ductile fracture phenomenon is as good as enabled by the scale transitions operated by means of homogenization. When scale separation does not apply, or still holds but with a size effect, recourse to higher resolution methods, eventually using direct numerical simulations may be necessary. In particular, dissipation in ductile fracture may occur at a multitude of length scales [341]. Crystallographic aspects can play a key role. For example, Crepin et al. [200] developed a void growth model that accounts for the enhancement in growth rates associated with the activation of prismatic slip and faceted void shapes. Kysar and co-workers [342] have developed slip-line solutions following the method of Rice [343] to determine discrete deformation/stress sectors around a void embedded in a single crystal. Yerra et al. [244] and Srivastava and Needleman [245] investigated the growth of voids in single crystals by means of crystal plasticity cell model calculations. Similar calculations were carried out by Borg et al. [344] but accounting for size effects. At even lower length scales, Segurado and Llorca [345] have employed discrete dislocation dynamics to study the initial growth rates of voids in FCC crystals; also see [346] for an overview of the literature on this topic. In the absence of size effects, it is of interest to determine whether the crystallographic aspects uncovered in theoretical [200, 342] or computational models [199, 244, 245] can be simply captured by net plastic anisotropy effects, as discussed in [244].

Remark 6: Because the constitutive relations of Section 4.5.2 contain a provision for rate-sensitivity effects, the damage laws above account for these through the flow stress $\bar{\sigma}$. As derived, however, these laws do not account for any intrinsic strain rate effects on the rate of void growth itself. It is therefore worth mentioning the works of Cocks and Ashby [347] among others [348–350]. In general, an increase in rate sensitivity decreases the net rate of void growth, everything else kept fixed.

4.5.4 Coalescence

If void growth could proceed, as modeled in the previous section, until failure (complete loss of stress carrying capacity) then a good estimate of void coalescence would be when

the lateral void diam, $2R$, has reached the lateral void spacing, $2S$. This approach would lead to a considerable overestimation of ductility and other fracture properties, even with clustering accounted for. The void size relative to its initial value is typically what a void growth model delivers. Integrating the damage law (59) for proportional loading (constant T and ν) up to some critical state, then inverting leads to:

$$\varepsilon_i = \frac{1}{\alpha(\nu)} \ln \left(\frac{R}{R_0} \right)_c \exp \left(-\frac{3}{2}T \right) \quad (79)$$

The current void spacing can directly be inferred from the initial void spacing and deformation history²:

$$S = S_0 \exp(-\zeta \varepsilon_{eq}) \quad (80)$$

where $\zeta = 1/2$ for a periodic array of voids in tension, more generally a factor characterizing the nature of void distribution. Substituting in (79) the estimate $R = S$ for mere impingement gives the following estimate of the fracture strain:

$$\varepsilon_i = \frac{\ln \frac{S_0}{R_0}}{\alpha(\nu) \exp \left(\frac{3}{2}T \right) + \zeta} \quad (81)$$

Fig. 47 illustrates how large the fracture strains are for two values of S_0/R_0 . The corresponding void growth ratio at impingement $(R/R_0)_c$ is either a constant (random case) or a function of triaxiality (periodic or clustered) varying between 4 and 25 in the case $S_0/R_0 = 25$. Note that the periodic assumption leads to the lowest bound and that the nature of void distribution, according to this rough estimate, plays no role at high triaxiality.

In order to predict lower fracture strains, lower values of some critical $(R/R_0)_c$ had to be used. First suggested by McClintock [314], this approach was followed by the Beremin group [24, 351]. In the Beremin model, as well as the Johnson–Cook model [138], the Rice–Tracey void growth equation is used. In principle, other equations accounting for plastic anisotropy or void shape effects can also be used. Typical values of the critical void growth ratio, $(R/R_0)_c$, needed in failure models fall between 1.2 and 2.0 [24, 56]. Similarly, in approaches based on porosity growth, a critical porosity f_c is used which is lower than that required for impingement (~ 0.6) [10, 352].

The $(R/R_0)_c$ (or f_c) method to modeling final failure is simple and attractive. It underlies much of the early local approach to ductile fracture as it enables calibration of the fracture process at the smallest scale of relevance [353]. In practice, one issue that faces the implementation of this method is that the critical parameters, $(R/R_0)_c$ or f_c , generally vary with the stress state [202, 329], hence do not constitute material parameters per se. This hints at a more fundamental issue, that of what physical process determines criticality, an issue that was eluded in earlier models. While the later cell model calculations [243] supported the idea of void growth acceleration past some critical growth ratio, the accumulated evidence for a stress-state dependent $(R/R_0)_c$ or f_c raised questions about model calibration, validation for a wide range of stress states, and transferability of model parameters from laboratory specimens to structural components.

²This is obvious for periodic or random Poisson-like distributions of void centroids, not so for clustered ones [216].

Void growth theory alone, including the main enhancements of Section 4.5.3, cannot in general deliver quantitative predictions of ductility. The reason for this is that void growth models assume that plastic flow takes place in the whole elementary volume. It is now established that certain modes of localized plastic deformation within that same volume would deliver lower values of plastic dissipation, hence are more likely to prevail after sufficient microstructural evolution [7]. In this context, microstructure evolution refers to changes in the damage state variables, D_i ; in other words changes in the geometrical configuration of voids, as described by their relative size, orientation and spacing. [Associating incipient failure with strain localization at the microscale is appealing as this unequivocally defines the notion of criticality underlying the phenomenological criteria based on \$\(R/R_0\)_c\$ and \$f_c\$.](#) Internal necking or internal shearing are possible mechanisms of strain localization.

Void coalescence is an inherently directional void growth process. When it occurs through internal necking in relatively ordered void distributions, this process begins with the onset of necking of the intervoid ligament, while the voids are quite distant from each other, and ends with the voids actually impinging on each other. This process was described experimentally in Section 4.2.4 and computationally in Section 4.3. It bears emphasis that for rate-sensitive materials, the transition from diffuse to localized flow is more gradual. Although the onset of internal necking in real materials is not as abrupt as in the rate-independent case, the below models provide a simple way to model the phenomenon. Also, the issue of determining the direction of internal necking or shearing is not addressed in detail. In practice, possible coalescence directions are affected by the nature of void distribution.

Coalescence under predominately tensile loads Attempts to capture the physics of strain localization in voided materials were concurrent with the early days of void growth modeling. Thomason [212] sought an approximate limit load causing plastic flow in the intervoid ligaments alone by considering a periodic array of square prismatic voids in plane strain loading. Brown and Embury [213] required that a local slip-line field develop between neighboring voids. In their treatment, this condition was met when the void height $2h$ is about equal to its separation from adjacent voids, $2h = 2S - 2R$, where R now denotes the in-plane void radius. Both models used a critical distance of approach of the growing voids, with the void configuration in [213] being more realistic. However, the Brown-Embury criterion is purely geometric and does not contain any provision for an applied stress state.

The early attempts in [212,213] were limited in scope. Nearly two decades later, Thomason [354] extended his earlier model to three-dimensional periodic arrays of square-prismatic voids. Plastic flow was described by the associated von Mises criterion and restricted to intervoid ligaments, the regions above and below the voids being modeled as rigid. With internal necking assumed to take place in the x_1 - x_2 plane, normal to the largest tensile stress, Thomason's coalescence condition reads³

$$\Phi^T(\boldsymbol{\sigma}; \chi, w) = \sigma_{33} - \sigma^T = 0 \quad (82)$$

where $\chi \equiv R/S$ is the ligament parameter (ratio of void breadth to cell breadth, the latter representing the in-plane void spacing), w is the void aspect ratio (height to breadth ratio

³Thomason hardly offered any details, neither on the analytical derivations nor the numerical method used to arrive at his proposed fit (83). For further background see [7,355].

for the prismatic void) and

$$\sigma^T(\chi, w) \equiv \bar{\sigma}(1 - \chi^2) \left[\alpha^T \left(\frac{\chi^{-1} - 1}{w} \right)^2 + \beta^T \sqrt{\chi^{-1}} \right] \quad (83)$$

with $\alpha^T = 0.1$ and $\beta^T = 1.2$. In the limit $w \rightarrow 0$ (flat void) $\sigma^T \rightarrow \infty$ and the criterion is never met. This deficiency is believed to have limited consequences in materials failing after some significant void growth. Later, Pardoen and Hutchinson [202] and Benzerga [313] have proposed additional heuristic extensions of the above criterion. Benzerga's extension of (83) focused on removing the singularity in the limit of flat voids ($w \rightarrow 0$) [313]. On the other hand, Pardoen and Hutchinson's extension focused on incorporating the effect of strain hardening by treating $\alpha^T(n)$ and $\beta^T(n)$ as functions of the strain hardening exponent n , to be fit on cell model calculations [202].

Recently, Benzerga and Leblond [356] have revisited Thomason's analysis by considering a circular cylindrical geometry⁴. They obtained a fully analytical expression for the coalescence criterion, which takes the form (82) with σ^T replaced by $\sigma^{\text{BL}} = \sigma^{\text{vol}} + \sigma^{\text{surf}}$ where:

$$\begin{aligned} \sigma^{\text{vol}}(\chi) &= \frac{\bar{\sigma}}{\sqrt{3}} \left[2 - \sqrt{1 + 3\chi^4} + \ln \frac{1 + \sqrt{1 + 3\chi^4}}{3\chi^2} \right] \\ \sigma^{\text{surf}}(\chi, w) &= \frac{\bar{\sigma}}{3\sqrt{3}} \frac{\chi^3 - 3\chi + 2}{\chi w} \end{aligned} \quad (84)$$

specify the dependence of the critical condition for coalescence $\sigma_{33} = \sigma^{\text{BL}}$ upon the internal state variables. It was shown in [356] that (83) is an imperfect fit to Thomason's own data. The closed form solution (84) constitutes a first step toward some needed generalizations to other geometries and general loadings. It has already paved the way to improved criteria by Morin et al. [357], to incorporating micro-inertia effects by Molinari et al. [358] and to accounting for shear loadings by Torki et al. [359].

Thomason did not supplement his limit load criterion with evolution equations for the microstructural variables w and χ . As a consequence, his criterion has essentially been used for the *onset* of void coalescence, which is often sufficient to estimate strains to failure as a function of loading parameters [162, 360]. For ductile fracture simulations, however, criterion (83) must be supplemented with evolution laws for χ , w and the void orientation. This task was undertaken by Benzerga [216, 313] and Pardoen and Hutchinson who independently derived evolution equations for the damage state variables w and χ on the basis of matrix incompressibility, boundary conditions and cell model phenomenology; also see [361]. For example, in [313] a shape factor γ was introduced in addition to w . The void shape was taken to evolve from spheroidal ($\gamma = 1/2$) at the onset of internal necking ($\chi = \chi_c$) to conical ($\gamma = 1$) at complete coalescence ($\chi = 1$). The evolution equations are:

$$\dot{\chi} = \frac{3}{4} \frac{\lambda}{w} \left[\frac{3\gamma}{\chi^2} - 1 \right] \dot{\epsilon}_{\text{eq}} + \frac{\chi}{2\gamma} \dot{\gamma}, \quad (85)$$

$$\dot{w} = \frac{9}{4} \frac{\lambda}{\chi} \left[1 - \frac{\gamma}{\chi^2} \right] \dot{\epsilon}_{\text{eq}} - \frac{w}{2\gamma} \dot{\gamma}, \quad (86)$$

⁴Thomason's fit (83) is actually better suited for this geometry.

$$\dot{\gamma} = \frac{1}{2(1 - \chi_c)} \dot{\chi} \quad (87)$$

where λ represents the current value of the void spacing ratio, which is updated through:

$$\dot{\lambda} = 3\zeta\lambda\dot{\epsilon}_{\text{eq}}, \quad (88)$$

where ζ is the same factor introduced in (80) to describe the nature of the void distribution (periodic ($\zeta = 1/2$), random ($\zeta = 0$) or clustered (presumably $0 \leq \zeta \leq 1/2$)). The void and cell axes were tacitly taken to rotate with the material as per (76). In actual implementations of the model [221,331] only (86), (88) and (76) are needed if the porosity is retained as a primary variable post-localization and χ (hence γ) is updated explicitly through a relationship to f , w and λ .

A simple geometric interpretation of coalescence criteria is possible when considering the rate-independent limit, as illustrated in Fig. 48. In such a multisurface representation there is provision for void growth (curved parts in the figure) and for void coalescence (straight parts). The multisurface model is said to be *hybrid* because the elementary volumes used to derive the void growth and void coalescence models are different. The (nonproportional) loading path (i)-(c)-(c⁺) shown in gray is typically encountered inside the neck of a tensile specimen in plane strain. At initial yield, indicated by (i) in the figure, the material has initial porosity (a fraction of a percent) and so the yield domain closes on the hydrostatic axis. This part is defined by a Gurson-like criterion, here actually an equation of the type (71) for spheroidal voids [323]. When a coalescence condition of the type (83) is met, the current loading point (denoted by (c) in the figure) is at a vertex in the hybrid model. The directions of plastic flow just prior and immediately after the onset of coalescence are indicated by $\mathbf{N}^{c-}$ and $\mathbf{N}^{c+}$. Upon continued plastic loading, yielding is governed by the straight portion of the surface. The equation of the latter is given by (83) which may be rewritten for arbitrary orientations of the localization band as

$$\Phi^C(\boldsymbol{\sigma}, \chi, w) = \frac{\sigma_{\text{eq}}}{\bar{\sigma}} + \frac{3}{2} \frac{|\sigma_m|}{\bar{\sigma}} - \frac{3}{2} \frac{\sigma^{\text{BL}}}{\bar{\sigma}} = 0 \quad (89)$$

using the more precise upper-bound estimate (84). Hence, the slope of the straight part in Fig. 48 is -3/2. It bears emphasis that the format (89) is valid for axisymmetric loadings and one system of localization bands. For more general loadings, the -3/2 slope should be replaced with a Lode-dependent factor $s(\nu)$ (or $s(L)$). The number of localization systems within the elementary volume is far less trivial. More research is needed in this area for void distributions that are neither periodic nor uniformly random.

Coalescence under combined tension and shear If applied under general loadings, the coalescence criteria (83) and (84) as well as those in [202,313,357,358] predict no effect of shear on coalescence. This is clearly an approximation. Earlier efforts addressing this issue include [362,363]. Leblond and Mottet [364] addressed this problem and developed a limit-load model in the spirit of earlier work by Gologanu and co-workers [365,366] where voids are smeared out through a two-level homogenization procedure. Tekoglu et al. [367] have recently improved upon this model through some suitable extension of Thomason's treatment of coalescence to non-axisymmetric loadings. In doing so, they accounted for

both the extensional and shear components of the microscopic deformation field. However, only that corresponding to shear was expressed in explicit, analytical form. Their coalescence criterion is quadratic and given by

$$\Phi^{\text{TLP}}(\boldsymbol{\sigma}; \chi, w, \mathbf{e}_3) = \frac{\sigma_{33}^2}{\sigma^A{}^2} + 4 \frac{\sigma_{31}^2 + \sigma_{32}^2}{\tau^2} - 1 = 0 \quad (90)$$

where σ^A refers to either σ^T in (83) or Benzerga's heuristic limit load [313] and $\tau(\chi)$ is a function of the ligament parameter defined through:

$$\tau(\chi) = \frac{2\bar{\sigma}}{\sqrt{3}}(1 - \chi^2) \quad (91)$$

Hence, in the absence of any shear loading, criterion (90) reduces to either Thomason's (83) or its variant by Benzerga [313].

More recently, Torki et al. [359] tackled the problem of void coalescence under combined tension and shear by explicitly considering the microscopic deformation field around the void in fully analytical form. This enabled a rigorous extension of the model of Benzerga and Leblond [356]. Their coalescence criterion is given by

$$\Phi^{\text{TBL}}(\boldsymbol{\sigma}; \chi, w, \mathbf{e}_3) = \begin{cases} \frac{(|\sigma_{33}| - t\sigma^{\text{surf}})^2}{b\sigma^{\text{vol}2}} + 4 \frac{\sigma_{31}^2 + \sigma_{32}^2}{l\tau^2} - 1 & \text{for } |\sigma_{33}| \geq \sigma^{\text{surf}} \\ 4 \frac{\sigma_{31}^2 + \sigma_{32}^2}{l\tau^2} - 1 & \text{for } |\sigma_{33}| \leq \sigma^{\text{surf}} \end{cases} \quad (92)$$

where $\sigma^{\text{vol}}(\chi)$, $\sigma^{\text{surf}}(\chi, w)$ and $\tau(\chi)$ are functions of microstructural parameters χ and w given by (84) and (91). Torki et al. have also introduced heuristic parameters t , b and l based on comparisons with cell model calculations. In general b and l are close to unity and the primary motivation for t was to obtain a finite limit load, hence coalescence, under pure tension of penny-shaped cracks, as would be relevant to materials failing after limited void growth, e.g., [117, 181, 219].

4.5.5 Dynamic aspects

Temperature and strain rate effects are readily incorporated in the formulation of Section 4.5.2. For dynamic loadings, the need may arise to solve fully coupled thermomechanical problems. In practice, the assumption of adiabatic heating is often invoked to decouple the mechanical and thermal boundary-value problems [368]. Other aspects may become important in modeling dynamic ductile failure, which are not considered in the above. For example, void nucleation by vacancy aggregation and vacancy-cluster coarsening has been simulated by means of molecular dynamics to guide the development of a nanovoid nucleation model [369]. More recently, Wilkerson and Ramesh [370] extended a dislocation based model by Lubarda et al. [371] to derive a criterion in stress space for void nucleation by dislocation emission. On another front, Molinari and Mercier [372] developed a porous metal plasticity model that takes into account the effect of microscale inertia on void growth. Such effects are found to be small at strain rates below 10^4s^{-1} but become quite important at higher

rates relevant to shock loadings [373]. More recently, Molinari et al. [358] have extended the void coalescence model of Benzerga and Leblond [356] to incorporate the effect of microscale inertia. Just like for void growth, such effects are expected to play an important role under high strain rates, in particular for predicting fragmentation by microcrack coalescence in ductile spall experiments.

4.6 Applications

In the local approach to fracture, material characteristics such as the fracture strain and fracture toughness are natural outcomes of its implementation. In particular, the stress state dependence of the fracture strain and geometry dependence of the fracture toughness are part of an emergent behavior. An attractive aspect that was already evident in the early days of the local approach is its “reliance on experiments to provide calibration of the failure process at the smallest scale” which is common to other top-down approaches to fracture [353]. In fact, as the micromechanics-based models have gained in accuracy, the application of the approach has gone more toward restricting the calibration to deformation related quantities. In this way, the local approach can be utilized to its full potential in terms of predictive capability. Clearly, applications of the advanced models remain limited, particularly when they involve full-field boundary-value problem solutions. In this section, we address the prediction of the strain to failure, fracture in notched bars as well as fracture toughness. In the first, we illustrate the application of uncoupled models, after briefly describing their formulation, as well as fully coupled models to obtain trends in fracture loci. In the second, we illustrate a typical application to real materials with elements of model identification provided in Section 4.6.2 below. In the third, trends in fracture toughness are discussed.

4.6.1 Uncoupled models

The porous metal plasticity models whose general structure was described in Section 4.5 deliver constitutive relations that seamlessly couple damage and plasticity. The use of coupled models in engineering failure analysis is key if the competition among the principal modes of failure is sought as emergent behavior. For instance, failure by mechanical instability would not emerge as an outcome of simulations without a fully coupled model, unless the instability is caused by non-damage related physics. Here, we present the general, much simpler structure of uncoupled models. These have proven quite useful and shall continue to be so, for example in predicting fracture loci, including under complex loading paths, and even for simulations of crack initiation and growth under certain circumstances.

