Transient interlaminar thermal stress in multi-layered thermoelectric materials

B. L. Wang, Y. J. Cui (*)

Graduate School at Shenzhen, Harbin Institute of Technology, Harbin 150001, P.R. China

Abstract: This paper studies the transient interlaminar stress in a multilayered thermoelectric material (TEM), which consists of a N-type and a P-type thermocouple, separated by an insulating layer. Analytical solution for one-dimensional temperature and the associated interlaminar stresses at the steady-state and the transient state are obtained. The influence of insulating layer's thickness and material properties on the peeling stress, which is the key reason for delamination, has also been investigated. Distribution of the temperature and interlaminar stress are presented graphically. The interlaminar stress at the free ends of the TEM shows significant stress concentration. A thinner insulating layer results in a smaller interlaminar stress. The interlaminar stress also reduces if the insulting layer has a smaller Young's modulus. The value of the transient interlaminar stress is found to be very different from that of the steady-state. Overall, the interlaminar stress level at the transient-state is higher than that at the steady-state.

Keywords: Thermoelectric material, Multilayered material, Interlaminar stress, Transient heat conduction

***Corresponding author**

Email: liuyeqingcyj@126.com

1. Introduction

Thermoelectric materials (TEMs) can mutually convert energy between heat and electricity. They can be used either for thermoelectric generators [1-3] and thermoelectric coolers [4-5]. On the other hand, multilayered materials are widely used in engineering due to their excellent properties [6-7]. Employing multilayered TEMs can improve their properties, such as strength, stiffness, thermostability and enhance the thermoelectric converting efficiency [8].

Extensive research of multilayered TEMs have been carried out. For example, Jin [7] investigated the thermal stress and deformations of a multilayered thin film thermoelectric (TE) structure with the theory of laminated composites. He also investigated the buckling behavior with the same method and module [8]. With the consideration of coupling of temperature field and electric potential field, Wang et al. [9] developed a three-dimensional numerical model of TE devices to figure out the performance of TE coolers. Huang and Duang [10] derived a linear dynamic model of the TE cooler consisted of the heat sink and the cooling-load heat exchanger with small-signal linearization method. Naji et al. [11] described the dynamic thermal behavior of thermoelectric generators and refrigerators under transient operating conditions. Generally, TEMs are brittle in nature and prone to damage by cracking. Therefore, understanding the fracture properties of multilayered TEMs is important for ensuring their reliable applications. Zhang and Wang [12] studied the fracture properties of a two-layer TEM with the complex variable method. In addition to theoretical research, mechanical behaviors of multilayered TEMs have been studied numerically using commercial finite element software. Merbati et al. [13] carried out a thermodynamics and thermal stress analysis of thermoelectric power generator. Gao et al. [14] presented the thermal stress distribution of a TEM. Huang et al. [15] investigated the two-dimensional thermal stresses induced in a four-layered thin film thermoelectric cooler. Wu et al. [16] performed a numerical analysis on the thermodynamics and thermal stress in a thermoelectric module.

Due to non-homogeneity in nature, nonhomogeneous or multilayered materials may result in obvious interlaminar thermal and residual stresses [17-26]. The stresses are very significant at the free ends of the multilayered materials [27-29]. Naturally, interlaminar thermal stress of multilayered TEMs is a research interest among materials and mechanical communities.

However, the research in this field is very limited. Yang et al. [30] and Huang et al. [31] presented a steady-state analytical solution of thermally induced axial stress and shear stress in a thin-film TEM cooler. Liu et al. [32] found that the steady-state shear stress exhibits singularity at the ends of the TEM thin film bonded to an elastic substrate. These studies have been focused on the steady-state problems and only considered the interlaminar shear stress.