Isolated-void growth models usually form the basis of uncoupled models. For example, the well known Beremin [351] and Johnson–Cook [138] models use the Rice–Tracey void growth rate equation (59) as basis⁵ along with a critical void growth ratio. More generally, the more accurate void growth laws (61), (65), (72) or (77) could be used under the assumption that the effective stress (σ_{eq} in the isotropic case and σ_{H} in the anisotropic case) is approximately equal

⁵In the Johnson–Cook model, the Rice–Tracey equations are used in integrated form, not in rate form. By way of consequence, they are strictly valid under proportional loading, a restriction that is often not observed in applications.

to the yield stress $\bar{\sigma}$ after relating the porosity rate to $\dot{R}/R$ in the dilute limit. Under these circumstances, the porosity-dependent yield criteria are not needed and fracture initiation is described by a criterion of the type:

$$\int_0^{\varepsilon_i} g(T, L) d\varepsilon_{eq} = D_c \quad (93)$$

where D_c is a constant, ε_{eq} and ε_i are the current effective strain and its value at failure, respectively, and g is a memory function that carries the signature of the damage process, viewed as a functional of loading parameters $T(\varepsilon_{eq})$ and $L(\varepsilon_{eq})$. For example, in the Beremin model

$$g(T, \nu) = 0.846 \sinh\left(\frac{3}{2}T\right) + 0.008\nu \cosh\left(\frac{3}{2}T\right); \quad D_c = \ln\left(\frac{R}{R_0}\right)_c \quad (94)$$

where Huang's correction has been accounted for. Alternatively, Thomas et al. [374] have proposed the following form:

$$g(T) = k \sinh[\kappa(T - \beta)] \quad (95)$$

which is based on (77) to the neglect of the first term. This form works better at low triaxialities using $k \approx 1$, $\kappa = \sqrt{3}$ and $\beta = 1/3$. The format (93) also encompasses a class of recent macroscopic models; see [375] and references therein. These models are truly uncoupled in the sense that they lack a damage growth law. Note that the Beremin model and its extensions are semi-coupled in the sense that the damage growth law is derived based on plasticity considerations. In particular, they provide provision for making the critical damage parameter, D_c , which appears on the right-hand side of (93), dependent upon stress state. This can be done by employing the void coalescence criteria of the previous section. In this case the damage growth law must be supplemented by a rate equation for the void aspect ratio, as was initiated by Thomason [360]; also see [250, 376].

For radial loadings, T and L are history independent and a well-defined two-dimensional fracture surface may be expressed as

$$\varepsilon_i = D_c / g(T, L) \quad (96)$$

so that the inverse of the integrand g in (93) is the expression of the fracture locus in closed form. If the critical damage parameter is not constant and depends on stress state then the above closed-form expression is not valid. [Criteria of this kind have been extensively used, including in bulk metal forming applications; see for example \[377\] and references therein.](#)

4.6.2 Material parameter identification

Uncoupled model The advantage of an uncoupled model of the type (93) is that it only involves one fracture parameter, D_c , which may be associated with a critical void growth ratio in certain regimes of stress state. Assuming a constant value of D_c then it can be calibrated on any notched bar experiment. In more elaborate variants using a limit-load coalescence criterion [250, 376], parameter D_c is actually replaced with the initial ligament parameter χ_0 , which can be identified using quantitative metallography or used as a free parameter. In this sense, there can be more parameters in uncoupled models, typically two when a void nucleation criterion is introduced; see e.g., [250].

GTN model The following parameters enter the GTN model: q_1 , q_2 , f_c , f_f , f_N , s_N and ϵ_N . There is no straightforward procedure for identifying these parameters using standard experiments. In addition, some of the above parameters play interdependent roles. In practice, the first four of these parameters could be fixed based on micromechanical models or cell model calculations. Other model parameters pertain to the elasto-plastic behavior of the matrix. The hardening response of the matrix is determined using uniaxial testing with appropriate large-strain corrections.

Hybrid model A hybrid model is a multi-surface model that combines a Gurson-like void growth model with a void coalescence model in the form of a different flow potential. One advantage of using the hybrid model is that it involves fewer parameters, void nucleation set aside. Examination of the constitutive equations of Sections 4.5.3 and 4.5.4 reveals actually no adjustable fracture parameter, unless $q \approx 1.6$ is treated as a free parameter. When this model is used to predict crack initiation, only deformation related parameters need to be calibrated on experiments. Account for plastic anisotropy may be necessary depending on the material. Anisotropic response is common in metal forming applications although it is often restricted to two-dimensional measurements. Within the confines of the integrated model above, this first step will deliver the basic hardening curve $\bar{\sigma}(\bar{\epsilon})$ as well as the anisotropy characteristics.

With the basic flow properties of the matrix calibrated, the next step is to determine the volume fraction, aspect ratio and relative spacing of damage initiation sites (inclusions, precipitates, etc.) in the average sense. Practically, this can be achieved by examining three perpendicular cross-sections in optical microscopy, carrying out the needed two-dimensional measurements using digital image analysis, and finally operating standard stereology transformations to infer their 3D counterparts. The outcome of this step is the set of parameters f_0 , W_0 and λ_0 needed to initialize the state of the microstructure in the constitutive equations. Other details on how to account for 3D aspects and void nucleation may be found in the review by [7].

4.6.3 Intrinsic fracture loci

As described in Section 4.6.1, uncoupled models are readily integrated to obtain intrinsic fracture loci using equation (93) in the simplest case. Figure 49 illustrates loci obtained using either the Beremin model or its extension (95) for various values of the critical void growth ratio and axisymmetric loadings. Both models provide the well known trend of a decreasing strain-to-failure with increasing triaxiality. Use of the original model should be limited to the moderate to high triaxiality regime. As noted under Remark 3 about equation (77), the behavior of voids, free of particle locking effects, under uniaxial tension is peculiar in that no coalescence condition could be met. This behavior is captured by the extended model. Neither model would predict cell model results over the entire range of triaxiality [374] because in fact the parameters k , κ and β in (95) depend on the current void shape, which evolves with straining.

The above loci are said to be intrinsic in that they are obtained for a material volume that is homogeneously deformed up to failure under proportional loading. If the same homogeneous deformation is imposed under nonproportional loading then the fracture strain may

vary significantly depending on the severity of the nonproportional (or nonradial) loading. Fig. 50a illustrates results obtained by Benzerga et al. [378] using finite-element cell model calculations for a step-jump in triaxiality with $T_0 = 1/3$ initially. Here, the strain to failure is plotted against the strain-weighted average of the triaxiality. Under radial loading, a unique fracture locus may be constructed, given a material characteristic $(R/R_0)_c$ (Fig. 49). Under nonradial loadings, however, an infinite number of fracture loci can be constructed, the one shown in Fig. 50a being just one example. What is of particular importance is that the deviation from the intrinsic fracture strain can be quite large. Interestingly, an uncoupled model such as the extended Beremin model can capture the trends of cell model calculations quite well (Fig. 50b).

Usually, when using uncoupled models the parameters k , κ and β are fixed and the only adjustable material parameter is $(R/R_0)_c$. The use of coupled models enables to investigate the extent to which the loading and microstructural parameters affect the fracture strain. Integrated models were developed for instance which account for void nucleation, growth and distortion as well as void coalescence [162, 221, 331, 379] on the basis of the models presented in Section 4.5.2. The intrinsic fracture loci predicted by these models will be presented and compared with experimental data in order to discuss successively the effects of (i) initial void shape, (ii) void distribution, (iii) flow properties, and (iv) resistance to void nucleation. The results to follow in this section were obtained using the Beremin model for void nucleation [23] combined with the GLD model for void growth [323] and void coalescence criterion (83) as modified in [202]. For simplicity, plastic flow in the undamaged material is represented using J_2 flow theory with power-law isotropic hardening characterized by initial yield strength σ_0 and hardening exponent n .

Figure 51a shows the effect of initial void shape on the fracture locus $\varepsilon_i(T)$. At low stress triaxiality, the effect is such that coalescence never occurs except for flat voids due to the important difference in the values of relative void spacing $\chi = R/L$ (L is the same but R varies). Note the low values of ductility due to the high initial value of porosity assumed (no nucleation here). In general, the effect of initial void shape becomes unimportant at sufficiently high triaxiality [202, 329].

The effect of the ‘void distribution’ parameter λ_0 is shown in Fig. 51b for various values of the particle volume fraction f_p . Varying λ_0 for a given value of f_p is equivalent to varying the relative ligament size χ_0 (or more precisely $1 - \chi_0$). This plot roughly estimates the maximum level of anisotropy in ductility that can be expected for a material with given λ_0 . When loaded in the perpendicular direction, this parameter changes roughly to $1/\lambda_0$. For instance, a metal with 1% of particles exhibiting an anisotropic particle distribution characterized by $\lambda_0 = 2$ presents a ductility that might change by more than a factor of 2 when tested in the other two orthogonal directions. Examples of anisotropic particle distributions can be found after severe plastic deformation typical of many forming operations. Note that the effect of particle clustering can be approximately captured by prescribing higher local values of λ_0 .

Specific results related to flat initial voids are shown in Fig. 51c providing the variation of ductility with particle volume fraction for various initial void shapes, considering uniaxial tension conditions only ($T = 1/3$) [250]. The threshold porosity f_p^* below which coalescence never occurs is indicated. This figure clearly shows that, for initially flat voids, a wide range of particle volume fractions gives way to failure by void coalescence at this low stress triaxiality. In contrast, more rounded voids involve a much more abrupt transition: below f_p^*

no coalescence takes place while above it, coalescence is almost immediate without any stable void growth stage. Another interesting outcome of the calculations is that void coalescence, and thus ductile fracture, is possible, if the volume fraction of particles is sufficiently large, before the onset of necking, that is, under purely uniaxial tension conditions. Recall that in rate insensitive materials and in the absence of geometric or material imperfections, necking sets in for a strain equal to n . Hence, for $n = 0.1$ and $n = 0.3$ fracture takes place before necking if $f_p > 23\%$ and $f_p > 17\%$, respectively.

Fig. 52 depicts the effect of the ‘flow properties’ of the material, namely yield strength and strain-hardening exponent, on the fracture strain for initially penny-shape voids, considering various initial volume fractions of particles (the results are similar with initially rounded voids). Here, we assume that the voids are present from the outset. The strain-hardening exponent has a moderate effect on the ductility while the yield strength has no effect. Changing for instance the strain-hardening exponent n from 0.1 to 0.2 by proper thermal treatment does not improve ductility as such. However, in practical structural loading conditions, an enhanced strain-hardening capacity, delays necking and significantly contributes to improving the ductility by postponing the rise of the stress triaxiality induced by necking. It bears emphasis that such effects are geometric and do not represent an intrinsic influence of strain hardening on damage evolution. Note also that, in practice, strain hardening can hardly be changed without affecting the strength (one important exception is given by aluminum alloys which show, at low temperature, a change of the strain-hardening capacity without much variation of the yield stress).

The effect of the flow properties changes quite a bit when void nucleation does not occur immediately but takes place when a critical stress is reached in the particle (σ^d) or along the interface (σ^c), thus requiring moderate to large amount of plastic deformation prior to nucleation. Fig. 53 shows the variations of the nucleation strain ε_c and of the fracture strain ε_i as a function of stress triaxiality for a material involving 1% of spherical hard particles giving rise to penny-shaped voids ($w_0 = 0.01$). Let us define the ‘void growth strain’ ε_g as the strain increment required to bring freshly nucleated voids to coalescence, that is, $\varepsilon_g = \varepsilon_i - \varepsilon_c$. Various values of the void nucleation stress σ^d/σ_0 were analyzed. The increase in the ductility with increasing nucleation stress is smaller than the increase of the void nucleation strain ε_c . In other words, the void growth strain decreases with increasing resistance to void nucleation. The reason for this effect is that the mean spacing between particles in the plane normal to the principal loading direction decreases before void nucleation. The main effect of the strain-hardening capacity is to accelerate the attainment of the nucleation condition by raising the stress in the particle. Hence, improving the strain-hardening capacity is beneficial for the ductility only when the ductility is not nucleation-controlled.

Effects of temperature and strain rate, as reported in Section 4.1.8, can be modeled provided that their effects on flow stress are well represented. For example, Lassance et al. [162] employed a rate-dependent version of the Gurson model supplemented with a limit-load void coalescence criterion to rationalize the increase of the tensile fracture strain with increasing strain rate (see Fig. 24b). There are three possible explanations for this effect: (i) the strain rate indirectly affects void nucleation by changing the stress in the particle; (ii) the strain rate intrinsically affects the void growth rate; and (iii) the strain rate affects, in a purely extrinsic way, the evolution of the geometry of the specimen. An effect of strain rate on void nucleation is ruled out, because it should induce an increase of the fracture strain

when decreasing the strain rate, whereas the opposite is observed. Regarding explanation (ii), the strain rate has a weak effect on the void growth rate [380]. Due to plastic flow localization in the neck, the strain rate increases during the test, thus leading to an effect similar to strain hardening. An extra hardening tends, with increasing deformation, to smooth out the strain gradients in plastic localization zones leading locally to lower stress triaxiality levels, hence to lower void growth rate. This effect has been discussed in detail in Ref. [330].

4.6.4 Fracture in notched bars

In the above illustrations, the stress and strain fields are supposed uniform throughout the loading history. In more general applications of the local approach, experimental data on laboratory specimens or even structural components can be directly simulated using full-field boundary-value problem solutions using, for example, the finite element method. In particular, notched bars have been central to the local approach over the years. Simulations using uncoupled models and the GTN model are now routine, following the pioneering works of Beremin [137] and Needleman and Tvergaard [352], respectively. Here, we present an application of the hybrid model with a focus on anisotropy.

Metal forming processes impart (initial) anisotropies of varying extent in both plastic flow characteristics and damage initiation sites. A notable example is that of hot-worked C-Mn steels containing MnS inclusions [112, 136, 198]. Softer than the metal at typical metalworking temperatures, these inclusions that form during casting develop into complex three-dimensional, often non-connected domains during processing. Typical shapes are disks in forging, plate stringers in rolling and cylinders in extrusion. The inclusions are embedded in an otherwise anisotropic matrix either due to processing-induced texture, grain elongation or both. Elongated inclusions such as sulfides are ubiquitous in steels of previous generations, which constitute the backbone of existing infrastructure (tens of thousands of miles of oil and gas transmission lines; structural parts in nuclear reactors; high-strength structures of naval ships; steel connections in buildings and civil infrastructure, etc.) These steels typically possess a ferrite-pearlitic or martensitic microstructure. The latest generations of steels do not contain MnS inclusions but often contain aligned particle clusters, which may result in similar effects, as in high-strength steels for line pipes [381, 382]. Other examples include oxide columns in extruded copper bars, potassium bubble columns in tungsten wires, stringers of sulfides in steels, and columns of broken intermetallic particles in rolled aluminum sheets. These steels as well as other high-specific-strength structural materials, such as aluminum and magnesium alloys, are often characterized by anisotropic fracture properties.

The extreme case of older generation steels with MnS inclusions, illustrated in Fig. 54 presents an ideal case for assessing fracture models. Using an implementation of the fully coupled formulation accounting for void shape effects, plastic anisotropy and void coalescence, Benzerga et al. [331] presented a set of predictions of anisotropic fracture in round notched bars of various notch radii. Fig. 55 shows examples of global force versus diameter reduction along two perpendicular directions (different due to plastic anisotropy) for two loading orientations. In the specific examples shown, the calculations were run without any adjustable parameter thus enabling a blind assessment of the predictive abilities of the constitutive model. As indicated in Section 4.6.2, the model predictions are only sensitive to

(i) the material's flow properties, identified on the deformation behavior of the undamaged material; and (ii) the attributes of the particle populations (i.e. initial state of equivalent microstructure in Fig. 54 described by f_0 , w_0 , λ_0 and initial particle/void orientation).

Fig. 56 illustrates the type of microscopic information provided by the model. In Fig. 56a for example, distributions of the void spacing ratio parameter λ are shown after the onset of cracking. Fig. 56b depicts a comparison between the critical void volume fraction, f_c , predicted by the coalescence model and measured values using various definitions (see details in [331]). The value of f_c at the center of the specimen correlates quite well with the maximum measured value, $f_{\max}$ (with a 10% frequency). Fig. 56c shows that for a more extreme form of plastic anisotropy but mild morphological anisotropy (relevant to a Mg alloy) the development of damage in the specimen can be quite anisotropic.

The use of such enhanced models of ductile fracture enables hypothesis-driven investigations and microstructure-dependent predictions. While finite-element simulations of laboratory specimens using the hybrid model remain limited, applications to the prediction of the fracture strain have been reported for low carbon steel, as above, dual-phase (DP) steels [274], cast Al alloys [260], wrought Al alloys [162,383], including for friction-stir welds [262], Cu alloys [384], and Mg alloys [117,385]. Also, the calculations presented in [331] and subsequent works in the literature are limited to microstructural states that are amenable to axial symmetry of void shapes and distributions, and this requires an identification of an "equivalent initial microstructure". The predictions labeled '1' and '2' in Fig. 55 correspond to different choices of the initial equivalent microstructure. The advent of three-dimensional models of void growth (see Section 4.5.3) paves the way to relaxing the arbitrariness of certain assumptions made in prior research. Steps in this direction have recently been reported [328,386]. On the one hand, implementation of such models would require the development of 3D void coalescence models. On the other hand, the mathematical complexity of the resulting constitutive relations warrants the development of simpler approaches that retain essential features of micromechanical models. One step in this direction has recently been proposed by Burlot [382].

4.6.5 Size effect

Predicting the failure of large structures typically depends on the use of failure criteria based on experimental testing of laboratory sized specimens. Therefore, the question of transferability to structural components arises in the presence of a possible size effect, as reported in Section 4.1.9. In brief, several explanations can be advanced to this size effect. We defer to [170–172] for further details. In the high toughness steel HY100 [172], the small specimen shows increased ductility accompanied by suppression of void-sheet coalescence observed in bigger specimens. These results can be readily understood on the basis that elongated inclusion clusters (200–300 μm) in length must be located in the highly stressed volume element in order to initiate failure by the void-sheet coalescence process. In the low strength steel investigated by Decamp et al. [170], and containing a relatively high volume fraction of MnS inclusions, the probability of finding a volume element containing a large volume fraction of inclusions and thus a low ductility is an increasing function of the specimen's cross-sectional area. This was the basis of a locally coupled model proposed by these authors to predict the size effect. In the duplex stainless steel studied in [171]

results of finite element calculations integrating random damage nucleation showed that a Gurson type model predicted both mean values and scatter of measured ductilities and the size effects that were evidenced (see Section 4.3.2 for related mechanisms).

The size effect on both the upper-shelf energy and the ductile to brittle transition was also investigated by means of the GTN model and a deterministic RKR type cleavage model [174,387]. When inertial effects are small, as encountered either at moderate impact velocities or in sufficiently small specimens, the calculations showed that larger specimens are more brittle. The extent of the size-dependence varies with material parameters [174]. The reason for this is that the size of the high-stress region at the notch root scales with the specimen size while the characteristic volume over which brittle fracture is evaluated is fixed for a given material. On the other hand, when dynamic effects are important, an additional material length scale $\sqrt{\sigma_0/\rho}/\dot{\epsilon}_0$ enters, which is related to inertia, with σ_0 the yield strength as above, ρ being the density and $\dot{\epsilon}_0$ a reference strain rate. If the specimen size is smaller than this length scale no size effect is observed. However, size effects emerge when the specimen size becomes comparable with it [387].

4.6.6 Fracture toughness

Modeling fracture toughness and ductile tearing involves the various contributions to the resistance to cracking described in Section 4.1.5 by means of equation (38). Often only crack initiation, through J_{Ic} , is of interest as for materials science driven questions regarding microstructure optimization or in some critical structural applications. In other applications, for instance in pipeline applications, ductile tearing must be simulated for long distances, involving questions of crack growth stability. Many different modeling approaches have been developed over the years to tackle this complex multiscale problem starting at the size of the void nucleation sites. These approaches are summarized in Fig. 57 and involve various assumptions that are ultimately based on a description of the mechanisms of nucleation, growth and coalescence of voids in the vicinity of the crack tip. Several studies have shown that void nucleation plays a minor role in setting J_{Ic} , e.g. [388], in contrast with its potential role in setting the ductility (see Fig. 53 above).

1) Approximate predictions of the fracture toughness for plane strain conditions can be formulated based on the elastoplastic crack tip fields introduced in Section 4.1.3 and the mechanisms of Fig. 37. A very simple argument states that the $\delta_c \sim X_0$ at crack initiation, meaning that the zone of large plastic strains, which scales with δ , must at least encompass one void nucleation site on average to trigger the damage mechanisms (Fig. 57a), hence

$$J_{Ic} = \alpha_n \sigma_0 X_0 \quad (97)$$

where α_n is the Shih factor (Section 4.1.2). Another closed form model has been proposed by Mudry and Pineau based on FE analysis [143,389] giving

$$J_{Ic} = \beta_M \sigma_0 X_0 \ln \left(\frac{R}{R_0} \right)_c \quad (98)$$

where $(R/R_0)_c$ is the critical void growth ratio, which can be experimentally determined (Section 4.6.2), and β_M is a factor typically equal to 4 [390], giving predictions in qualitative agreement with equation (97).

In thin sheets, Pardoen et al. [118] provided a closed form estimate of Γ_0 and Γ_n for steady state propagation (i.e. after the crack has propagated over a length equal to several times the thickness) as a function of damage parameters (f_0 , χ_0 , w_0 , X_0) and flow properties (initial yield stress σ_0 , strain hardening exponent n). The GLD model (specialization of (71) to spheroidal voids) combined with Thomason's coalescence model was used to predict Γ_0 assuming plane strain conditions (in the ligament direction) with plane stress in the out of plane near the crack tip. Dimensional analysis shows that

$$\frac{\Gamma_0}{\sigma_0 X_0} = F\left(n, \frac{\sigma_0}{E}, \nu, f_0, w_0, \chi_0\right). \quad (99)$$

For a given geometry, loading configuration and a constant product $\sigma_0 X_0$, the two most important parameters affecting ductile fracture are the initial void volume fraction f_0 and the strain-hardening exponent n . For low values of f_0 , typically below 10^{-4} , and high values of n (larger than 0.3), the function F takes values larger than 100 (for thin metallic sheets involving low triaxiality). The “plane stress” Γ_0 is typically three times larger than the value predicted for $T \sim 3$ typical of plane strain conditions, see also [391]. The second contribution to the fracture toughness Γ_n , was estimated using a FE based approach assuming plane strain condition in the crack direction and plane stress in the out of plane direction, and using the fracture strain predicted when computing Γ_0 . The contribution Γ_n is proportional to the thickness in the near plane stress regime, with an amplitude sometimes several times larger than Γ_0 .

Note finally that uncoupled approaches, neglecting the coupling between the mechanical fields and the damage evolution, can be used also to propagate cracks; see [392] who used the Rice-Tracey model and the $(R/R_0)_c$ criterion to be attained at a distance X_0 from the crack tip in conjunction with a node release technique.

2) Fig. 57b presents a strategy based on finite element calculations with the voids explicitly modeled using a refined FE mesh assuming either 2D cylindrical geometry or 3D representation, with regular or random distributions, e.g. [68, 225, 230, 393–398]. These analyses accurately model the growth and coalescence process while properly accounting for the length scale introduced by the void spacing. Such simulations are computationally intensive but quite useful for assessing other approaches to be discussed below.

3) A third approach, pursued for example in [143, 231, 261, 283, 379, 391, 399–402] relies on constitutive models of porous metal plasticity, such as the GTN model, Gurson-like models or the Rousselier model, which account for damage induced softening. This approach requires the introduction of a length scale by tying the element size to the void spacing (Fig. 57c) eventually calibrated to crack growth resistance data. Quite recently, Besson et al. [402] have developed a strategy based on this approach for modeling in 3D the transition from flat to slant fracture in metallic sheets.

4) Non-local constitutive models of ductile fracture have been formulated in order to explicitly introduce a length scale into porous plasticity models (Fig. 57d) and avoid the artificial connection with mesh size e.g. [403–408]. There are several methods for introducing internal lengths into the model and no consensus has emerged yet about the best approach.

5) Yet another strategy, schematically shown in Fig. 57e and initiated by Tvergaard and Hutchinson [228] for ductile failure, makes use of cohesive zone surfaces to simulate ductile

fracture, see also e.g. [409–412] The response of the fracture process zone is modeled by a traction-separation law, characterized by the work of separation Γ_0 and the cohesive (or peak) stress $\hat{\sigma}$ [288]. The value of Γ_0 is usually inferred either from experiments or an independent prediction through a damage model. One advantage of this method is that it introduces the cohesive length as a characteristic length scale. Some issues arise when introducing dependencies of the cohesive zone parameters on the mechanical fields next to the cohesive surfaces in order to artificially account for constraint effects on damage evolution. The most recent applications have focused on ductile tearing of thin sheets involving the problem of properly accounting for necking and/or slant mechanism with such a formalism [412].

6) As depicted in Fig. 57f, hybrid approaches have been developed in order to preserve the relative simplicity and physical relevance of the local formulation of damage constitutive laws while introducing a length scale with another strategy. For instance, Huespe et al [413] use a Gurson type model (strategy 3) until the incremental problem loses ellipticity, at which point they use a traction-separation law (strategy 5) that is embedded as a weak discontinuity. A similar approach has been pursued in [261] but the separation model is also introduced at the onset of coalescence. Similarly, Needleman and Tvergaard [230, 414] model one population of voids by discrete voids (strategy 2) occupying regions of very high porosity modeled using the Gurson model (strategy 3), while a second population of voids is introduced at the lower scale using the Gurson model again in the matrix (strategy 3), see also [415, 416] where a length scale is introduced at the scale of the second population (strategy 4). The approach in [398] is similar but with the primary voids discretized in the 3D FE mesh.

Fig. 58 illustrates the predictions of strategy 3, in terms of the link between fracture toughness and the microstructure and flow properties [379]. Among the various parameters indicated by the dimensional analysis underlying equation (99) the strain hardening exponent has a major influence on fracture toughness. The magnitude of function F agrees with the simple closed form solutions (97) and (98). However, this model does not capture some experimental results associated with mechanism 4 of Fig. 37 that would lead to much larger values of J_{Ic} . The fact that the fracture toughness is proportional to the yield stress seems to contradict the experimental evidence that toughness usually decreases with increasing yield stress. However, an increase of σ_0 by metallurgical means is usually accompanied by a decrease in strain-hardening capacity, which has the opposite effect on fracture toughness. These counteracting trends occur in precipitation hardening of aluminum alloys, where the precipitates do not, in general, take part in the failure process.

According to the scaling relationship (99), with voids already present, a higher yield stress directly implies a higher fracture toughness, all other parameters being kept constant, including the strain hardening exponent. The effect of temperature and strain rate on the fracture toughness of metals can thus be addressed in part under this perspective. A good example is the decrease of the fracture toughness of ferritic steel with increasing temperature in the upper shelf region. This decrease must be associated with thermal softening. Indeed, for the typical temperature range covered when determining a ductile-brittle transition curve, no modification of microstructure and strain hardening mechanisms is expected. On the other hand, at sufficiently elevated temperatures, viscoplastic effects become dominant. The increase in rate sensitivity plays a role similar to an increase in strain hardening. Also, in some materials exhibiting dynamic strain aging a minimum in ductility and toughness has

been established. For instance, Amar and Pineau [165] have demonstrated the applicability of an uncoupled model to predict this phenomenon; also see more recent work by Berdin and Wang [166]. Implications of dynamic strain aging in failure by instability has also been addressed, e.g., [417–419].

The fracture toughness of metallic materials may be orientation dependent [216,420,421], just like ductility (see Section 4.6.4 above). The anisotropy of fracture toughness may be rooted in the shape of second-phase particles, their spacing, or both (see Fig. 54). The effect of plastic flow anisotropy on the orientation dependence of toughness is believed to be small [422], although it may affect the absolute magnitude of fracture toughness.

Strong fracture anisotropy is typical of wrought products, such as rolled or forged C-Mn steels. For instance, Lautridou and Pineau [233] performed fracture toughness tests in the T–L, L–T and S–T orientations, the first letter indicating the direction of loading in a CT specimen, and the second the direction of crack growth. The initiation toughness J_{Ic} was found to vary by a factor of four depending on the orientation. These authors discussed the relationship between J_{Ic} , as well as the tearing modulus, and the mean inclusion spacing in the plane containing the loading direction and the crack growth direction. Similarly, Bauvineau [421] characterized the out-of-plane anisotropy and showed that the initiation toughness of the S–L orientation was about half that of the T–L orientation. Also, Benzerga [216] characterized the in-plane anisotropy of a low alloy Mn steel and showed that the initiation toughness of the T–L orientation was about one third that of the L–T orientation, although the small CT specimen thickness did not enable small-scale yielding conditions to be met in the L–T case.

Based on equation (99), Pardoen and Hutchinson [379] have demonstrated the predictive capability of a model accounting for void shape effects and limit-load coalescence in capturing the anisotropy of fracture toughness. In modern steels, elongated sulfide inclusions are generally avoided, for example by gettering sulfur through alloying [423]. In such cases, any residual anisotropy in fracture toughness is believed to be associated with the orientation dependence of mean inclusion spacings [382].

The prediction of initiation toughness is important in a material science context, but from a structural integrity viewpoint, the approaches described above can be used too. For instance, the capacity to predict crack resistance curves in the case of loss of constraint has granted, over the years, a considerable success to the local approach to fracture. The success of the approach is underpinned by the requirement that the microstructural parameters (the void volume fraction, void spacing, etc.) or the more phenomenological parameters of traction separation laws must be set such that the model reproduces experimental crack data for specific specimens [56].

5 Concluding Remarks

The local approach to fracture is far more complex than the global approach, which assumes that fracture can be described by a single (eventually two) loading parameter, such as K_{Ic} or J_{Ic} . In particular the local approach applied to metallic alloys requires detailed metallographic measurements (grain size, grain or packet orientation, second-phase volume fraction, particle shape and spacing, grain boundary composition, etc.) as well as advanced FEM cal-

culations. Contrary to the global approach to fracture in which the material is considered as a “black box” to which macroscopic, statistically based failure criteria are applied, the local approach to fracture has largely contributed to the issues related to the transferability of laboratory test results to components in the presence of size and constraint effects. It has allowed modeling complex macroscopic phenomena – a small sample of which has been highlighted in the present overview – such as the beneficial effect of warm pre-stressing observed when a piece of steel is pre-stressed above the ductile-to-brittle transition curve and then loaded below this transition, [the scatter and the specimen-size effect observed not only in brittle fracture but also in ductile rupture, the anisotropy of ductility and fracture toughness, etc.](#)

Many results derived from the micromechanical approach to fracture still remain speculative in the absence of a sufficiently large basis of experimental verifications and more extensive studies. In brittle fracture, for instance, this is the situation with the temperature dependence of the stress σ_u in Eq. (29). The increase of this parameter with temperature which has been reported in a number of studies may simply reflect a change in the micro-mechanisms of failure with temperature, as already discussed. This is also the situation with intergranular fracture for which there are still very few quantitative studies relating the intergranular fracture toughness to the amount of impurities segregated at grain boundaries. Similarly, in spite of a number of recent works, there are still very few studies devoted to the transition of ductile tearing to brittle cleavage fracture. The effect of inhomogeneities should also be investigated in much more detail. The presence of macro-segregation introduces another source of scatter and therefore of size dependence which may be such that a difference with the Beremin law $K_{Ic}^4 B$ is observed. To date, crack arrest, which is also another important phenomenon, has been poorly investigated in the frame of the local approach to fracture in spite of its importance for engineering applications. Further investigations in this domain are also much needed.

In the area of ductile fracture, it is fair to say that a universal theory is still missing, although the fundamental mechanisms are rather well characterized and understood. The models that were critically overviewed and generally applied to structural materials are, by mere inspection, different from those commonly employed in sheet or bulk metal forming [142]. The distinction between failure by instability and failure by void coalescence may be a ground for the difference in approach, as the former mode of failure is ubiquitous in metalworking. On the other hand, when the fundamental mechanisms of the various modes of failure are well characterized, the hope is that a unifying or converged description of failures in service and metal forming becomes possible. Attempts at this have emerged in recent years guided by macroscopic models, for example using critical strain criteria. Such models have not been included in this overview. The local approach to fracture certainly provides a fertile ground for a unified description but its applicability in metalworking remains to be demonstrated.

It should be emphasized that the actual, or apparent, fracture locus of a ductile material is path-dependent. Since the stress triaxiality T and Lode parameter L can be viewed as functions of some monotonically increasing variable, such as time or effective plastic strain p , a locus can be defined only if, say initial values, terminal values or strain-weighted averages of T and L are introduced. Under proportional loadings, both T and L remain constant and a unique fracture locus is defined, which may be thought of as *intrinsic* to the material. An

important corollary is that the intrinsic locus is *not* accessible to experimental measurement for it is impossible, in general, to impose constant- T and L loadings up to failure over the full range of stress states. This issue hints at the key role of predictive mechanism-based modeling in fracture. One cannot emphasize enough that what is measured in experimental reports is some apparent, not the *intrinsic*, fracture locus. The apparent locus may be affected by extrinsic factors such as the occurrence of plastic instabilities and other strong deviations from proportionality. Plastic instabilities, such as shear bands, often involve strain localization at a length scale greater than the mean spacing between material inhomogeneities (voids). Plastic instabilities such as necks and shear bands are strongly dependent upon the states of stress and strain as well as boundary conditions. As such, they should be viewed as structural, hence non-intrinsic effects.

The most recent micromechanical models that underly the local approach may appear to be sophisticated. Yet, those on void coalescence are in their infancy. How to incorporate void nucleation models, beyond the phenomenological ones, has drawn no consensus yet. The extent to which particle/void clustering affects ductility and toughness is unclear. While it is generally believed that clustering decreases the fracture characteristics, and plays first-order effects under certain circumstances, there are conflicting reports on its effects, even in the absence of gradients in the mechanical fields. Strategies for dealing with situations where scale separation breaks down are yet to be established. Research in this area is mostly reduced to incorporating a length scale by various means in the constitutive relations. However, specific connections of the length scales to the actual origins of scale separation breakdown are rarely made. Such issues arise for example in the presence of strong heterogeneities, or when the second-phase particles responsible for the flow characteristics are the very damage initiation sites. An even more limiting case is when damage initiation is merely deformation induced, such as at mechanical twins.

The current challenges in the area of modeling ductile crack growth in engineering metals are related to a better representation of the heterogeneities in the material, especially when dealing with multiphase alloys, to the simulation of thin metallic sheets involving necking and/or a transition into a slant mode, to a better account for anisotropy effects and failure mode competition and to the proper modeling of cracking in materials exhibiting a large fracture process zone, the size of which is two to three orders of magnitude larger than the void spacing. [Recent progress in 3D *in situ* microtomography has provided a wealth of data that allow unraveling complex mechanisms and critical assessment of models.](#)

In closing, the local approach to fracture presents key advantages for both structural integrity assessment and material design. Incorporating physics/micromechanics based models provides not only a framework to metallurgists for understanding how materials fail, but also to the mechanics community for building more reliable approaches for structural integrity. A major challenge in material design is to translate the know-how underlying the local approach into specific guidelines for processing fracture resistant metallic alloys. This involves, among other things, identifying then optimizing such microstructural features as grain size, grain-boundary character, second-phase composition and size, interface characteristics, precipitation hardening, graded structures or twinning characteristics. Some of these endeavors are currently being pursued in the area of nanostructured metals where failure is a key issue, as overviewed in part III of this series [424].

Acknowledgments

A. Pineau would like to acknowledge the Ecole des Mines (now Mines ParisTech), the French Ministry of Industry, the French Ministry of Research (through CNRS), all the industrial firms, in particular those involved in Nuclear Industry (AREVA, EDF) and Steel Making Industries (ArcelorMittal), the Safran-Snecma group, the French research organizations, such as CEA, which have largely contributed to the support of his research. He would like to thank also all his former PhD students. Without them it would have been impossible to develop such research. AAB acknowledges support from the National Science Foundation under CMMI grant # 1405226. TP acknowledges support from the Belgian Science Policy through the IAP 7/21 project.

Structure	Cleavage plane	Some Materials
BCC	{100}	Ferritic steels, Mo ; Nb, W
FCC	{111}	Very rarely observed
HCP	{0002}	Be, Mg, Zn
Diamond	{111}	Diamond, Si, Ge
NaCl	{100}	NaCl, LiF, MgO, AgCl
ZnS	{110}	ZnS, BeO
CaF ₂	{111}	CaF ₂ , UO ₂ , ThO ₂

Table 1: Cleavage planes in various materials.

Metal	Au	Ag	Cu	Pt	Ni	Rh	Ir	Nb	Ta	V	Fe	Mo	W	Cr
μ/K	0.11	0.19	0.22	0.24	0.34	0.52	0.52	0.25	0.31	0.32	0.33	0.48	0.52	0.82
R_{CI}	1.09	1.02	0.99	0.97	0.87	0.71	0.70	0.97	0.91	0.89	0.88	0.75	0.71	0.42
$(\mu/K)_{\text{CD}}$	0.36	0.43	0.57	0.38	0.49	0.39	0.32	0.59	0.55	0.65	0.56	0.35	0.45	0.68

Table 2: Transition parameters for fracture. μ/K is the ratio of the shear modulus to bulk modulus; R_{CI} quantifies the risk of intergranular fracture versus cleavage; $(\mu/K)_{\text{CD}}$ is the ratio required for the transition between cleavage and ductile fracture.

Material	γ_S ($T = 0$) (J/m ²)	μ_{slip} (GPa)	b (nm)	$\gamma_S/\gamma_{\text{uS}}$ (Frenkel)	$\gamma_S/\gamma_{\text{uS}}$ (EAM)
FCC metals					
Ag	1.34	25.6	0.166	8.8	12.5
Al	1.20	25.1	0.165	8.1	11.5
Au	1.56	23.7	0.166	11.0	15.7
Cu	1.79	40.8	0.147	8.3	11.8
Ir	2.95	198	0.156	2.7	3.8
Ni	2.27	74.6	0.144	5.9	8.4
Pb	0.61	7.27	0.201	11.6	16.6
Pt	2.59	57.5	0.160	7.8	11.2
BCC metals					
Cr	2.32	131	0.250	1.1	1.6
Fe	2.37	69.3	0.248	2.2	3.2
K	0.13	1.15	0.453	4.0	5.7
Li	0.53	3.90	0.302	7.3	10.4
Mo	2.28	131	0.273	1.0	1.5
Na	0.24	2.43	0.366	4.4	6.2
Nb	2.57	46.9	0.286	3.1	4.4
Ta	2.90	62.8	0.286	2.6	3.7
V	2.28	50.5	0.262	2.8	4.0
W	3.07	160	0.274	1.1	1.6
Diamond Cubic					
C	5.79	509	0.145	2.2	3.1
Ga	1.2	49.2	0.231	2.9	4.2
Si	1.56	60.5	0.195	3.7	5.2

Table 3: Materials properties to evaluate the ratio $\gamma_{\text{u}}/\gamma_{\text{us}}$ [17].

Parameter	Lambert-Perlade et al. [35]	Literature Data		
		Value	Microstructural Unit	References
σ_{Ic} (MPa)	2112	-	-	-
$K_{Ia}^{c/f}$ (MPa m ^{1/2})	7.8	2.5 to 5.0 2.5 1.8	Carbides Globular carbides TiN particles	Martin-Meizoso et al. [42] Hahn [425] Rodrigues-Ibabe [426]
$K_{Ia}^{f/f}$ (MPa m ^{1/2})	CGHAZ-25: 28 ICCGHAZ-25: 18	5.0 to 7.0 7.0 7.5 4.8 15.2	Bainite packets Bainite packets Ferrite grains Bainite packets Bainite packets	Martin-Meizoso et al. [42] Martin-Meizoso et al. [42] Hahn [425] Rodrigues-Ibabe [426] Rodrigues-Ibabe [426]

Table 4: Parameters of multiple barriers models [35].

Stress state	Stress ratio	Expression	Minimum	Maximum
Axisymmetric	$\rho = \sigma_r/\sigma_z$	$\frac{1}{3} \text{sign}(\sigma_z) \frac{2\rho + 1}{ 1 - \rho }$	$-\frac{2}{3}$	$+\infty$
Plane strain	$\rho = \sigma_1/\sigma_3$	$\frac{1}{\sqrt{3}} \text{sign}(\sigma_3) \frac{1}{ 1 - \rho }$	0	$+\infty$
Plane stress	$\rho = \sigma_2/\sigma_1$	$\frac{1}{3} \text{sign}(\sigma_1) \frac{\rho + 1}{\sqrt{\rho^2 - \rho + 1}}$	$-\frac{1}{3}$	$1/\sqrt{3}$

Table 5: Typical expressions, values and bounds of the triaxiality.