While many engineering materials operate under the transient mechanical loads [33, 34] and thermal loads [35-37], understanding of the transient behavior of changing of materials is of great importance. Past research on TEMs has confirmed that the thermal stress at transient state is significantly higher than that at steady-state [38]. Consequently, it is very necessary to analyze the transient interlaminar stress for multilayered TEMs. This paper studies the transient interlaminar shear stress as well as peeling stress in multilayered TEMs. An analytical-form temperature fields for a multilayered TEMs are derived by using the energy balance method and the technique of separation-of-variables. Based on the obtained temperatures and the concept of the interface compliance [39], the associated interlaminar stresses are formulated by considering mechanical boundary and the displacement conditions at the interface. Some typical numerical examples of the transient temperature and interlaminar stress are given.

2. The temperature field

Considered is a three-layer thermoelectric material which is free of any mechanical loads as shown in Fig.1. The structure configuration consists a N-type thermoelectric layer and a P-type thermoelectric layer, sandwiched by an insulating layer. It is assumed that all layers are perfectly bonded together. The thickness of the multilayered thermoelectric material is considerably smaller than its length. Furthermore, the two thermoelectric layers have the same material properties and thickness. The initial temperature of the entire system is T_0 and the temperature at $x=L$ end is suddenly changed to T_h . This means that the temperature and electric potential at the cold and hot end are prescribed as (T_0, V_0) and (T_h, V_h) , respectively. Thus, the thermal boundary conditions for the multilayered TEM of Fig. 1 are specified at both ends. These are $T=T_0$ at $x=-L$ and $T=T_h$ at $x=L$. The electric boundary conditions for the

multilayered TEM of Fig. 1 are also specified at both ends. These are $V=V_0$ at $x=-L$ and $V=V_h$ at $x=L$.

Since the thickness of each layer is very small comparing to the length of the TEM, the electric current and heat flux are both considered as one-dimensional quantities, which are continuous function of coordinate x . The basic governing equations for one-dimensional TEMs include [7, 40-42], constitutive equations:

$$J = -\sigma \frac{\partial V}{\partial x} - \sigma s \frac{\partial T}{\partial x} \quad (1)$$

$$q = -\sigma s T \frac{\partial V}{\partial x} - (k + \sigma s^2 T) \frac{\partial T}{\partial x} \quad (2)$$

and equilibrium equations

$$-\frac{\partial J}{\partial x} = 0 \quad (3)$$

$$-\frac{\partial q}{\partial x} - J \frac{\partial V}{\partial x} = \rho c \frac{\partial T}{\partial t} \quad (4)$$

where T and V are, respectively, temperature and electric potential, q and J are, respectively, the heat flux and electric current density, ρ , c , k , s and σ , denote, respectively, mass density, specific heat, thermal conductivity, seebeck coefficient and electrical conductivity, t is time.

The two thermoelectric layers and the insulating layer have different material properties. The subscripts N , P and I will be used to denote the N-type thermoelectric layer, the P-type thermoelectric layer and the insulating layer, respectively. The heat transfer by radiation and convection at the structure surfaces is ignored. The governing equation of transient temperature of the multilayered TEM can be derived by considering energy balance (details are given in Appendix A). This gives

$$k_{eq} \frac{\partial^2 T}{\partial x^2} + \frac{J^2}{\sigma} = (\rho c)_{eq} \frac{\partial T}{\partial t} \quad (5)$$

where k_{eq} is the effective thermal conductivity defined by $k_{eq} = k_N + k_I H / (2h)$, $(\rho c)_{eq} = \rho_N c_N + \rho_I c_I H / (2h)$. Eq. (5) clearly show that the temperature field is governed by a nonlinear equation, which is a quadratic function of the applied thermal flux.