Stress state	Stress ratio	Expression	Minimum	Maximum
Axisymmetric	$\rho = \sigma_r / \sigma_z$	± 1	-1	+1
Plane strain	$\rho = \sigma_1 / \sigma_3$	0	0	0
Plane stress	$\rho = \sigma_2 / \sigma_1$	$\frac{\rho + 1}{\rho - 1}$ for $\rho \leq 0$		
		$2\rho - 1$ for $0 \leq \rho < 1$	-1	+1
		$\frac{\rho - 2}{\rho}$ for $\rho > 1$		

Table 6: Typical expressions, values and bounds of the Lode parameter.

Parameter	Type	Trend	
		Decohesion	Cracking
Matrix yield strength		↘	↗
Matrix hardening exponent		↘	↗
Particle elongation		↘	↗
Particle stiffness		↗	↗
Load orientation	axial	↘	↗
	transverse	↗	↘
Load triaxiality		↗	↘

Table 7: Key parameters in void nucleation and relative trends upon increasing the parameter for the microscopic mechanism.

Aspect triggering shear band	Investigation type	References
Vertex in yield surface	Analytical Computational	[113] [427]
Non-associated flow	Analytical	[428]
High yield surface curvature	Analytical Computational	[429] [430]
Plastic dilatancy	Analytical	[431]
Porosity induced softening	Analytical Computational	[403, 432] [403, 404]
Void nucleation	Analytical Computational	[433] [434]
Void coalescence*	Computational	[435]
Plastic anisotropy	Analytical Computational	[436] [437]
Plastic anisotropy & coalescence	Computational	[221]
Thermal softening	Computational	[438]

Table 8: Some potentially sufficient conditions for shear band formation in metals, as investigated in the literature. * Modeled using the f^* approach [10] and illustrating the stabilization of plastic flow for low critical porosities.

Figure 1: Cleavage fracture surfaces (a) low alloy steel [439] – Rivers; (b) Polycrystalline zinc rivers originating from a grain boundary; (c) SEM micrograph showing the presence of “tongues” (indicated by arrows) formed at the intersection of the main (001) fracture plane with mechanical twins.

Figure 2: A representative volume element containing an inclusion and bilinear constitutive equation for the matrix.

Figure 3: Cleavage fracture of an oxide inclusion in a low alloy steel. Loading is horizontal.

(a)

(b)

Figure 4: Variation of the ratio E_V/E_S (see text) with stress triaxiality Σ_m/Σ_{eq} for $f = 0.10$, $E_p = 2000\text{MPa}$ (tangent modulus), $\sigma_{1c} = 2000\text{MPa}$, $\gamma_S = 2\text{J/m}^2$, $h = 1\mu\text{m}$, and for various values of (a) the particle aspect ratio k with $\sigma_0 = 500\text{MPa}$; (b) σ_0 with $k = 1/2$.

Figure 5: Schematic representation of the role of microstructural barriers on fracture micromechanisms. The crack is assumed to nucleate from an intragranular particle: (a) undamaged material; (b) microcrack initiation and propagation in the particle, (c) microcrack propagation across the particle/matrix interface and through the grain; (d) crossing of grain boundary leading to final fracture [35].

Figure 6: Initiation of a cleavage microcrack from a particle (e.g. martensite/austenite (M-A) constituents in welds). The crack may eventually be arrested at the interface c/f; then propagates through the matrix and is arrested at the grain boundary. See the definition of $K_{Ia}^{c/f}$ and $K_{Ia}^{f/f}$ in [6].

Figure 7: Multiple barrier model. Two events, i.e. (i) crack propagation across the carbide/matrix interface and (ii) grain boundary crossing, are indicated with their probability of occurrence. Here and in Eq. (9) δ is a particle- or grain-shape dependent numerical constant.

Figure 8: (a) A cracked grain boundary carbide in ferrite, (b) axisymmetric model with a carbide embedded in ferrite [43].

Figure 9: Non-dimensional overall stress as a function of the critical largest carbide size, C , for four levels of stress triaxiality, T . Comparison with the Griffith criterion [43].

Figure 10: Steel A508. Fracture surfaces observed on CT specimens tested at -60°C . (a) The tilt and twist angles between adjacent (001) planes on the fracture surface are shown along the crack path indicated by black arrows. The white arrow designates ductile fracture on a riser between tilted cleavage planes. (b) Fracture surface showing the segmentation of the cleavage crack front into four facets separated by risers in the grain. Illustration of the formation of an intergranular triangle along the grain boundary crossed by a cleavage plane. Tongues are observed on the cleavage facets [12].

Figure 11: The four steps for the crossing of a twist grain boundary by a cleavage crack.

Figure 12: Calculated fracture stress as a function of twist angle [40].

Figure 13: Schematic variation of the probability to failure as a function of loading parameter, J/σ_0 , at various increasing temperatures, $\theta_1, \theta_2, \theta_3$. The condition for the small scale yielding (SSY) to large scale yielding (LSY) transition is indicated by a dotted line. The difference between long cracks (LC) and short cracks (SC) is also indicated.

Figure 14: Relationship between critical fracture stress and phosphorus monolayer coverage. Results obtained on a low alloy steel with various heat treatments (OQ, oil quench; WQ, water quench; AC, air cooled; FC furnace cooled, 20°C/h cooling rate after austenitization; ST step cooling [105]).

Figure 15: The two generic modes of ductile failure (a) by plastic instability; and (b) by void coalescence.

Figure 16: Triaxiality versus the ratio of (a) radial-to-axial stress in axially symmetric loading; (b) any in-plane to out-of-plane stress in plane strain; (c) in-plane stresses in plane stress. For a compressive σ_3 , the plots would be symmetric of those shown with respect to the abscissa axis.

Figure 17: Lode parameter in plane stress loading versus the ratio of in-plane stresses.

Figure 18: Triaxiality T versus the distance r to the crack tip normalized by the crack tip opening displacement δ in (a) plane strain; (b) near plane stress.

Figure 19: Cross-sections of the intrinsic fracture locus (a) at fixed Lode parameter L ; (b) at fixed triaxiality T . Section (a) is known with high-fidelity for materials failing by void growth to coalescence and $T \in]1/3, \infty[$. That in (b) is only illustrative, based on Gao et al.'s work [440] for $T \sim 0.5$.

Figure 21: Various nonlinear strain paths to fracture in compression. Adapted from [116].

Figure 22: Schematic J_R curve and plastic zone extension during crack propagation.

Figure 23: Fracture toughness, J_{Ic} , normalized by its plane strain value versus specimen thickness normalized by the plane strain plastic zone size, r_{SSY} , for two typical classes of materials.

Figure 24: (a) Tensile fracture strain, $\bar{\epsilon}_f$, versus strain rate for various temperatures in a magnesium alloy [161]. (b) Tensile fracture strain versus temperature for various strain rates in an aluminum alloy [162]. (c) Initiation fracture toughness, J_{IC} , versus temperature in steel [165].

Figure 25: Fracture surface of ultra-fine grained IF steel [178].

Figure 26: Mechanisms of void nucleation in simple tension of model Al metal matrix composites. (a) Particle-matrix decohesion in a soft matrix. (b) Particle cracking in a hard matrix. Printed with permission from Elsevier from [181].

Figure 27: Mechanism of void nucleation in shear loading of HSLA steel. Percentages are of remote displacement to fracture. Adapted from [191].

Figure 28: Void growth after 0.5 strain in high-purity copper within a triaxial stress field induced by specimen necking. Adapted from [195].

Figure 29: Void growth in low carbon steel. Voids nucleate at large inclusions (sulfides and oxides) while carbides and cementite particles remain intact. Loading is normal to plane of view.

Figure 30: Void elongation with negligible net porosity growth at low triaxiality in Glidcop. Strain is (a) 0.0; (b) 0.35; (c) 0.45. Adapted from [195].

Figure 31: (a) Void rotation near a free surface of a notched steel bar [112]. (b) Simulated void distortion in a shear field [206]. (c) Void shearing by a shear band [441].

Figure 32: Void locking at low stress triaxiality (a) in uniaxial tension (Al_2O_3 in pure Al) [181]; (b) in shear (Al_2O_3 in HSLA steel) [191]; (c) in notch tension (MnS in low carbon steel) [112].

Figure 33: Modes of void coalescence by (a) internal necking [442]; (b) internal shearing [442]; and (c) “necklace” formation [216, 221]. Tomography observations of internal necking (d) regular; (e) irregular [195]. Loading is vertical in all with different amounts of triaxiality. See text for details.

Figure 34: Mechanisms of void coalescence in simple tension of model Al metal matrix composites. (a) after significant void growth (soft matrix). (b) after limited void growth (hard matrix). Adapted from [218].

Figure 35: Transition from diffuse damage to cracking in round notched bars of steel. The loading curve corresponds to the specimen shown at the near-final stage (f). Adapted from [112,216].

Figure 36: Fracture surface of cast Ti-6Al-4V alloy after torsion [224]. 1500x.

Figure 37: Mechanisms of crack initiation and propagation in ductile metals.

Figure 38: Doubly periodic array of voids or particles and axisymmetric cell model.

Figure 39: Typical results from direct numerical simulations of void growth and coalescence for ordered void distributions by means of the cell model and three values of the stress triaxiality T . Plotted against the effective strain, E_e are: (a) the effective stress, Σ_e , normalized by the matrix yield stress; (b) the void volume fraction, f ; (c) the void aspect ratio, w ; and (d) the net lateral strain, E_r . The onset of internal necking is marked by x. Voids are initially spherical with porosity $f_0 = 0.001$. Adapted from [7].

Figure 40: Transition from diffuse to localized plasticity mediated void growth revealed by: (a) direct numerical simulations as in Fig. 39; (b) tomography observations of model void distributions [195], see Fig. 28.

Figure 41: Schematic description of a typical microstructure of a multiphase polycrystalline metal in which voids nucleate, grow and coalesce in a heterogeneous way.

Figure 42: Transition from distributed damage to cracking in a round notched bar of Al 6056 at a strain $\bar{\epsilon}$ of (a) 0.22, (b) 0.3, (c) 0.35, and (d) 0.55.

Figure 43: Particle–matrix interface stress concentration factor κ^I in (48) versus effective strain E_e for an elasto-plastic matrix with $N = 0.2$ under remote uniaxial loading ($T = 1/3$) and two values of the modulus mismatch E^P/E (a) Spherical particle, $w^p = 1$; and (b) Elongated particle with $w^p = 7$. Adapted from [207].

Figure 44: (a) Interface stress concentration factor κ^I versus effective strain E_e for an elastoplastic matrix containing a prolate particle with $w^P = 7$ using $N = 0.2$ and $E^P/E = 2$ and two values of stress triaxiality T . (b) Ratio of the particle to interface stress concentration factors κ^P/κ^I in (48) versus E_e for uniaxial loading ($T = 1/3$), $N = 0.1$ and $E^P/E = 2$ and two particle aspect ratios. Adapted from [207].

Figure 45: Interfacial normal traction, normalized by remote axial stress Σ_{33} , versus the angle ϕ measured from the x_3 -axis. Results are shown for two particle aspect ratios, $N = 0.1$, $E^P/E = 2$ at an effective strain of $E_e = 0.03$. (a) Uniaxial loading ($T = 1/3$); and (b) $T = 10/3$. Adapted from [207].

Figure 46: Sketch showing the distribution of the interfacial normal traction for elastic particles in an elasto-plastic matrix with hardening under (a) uniaxial loading ($T = 1/3$) and (b) triaxial loading ($T = 10/3$) for two particle shapes. Special angles indicate either a maximum value, a transition to compressive tractions or a saturation of the traction, consistent with the results in Fig. 45. Traction vectors may not be drawn normal to interfacial regions of high curvature. Adapted from [216].

Figure 47: Strain to failure versus triaxiality T when void coalescence is by mere impingement for two values of the initial void size to spacing ratio R_0/S_0 assuming either a periodic (solid) or random (dashed) void distribution.

Figure 48: Hybrid model for multisurface representation of void growth and coalescence. After [221].

(a)

(b)

Figure 49: Typical fracture loci obtained using uncoupled models: (a) the Beremin model; (b) the extended model using $k = 0.854$, $\kappa = \sqrt{3}$ and $\beta = 1/3$. After [374].

(a)

(b)

Figure 50: Strain to failure versus average triaxiality for proportional and one type of non-proportional loadings (see text for details) obtained using: (a) finite-element cell model calculations [378]; (b) the uncoupled model (95) using $(R/R_0)_c = 2.7$, $k = 0.854$, $\kappa = \sqrt{3}$ and $\beta = 1/3$ [374].

Figure 51: Intrinsic fracture loci for various values of (a) initial void aspect ratio w_0 (labeled W_0); and (b) initial void spacing ratio λ_0 . (c) Fracture strain versus particle volume fraction under uniaxial loading ($T = 1/3$) for various values of $w_0 < 1$ (see text).

Figure 52: Intrinsic fracture loci for various values of (a) the strain hardening exponent n and particle volume fraction f_p ; and (b) elastic modulus to initial yield ratio E/σ_0 .

Figure 53: Intrinsic fracture loci accounting for void nucleation for various values of the critical nucleation stress σ_d .

Figure 54: Representation of initial anisotropy in models assuming axial symmetry in void shape and distribution. After [331].

Figure 55: Prediction of anisotropic damage progression, crack initiation and growth in round notched bars. Details in [331].

(a)

(b)

(c)

Figure 56: (a) Contours of void spacing ratio, λ , for “prediction 2” in Fig. 55a. (b) Calculated versus measured critical void volume fractions [331]. (c) Contours of void volume fraction in notched bar of AZ31.

Figure 57: Various modeling approaches to predict fracture toughness and simulate ductile tearing in ductile metals.

Figure 58: Variation of the fracture toughness with the initial porosity for various values of the strain hardening exponent n .

References

- [1] J. Knott, *Fundamentals of Fracture Mechanics*, Butterworths, London, 1973.
- [2] J. Besson, S. Bugat, C. Berdin, R. Desmorat, F. Feyel, S. Forest, E. Lorentz, E. Maire, T. Pardoen, A. Pineau, B. Tanguy, *Local Approach to Fracture*, Les Presses de l'Ecole des Mines, Paris, 2004.
- [3] D. François, A. Pineau, A. Zaoui, *Mechanical Behaviour of Materials, Volume II: Fracture Mechanics and Damage*, Springer, Netherlands, 2013.
- [4] A. Pineau, Review of fracture micromechanisms and a local approach to predicting crack resistance in low strength steels, in: D. Francois (Ed.), *Advances in Fracture Research*, Proceedings of ICF5, Pergamon press, 1981, pp. 553–577.
- [5] F. Beremin, A local criterion for cleavage fracture of a nuclear pressure vessel steel, *Met. Trans. A* 14A (1983) 2277–2287.
- [6] A. Pineau, T. Pardoen, *Failure of Metals*, *Comprehensive Structural Integrity* 2 (2007) 684–797, Chapter 2.06.
- [7] A. A. Benzerga, J.-B. Leblond, Ductile fracture by void growth to coalescence, *Adv. Appl. Mech.* 44 (2010) 169–305.
- [8] J. Besson, Continuum Models of Ductile Fracture: A Review, *Int. J. Damage Mech.* 19 (2010) 3–52.
- [9] A. L. Gurson, Continuum Theory of Ductile Rupture by Void Nucleation and Growth: Part I– Yield Criteria and Flow Rules for Porous Ductile Media, *J. Eng. Mat. Tech.* 99 (1977) 2–15.
- [10] V. Tvergaard, A. Needleman, Analysis of the cup–cone fracture in a round tensile bar, *Acta Metall.* 32 (1984) 157–169.
- [11] L. Xia, C. F. Shih, J. W. Hutchinson, A computational approach to ductile crack growth under large scale yielding conditions, *J. Mech. Phys. Solids* 43 (1995) 389–413.
- [12] A. Andrieu, Mécanisme et modélisation multi-échelle de la rupture fragile trans- et inter-granulaire des aciers rep en lien avec le vieillissement thermique, Ph.D. thesis, Mines ParisTech, Paris (2013).
- [13] A. Cottrell, Strengths of grain boundaries in pure metals, *Mater. Sci. Tech.* 5 (1989) 1165–67.
- [14] A. Cottrell, Strengthening of grain-boundaries by segregated interstitials in iron, *Mater. Sci. Tech.* 6 (1990) 121–23.
- [15] A. Cottrell, Strengths of grain boundaries in impure metals, *Mater. Sci. Tech.* 6 (1990) 325–29.

- [16] J. R. Rice, R. Thomson, Ductile versus brittle behavior of crystals, *Phil. Mag.* 29 (1974) 73–97.
- [17] J. R. Rice, Dislocation nucleation from a crack tip - An analysis based on the Peierls concept, *J. Mech. Phys. Solids* 40 (1992) 239–271.
- [18] C. Zener, *Fracturing of metals*, American Society for Metals (Cleveland), 1948.
- [19] C. Stroh, The formation of cracks as a result of plastic flow, *Proc. Roy. Soc. London A* 223 (1954) 404–414.
- [20] A. H. Cottrell, Theory of brittle fracture in steel and similar metals, *Trans. AIME* 212 (1958) 192–203.
- [21] E. Smith, The nucleation and growth of cleavage microcracks in mild steel, in: *Proceedings of the conference on physical basis of yield and fracture*, Institute of Physics & Physics Society, 1966, pp. 36–46.
- [22] M. Berveiller, A. Zaoui, An extension of the self-consistent scheme to plastically-flowing polycrystals, *J. Mech. Phys. Solids* 26 (1979) 325–344.
- [23] Beremin, F. M. and Pineau, A. and Mudry, F. and Devaux, J.C. and DEscatha, Y. and Ledermann, P., Cavity formation from inclusions in ductile fracture, *Met. Trans. A* 12A (1981) 723–731.
- [24] A. Pineau, Global and local approaches of fracture. transferability of laboratory test results to components, in: A. S. Argon (Ed.), *Topics in Fracture and Fatigue*, Springer-Verlag, 1992, pp. 197–234.
- [25] J. Chen, G. Wang, H. Wang, A statistical model for cleavage fracture of low alloy steel, *Acta Mater.* 44 (1996) 3979–89.
- [26] J. Chen, R. Cao, *Micromechanism of Cleavage Fracture of Metals: A Comprehensive Microphysical Model for Cleavage Cracking in Metals*, Butterworth-Heinemann, 2014.
- [27] M. Linaza, J. Rodriguez-Ibabe, J. Urcola, Determination of the energetic parameters controlling cleavage fracture initiation in steels, *Fat. Frac. Eng. Mat. Struct.* 20 (1997) 619–32.
- [28] J. San Martin, J. Rodriguez-Ibabe, Determination of energetic parameters controlling cleavage fracture in a Ti-V microalloyed ferrite-pearlite steel, *Scr. Mater.* 40 (1999) 459–64.
- [29] K. Shibamura, S. Aihara, M. Matsubara, H. Shirahata, T. Handa, Prediction model of cleavage fracture toughness of ferrite steel, in: *13th International Conference on Fracture*, S34-009, Jun.16-21, 2013. Beijing, China, 2013.
- [30] S. H. Goods, L. M. Brown, The nucleation of cavities by plastic deformation, *Acta Metall.* 27 (1979) 1–15.

- [31] B. Margolin, A. Gulenko, V. Shvetsova, Improved probabilistic model for fracture toughness prediction for nuclear pressure vessel steels, *Int. J. Pres. Ves. Pip.* 75 (1998) 843–55.
- [32] S. Deyber, F. Alexandre, J. Vaissaud, A. Pineau, Probabilistic life of da718 for aircraft engine disks, in: *Superalloys 718, 625, 706 and Derivatives*, Proceedings, 2005, pp. 97–110.
- [33] E. Bouyne, H. Flower, T. Lindley, A. Pineau, Use of EBSD technique to examine microstructure and cracking in a bainitic steel, *Scr. Mater.* 39 (1998) 295–300.
- [34] A. Gourgues, H. Flower, T. Lindley, Electron backscattering diffraction study of acicular ferrite, bainite, and martensite steel microstructures, *Mater. Sci. Tech.* 16 (2000) 26–40.
- [35] A. Lambert-Perlade, A. Gourgues, J. Besson, T. Sturel, A. Pineau, Mechanisms and modeling of cleavage fracture in simulated heat-affected zone microstructures of a high-strength low alloy steel, *Metall. Mater. Trans. A* 35 (2004) 1039–53.
- [36] M. Gell, E. Smith, Propagation of cracks through grain boundaries in polycrystalline 3 percent silicon-iron, *Acta Metall.* 15 (1967) 253–58.
- [37] Y. Qiao, A. Argon, Cleavage crack-growth-resistance of grain boundaries in polycrystalline Fe-2%Si alloy: experiments and modeling, *Mech. Mater.* 35 (2003) 129–154.
- [38] Y. Qiao, A. Argon, Cleavage cracking resistance of high angle grain boundaries in Fe-3%Si alloy, *Mech. Mater.* 35 (2003) 313–331.
- [39] Y. Qiao, X. Kong, An energy analysis of the grain boundary behavior in cleavage cracking in Fe-3 wt % Si alloy, *Mater. Lett.* 58 (2004) 3156–3160.
- [40] A. Pineau, Crossing grain boundaries in metals by slip bands, cleavage and fatigue cracks, *Phil. Trans. R. Soc. A* 373 (2015) 20140131.
- [41] A. A. Griffith, The Phenomena of Rupture and Flow in Solids, *Phil. Trans. R. Soc. London A* 221 (1921) 163–198.
- [42] A. Martin-Meizoso, I. Ocana-Arizcorreta, J. Gil-Sevillano, M. Fuentes-Prez, Modeling cleavage fracture of bainitic steels, *Acta Metall. Mater.* 42 (1994) 2057–68.
- [43] M. Kroon, J. Faleskog, Micromechanics of cleavage fracture initiation in ferritic steels by carbide cracking, *J. Mech. Phys. Solids* 53 (2005) 171–96.
- [44] G. Hahn, B. Averbach, W. Owen, M. Cohen, Initiation of cleavage microcracks in polycrystalline iron and steel, in: *Proceedings International Conference on Atomic Mechanisms of Fracture*, Swamscott, 1959, pp. 91–116.
- [45] Y. Qiao, Modeling of resistance curve of high-angle grain boundary in Fe-3 wt. % Si alloy, *Mater. Sci. Eng.:A* 361 (2003) 350–57.

- [46] A. Andrieu, A. Pineau, D. Ryckelynck, B. O., Extension of beremin model to bi-modal brittle fracture, in: 20th French Congress on Mechanics, Besançon, 2011.
- [47] A. Andrieu, A. Pineau, P. Joly, F. Roch, D. Ryckelynck, Influence of P and C intergranular segregation during manufacturing and ageing on the fracture toughness of nuclear pressure vessel steels, *Proc. Mater. Sci.*, 20th European Conference on Fracture, Trondheim, Norway 3 (2014) 655–660.
- [48] A. Pineau, Modeling ductile to brittle fracture transition in steels — micromechanical and physical challenges, *Int. J. Frac.* 150 (2008) 129–156.
- [49] K. Wallin, T. Saario, K. Törrönen, Statistical-model for carbide induced brittle-fracture in steel, *Met. Sci.* 18 (1984) 13–16.
- [50] K. Wallin, Fracture toughness transition curve shape for ferritic structural steels, in: *Proceedings of the joint FEFG, ICF international conference on fracture of engineering materials and structures*, Elsevier, London, 1991, pp. 83–88.
- [51] K. Wallin, Statistical modelling of fracture in the ductile-to-brittle transition region, in: *Defect assessment in components-fundamentals and applications ESIS/ECF 9*, 1991. Mechanical Engineering Publications, London, 1991, pp. 415–445.
- [52] W. Weibull, A statistical theory of the strength of materials, in: *Proceedings of Roy Swedish Inst. Eng. Res.*, Vol. 151, 1939, pp. 1–53.
- [53] W. Weibull, A statistical distribution function of wide applicability, *J. Appl. Mech.* 18 (1951) 293–297.
- [54] F. Mudry, A local approach to cleavage fracture, *Nucl. Eng. Des.* 105 (1987) 65–76.
- [55] A. Pineau, Practical application of local approach methods, in: *Comprehensive Structural Integrity*, Elsevier, Amsterdam, Vol. 7, 2003, pp. 177–225.
- [56] A. Pineau, Development of the local approach to fracture over the past 25 years: theory and applications, *Int. J. Frac.* 138 (2006) 139–166.
- [57] H. E. Evans, *Mechanisms of Creep Fracture*, Elsevier Applied Science Publishers Ltd., 1984.
- [58] B. Tanguy, J. Besson, A. Pineau, Comment on “effect of carbide distribution on the fracture toughness in the transition temperature region of an sa 508 steel”, *Scr. Mater.* 49 (2003) 191–97.
- [59] A. Bakker, R. W. I. Koers, Prediction of cleavage fracture events in the brittle-ductile transition region of a ferritic steel, in: J. G. Blauel, K. H. Schwalbe (Eds.), *Defect Assessment in Components – Fundamentals and Applications*, ESIS, European Group on Fracture Publication, 1991, pp. 613–32.
- [60] L. Xia, L. Cheng, Transition from ductile tearing to cleavage fracture: A cell-model approach, *Int. J. Frac.* 87 (1997) 289–306.

- [61] X. Gao, J. Faleskog, C. Shih, Analysis of ductile to cleavage transition in part-through crack using a cell model incorporating statistics, *Fat. Frac. Eng. Mat. Struct.* 22 (1999) 239–250.
- [62] X. Gao, J. Faleskog, C. F. Shih, Cell model for nonlinear fracture analysis - II. Fracture-process calibration and verification, *Int. J. Frac.* 89 (1998) 375–398.
- [63] X. Gao, C. Ruggieri, R. H. Dodds, Calibration of weibull stress parameters using fracture toughness data, *Int. J. Frac.* 92 (1998) 175–200.
- [64] X. Gao, R. H. Dodds, Constraint effects on the ductile-to-brittle transition temperature of ferritic steels: A weibull stress model, *Int. J. Frac.* 102 (2000) 43–69.
- [65] R. O. Ritchie, J. F. Knott, J. R. Rice, On the relationship between critical tensile stress and fracture toughness in mild steel, *J. Mech. Phys. Solids* 21 (1973) 395–410.
- [66] F. Minami, M. Iida, W. Takahara, N. Konda, K. Arimochi, Fracture mechanics analysis of charpy test results based on the weibull stress criterion, in: *In:From Charpy to present impact testing 2002*, Elsevier/ESIS, London, 2002, pp. 411–18.
- [67] J. Petti, R. H. Dodds, Calibration of the weibull stress scale parameter, $\sigma(u)$, using the master curve, *Eng. Frac. Mech.* 72 (2005) 91–120.
- [68] J. Petti, R. H. Dodds, Ductile tearing and discrete void effects on cleavage fracture under small-scale yielding conditions, *Int. J. Solids Struct.* 42 (2005) 3655–3676.
- [69] L. Kaechele, A. Tetelman, A statistical investigation of microcrack formation, *Acta Metall.* 17 (1969) 463–475.
- [70] S. Bordet, A. Karstensen, D. Knowles, C. Wiesner, A new statistical local criterion for cleavage fracture in steel. part i: model presentation, *Eng. Frac. Mech.* 72 (2005) 435–52.
- [71] S. Bordet, A. Karstensen, D. Knowles, C. Wiesner, A new statistical local criterion for cleavage fracture in steel. part ii: application to an offshore structural steel, *Eng. Frac. Mech.* 72 (2005) 453–74.
- [72] S. R. Bordet, B. Tanguy, J. Besson, S. Bugat, D. Moinereau, A. Pineau, Cleavage fracture of rpv steel following warm pre-stressing: micromechanical analysis and interpretation through a new model, *Fat. Frac. Eng. Mat. Struct.* 29 (2006) 799–816.
- [73] G. Bernauer, W. Brocks, W. Schmitt, Modifications of the Beremin model for cleavage fracture in the transition region of a ferritic steel, *Eng. Frac. Mech.* 64 (1999) 305–25.
- [74] A. Andrieu, A. Pineau, J. Besson, D. Ryckelynck, O. Bouaziz, Bimodal Beremin-type model for brittle fracture of inhomogeneous ferritic steels: Theory and applications, *Eng. Frac. Mech.* 95 (2012) 84–101.

- [75] A. E1921-02, Standard Test Method for Determination of Reference Temperature, T_0 , for Ferritic Steels in the Transition Range, in: ASTM International, West Conshohocken, PA, 2002.
- [76] J. W. Hutchinson, Singular behaviour at the end of a tensile crack in a hardening material, *J. Mech. Phys. Solids* 16 (1968) 13–31.
- [77] J. R. Rice, G. Rosengren, Plane strain deformation near a crack tip in a power-law hardening material, *J. Mech. Phys. Solids* 16 (1968) 1–12.
- [78] Y. Lei, N. P. O'Dowd, E. P. Busso, G. A. Webster, Weibull stress solutions for 2-d cracks in elastic and elastic-plastic materials, *Int. J. Frac.* 89 (1998) 245–68.
- [79] N. P. O'Dowd, Y. Lei, E. P. Busso, Prediction of cleavage failure probabilities using the weibull stress, *Eng. Frac. Mech.* 67 (2000) 87–100.
- [80] A. Andrieu, A. Pineau, J. Besson, D. Ryckelynck, O. Bouaziz, Beremin model: Methodology and application to the prediction of the Euro toughness data set, *Eng. Frac. Mech.* 95 (2012) 102–17.
- [81] J. D. G. Sumpter, An experimental investigation of the t-stress approach, in: E. M. Hackett, K. H. Schwalbe, R. H. Dodds (Eds.), *Constraint Effects in Fracture*, Vol. 1171 of American Society for Testing and Materials Special Technical Publication, Amer Soc Testing & Mat, Comm Fracture Testing; European Struct Integr Soc, 1993, pp. 492–502, symp on Constraint Effects in Fracture, Indianapolis, IN, May 08-09, 1991.
- [82] M. Henry, B. Marandet, F. Mudry, A. Pineau, Effets de la température et de la vitesse de chargement sur la tenacité à rupture d'un acier faiblement allié. Interprétation par des critères locaux, *J. Méc. Théor. Appl.* 4 (1986) 741–768.
- [83] M. Ohata, F. Minami, M. Toyoda, Local approach to strength mis-match effect on cleavage fracture of notched material, *J. Phys. IV* 6 (1996) 269–278.
- [84] T. L. Briggs, J. Campbell, The effect of strain rate and temperature on the yield and flow of polycrystalline niobium and molybdenum, *Acta Metall. Mater.* 20 (1972) 711–24.
- [85] A. Y. Koval, A. D. Vasilev, S. A. Firstov, Fracture toughness of molybdenum sheet under brittle-ductile transition, *Inter. J. Ref. Met. Hard Mater.* 15 (1997) 223–226.
- [86] A. V. Samant, J. J. Lewandowski, Effects of test temperature, grain size, and alloy additions on the cleavage fracture stress of polycrystalline niobium, *Metall. Mater. Trans. A* 28 (1997) 389–99.
- [87] A. V. Samant, J. J. Lewandowski, Effects of test temperature, grain size, and alloy additions on the low-temperature fracture toughness of polycrystalline niobium, *Metall. Mater. Trans. A* 28 (1997) 2297–2307.

- [88] D. Padhi, J. J. Lewandowski, Resistance curve behavior of polycrystalline niobium failing via cleavage, *Mater. Sci. Eng.:A* 366 (2004) 56–65.
- [89] Z. Wu, W. Curtin, Brittle and ductile crack-tip behavior in magnesium, *Acta Mater.* 88 (2015) 1–12.
- [90] T. Kubo, Y. Wakashima, K. Amano, M. Nagai, Effects of crystallographic orientation on plastic-deformation and scc initiation of zirconium alloys, *J. Nuc. Mater.* 132 (1985) 1–9.
- [91] I. Schuster, C. Lemaignan, Characterization of zircaloy corrosion fatigue phenomena in an iodine environment part i: Crack growth, *J. Nuc. Mater.* 166 (1989) 348–56.
- [92] B. Cox, Environmentally-induced cracking of zirconium alloys - a review, *J. Nuc. Mater.* 170 (1990) 1–23.
- [93] G. M. Hughes, G. Smith, A. G. Crocker, P. E. J. Flewitt, An examination of the linkage of cleavage cracks at grain boundaries, *Mater. Sci. Tech.* 21(11) (2005) 1268 – 1274.
- [94] J. J. Gilman, Fracture of zinc monocrystals and bicrystals, *Trans. AIME* 212 (1958) 783–91.
- [95] A. Deruyttere, G. Greenough, The criterion for the cleavage fracture of zinc single crystals, *J. Inst. Met.* 84 (1956) 337.
- [96] R. Parisot, S. Forest, A. Pineau, F. Grillon, X. Demonet, J. Maigne, Deformation and damage mechanisms of zinc coatings on hot-dip galvanized steel sheets: Part 1. deformation modes, *Metall. Mater. Trans. A* 35A (2004) 797–811.
- [97] R. Parisot, S. Forest, A. Pineau, F. Grillon, X. Demonet, J. Maigne, Deformation and damage mechanisms of zinc coatings on hot-dip galvanized steel sheets: Part 2. damage modes, *Metall. Mater. Trans. A* 35A (2004) 813–823.
- [98] F. Lemant, A. Pineau, Mixed-mode fracture of a brittle orthotropic material - example of strongly textured zinc sheets, *Eng. Frac. Mech.* 14 (1981) 91–105.
- [99] M. P. Seah, Adsorption-induced interface decohesion, *Acta Mater.* 28 (1980) 955–962.
- [100] M. Guttman, P. Dumoulin, M. Wayman, The thermodynamics of interactive co-segregation of phosphorus and alloying elements in iron and temper-brittle steels, *Metall. Mater. Trans. A* 13 (1982) 1693–1711.
- [101] P. Gas, M. Guttman, J. Bernardini, The interactive co-segregation of sb and ni at the grain-boundaries of ultrahigh purity fe-base alloys, *Acta Mater.* 30 (1982) 1309–1316.
- [102] H. Erhart, H. J. Grabke, Equilibrium segregation of phosphorus at grain boundaries in Fe-Cr-P, and Fe-Cr-C-P alloys, *Met. Sci.* 15 (1981) 401–408.
- [103] M. Militzer, J. Wieting, Interfacial two-dimensional phase transitions and impurity segregation, *Acta Metall. Mater.* 35 (1987) 2765–77.

- [104] J. Kameda, C. J. McMahon, Solute segregation and brittle-fracture in an alloy-steel, *Metall. Mater. Trans. A* 11 (1980) 91–101.
- [105] C. Naudin, J. M. Frund, A. Pineau, Intergranular fracture stress and phosphorus grain boundary segregation of a Mn-Ni-Mo steel, *Scr. Mater.* 40 (1999) 1013–1019.
- [106] S. J. Wu, J. F. Knott, On the statistical analysis of local fracture stresses in notched bars, *J. Mech. Phys. Solids* 52 (2004) 907–24.
- [107] E. Kantidis, B. Marini, A. Pineau, A criterion for intergranular brittle-fracture of a low-alloy steel, *Fat. Frac. Eng. Mat. Struct.* 17 (1994) 619–33.
- [108] S. Raoul, B. Marini, A. Pineau, Intergranular brittle fracture of a weakly alloyed steel induced by segregation of grain boundary impurities, *J. Phys. IV* 9 (1999) 179–184.
- [109] O. Yahya, F. Borit, R. Piques, A. Pineau, Statistical modelling of intergranular brittle fracture in a low alloy steel, *Fat. Frac. Eng. Mat. Struct.* 21 (1998) 1485–1502.
- [110] C. A. Hippsley, S. G. Druce, The influence of phosphorus segregation to particle matrix interfaces on ductile fracture in a high-strength steel, *Acta Mater.* 31 (1983) 1861–72.
- [111] ASTM E.1921-02-2002 Standard test method for determination of reference temperature, T_0 , for ferritic steels in the transition range, in: *Annual Book of ASTM Standards*, 2002.
- [112] A. A. Benzerga, J. Besson, A. Pineau, Anisotropic ductile fracture. Part I: experiments, *Acta Mater.* 52 (2004) 4623–4638.
- [113] K. Anand, W. A. Spitzig, Initiation of localized shear bands in plane strain, *J. Mech. Phys. Solids* 28 (1980) 113–128.
- [114] U. S. Lindholm, A. Nagy, G. R. Johnson, J. M. Hoegfeldt, Large strain, high strain rate testing of copper., *J. Eng. Mat. Tech.* 102 (1980) 376–381.
- [115] B. Dodd, A. Atkins, Flow localization in shear deformation of void-containing and void-free solids, *Acta Metall.* 31 (1983) 9–15.
- [116] H. Kudo, K. Aoi, Effect of compression test condition upon fracturing of a medium carbon steel – Study on cold forgeability test, *J. Jap. Soc. Tech. Plast.* 8 (1967) 17–27.
- [117] B. Kondori, Ductile fracture of magnesium alloys: characterization and modeling, Ph.D. thesis, Texas A&M University, USA (2015).
- [118] T. Pardoen, F. Hachez, B. Marchioni, H. Blyth, A. G. Atkins, Mode I fracture of sheet metal, *J. Mech. Phys. Solids* 52 (2004) 423–452.
- [119] K. L. Nielsen, J. Dahl, V. Tvergaard, Collapse and coalescence of spherical voids subject to intense shearing: studied in full 3D, *Int. J. Frac.* 177 (2012) 97–108.

- [120] B. Simonsen, R. Törnqvist, Experimental and numerical modelling of ductile crack propagation in large-scale shell structures, *Marine Struct.* 17 (2004) 1–27.
- [121] S. A. El-Naaman, K. L. Nielsen, Observations on mode i ductile tearing in sheet metals, *Eur. J. Mech. A* 42 (2013) 54–62.
- [122] F. Rivalin, J. Besson, A. Pineau, M. Di Fant, Ductile tearing of pipeline-steel wide plates. i. dynamic and quasi-static experiments, *Eng. Frac. Mech.* 68 (2001) 329–345.
- [123] G. R. Johnson, J. M. Hoegfeldt, U. S. Lindholm, A. Nagy, Response of various metals to large torsional strains over a large range of strain rates – Part 1: Ductile metals, *J. Eng. Mat. Tech.* 105 (1983) 42–47.
- [124] P. W. Lee, H. A. Kuhn, Fracture in Cold Upset Forging – A Criterion and Model, *Met. Trans.* 4 (1973) 969–974.
- [125] J. W. Hutchinson, Fundamentals of the phenomenological theory of nonlinear fracture mechanics, *J. Appl. Mech.* 50 (1983) 1042–1051.
- [126] C. Shih, M. German, Requirements for a one parameter characterization of crack tip fields by the hrr singularity, *Int. J. Frac.* 17.
- [127] B. Cotterell, J. K. Reddel, The essential work of plane stress ductile fracture, *Int. J. Frac.* 13 (1977) 267–77.
- [128] Y. W. Mai, P. Powell, Essential work of fracture and j-integral measurements for ductile polymers, *J. Polymer Sci. B* 29 (1991) 785–93.
- [129] T. Pardoen, Y. Marchal, F. Delannay, Essential work of fracture versus fracture mechanics - towards a thickness independent plane stress toughness, *Eng. Frac. Mech.* 69 (2002) 617–31.
- [130] T. Pardoen, Y. Marchal, F. Delannay, Thickness dependence of cracking initiation criteria in thin aluminum plates, *J. Mech. Phys. Solids* 47 (1999) 2093–2123.
- [131] W. Lode, The influence of the intermediate principal stress on yielding and failure of iron, copper and nickel, *Eng. Frac. Mech.* 5 (1925) 142–144.
- [132] J. Rice, E. P. Sorensen, Continuing crack-tip deformation and fracture for plane-strain crack growth in elastic plastic solids, *J. Mech. Phys. Solids* 26 (1978) 163–186.
- [133] L. Xia, C. F. Shih, Ductile crack growth: I. A numerical study using computational cells with microstructurally based length scales, *J. Mech. Phys. Solids* 43 (1995) 233–259.
- [134] L. Xia, C. F. Shih, Ductile crack growth: Iii. transition to cleavage fracture incorporating statistics, *J. Mech. Phys. Solids* 44 (1996) 603–39.