The solution of Eq. (5) can be expressed as the sum of the steady part and the transient part [38, 40]: $T(x, t) = W(x) + U(x, t)$, where $W(x)$ and $U(x, t)$ are, respectively, the steady part and the transient part of the temperature field satisfying:

$$k_{eq} \frac{d^2 W}{dx^2} + \frac{J^2}{\sigma} = 0 \quad (6)$$

under the boundary conditions $W(-L) = T_0$ and $W(L) = T_h$, and

$$k_{eq} \frac{\partial^2 U}{\partial x^2} = (\rho c)_{eq} \frac{\partial U}{\partial t} \quad (7)$$

under the initial condition $U(x, 0) = T_0 - W(x)$ and the boundary conditions $U(-L, t) = 0$ and $U(L, t) = 0$. From Eq. (3) we see that the electric current in one-dimensional case is a constant. Thus, the solution of Eq. (6) for the steady part of the temperature field is obtained as

$$W(x) - T_0 = \frac{J^2}{2k_{eq}\sigma} (L^2 - x^2) + \frac{\Delta T}{2} \left(\frac{x}{L} + 1 \right) \quad (8)$$

where ΔT is the temperature difference between the hot and cold ends of the multilayered TEM, namely, $\Delta T = T_h - T_0$. With the technique of separation of variable used in Ref. [38, 40], the solution of $U(x, t)$ of Eq. (7) can be obtained in a straightforward way (details are given in Appendix B). The result is

$$U(x, t) = \sum_{n=1}^{\infty} D_n \cos(\beta_n x) \exp \left(\frac{-k_{eq}}{(\rho c)_{eq}} \beta_n^2 t \right) \quad (9)$$

where $D_n = \frac{(-1)^n}{L} \left(\frac{J^2}{k_{eq}\sigma\beta_n^3} + \frac{\Delta T}{\beta_n} \right)$, $\beta_n = \frac{(2n-1)\pi}{2L}$, $n = 1, 2, 3, \dots$. Finally, the transient temperature field is

$$T(x, t) - T_0 = \frac{-J^2 L^2}{2k_{eq}\sigma} \left[\left(\frac{x}{L} \right)^2 - 1 \right] + \frac{\Delta T}{2} \left(\frac{x}{L} + 1 \right) + \sum_{n=1}^{\infty} D_n \cos(\beta_n x) \exp \left(\frac{-k_{eq}}{(\rho c)_{eq}} \beta_n^2 t \right) \quad (10)$$

Substitution of Eq. (1) into Eq. (3) gives $\partial^2 V / \partial x^2 + s \partial^2 T / \partial x^2 = 0$ whose solution under the boundary conditions $V = V_0$ at $x = -L$ and $V = V_h$ at $x = L$ are

$$V(x, t) - V_0 = -s [T(x, t) - T_0] + \frac{1}{2} (\Delta V + s \Delta T) \left(\frac{x}{L} + 1 \right) \quad (11)$$

where ΔV is the electric potential difference between the hot and cold ends of the multilayered TEM, namely, $\Delta V = V_h - V_0$.

Note that J , V and T are not independent. Their relationship can be obtained from Eqs. (1) and (11), to give $J = -\sigma(\Delta V + s\Delta T)/(2L)$. As a result, Eq. (11) can also be written as

$$V(x,t) - V_0 = -s[T(x,t) - T_0] - \frac{JL}{\sigma} \left(\frac{x}{L} + 1 \right) \quad (12)$$

This means that once the temperature and electric current density are given, the distribution of the electric potential is determined.

3. The interlaminar stress

We can establish some differential relationships between force and associated stress of the multilayered TEM by considering a small differential element in the axial direction as shown in Fig. 2. Because the thickness of structure is very small, thus the moment exists in each layer can be ignored. Due to symmetry of structure configuration and material properties, it's necessary and sufficient to analyze half of the structure. Utilizing force and moment balance condition for the N-type film, we can derive the following differential equations

$$-N_N + \sigma_{NI}dx + (dN_N + N_N) = 0 \Rightarrow \sigma_{NI} = -\frac{dN_N}{dx} \quad (13)$$

$$-F_N + \tau_{NI}dx + F_N + dF_N = 0 \Rightarrow \tau_{NI} = -\frac{dF_N}{dx} \quad (14)$$

$$\int_{-L}^x \sigma_{NI}(\zeta)(x+L-\zeta)d\zeta - F_N \frac{h}{2} = 0 \quad (15)$$

where N_N and F_N are transverse shear force and axial normal force in N-type layer, σ_{NI} and τ_{NI} are interlaminar transverse normal stress (peeling stress) and interlaminar shear stress at the interface of N-type and insulating layer. The longitudinal displacements of the upper surface of the N-type layer (i.e., the $z=-H/2$ surface), the lower and upper surfaces of the insulating layer (i.e., the $z=-H/2$ surface and $z=H/2$ surface, respectively), and the lower surface of the P-type layer (i.e., the $z=H/2$ surface) are respectively denoted by u_N^+, u_I^-, u_I^+ and u_P^- , which can be expressed by the following formulas [39]

$$u_N^+ = \alpha_N(T - T_0)x + \lambda_N \int_0^x F_N(\xi) d\xi + \kappa_N \tau_{NI} \quad (16)$$

$$u_I^- = \alpha_I(T - T_0)x + \lambda_I \int_0^x F_I(\xi) d\xi - \kappa_I \tau_{NI} \quad (17)$$

$$u_I^+ = \alpha_I(T - T_0)x + \lambda_I \int_0^x F_I(\xi) d\xi + \kappa_I \tau_{IP} \quad (18)$$

$$u_P^- = \alpha_P(T - T_0)x + \lambda_P \int_0^x F_P(\xi) d\xi - \kappa_P \tau_{IP} \quad (19)$$

where α_i, λ_i and $\kappa_i (i = I, N, P)$ respectively denote the coefficient of thermal expansion, coefficient of axial compliance and coefficient of interfacial compliance for each layer, λ_i and κ_i are respectively defined as $\lambda_i = (1 - \nu_i) / (E_i h_i)$ and $\kappa_i = 2h_i(1 - \nu_i) / (3E_i)$, in which ν_i presents the Poisson's ratio.

With the displacement continuity condition at the interface ($u_N^+ = u_I^-, u_I^+ = u_P^-$), Eqs. (16)-(19) can be used to give

$$(\alpha_N - \alpha_I)(T - T_0)x + \lambda_N \int_0^x F_N(\xi) d\xi - \lambda_I \int_0^x F_I(\xi) d\xi + (\kappa_N + \kappa_I) \tau_{NI} = 0 \quad (20)$$

$$(\alpha_I - \alpha_P)(T - T_0)x + \lambda_I \int_0^x F_I(\xi) d\xi - \lambda_P \int_0^x F_P(\xi) d\xi + (\kappa_I + \kappa_P) \tau_{IP} = 0 \quad (21)$$

The multilayered TEM considered here is symmetric with respect to the $z=0$ plane and does not subjected to any external mechanical load. Therefore, Eqs. (20) and (21) will be equivalent to each other and the axial forces F_I, F_P and F_N must be such that $F_P = F_N$ and $F_P + F_N + F_I = 0$. Thus, by differentiating Eq. (20) with respect to x and using Eq. (14), we can obtain the differential equation governing the axial force for the N-type thermoelectric layer as

$$\frac{d^2 F_N(x)}{dx^2} - \eta^2 F_N(x) = \gamma \frac{d[(T - T_0)x]}{dx} \quad (22)$$

where η^2 and γ are the material properties of the multilayered TEM and are defined as

$$\eta^2 = (\lambda_N + 2\lambda_I) / (\kappa_I + \kappa_N), \gamma = (\alpha_N - \alpha_I) / (\kappa_I + \kappa_N).$$

3.1 Steady-state interlaminar stress

Substitute the steady part of the temperature field, Eq. (8), into Eq. (22), we can get the steady-state expression of axial force for the N-type thermoelectric layer

$$\frac{d^2 F_N(x)}{dx^2} - \eta^2 F_N(x) = \gamma \left[\frac{J^2}{2k_{eq}\sigma} (L^2 - 3x^2) + \Delta T \left(\frac{x}{L} + \frac{1}{2} \right) \right] \quad (23)$$