- [135] B. Tanguy, J. Besson, R. Piques, A. Pineau, Ductile to brittle transition of an a508 steel characterized by charpy impact test. part ii: Modeling of the charpy transition curve, *Eng. Frac. Mech.* 72 (2005) 413–434.
- [136] J. W. Hancock, A. C. MacKenzie, On the mechanisms of ductile failure in high-strength steels subjected to multi-axial stress states, *J. Mech. Phys. Solids* 24 (1976) 147–169.
- [137] F. M. Beremin, Elasto-plastic calculations of circumferentially notched specimens using the finite element method, *J. Mécanique Appliquée* 4 (1980) 307.
- [138] G. R. Johnson, W. H. Cook, Fracture characteristics of three metals subjected to various strains, strain rates, temperatures and pressures, *Eng. Frac. Mech.* 21 (1985) 31–48.
- [139] Y. Bao, T. Wierzbicki, On fracture locus in the equivalent strain and stress triaxiality space, *Int. J. Mech. Sci.* 46 (2004) 81–98.
- [140] I. Barsoum, J. Faleskog, Rupture mechanisms in combined tension and shear-Experiments, *Int. J. Solids Struct.* 44 (2007) 1768–1786.
- [141] B. Kondori, A. A. Benzerga, Effect of Stress Triaxiality on the Flow and Fracture of Mg Alloy AZ31, *Metall. Mater. Trans. A* 45 (2014) 3292–3307.
- [142] P. A. F. Martins, N. Bay, A. E. Tekkaya, A. G. Atkins, Characterization of fracture loci in metal forming, *Int. J. Mech. Sci.* 83 (2014) 112–123.
- [143] F. Mudry, F. di Rienzo, A. Pineau, Numerical Comparison of Global and Local Fracture Criteria in Compact Tension and center-crack panel specimens, *Non linear Fracture Mechanics, Volume II*, ASTM STP 995, American Society for Testing and Materials, Philadelphia, 1989.
- [144] W. Brocks, W. Schmitt, The second parameter in j-r curve : Constraint or triaxiality, In: *Proceedings of the Second Symposium on Constraint Effects*, ASTM STP 1244, American Society for testing and materials, Philadelphia, 1994, pp. 209–231.
- [145] T. L. Anderson, *Fracture Mechanics - Fundamentals and Applications*, CRC Press, Boca Raton, 1995.
- [146] N. P. O’Dowd, C. F. Shih, Family of crack tip fields characterized by a triaxiality parameter - I. Structure of fields, *J. Mech. Phys. Solids* 39 (1991) 898–915.
- [147] N. P. O’Dowd, C. F. Shih, Family of crack tip fields characterized by a triaxiality parameter II. Fracture applications, *J. Mech. Phys. Solids* 40 (1991) 939–963.
- [148] G. Johnson, J. Hoegfeldt, U. Lindholm, A. Nagy, Response of various metals to large torsional strains over a large range of strain rates – Part 2: Less ductile metals, *J. Eng. Mat. Tech.* 105 (1983) 48–53.
- [149] S. S. Haltom, S. Kyriakides, K. Ravi-Chandar, Ductile failure under combined shear and tension, *Int. J. Solids Struct.* 50 (2013) 1507–1522.

- [150] D. Grady, The spall strength of condensed matter, *J. Mech. Phys. Solids* 36 (1988) 353–384.
- [151] M. A. Meyers, C. T. Aimone, Dynamic fracture (spalling) of metals, *Prog. Mater. Sci.* 28 (1983) 1–96.
- [152] R. J. Clifton, Response of materials under dynamic loading, *Int. J. Solids Struct.* 37 (2000) 105–113.
- [153] J. Field, S. Walley, W. Proud, H. Goldrein, C. Siviour, Review of experimental techniques for high rate deformation and shock studies, *Int. J. Impact Eng.* 30 (2004) 725–775.
- [154] H. Zhang, K. Ravi-Chandar, Dynamic fragmentation of ductile materials, *J. Phys. D: Appl. Phys.* 42 (2009) Art. No 214010.
- [155] N. A. Pedrazas, D. L. Worthington, D. A. Dalton, P. A. Sherek, S. P. Steuck, H. J. Quevedo, A. C. Bernstein, E. M. Taleff, T. Ditmire, Effects of microstructure and composition on spall fracture in aluminum, *Mater. Sci. Eng.:A* 536 (2012) 117–123.
- [156] T. Borvik, M. Langseth, O. S. Hopperstad, K. A. Malo, Ballistic penetration of steel plates, *Int. J. Impact Eng.* 22 (2002) 855–885.
- [157] M. Dolinski, D. Rittel, Experiments and modeling of ballistic penetration using an energy failure criterion, *J. Mech. Phys. Solids* 83 (2015) 1–18.
- [158] H. Zhang, K. Ravi-Chandar, On the dynamics of necking and fragmentation – I. Real-time and post-mortem observations in Al6061-O, *Int. J. Frac.* 142 (2006) 183–217.
- [159] R. L. Woodward, The interrelation of failure modes observed in the penetration of metallic targets, *Int. J. Impact Eng.* 2 (1984) 121–129.
- [160] T. Borvik, O. S. Hopperstad, M. Langseth, K. A. Malo, Perforation of 12mm thick steel plates by 20mm diameter projectiles with flat, hemispherical and conical noses Part I: Experimental study, *Int. J. Impact Eng.* 27 (2002) 19–35.
- [161] A. K. Rodriguez, G. A. Ayoub, B. Mansoor, A. A. Benzerga, Effect of strain rate and temperature on fracture of AZ31B magnesium alloy. Submitted.
- [162] D. Lassance, D. Fabrègue, F. Delannay, T. Pardoen, Micromechanics of room and high temperature fracture in 6xxx Al alloys, *Prog. Mater. Sci.* 52 (2007) 62–129.
- [163] J. Harding, Effect of temperature and strain rate on strength and ductility of four alloy steels., *Met. Technol.* 4 (1977) 6–16.
- [164] V. K. Sikka, Elevated temperature ductility of types 304 and 316 stainless steel, Tech. rep., Oak Ridge National Laboratory, Tennessee (1978).

- [165] E. Amar, A. Pineau, Interpretation of ductile fracture toughness temperature dependence of a low strength steel in terms of a local approach, *Eng. Frac. Mech.* 22 (1985) 1061–1071.
- [166] C. Berdin, H. Wang, Local approach to ductile fracture and dynamic strain aging, *Int. J. Frac.* 182 (2013) 39–51.
- [167] G. R. Irwin, A. A. Wells, Variation of fracture toughness with loading rate in a semi-killed steel, *Met. Rev.* 10 (1965) 223–270.
- [168] C. S. Wiesner, H. MacGillivray, Loading rate effects on tensile properties and fracture toughness of steel, in: P. Hirsch, D. Lidbury (Eds.), *Fracture, Plastic Flow and Structural Integrity*, IOM Communications Ltd, The Institute of Materials, 2000, pp. 149–173.
- [169] D. F. Hasson, J. A. Joyce, Effect of a higher loading rate on the J_{Ic} fracture toughness transition temperature of HY steels., *J. Eng. Mat. Tech.* 103 (1981) 133–141.
- [170] K. Decamp, L. Bauvineau, J. Besson, A. Pineau, Size and geometry effects on ductile rupture of notched bars in a C–Mn steel: Experiments and modelling, *Int. J. Frac.* 88 (1997) 1–18.
- [171] L. Devillers-Guerville, J. Besson, A. Pineau, Notch fracture toughness of a cast duplex stainless steel: modelling of experimental scatter and size effect, *Nucl. Eng. Des.* 168 (1997) 211–225.
- [172] C. J. Young, D. A. Koss, R. K. Everett, Specimen size effects and ductile fracture of HY-100 steel, *Metall. Mater. Trans. A* 33 (2002) 3293–3295.
- [173] B. S. Loudon, A. S. Kumar, F. A. Garner, M. L. Hamilton, W. L. Hu, The influence of specimen size on Charpy impact testing of unirradiated HT-9, *J. Nuc. Mater.* 155–157 (1988) 662–667.
- [174] A. A. Benzerga, V. Tvergaard, A. Needleman, Size effects in the Charpy V-notch test, *Int. J. Frac.* 116 (2002) 275–296.
- [175] J. Plateau, G. Henry, C. Crussard, Quelques nouvelles applications de la microfractographie, *Revue de Métallurgie* 54 (1957) 200–216.
- [176] R. H. Van Stone, T. B. Cox, J. R. Low, J. A. Psioda, Microstructural aspects of fracture by dimpled rupture, *Int. Metals Rev.* 30 (4) (1985) 157–179.
- [177] W. M. Garrison Jr., A. L. Wojcieszynski, L. E. Iorio, Effects of inclusion distributions on the fracture toughness of structural steels, in: R. K. Mahidhara, A. B. Geltmacher, P. Matic, K. Sadananda (Eds.), *Recent advances in Fracture*, TMS, 1997, pp. 125–136.
- [178] N. R. AbdelAl, An Experimental Study of Deformation and Fracture of a Nanostructured Metallic Material, M.S. thesis, Texas A&M University (2009).

- [179] C. F. Tipper, The fracture of metals, *Metallurgia* 39 (1949) 133–137.
- [180] B. Marini, F. Mudry, A. Pineau, Experimental study of cavity growth in ductile rupture, *Eng. Frac. Mech.* 22 (1985) 989–996.
- [181] L. Babout, Y. Bréchet, E. Maire, R. Fougères, On the competition between particle fracture and particle decohesion in metal matrix composites, *Acta Mater.* 52 (2004) 4517–4525.
- [182] A. S. Argon, J. Im, Separation of Second Phase Particles in Spheroidized 1045 Steel, Cu-0.6pct Cr Alloy, and Maraging Steel in Plastic Straining, *Met. Trans.* 6A (1975) 839–851.
- [183] G. Le Roy, J. D. Embury, G. Edward, M. F. Ashby, A model of ductile fracture based on the nucleation and growth of voids, *Acta Metall.* 29 (1981) 1509–1522.
- [184] M. F. Ashby, Work hardening of dispersion-hardened crystals, *Phil. Mag.* 14 (1966) 1157–1178.
- [185] P. Joly, R. Cozar, A. Pineau, Effect of crystallographic orientation of austenite on the formation of cleavage cracks in ferrite in an aged duplex stainless steel, *Scripta metall. mater.* 24 (1990) 2235–2240.
- [186] G. Lacroix, T. Pardoen, P. Jacques, The fracture toughness of trip-assisted multiphase steels, *Acta Mater.* 56 (2008) 3900–3913.
- [187] C. Tasan, M. Diehl, D. Yan, M. Bechtold, F. Roters, L. Schemmann, C. Zheng, N. Peranio, D. Ponge, M. Koyama, K. Tsuzaki, D. Raabe, An Overview of Dual-Phase Steels: Advances in Microstructure-Oriented Processing and Micromechanically Guided Design, *Annual Rev. Mater. Res.* 45 (2015) 391–431.
- [188] Q. Lai, O. Bouaziz, M. Gouné, L. Brassart, G. Verdier, M. Parry, A. Perlade, Y. Bréchet, T. Pardoen, Damage and fracture of dual-phase steels: influence of martensite volume fraction, *Mater. Sci. Eng.:A* 646 (2016) 322–331.
- [189] M. R. Barnett, Twinning and the ductility of magnesium alloys Part II. "contraction" twins, *Mater. Sci. Eng.:A* 464 (2007) 8–16.
- [190] B. Kondori, A. A. Benzerga, Fracture strains, damage mechanisms and anisotropy in a magnesium alloy across a range of stress triaxialities, *Exp. Mech.* 54 (2014) 493–499.
- [191] M. Achouri, G. Germain, P. Dal Santo, D. Saidane, Experimental characterization and numerical modeling of micromechanical damage under different stress states, *Materials & Design* 50 (2013) 207–222.
- [192] J. R. Rice, D. M. Tracey, On the enlargement of voids in triaxial stress fields, *J. Mech. Phys. Solids* 17 (1969) 201–217.
- [193] R. Becker, A. Needleman, O. Richmond, V. Tvergaard, Void growth and failure in notched bars, *J. Mech. Phys. Solids* 36 (1988) 317–351.

- [194] V. Tvergaard, On localization in ductile materials containing spherical voids, *Int. J. Frac.* 18 (1982) 237–252.
- [195] A. Weck, D. S. Wilkinson, E. Maire, H. Toda, Visualization by X-ray tomography of void growth and coalescence leading to fracture in model materials, *Acta Mater.* 56 (2008) 2919–2928.
- [196] E. Maire, P. Withers, Quantitative X-ray tomography, *Inter. Mater. Rev.* 59 (2014) 1–43.
- [197] T. Pardoen, F. Delannay, Assessment of Void Growth Models from Porosity Measurements in Cold-Drawn Copper Bars, *Met. Trans. A* 29A (1998) 1895–1909.
- [198] D. Chae, D. Koss, Damage accumulation and failure of hsla-100 steel, *Mater. Sci. Eng.:A* 366 (2) (2004) 299–309.
- [199] Y. X. Gan, J. W. Kysar, T. L. Morse, Cylindrical void in a rigid-ideally plastic single crystal II: Experiments and simulations, *Int. J. Plast.* 22 (2006) 39–72.
- [200] Crépin, J. and Bretheau, T. and Caldemaison, D., Cavity growth and rupture of β -treated zirconium: A crystallographic model, *Acta Mater.* 44 (1996) 4927–4935.
- [201] D. C. Ahn, P. Sofronis, R. Minich, On the micromechanics of void growth by prismatic-dislocation loop emission, *J. Mech. Phys. Solids* 54 (2006) 735–755.
- [202] T. Pardoen, J. W. Hutchinson, An extended model for void growth and coalescence, *J. Mech. Phys. Solids* 48 (2000) 2467–2512.
- [203] F. Scheyvaerts, P. R. Onck, C. Tekoglu, T. Pardoen, The growth and coalescence of ellipsoidal voids in plane strain under combined shear and tension, *J. Mech. Phys. Solids* 59 (2011) 373–397.
- [204] F. A. McClintock, Ductile fracture by hole growth in shear bands, *Int. J. Frac. Mech.* 2 (4) (1966) 614–627.
- [205] N. A. Fleck, J. W. Hutchinson, Void growth in shear, *Proc. Roy. Soc. London A* 407 (1986) 435–458.
- [206] V. Tvergaard, Effect of stress-state and spacing on voids in a shear-field, *Int. J. Solids Struct.* 49 (2012) 3047–3054.
- [207] B. J. Lee, M. E. Mear, Stress concentration induced by an elastic spheroidal particle in a plastically deforming solid, *J. Mech. Phys. Solids* 47 (1999) 1301–1336.
- [208] K. Siruguet, J.-B. Leblond, Effect of void locking by inclusions upon the plastic behavior of porous ductile solids—I: theoretical modeling and numerical study of void growth, *Int. J. Plast.* 20 (2004) 225–254.
- [209] M. Ashby, Mechanisms of Deformation and Fracture, *Adv. Appl. Mech.* 23 (1983) 117–177.

- [210] J. Henry, , J. Proc. Ass. Adv. Sci 9 (1855) 102.
- [211] A. H. Cottrell, Theoretical aspects of fracture, in: B. L. Averbach (Ed.), Fracture, Chapman and Hall, London, 1959, pp. 20–53.
- [212] P. F. Thomason, A theory for ductile fracture by internal necking of cavities, J. Inst. Metals 96 (1968) 360.
- [213] L. M. Brown, J. D. Embury, The initiation and growth of voids at second phase particles, in: Proceedings of the 3rd International Conference on the Strength of Metals and Alloys, Institute of Metals, London, 1973, pp. 164–169.
- [214] A. S. Argon, J. Im, R. Safoglu, Cavity Formation from Inclusions in Ductile Fracture, Met. Trans. A 6A (1975) 825–837.
- [215] T. B. Cox, J. R. Low, An Investigation of the Plastic Fracture of AISI 4340 and 18 Nickel–200 Grade Maraging Steels, Met. Trans. 5 (1974) 1457–1470.
- [216] A. A. Benzerga, Rupture ductile des tôles anisotropes, Ph.D. thesis, Ecole Nationale Supérieure des Mines de Paris (2000).
- [217] T. Pardoen, Ductile Fracture of Cold-Drawn Copper Bars: Experimental Investigation and Micromechanical Modeling, Ph.D. thesis, Université Catholique de Louvain (1998).
- [218] L. Babout, E. Maire, R. Fougères, Damage initiation in model metallic materials: X-ray tomography and modelling, Acta Mater. 52 (2004) 2475–2487.
- [219] Y. Shen, T. F. Morgeneyer, J. Garnier, L. Allais, L. Helfen, J. Crepin, Three-dimensional quantitative in situ study of crack initiation and propagation in AA6061 aluminum alloy sheets via synchrotron laminography and finite-element simulations, Acta Mater. 61 (2013) 2571–2582.
- [220] B. Kondori, T. Morgeneyer, A. A. Benzerga, High-resolution tomography analysis of failure with limited void growth: application to Mg-Al-Zn alloy In preparation.
- [221] A. A. Benzerga, J. Besson, R. Batisse, A. Pineau, Synergistic effects of plastic anisotropy and void coalescence on fracture mode in plane strain, Modell. Simul. Mater. Sci. Eng. 10 (2002) 73–102.
- [222] G. I. Taylor, Plastic strain in metals, J. Inst. Metals 62 (1938) 307–324.
- [223] A. A. Benzerga, Y. Bréchet, A. Needleman, E. Van der Giessen, The stored energy of cold work: predictions from discrete dislocation plasticity, Acta Mater. 53 (2005) 4765–4779.
- [224] B. A. Miller, ASM Handbook Volume 11, Failure Analysis and Prevention, ASM International, 2002, Ch. Ductile Overload Failures, pp. 671–699.
- [225] V. Tvergaard, J. W. Hutchinson, Two mechanisms of ductile fracture : void by void growth versus multiple void interaction, Int. J. Solids Struct. 39 (2002) 3581–3597.

- [226] J. R. Rice, M. A. Johnson, Inelastic behavior of solids, Mc Graw Hill, 1970, Ch. The role of large crack tip geometry changes in plane strain fracture, pp. 641–672.
- [227] L. G. Luo, A. Ryks, J. D. Embury, On the development of a metallographic method to determine the strain distribution ahead of a crack tip, *Metallography* 23 (2) (1989) 101–117.
- [228] V. Tvergaard, J. W. Hutchinson, The relation between crack growth resistance and fracture process parameters in elastic-plastic solids, *J. Mech. Phys. Solids* 40 (1992) 1377–1397.
- [229] J. Clayton, J. Knott, Observations of fibrous fracture modes in a prestrained low-alloy steel, *Met. Sci.* 10 (1976) 63–71.
- [230] A. Needleman, V. Tvergaard, An analysis of ductile rupture modes at a crack tip, *J. Mech. Phys. Solids* 35 (1987) 151–183.
- [231] L. Xia, C. Shih, J. W. Hutchinson, A computational approach to ductile crack growth under large scale yielding conditions, *J. Mech. Phys. Solids* 43 (1995) 389–413.
- [232] W. M. Garrison Jr., A. L. Wojcieszynski, A discussion of the effect of inclusion volume fraction on the toughness of steel, *Mater. Sci. Eng.:A* 464 (2007) 321–329.
- [233] J.-C. Lautridou, A. Pineau, Crack initiation and stable crack growth resistance in A508 steels in relation to inclusion distribution, *Eng. Frac. Mech.* 15 (1981) 55–71.
- [234] P. P. Darcis, C. N. McCowan, H. Windhoff, J. D. McColskey, T. A. Siewert, Crack tip opening angle optical measurement methods in five pipeline steels, *Eng. Frac. Mech.* 75 (8) (2008) 2453–2468.
- [235] D. Curran, L. Seaman, D. Shockey, Dynamic failure of solids, *Physics Reports* 147 (1987) 253–388.
- [236] D. Rittel, Z. Wang, M. Merzer, Adiabatic shear failure and dynamic stored energy of cold work, *Phys. Rev. Lett.* 96 (2006) Art. No. 075502.
- [237] W.-S. Lee, C.-Y. Liu, The effects of temperature and strain rate on the dynamic flow behaviour of different steels, *Mater. Sci. Eng.:A* 426 (2006) 101–113.
- [238] M. F. Horstemeyer, M. M. Matalanis, A. M. Sieber, M. L. Botos, Micromechanical finite element calculations of temperature and void configuration effects on void growth and coalescence, *Int. J. Plast.* 16 (2000) 979–1013.
- [239] L. Kubin, Y. Estrin, The Portevin-Le Chatelier effect in deformation with constant stress rate, *Acta Metall.* 33 (1985) 397–407.
- [240] P. McCormick, Theory of flow localisation due to dynamic strain ageing, *Acta Metall.* 36 (1988) 3061–3067.