To simultaneously satisfy Eq. (23) and the mechanical boundary conditions ($x = \pm L, F_N(x) = 0$), the solution F_N can be given as

$$F_N(x) = A \sinh(\eta x) + B \cosh(\eta x) + a_1 x^2 + a_2 x + a_3 \quad (24)$$

in which the constants a_1, a_2, a_3, A and B are

$$a_1 = \frac{3\gamma J^2}{2\eta^2 k_{eq}\sigma}, a_2 = -\frac{\gamma \Delta T}{\eta^2 L}, a_3 = -\frac{\gamma}{2\eta^2} \left(\frac{J^2 L^2}{k_{eq}\sigma} + \Delta T \right) + \frac{2a_1}{\eta^2},$$

$$A = -\frac{a_2 L}{\sinh(\eta L)}, B = -\frac{a_1 L^2 + a_3}{\cosh(\eta L)} \quad (25)$$

To obtain the lateral force N_N , we differentiate Eq. (15) with respect to x twice and using Eq. (13). This yields the following differential governing equation of steady-state transverse shear force

$$\frac{d^2 N_N(x)}{dx^2} + \frac{1}{L} \frac{dN_N(x)}{dx} = -\frac{h}{2L} \frac{d^2 F_N(x)}{dx^2} \quad (26)$$

Noting that F_N has been given by Eq. (24). The solution of Eq. (26) satisfying the mechanical boundary conditions $N_N=0$ for $x=L$ and $x=-L$ are

$$N_N(x) = C_1 \exp\left(-\frac{x}{L}\right) + b_1 \sinh(\eta x) + b_2 \cosh(\eta x) + b_3 x + C_2 \quad (27)$$

in which the constants b_1, b_2, b_3, C_1 and C_2 are

$$b_1 = -\frac{h\eta(B - A\eta L)}{2(1 - \eta^2 L^2)}, b_2 = \frac{h\eta(BL\eta - A)}{2(1 - \eta^2 L^2)}, b_3 = -ha_1$$

$$C_1 = \frac{2e}{e^2 - 1} [b_1 \sinh(\eta L) + b_3 L], C_2 = -b_2 \cosh(\eta L) - \frac{e^2 + 1}{2e} C_1 \quad (28)$$

The shearing stress τ_{NI} and the transverse peeling stress σ_{NI} can be obtained directly by applying Eqs. (24) and (27) to Eqs. (13) and (14). Equation (22) shows that a one-dimensional temperature field will result in a one-dimensional axial normal stress in each layer. However, due to the mismatching of the material properties among the layers, the out-of-plan stress will develop at the interfaces of the layers. Equation (27) clearly confirms this fact.

3.2 Transient-state interlaminar stress

The transient interlaminar stresses can also be obtained in a similar way as the steady-state stresses. By substituting the transient thermal stress of Eq. (10) into Eq. (22), we can get the transient-state expression of axial force for the N-type thermoelectric layer as follows

$$\begin{aligned} \frac{d^2 F_N(x)}{dx^2} - \eta^2 F_N(x) = & \gamma \left[\frac{J^2}{2k_{eq}\sigma} (L^2 - 3x^2) + \Delta T \left(\frac{x}{L} + \frac{1}{2} \right) \right] \\ & + \gamma \sum_{n=1}^{\infty} D_n [\cos(\beta_n x) - \beta_n x \sin(\beta_n x)] \exp \left(\frac{-k_{eq}}{(\rho c)_{eq}} \beta_n^2 t \right) \end{aligned} \quad (29)$$

The solution of Eq. (29) must have the following form

$$\begin{aligned} F_N(x, t) = & A \sinh(\eta x) + B \cosh(\eta x) + a_1 x^2 + a_2 x + a_3 \\ & + \sum_{n=1}^{\infty} [A_1 \sinh(\eta x) + B_1 \cosh(\eta x) + c_1 \cos(\beta_n x) + c_2 x \sin(\beta_n x)] \exp \left(-\frac{k_{eq}}{(\rho c)_{eq}} \beta_n^2 t \right) \end{aligned} \quad (30)$$

in which the constants A , B , a_1 , a_2 and a_3 have been given in Eq. (25). To satisfy Eq. (29), the constants c_1 and c_2 must be:

$$c_2 = \frac{\gamma D_n \beta_n}{\eta^2 + \beta_n^2}, \quad c_1 = \frac{2\beta_n c_2 - \gamma D_n}{\eta^2 + \beta_n^2} \quad (31)$$

The remaining constants A_1 and B_1 can be determined by applying Eq. (30) to the axial force free boundary conditions at both ends of the multilayered TEM (these are: $F_N(-L, t) = 0$ and $F_N(L, t) = 0$). This yields:

$$A_1 = 0, \quad B_1 = \frac{(-1)^n L c_2}{\cosh(\eta L)} \quad (32)$$

The transient-state transverse lateral force N_N can be obtained in a similar way as the steady-state. Note that the differential governing equation for transverse shear force at transient-state is the same as that at the steady-state, Eq. (26), provided that F_N is given by Eq. (30). The solution of Eq. (26) must have the following form

$$N_N(x, t) = C_1 \exp \left(-\frac{x}{L} \right) + b_1 \sinh(\eta x) + b_2 \cosh(\eta x) + b_3 x + C_2$$

$$\begin{aligned}
& + \sum_{n=1}^{\infty} \left[H_1 \exp\left(-\frac{x}{L}\right) + D_1 \sinh(\eta x) + D_2 \cosh(\eta x) + H_2 \right] \exp\left(-\frac{k_{eq}}{(\rho c)_{eq}} \beta_n^2 t\right) \\
& + \sum_{n=1}^{\infty} [(d_1 + d_2 x) \sin(\beta_n x) + (d_3 + d_4 x) \cos(\beta_n x)] \exp\left(-\frac{k_{eq}}{(\rho c)_{eq}} \beta_n^2 t\right) \quad (33)
\end{aligned}$$

in which the constants C_1 , b_1 , b_2 , b_3 and C_2 have been given in Eq. (28). To simultaneously satisfy Eq. (26) and the shearing force free boundary conditions $N_N(-L, t)=0$ and $N_N(L, t)=0$, the remaining constants appearing in Eq. (33) must be:

$$\begin{aligned}
D_1 &= -\frac{h}{2} \frac{\eta B_1}{1 - \eta^2 L^2}, \quad D_2 = -L \eta D_1, \\
d_2 &= -\frac{h}{2} \frac{\beta_n^2 L c_2}{1 + \beta_n^2 L^2}, \quad d_4 = -\frac{h}{2} \frac{\beta_n c_2}{1 + \beta_n^2 L^2}, \quad d_3 + \beta_n d_1 L = -d_4 L, \\
d_1 &= -\frac{1}{\beta_n (1 + \beta_n^2 L^2)} \left[\frac{h}{2} (2\beta_n c_2 - \beta_n^2 c_1) - 2(1 + \beta_n^2 L^2) d_4 \right], \\
H_1 &= \frac{2e}{e^2 - 1} \left[D_1 \sinh(\eta L) + (-1)^{n-1} d_1 \right], \\
H_2 &= -\left\{ \frac{e^2 + 1}{e^2 - 1} \left[D_1 \sinh(\eta L) + (-1)^{n-1} d_1 \right] + \left[D_2 \cosh(\eta L) + (-1)^{n-1} d_2 L \right] \right\} \quad (34)
\end{aligned}$$

Now that the axial force F_N and the lateral force N_N are obtained, the shearing stress τ_{NI} and the transverse peeling stress σ_{NI} can be obtained directly by applying Eqs. (30) and (33) to Eqs. (13) and (14). To this end, the interlaminar thermal stresses in steady state and transient state are obtained. The numerical results will be presented subsequently.