- [241] A. Needleman, Void Growth in an Elastic–Plastic Medium, *J. Appl. Mech.* 39 (1972) 964–970.
- [242] V. Tvergaard, Influence of voids on shear band instabilities under plane strain conditions, *Int. J. Frac.* 17 (1981) 389–407.
- [243] J. Koplik, A. Needleman, Void growth and coalescence in porous plastic solids, *Int. J. Solids Struct.* 24 (1988) 835–853.
- [244] S. K. Yerra, C. Tekoglu, F. Scheyvaerts, L. Delannay, P. Van Houtte, T. Pardoen, Void growth and coalescence in single crystals, *Int. J. Solids Struct.* 47 (2010) 1016–1029.
- [245] A. Srivastava, A. Needleman, Void growth versus void collapse in a creeping single crystal, *J. Mech. Phys. Solids* 61 (2013) 1169–1184.
- [246] A. A. Benzerga, J. Besson, Plastic potentials for anisotropic porous solids, *Eur. J. Mech. A* 20A (2001) 397–434.
- [247] S. M. Keralavarma, S. Hoelscher, A. A. Benzerga, Void growth and coalescence in anisotropic plastic solids, *Int. J. Solids Struct.* 48 (2011) 1696–1710.
- [248] W. Brocks, D. Z. Sun, A. Hönl, Verification of the transferability of micromechanical parameters by cell model calculations with visco–plastic materials, *Int. J. Plast.* 11 (8) (1995) 971–989.
- [249] D. Fabrègue, T. Pardoen, A constitutive model for elastoplastic solids containing primary and secondary voids, *J. Mech. Phys. Solids* 56 (2008) 719–41.
- [250] D. Lassance, F. Scheyvaerts, T. Pardoen, Growth and coalescence of penny-shaped voids in metallic alloys, *Eng. Frac. Mech.* 73 (2006) 1009–1034.
- [251] C. L. Hom, R. M. McMeeking, Void Growth in Elastic–Plastic Materials, *J. Appl. Mech.* 56 (1989) 309–317.
- [252] M. J. Worswick, R. J. Pick, Void growth and constitutive softening in a periodically voided solid, *J. Mech. Phys. Solids* 38 (5) (1990) 601–625.
- [253] A. B. Richelsen, V. Tvergaard, Dilatant plasticity or upper bound estimates for porous ductile solids, *Acta Metall. Mater.* 42 (8) (1994) 2561–2577.
- [254] X. Gao, J. Kim, Modeling of ductile fracture: Significance of void coalescence, *Int. J. Solids Struct.* 43 (2006) 6277–6293.
- [255] I. Barsoum, J. Faleskog, Rupture mechanisms in combined tension and shear–Micromechanics, *Int. J. Solids Struct.* 44 (2007) 5481–5498.
- [256] M. Dunand, D. Mohr, Effect of Lode parameter on plastic flow localization after proportional loading at low stress triaxialities, *J. Mech. Phys. Solids* 66 (2014) 133–153.

- [257] W. Wong, T. Guo, On the energetics of tensile and shear void coalescences, *J. Mech. Phys. Solids* 82 (2015) 259–286.
- [258] C. Tekoglu, J. W. Hutchinson, T. Pardoen, On localization and void coalescence as a precursor to ductile fracture, *Phil. Trans. R. Soc. A* 373 (2015) 20140121.
- [259] M. Dighe, A. Gokhale, M. Horstemeyer, Effect of loading condition and stress state on damage evolution of silicon particles in an al-si-mg-base cast alloy, *Metall. Mater. Trans. A* 33.
- [260] G. Huber, Y. Bréchet, T. Pardoen, Void growth and void nucleation controlled ductility in quasi eutectic cast aluminium alloys, *Acta Mater.* 53 (2005) 2739–2749.
- [261] T. Pardoen, F. Scheyvaerts, A. Simar, C. Tekoglu, P. Onck, Multiscale modeling of ductile failure in metallic alloys, *C. R. Phys.* 11 (2010) 326–45.
- [262] A. Simar, Y. Brechet, B. de Meester, A. Denquin, C. Gallais, T. Pardoen, Integrated modeling of friction stir welding of 6xxx series Al alloys: Process, microstructure and properties, *Prog. Mater. Sci.* 57 (2012) 95–183.
- [263] S. Bugat, J. Besson, A. Pineau, Micromechanical modeling of the behavior of duplex stainless steels, *Compt. Mater. Sci.* 16 (1-4) (1999) 158–166.
- [264] H. Toda, H. Oogo, K. Horikawa, K. Uesugi, A. Takeuchi, Y. Suzuki, M. Nakazawa, Y. Aoki, M. Kobayashi, The true origin of ductile fracture in aluminum alloys, *Metall. Mater. Trans. A* 45A (2014) 765–776.
- [265] E. Maire, D. S. Wilkinson, P. Poruks, H. Henein, J. D. Embury, Damage mechanisms of model metal matrix composites with heterogeneous spatial distribution of the reinforcement, in: K. A. S. (Ed.), *Proceedings of the 6th international symposium on plasticity and its current applications: physics and mechanics of finite plastic and viscoplastic deformation*, Neat Press, Fulton, Maryland, 1999, pp. 207–208.
- [266] E. Roux, M. Shakoar, M. Bernacki, P. Bouchard, A new finite element approach for modelling ductile damage void nucleation and growth – analysis of loading path effect on damage mechanisms, *Modell. Simul. Mater. Sci. Eng.* 22 (7) (2014) 075001.
- [267] A. Melander, U. Støahler, The effect of void size and distribution on ductile fracture, *Int. J. Frac.* 16 (1980) 431–440.
- [268] A. Needleman, A. S. Kushner, An analysis of void distribution effects on plastic flow in porous solids, *Eur. J. Mech. A* 9A (1990) 193.
- [269] Y. Huang, The role of nonuniform particle distribution in plastic flow localization, *Mech. Mater.* 16 (1993) 265–79.
- [270] C. Thomson, M. Worswick, A. Pilkey, D. Lloyd, Void coalescence within periodic clusters of particles, *J. Mech. Phys. Solids* 51 (2003) 127–46.

- [271] P. Achon, Comportement et ténacité d'alliages d'aluminium à haute résistance, Ph.D. thesis, Ecole des Mines de Paris (1994).
- [272] V. Tvergaard, Interaction of very small voids with larger voids, *Int. J. Solids Struct.* 35 (1998) 3989–4000.
- [273] Z. Li, W. Guo, The influence of plasticity mismatch on the growth and coalescence of spheroidal voids on the bi-material interface, *Int. J. Plast.* 18 (2002) 249–79.
- [274] S. K. Yerra, G. Martin, M. Veron, Y. Brechet, J. D. Mithieux, L. Delannay, T. Pardoen, Ductile fracture initiated by interface nucleation in two-phase elastoplastic systems, *Eng. Frac. Mech.* 102 (2013) 77–100.
- [275] L. Lecarme, E. Maire, A. K. K. C., C. D. Vleeschouwer, L. Jacques, A. Simar, T. Pardoen, Heterogenous void growth revealed by in situ 3-d x-ray microtomography using automatic cavity tracking, *Acta Mater.* 63 (2014) 130–139.
- [276] F. Hannard, T. Pardoen, E. Maire, C. Le Bourlot, R. Mokso, A. Simar, Characterization and micromechanical modelling of microstructural heterogeneity effects on ductile fracture of 6xxx aluminium alloys, *Acta Mater.* 103 (2016) 558–572.
- [277] M. Worswick, Z. Chen, A. Pilkey, D. Lloyd, S. Court, Damage characterization and damage percolation modelling in aluminum alloy sheet, *Acta Mater.* 49 (2001) 2791–803.
- [278] J. Rice, The localization of plastic deformation, in: W. Koiter (Ed.), 14th int. cong. Theoretical and Applied Mechanics, North-Holland, Amsterdam, 1976, pp. 207–220.
- [279] L. Anand, K. Kim, T. Shawki, Onset of shear localization in viscoplastic solids, *J. Mech. Phys. Solids* 35 (1987) 407–429.
- [280] G. Rousselier, Stabilité locale et modes de rupture ductile, *C. R. Acad. Sci. Paris* 320 (Série IIB) (1995) 69–75.
- [281] G. R. Irwin, J. A. Kies, H. L. Smith, Fracture strengths relative to onset and arrest of crack propagation, *ASTM Transactions, American Society for testing and materials, Philadelphia* 58 (1958) 640–660.
- [282] M. A. James, J. C. Newman Jr., The effect of crack tunneling on crack growth: experiments and CTOA analyses, *Eng. Frac. Mech.* 70 (2003) 457–468.
- [283] O. Chabanet, D. Steglich, J. Besson, V. Heitman, D. Hellmann, W. Brocks, Predicting crack growth resistance of aluminium sheets, *Compt. Mater. Sci.* 26 (2003) 1–12.
- [284] A. Asserin-Lebert, J. Besson, A. F. Gourgues, Fracture of 6056 aluminum sheet materials : Effect of specimen thickness and hardening behavior on strain localization and toughness, *Mater. Sci. Eng.:A* 395 (2005) 186–194.

- [285] T. F. Morgeneyer, T. Taillandier-Thomas, L. Helfen, T. Baumbach, I. Sinclair, S. Roux, F. Hild, In situ 3-D observation of early strain localization during failure of thin Al alloy (2198) sheet, *Acta Mater.* 69 (2014) 78–91.
- [286] M. A. Sutton, D. S. Dawicke, J. C. Newman Jr., Orientation effects on the measurement and analysis of critical CTOA in an Aluminum alloy sheet, in: W. G. Reuter, J. H. Underwood, J. C. Newman Jr. (Eds.), *Fracture Mechanics: 26th Volume*, ASTM STP 1256, American Society for testing and materials, Philadelphia, 1995, pp. 243–255.
- [287] F. Bron, J. Besson, A. Pineau, Ductile rupture in thin sheets of two grades of 2024 aluminum alloy, *Mater. Sci. Eng.:A* 380 (2004) 356–364.
- [288] A. Needleman, A continuum model for void nucleation by inclusion debonding, *Journal of Applied Mechanics* 54 (1987) 525.
- [289] M. Li, S. Ghosh, O. Richmond, An experimental-computational approach to the investigation of damage evolution in discontinuously reinforced aluminum matrix composite, *Acta Mater.* 47 (1999) 3515–3532.
- [290] S. Hao, W. K. Liu, B. Moran, F. Vernerey, G. B. Olson, Multi-scale constitutive model and computational framework for the design of ultra-high strength, high toughness steels, *Comput. Methods Appl. Mech. Eng.* 193 (2004) 1865–1908.
- [291] C. Hu, S. Ghosh, Locally enhanced voronoi cell finite element model (LE-VCFEM) for simulating evolving fracture in ductile microstructures containing inclusions, *Int. J. Numer. Meths. Eng.* 76 (2008) 1955–1992.
- [292] F. J. Vernerey, W. K. Liu, B. Moran, , G. B. Olson, A micromorphic model for the multiple scale failure of heterogeneous materials, *J. Mech. Phys. Solids* 56 (2008) 1320–1347.
- [293] R. Tian, S. Chan, S. Tang, A. M. Kopacz, J.-S. Wang, H.-J. Jou, L. Siad, L.-E. Lindgren, G. B. Olson, W. K. Liu, A multiresolution continuum simulation of the ductile fracture process, *J. Mech. Phys. Solids* 58 (2010) 1681–1700.
- [294] S. Ghosh, J. Bai, D. Paquet, Homogenization-based continuum plasticity-damage model for ductile failure of materials containing heterogeneities, *J. Mech. Phys. Solids* 57 (2009) 1017–1044.
- [295] S. Ghosh, D. Paquet, Adaptive concurrent multi-level model for multi-scale analysis of ductile fracture in heterogeneous aluminum alloys, *Mech. Mater.* 65 (2013) 12–34.
- [296] V. Tvergaard, Behaviour of voids in a shear field, *Int. J. Frac.* 158 (2009) 41–49.
- [297] A. Srivastava, S. Gopagoni, A. Needleman, V. Seetharaman, A. Staroselsky, R. Banerjee, Effect of specimen thickness on the creep response of a ni-based single-crystal superalloy, *Acta Mater.* 60 (16) (2012) 5697–5711.

- [298] X. Gao, J. Faleskog, C. F. Shih, Cell model for nonlinear fracture analysis - II. Fracture-process calibration and verification, *Int. J. Frac.* 89 (1998) 375–398.
- [299] J. Gurland, J. Plateau, The mechanism of ductile rupture of metals containing inclusions, *Trans. Quartely of ASM* 56 (1963) 442–454.
- [300] A. S. Argon, Formation of Cavities from Nondeformable Second-Phase Particles in Low Temperature Ductile Fracture, *J. Eng. Mat. Tech.* 18 (1976) 60–68.
- [301] B. Wilner, Stress analysis of particles in metals, *J. Mech. Phys. Solids* 36 (2) (1988) 141–165.
- [302] J. Eshelby, The determination of the elastic field of an ellipsoidal inclusion, and related problems, *Proc. Roy. Soc A* 241 (1957) 357–396.
- [303] M. F. Ashby, The deformation of plastically non-homogeneous materials, *Phil. Mag.* 21 (1970) 399–424.
- [304] C. Tekoglu, T. Pardoen, A micromechanics based damage model for composite materials, *Int. J. Plast.* 26 (2010) 549–69.
- [305] K. Tanaka, T. Mori, T. Nakamura, Cavity formation at the interface of a spherical inclusion in a plastically deforming matrix, *Philosophical Magazine* 21 (1970) 267–279.
- [306] F. Montheillet, P. Gilormini, Amorçage de l'endommagement, in: F. Montheillet, F. Moussy (Eds.), *Physique et mécanique de l'endommagement*, Les éditions de physique, Les Ulis, 1986, pp. 122–181.
- [307] C. Chu, A. Needleman, Void nucleation effects in biaxially stretched sheets, *J. Eng. Mat. Tech.* 102 (1980) 249–256.
- [308] M. F. Horstemeyer, A. M. Gokhale, A void-crack nucleation model for ductile metals, *Int. J. Solids Struct.* 36 (1999) 5029–5055.
- [309] D. L. McDowell, Viscoplasticity of heterogeneous metallic materials, *Mater. Sci. Eng.:R* 62 (2008) 67–123.
- [310] H. J. Frost, M. F. Ashby, *Deformation-Mechanism Maps: The Plasticity and Creep of Metals and Ceramics*, Pergamon Press, 1982.
- [311] K. Danas, P. Ponte Castañeda, A finite-strain model for anisotropic viscoplastic porous media: I–Theory, *Eur. J. Mech. A* 28 (2009) 387–401.
- [312] K. Madou, J.-B. Leblond, L. Morin, Numerical studies of porous ductile materials containing arbitrary ellipsoidal voids —II: Evolution of the length and orientation of the void axes, *Eur. J. Mech. A* 42 (2013) 490–507.
- [313] A. A. Benzerga, Micromechanics of Coalescence in Ductile Fracture, *J. Mech. Phys. Solids* 50 (2002) 1331–1362.

- [314] F. A. McClintock, A criterion for ductile fracture by the growth of holes, *J. Appl. Mech.* 35 (1968) 363–371.
- [315] Y. Huang, Accurate Dilatation Rates for Spherical Voids in Triaxial Stress Fields, *J. Appl. Mech.* 58 (1991) 1084–1085.
- [316] G. Perrin, J.-B. Leblond, Analytical study of a hollow sphere made of plastic porous material and subjected to hydrostatic tension— application to some problems in ductile fracture of metals, *Int. J. Plasticity* 6 (6) (1990) 677–699.
- [317] S. Basu, E. Dogan, B. Kondori, I. Karaman, A. A. Benzerga, Towards Designing Anisotropy for Ductility Enhancement: A Theory-Driven Investigation in Mg-alloys, *Acta Mater.* In preparation.
- [318] P. Ponte Castañeda, M. Zaidman, Constitutive models for porous materials with evolving microstructure, *J. Mech. Phys. Solids* 42 (1994) 1459–1495.
- [319] M. Kailasam, P. Ponte Castaneda, A general constitutive theory for linear and non-linear particulate media with microstructure evolution, *J. Mech. Phys. Solids* 46 (3) (1998) 427–465.
- [320] K. Danas, P. Ponte Castañeda, A finite-strain model for anisotropic viscoplastic porous media: II—Applications, *Eur. J. Mech. A* 28 (2009) 402–416.
- [321] M. Gologanu, J.-B. Leblond, J. Devaux, Approximate models for ductile metals containing non-spherical voids – case of axisymmetric prolate ellipsoidal cavities, *J. Mech. Phys. Solids* 41 (11) (1993) 1723–1754.
- [322] M. Gologanu, J.-B. Leblond, J. Devaux, Approximate Models for Ductile Metals Containing Non-spherical Voids — Case of Axisymmetric Oblate Ellipsoidal Cavities, *J. Eng. Mat. Tech.* 116 (1994) 290–297.
- [323] M. Gologanu, J.-B. Leblond, G. Perrin, J. Devaux, Recent extensions of Gurson’s model for porous ductile metals, in: P. Suquet (Ed.), *Continuum Micromechanics*, CISM Lectures Series, Springer, New York, 1997, pp. 61–130.
- [324] K. Madou, J.-B. Leblond, A Gurson-type criterion for porous ductile solids containing arbitrary ellipsoidal voids – I: Limit-analysis of some representative cell, *J. Mech. Phys. Solids* 60 (2012) 1020–1036.
- [325] K. Madou, J.-B. Leblond, A Gurson-type criterion for porous ductile solids containing arbitrary ellipsoidal voids – II: Determination of yield criterion parameters, *J. Mech. Phys. Solids* 60 (2012) 1037–1058.
- [326] K. Madou, J.-B. Leblond, Numerical studies of porous ductile materials containing arbitrary ellipsoidal voids — I: Yield surfaces of representative cells, *Eur. J. Mech. A* 42 (2013) 480–489.

- [327] D. M. Tracey, Strain hardening and interaction effects on the growth of voids in ductile fracture, *Eng. Frac. Mech.* 3 (1971) 301–315.
- [328] L. Morin, Influence des effets de forme et de taille des cavités, et de l’anisotropie plastique sur la rupture ductile, Ph.D. thesis, Université Pierre et Marie Curie – Paris VI (2015).
- [329] A. A. Benzerga, J. Besson, A. Pineau, Coalescence–Controlled Anisotropic Ductile Fracture, *J. Eng. Mat. Tech.* 121 (1999) 221–229.
- [330] T. Pardoen, Numerical simulation of low stress triaxiality ductile fracture, *Comput. Struct.* 84 (2006) 1641–1650.
- [331] A. A. Benzerga, J. Besson, A. Pineau, Anisotropic ductile fracture. Part II: theory, *Acta Mater.* 52 (2004) 4639–4650.
- [332] S. M. Keralavarma, A Micromechanics based Ductile Damage Model for Anisotropic Titanium Alloys, Master’s thesis, Texas A&M University, USA (2008).
- [333] S. M. Keralavarma, A. A. Benzerga, An approximate yield criterion for anisotropic porous media, *C. R. Mecanique* 336 (2008) 685–692.
- [334] V. Monchiet, O. Cazacu, E. Charkaluk, D. Kondo, Macroscopic yield criteria for plastic anisotropic materials containing spheroidal voids, *Int. J. Plast.* 24 (2008) 1158–1189.
- [335] S. M. Keralavarma, A. A. Benzerga, A constitutive model for plastically anisotropic solids with non-spherical voids, *J. Mech. Phys. Solids* 58 (2010) 874–901.
- [336] S. M. Keralavarma, A. A. Benzerga, Numerical assessment of an anisotropic porous metal plasticity model, *Mech. Mater.* 90 (2015) 212–228.
- [337] S. Kweon, Investigation of shear damage considering the evolution of anisotropy, *J. Mech. Phys. Solids* 61 (2013) 2605–2624.
- [338] Y. F. Dafalias, Corotational rates for kinematic hardening at large plastic deformation, *J. Appl. Mech.* 50 (1983) 561–565.
- [339] H. Zbib, E. Aifantis, On the concept of relative and plastic spins and its implications to large deformation theories. Part I: Hypoelasticity and vertex-type plasticity, *Acta Mech.* 75 (1988) 15–33.
- [340] L. Morin, J.-B. Leblond, D. Kondo, A gurson-type criterion for plastically anisotropic solids containing arbitrary ellipsoidal voids, *Int. J. Solids Struct.* Accepted.
- [341] J. W. Kysar, Energy dissipation mechanisms in ductile fracture, *J. Mech. Phys. Solids* 51 (2003) 795–824.
- [342] J. W. Kysar, Y. X. Gan, G. Mendez-Arzuza, Cylindrical void in a rigid-ideally plastic single crystal. Part I: Anisotropic slip line theory solution for face-centered cubic crystals, *Int. J. Plast.* 21 (2005) 1481–1520.