4. Numerical examples and discussion

The material properties used in the numerical calculations are listed in Table 1 [7, 14, 43]. Unless otherwise specified, all layers are assumed to have the same thickness $h=H=3\mu\text{m}$. The semi-length of the structure is $L=150\mu\text{m}$. The temperature at the cold and hot end are, respectively, $T_0=290\text{K}$ and $T_h=330\text{K}$. The value of the reference electrical current density used in this section is determined such that the system approximately has the peak thermoelectric conversion efficiency [7, 8, 41]:

$$J_0 = -\frac{k_p \Delta T}{2sL[T_h - (\Delta T / 2)(k_p / k_{eq})]}(\sqrt{1 + Z[T_h - (\Delta T / 2)(k_p / k_{eq})]} - 1) \quad (35)$$

where $Z = s^2 \sigma / k_p$ is the figure-of-merit. Substitution of corresponding material property parameters gives the figure-of-merit $Z=0.0018\text{K}^{-1}$ and associated reference electric current density $J_0=-1.1789 \times 10^6 \text{A/m}^2$. Again, unless otherwise specified, the applied electric current density J is equal to the reference electric current density J_0 .

As presented in Eq. (10), the transient solution of the temperature field is given by the sum of an infinite series. All results in this section are for the number of series 500, which has resulted in the convergent results for all calculations.

Table 1 Material properties [7, 14, 41].

	TE layers (N-type and P-type)	Insulating layer
Coefficient of thermal expansion ($10^{-5}/\text{K}$)	1.68	0.65
Young's modulus (GPa)	47	380
Poisson's ratio	0.4	0.26
Thermal conductivity (W/(m K))	2.2	17
Electrical conductivity ($\Omega^{-1}\text{m}^{-1}$)	1×10^5	
Mass density (kg/m^3)	7720	3970
Specific heat (J/kg K)	170	800
Seebeck coefficient ($\mu\text{V/K}$)	200	

Figures 3 and 4, respectively, show the distributions of transient temperature $T(x, t)$ and electric potential difference $V(x, t)-V_0$ along the longitudinal direction of the multilayered TEM. It is found that as time increasing, the transient parts of the temperature and electric potential gradually decrease. As time is sufficiently large (e.g., 0.1s in the current case), the temperature field and electric potential field become steady-state. The transient-state temperature is smaller than that of steady state. However, the electric current in transient-state is higher than that in steady-state.

Figures 5-7 show, respectively, the transient axial stress σ_x of the N-type thermoelectric layer, the transient shear stress τ_{NI} and the transient peeling stress σ_{NI} at the interface between the N-type thermoelectric layer and the insulating layer. As expected, the values of the stresses at transient-state are different from those at the steady-state. Steady stresses retain only when time is sufficiently large. The overall stress level at the transient state is higher than that at the steady state. The results also clearly show that interlaminar stresses have significant concentrations at the free ends of the multilayered TEM.

Existing of the interlaminar peeling stress is the key reason for the delamination of the multilayered materials. For this, the influence of the thickness and material properties of the insulating layer on the distribution of the interlaminar peeling stress σ_{NI} is examined. Results for a typical time $t=0.002s$ are shown in Figs. 8-11. It is found that both the magnitude of the interlaminar peeling stress σ_{NI} and the region of the stress concentration gradually increase as the thickness and Young's modulus of the insulating layer increase. The magnitude of the interlaminar peeling stress σ_{NI} can decrease or increase with the coefficient of thermal expansion and the thermal conductivity of the insulating layer. In particular, when the thermal expansion of the thermoelectric layers and the insulating layer are same, the interlaminar peeling stress vanishes. This suggests that the interlaminar stress can develop only if the adjacent two layers have different material properties.