- [343] J. R. Rice, Tensile crack tip fields in elastic-ideally plastic crystals, *Mech. Mater.* 6 (1987) 317–335.
- [344] U. Borg, C. Niordson, J. W. Kysar, Size effects on void growth in single crystals with distributed voids, *Int. J. Plast.* 24 (2008) 688–701.
- [345] J. Segurado, J. Llorca, An analysis of the size effect on void growth in single crystals using discrete dislocation dynamics, *Acta Mater.* 57 (2009) 1427–1436.
- [346] S. M. Keralavarma, A Contribution to the Modeling of Metal Plasticity and Fracture: From Continuum to Discrete Descriptions, Ph.D. thesis, Texas A&M University, USA (2011).
- [347] A. C. F. Cocks, M. F. Ashby, On creep fracture by void growth, *Prog. Mater. Sci.* 27 (1982) 189–244.
- [348] J. M. Duva, J. W. Hutchinson, Constitutive potentials for dilutely voided nonlinear materials, *Mech. Mater.* 3 (1984) 41–54.
- [349] J.-B. Leblond, G. Perrin, P. Suquet, Exact results and approximate models for porous viscoplastic solids, *Int. J. Plast.* 10 (1994) 213–225.
- [350] H. Klöcker, F. Montheillet, Velocity, strain rate and stress fields around a spheroidal cavity in a linearly viscous material, *Eur. J. Mech., A* 15 (3) (1996) 397–422.
- [351] Beremin, F. M. and Pineau, A. and Mudry, F. and Devaux, J.C. and DEscatha, Y. and Ledermann, P., Experimental and numerical study of the different stages in ductile rupture: application to crack initiation and stable crack growth, in: Nemat-Nasser, S. (Ed.), *Three-Dimensional Constitutive relations of Damage and Fracture*, Pergamon press, North Holland, 1981, pp. 157–172.
- [352] A. Needleman, V. Tvergaard, An analysis of ductile rupture in notched bars, *J. Mech. Phys. Solids* 32 (1984) 461–490.
- [353] J. W. Hutchinson, A. G. Evans, *Mechanics of materials: Top-down approaches to fracture*, *Acta Mater.* 48 (2000) 125–135.
- [354] P. F. Thomason, Three-dimensional models for the plastic limit-loads at incipient failure of the intervoid matrix in ductile porous solids, *Acta Metall.* 33 (1985) 1079–1085.
- [355] A. A. Benzerga, *Handbook of Damage Mechanics*, Springer, New York, 2015, Ch. Micromechanical models of ductile damage and fracture, pp. 939–962.
- [356] A. A. Benzerga, J.-B. Leblond, Effective Yield Criterion Accounting for Microvoid Coalescence, *J. Appl. Mech.* 81 (2014) 031009.
- [357] L. Morin, J.-B. Leblond, A. A. Benzerga, Coalescence of voids by internal necking: theoretical estimates and numerical results, *J. Mech. Phys. Solids* 75 (2015) 140–158.

- [358] A. Molinari, N. Jacques, S. Mercier, , J.-B. Leblond, A. A. Benzerga, A micromechanical model for the dynamic behaviour of porous media in the void coalescence stage, *Int. J. Solids Struct.* 71 (2015) 1–18.
- [359] M. E. Torki, A. A. Benzerga, J.-B. Leblond, On Void Coalescence under Combined Tension and Shear, *J. Appl. Mech.* 82 (7) (2015) 071005.
- [360] P. F. Thomason, A three-dimensional model for ductile fracture by the growth and coalescence of microvoids, *Acta Metall.* 33 (6) (1985) 1087–1095.
- [361] F. Scheyvaerts, P. R. Onck, T. Pardoen, A new model for void coalescence by internal necking, *Int. J. Damage Mech.* 19 (2010) 95–126.
- [362] L. Xue, Constitutive modeling of void shearing effect in ductile fracture of porous materials, *Eng. Frac. Mech.* 75 (2008) 3343–3366.
- [363] C. Butcher, Z. T. Chen, A void coalescence model for combined tension and shear, *Modell. Simul. Mater. Sci. Eng.* 17 (2009) Art. No. 025007.
- [364] J.-B. Leblond, G. Mottet, A theoretical approach of strain localization within thin planar bands in porous ductile materials, *C. R. Mecanique* 336 (2008) 176–189.
- [365] M. Gologanu, J.-B. Leblond, G. Perrin, J. Devaux, Theoretical models for void coalescence in porous ductile solids – I: Coalescence in “layers”, *Int. J. Solids Struct.* 38 (2001) 5581–5594.
- [366] M. Gologanu, J.-B. Leblond, G. Perrin, J. Devaux, Theoretical models for void coalescence in porous ductile solids – II: Coalescence in “columns”, *Int. J. Solids Struct.* 38 (2001) 5595–5604.
- [367] C. Tekoglu, J.-B. Leblond, T. Pardoen, A criterion for the onset of void coalescence under combined tension and shear, *J. Mech. Phys. Solids* 60 (2012) 1363–1381.
- [368] V. Tvergaard, A. Needleman, An analysis of the brittle–ductile transition in dynamic crack growth, *Int. J. Frac.* 59 (1993) 53–67.
- [369] C. Reina, J. Marian, M. Ortiz, Nanovoid nucleation by vacancy aggregation and vacancy-cluster coarsening in high-purity metallic single crystals, *Phys. Rev. B* 84 (2011) 104117.
- [370] J. Wilkerson, K. Ramesh, A closed-form criterion for dislocation emission in nanoporous materials under arbitrary thermomechanical loading, *J. Mech. Phys. Solids* 86 (2016) 94–116.
- [371] V. Lubarda, M. Schneider, D. Kalantar, B. Remington, M. Meyers, Void growth by dislocation emission, *Acta Mater.* 52 (2004) 1397–1408.
- [372] A. Molinari, S. Mercier, Micromechanical modelling of porous materials under dynamic loading, *J. Mech. Phys. Solids* 49 (2001) 1497–1516.

- [373] N. Jacques, S. Mercier, A. Molinari, Effects of microscale inertia on dynamic ductile crack growth, *J. Mech. Phys. Solids* 60 (2012) 665–690.
- [374] N. Thomas, S. Basu, A. A. Benzerga, On fracture loci of ductile materials under nonproportional loading, *Int. J. Mech. Sci.*. In preparation.
- [375] D. Mohr, S. Marcadet, Micromechanically-motivated phenomenological hosford-coulomb model for predicting ductile fracture initiation at low stress triaxialities, *Int. J. Solids Struct.* 67-68 (2015) 40–55.
- [376] A. Hosokawa, D. S. Wilkinson, J. Kang, E. Maire, Onset of void coalescence in uniaxial tension studied by continuous X-ray tomography, *Acta Mater.* 61 (2013) 1021–1036.
- [377] T.-S. Cao, C. Bobadilla, P. Montmitonnet, P.-O. Bouchard, A comparative study of three ductile damage approaches for fracture prediction in cold forming processes, *J. Mat. Proc. Tech.* 216 (2015) 385–404.
- [378] A. A. Benzerga, D. Surovik, S. M. Keralavarma, On the path-dependence of the fracture locus in ductile materials – Analysis, *Int. J. Plast.* 37 (2012) 157–170.
- [379] T. Pardoen, J. W. Hutchinson, Micromechanics-based model for trends in toughness of ductile metals, *Acta Mater.* 51 (2003) 133–148.
- [380] H. Klöcker, V. Tvergaard, Growth and coalescence of non-spherical voids in metals deformed at elevated temperature, *Int. J. Mech. Sci.* 45 (2003) 1283–1308.
- [381] B. Tanguy, T. Luu, G. Perrin, A. Pineau, J. Besson, Plastic and damage behaviour of a high strength X100 pipeline steel: Experiments and modelling, *International Journal of Pressure Vessels and Piping* 85 (2008) 322–335.
- [382] P. Burlot, Effect du confinement plastique sur la stabilité mécanique des défauts dans les gazoducs, Ph.D. thesis, Mines ParisTech (2015).
- [383] T. Pardoen and D. Dumont and A. Deschamps and Y. Bréchet, Grain boundary versus transgranular ductile failure, *J. Mech. Phys. Solids* 51 (2003) 637 – 665.
- [384] T. Pardoen, I. Doghri, F. Delannay, Experimental and numerical comparison of void growth models and void coalescence criteria for the prediction of ductile fracture in copper bars, *Acta mater.* 46 (2) (1998) 541–552.
- [385] B. Kondori, A. A. Benzerga, Modeling damage accumulation to fracture in a magnesium-rare earth alloy, *Acta Mater.* In preparation.
- [386] L. Morin, J.-B. Leblond, V. Tvergaard, Application of a model of plastic porous materials including void shape effects to the prediction of ductile failure under shear-dominated loadings, *J. Mech. Phys. Solids* Submitted.
- [387] D. A. DeSandre, A. A. Benzerga, V. Tvergaard, A. Needleman, Material inertia and size effects in the Charpy V-notch test, *Eur. J. Mech. A* 23A (2004) 373–386.

- [388] L. Xia, C. F. Shih, Ductile crack growth - II. Void nucleation and geometry effects on macroscopic fracture behavior, *J. Mech. Phys. Solids* 43 (1995) 1953–1981.
- [389] F. Mudry, In elastic-plastic fracture mechanics, Fourth ISPRA Conference, Joint Research Center, Ispra, 1983, p. 263303.
- [390] J. C. Devaux, F. Mudry, A. Pineau, G. Rousselier, Experimental and numerical validation of a ductile fracture local criterion based on a simulation of cavity growth, *Fracture Mechanics Fracture*, ASTM STP 995, American Society for testing and materials, Philadelphia, 1989.
- [391] F. Rivalin, J. Besson, A. Pineau, M. Di Fant, Ductile tearing of pipeline-steel wide plates. ii. modeling of in-plane crack propagation, *Eng. Frac. Mech.* 68 (2001) 347–364.
- [392] D’Escatha, Y. and Devaux, J. C., ASTM STP 668, American Society for Testing and Materials, 1979, Ch. Numerical study of initiation, stable crack growth and maximum load with a ductile fracture criterion based on the growth of holes, pp. 229–248.
- [393] N. Aravas, R. M. McMeeking, Finite element analysis of void growth near a blunting crack tip., *J. Mech. Phys. Solids* 33 (1985) 25–37.
- [394] C. Ruggieri, T. L. Panontin, R. H. Dodds, Numerical modeling of ductile crack growth in 3-D using computational cell elements, *Int. J. Frac.* 82 (1996) 67–95.
- [395] J. Kim, X. S. Gao, T. S. Srivatsan, Modeling of crack growth in ductile solids: a three-dimensional study, *Int. J. Solids Struct.* 40 (2003) 7357–7374.
- [396] X. S. Gao, T. H. Wang, J. Kim, On ductile fracture initiation toughness: Effects of void volume fraction, void shape and void distribution, *Int. J. Solids Struct.* 42 (2005) 5097–5117.
- [397] V. Tvergaard, Discrete modeling of ductile crack growth by void growth to coalescence, *Int. J. Frac.* 148 (2007) 1–12.
- [398] G. Huetter, L. Zybelle, U. Muehlich, M. Kuna, Consistent simulation of ductile crack propagation with discrete 3D voids, *Comput. Mater. Sci.* 80 (2013) 61–70.
- [399] G. Rousselier, J. C. Devaux, G. G. Mottet, G. Devesa, A methodology for ductile fracture analysis based on damage mechanics : an illustration of a local approach of fracture. (1989) 332–354.
- [400] Z. L. Zhang, C. Thaulow, J. Odegard, A complete Gurson model approach for ductile fracture, *Eng. Frac. Mech.* 67 (2000) 155–168.
- [401] Z. Xue, M. Pontin, F. Zok, J. Hutchinson, Calibration procedures for a computational model of ductile fracture, *Eng. Frac. Mech.* 77 (2010) 492–509.
- [402] J. Besson, C. N. McCowan, E. S. Drexler, Modeling flat to slant fracture transition using the computational cell methodology, *Eng. Frac. Mech.* 104 (2013) 80–95.

- [403] J.-B. Leblond, G. Perrin, J. Devaux, Bifurcation Effects in Ductile Metals With Non-local Damage, *J. Appl. Mech.* 61 (1994) 236–242.
- [404] V. Tvergaard, A. Needleman, Effects of nonlocal damage in porous plastic solids, *Int. J. Solids Struct.* 32 (8/9) (1995) 1063–1077.
- [405] F. Reusch, B. Svendsen, D. Klingbeil, Local and non-local gurson-based ductile damage and failure modelling at large deformation, *Eur. J. Mech. A* 22 (2003) 770–792.
- [406] J. Mediavilla, R. H. J. Peerlings, M. G. D. Geers, Discrete crack modelling of ductile fracture driven by non-local softening plasticity, *Int. J. Numer. Meths. Eng.* 66 (2006) 661–688.
- [407] R. Bargellini, J. Besson, E. Lorentz, S. Michel-Ponnelle, A non-local finite element based on volumetric strain gradient: Application to ductile fracture, *Comput. Mater. Sci.* 45 (2009) 762–767.
- [408] K. Enakoutsa, J. B. Leblond, G. Perrin, Numerical implementation and assessment of a phenomenological nonlocal model of ductile rupture, *Comput. Methods Appl. Mech. Eng.* 196 (2007) 1946–1957.
- [409] T. Siegmund, W. Brocks, A numerical study on the correlation between the work of separation and the dissipation rate in ductile fracture, *Eng. Frac. Mech.* 67 (2000) 139–154.
- [410] Y. A. Roy, R. H. Dodds, Simulation of ductile crack growth in thin aluminium panels using 3-d surface cohesive elements, *Int. J. Frac.* 110 (2001) 21–45.
- [411] C. R. Chen, O. Kolednik, J. Heerens, F. Fischer, Three-dimensional modelling of ductile crack growth: Cohesive zone parameters and crack tip triaxiality, *Eng. Frac. Mech.* 72 (2005) 2072–2094.
- [412] P. B. Woelke, M. D. Shields, J. W. Hutchinson, Cohesive zone modeling and calibration for mode i tearing of large ductile plates, *Eng. Frac. Mech.* In press.
- [413] A. Huespe, A. Needleman, J. Oliver, P. Snchez, A finite strain, finite band method for modeling ductile fracture, *Int. J. Plast.* 28 (2012) 53–69.
- [414] A. Needleman, V. Tvergaard, A numerical study of void distribution effects on dynamic, ductile crack growth, *Eng. Frac. Mech.* 38 (1991) 157–173.
- [415] S. Tang, A. M. Kopacz, S. Chan O’Keeffe, G. B. Olson, W.-K. Liu, Three-dimensional ductile fracture analysis with a hybrid multiresolution approach and microtomography, *J. Mech. Phys. Solids* 61 (2013) 2108–2124.
- [416] A. Srivastava, L. Ponson, S. Osovski, E. Bouchaud, V. Tvergaard, A. Needleman, Effect of inclusion density on ductile fracture toughness and roughness, *J. Mech. Phys. Solids* 63 (2014) 62–79.

- [417] S. D. Mesarovic, Dynamic strain aging and plastic instabilities, *J. Mech. Phys. Solids* 43 (1995) 671–700.
- [418] A. Benallal, T. Berstad, T. Borvik, O. Hopperstad, I. Koutiri, R. de Codes, An experimental and numerical investigation of the behaviour of AA5083 aluminium alloy in presence of the Portevin-Le Chatelier effect, *Int. J. Plast.* 24 (2008) 1916–1945.
- [419] J. Belotteau, C. Berdin, S. Forest, A. Parrot, C. Prioul, Mechanical behavior and crack tip plasticity of a strain aging sensitive steel, *Mater. Sci. Eng.:A* 526 (2009) 156–165.
- [420] J. C. Lautridou, Etude de la déchirure ductile d’aciers à faible résistance. influence de la teneur inclusionnaire., Ph.D. thesis, Ecole des Mines de Paris (1980).
- [421] L. Bauvineau, Approche locale de la rupture ductile : Application à un acier Carbone–Manganèse, Ph.D. thesis, Ecole des Mines de Paris (1996).
- [422] Y. Shinohara, Y. Madi, J. Besson, Anisotropic ductile failure of a high-strength line pipe steel, *Int. J. Frac.* To appear.
- [423] L. E. Iorio, W. M. Garrison Jr., Effects of gettering sulfur as CrS or MnS on void generation behavior in ultra-high strength steel, *Scr. Mater.* 46 (2002) 863–868.
- [424] A. Pineau, A. A. Benzerga, T. Pardoen, Overview paper: Failure of metals III. Fracture and fatigue of nanostructured metallic materials, *Acta Mater.* In press.
- [425] G. T. Hahn, The influence of microstructure on brittle fracture toughness, *Metall. Trans.* 15A (1984) 947–959.
- [426] Rodriguez-Ibabe, The role of microstructure in toughness behaviour of microalloyed steels, *Mat. Sci. Forum* 284–286 (1998) 51–62.
- [427] V. Tvergaard, A. Needleman, K. K. Lo, Flow localization in the plane strain tensile test, *J. Mech. Phys. Solids* 29 (1981) 115–142.
- [428] K. Anand, W. A. Spitzig, Shear band orientations in plane strain, *Acta Metall.* 30 (1982) 553–561.
- [429] J. W. Hutchinson, V. Tvergaard, Shear band formation in plane strain, *Int. J. Solids Struct.* 17 (1981) 451–470.
- [430] R. Becker, A. Needleman, Effect of yield surface curvature on necking and failure in porous solids, *J. Appl. Mech.* 53 (1986) 491–499.
- [431] J. W. Rudnicki, J. R. Rice, Conditions for the localization of deformation in pressure-sensitive dilatant materials, *J. Mech. Phys. Solids* 23 (1975) 371–394.
- [432] H. Yamamoto, Conditions for shear localization in the ductile fracture of void-containing materials, *Int. J. Frac.* 14 (1978) 347–365.

- [433] A. Needleman, J. R. Rice, Limits to ductility set by plastic flow localization, in: D. P. Koistinen (Ed.), *Mechanics of sheet metal forming*, Plenum, 1978, pp. 237–267.
- [434] V. Tvergaard, Effect of yield surface curvature and void nucleation on plastic flow localization, *J. Mech. Phys. Solids* 35 (1) (1987) 43–60.
- [435] J. Besson, D. Steglich, W. Brocks, Modeling of plane strain ductile rupture, *Int. J. Plast.* 19 (2003) 1517–1541.
- [436] P. Steinmann, C. Miehe, E. Stein, On the localization analysis of orthotropic Hill-type elastoplastic solids, *J. Mech. Phys. Solids* 42 (12) (1994) 1969–1994.
- [437] M. Kuroda, V. Tvergaard, Effects of texture on shear band formation in plane strain tension/compression and bending, *Int. J. Plast.* 23 (2007) 244–272.
- [438] A. Needleman, Dynamic Shear Band Development in Plane Strain, *J. Appl. Mech.* 56 (1989) 1–9.
- [439] A. Lambert-Perlade, Rupture par clivage de microstructures daciens bainitiques obtenues en conditions de soudage, Ph.D. thesis, Ecole de Mines de Paris, Paris (2001).
- [440] X. Gao, G. Zhang, C. Roe, A study on the effect of the stress state on ductile fracture, *Int. J. Damage Mech.* 19 (2010) 75–94.
- [441] A. Weck, D. S. Wilkinson, Experimental investigation of void coalescence in metallic sheets containing laser drilled holes, *Acta Mater.* 56 (2008) 1774–1784.
- [442] A. A. Benzerga, Unpublished research.

Graphical Abstract

Macroscopic Crack Propagation Direction