Thermoelectric materials may operate under various values of the temperature and electric current density. Naturally, understanding the influence of the electric current density on the interlaminar stress is important for reliability design of thermoelectric devices. For this, the peeling stress σ_{NI} for a typical time $t=0.002s$ is examined for various values of the electric current density. The results are shown in Fig. 12. It is observed that the variation of peeling stress is not obvious if the electric current density is small (e.g., $J < J_0$). The effect becomes increasingly remarkable when the electric current density increases. In addition, the magnitude of the peeling stress σ_{NI} near the ends of the TEM increases as the electric current density increases.

5. Conclusions

The transient temperature and interlaminar thermal stress fields in a multilayered TEM are

examined. It is found that the distributions of thermal stresses at transient-state are very different from these at the steady state. Steady stresses retain only when time is sufficiently large. Significant interlaminar stress concentration at the free ends of the multilayered TEM is observed. The applied electric current also enhance the interlaminar stress concentration therefore can not be neglected. The region of the interlaminar stress concentration at transient-state is much bigger than that at the steady-state. This confirms the significance of the current research on the transient interlaminar stress of multilayered TEMs.

The influence of the thickness and properties of the insulating layer on the transient interlaminar stress is also investigated. A multilayered TEM with a thinner insulating layer will result in a smaller interlaminar stress. The interlaminar stress also reduces when the insulating layer has a smaller Young's modulus. Same as the multilayered elastic materials, for multilayered thermoelectric materials, the transient interlaminar stress vanishes if the coefficients of thermal expansion of all layers are identical.

Acknowledgements : The research was supported by Research Innovation Fund of Shenzhen City of China (project Nos. JCYJ20150805142729431, JCYJ20160427184645305), the Natural Science Foundation of Guangdong Province of China (project Nos. 2014A030313696, 2016A030311006) and the National Science Foundation of China (NSFC) (Project Nos. 11421091, 11672084 and 11372086).

Appendix A. Equivalent governing equation of temperature for the multilayered TEMs

Consider a small differential element in the axial direction as shown in Fig. 2(a), with the energy balance method, the energy of Fourier's heat conduction, Joule's heat and the energy accumulation in the body must satisfy the following relationship

$$\sum_{i=N,I,P} [\int_V (k_i \frac{\partial^2 T}{\partial x^2}) dV + \int_l I^2 dR - \int_V (\rho_i c_i \frac{\partial T}{\partial t}) dV] = 0 \quad (A1)$$

where l and V are length and volume of the analyzed small differential element, I and R are electric current and electric resistance, which are respectively defined by

$$I = Jhdy, R = \frac{dx}{\sigma hdy} \quad (A2)$$

in which dy is the width of the multilayered thermoelectric material along the y direction. For the insulating layer, the term involves the electric current should be dropped in Eq. (A1). Substitution of Eq. (A2) into Eq. (A1), gives

$$(2k_N h + k_I H) \frac{\partial^2 T}{\partial x^2} dy dx + 2(Jhdy)^2 \frac{dx}{\sigma hdy} - (2\rho_N c_N h + \rho_I c_I H) \frac{\partial T}{\partial t} dy dx = 0 \quad (A3)$$

Setting $k_{eq} = k_N + k_I H / (2h)$, $(\rho c)_{eq} = \rho_N c_N + \rho_I c_I H / (2h)$, Eq. (A3) can be rewritten as

$$[k_{eq} \frac{\partial^2 T}{\partial x^2} + \frac{J^2}{\sigma} - (\rho c)_{eq} \frac{\partial T}{\partial t}] dy dx = 0 \quad (A4)$$

For an arbitrary small differential element, $dy dx \neq 0$, thus we can obtain the governing equation of transient-state temperature from Eq. (A4) as follows

$$k_{eq} \frac{\partial^2 T}{\partial x^2} + \frac{J^2}{\sigma} = (\rho c)_{eq} \frac{\partial T}{\partial t} \quad (A5)$$

For the case of steady state, the governing equation can be easily acquired if $\partial T / \partial t = 0$ is used in Eq. (A5).

Appendix B. Solution of the transient part of the temperature field

The transient part of the temperature field, equation (7), can be solved in a straightforward way by using the technique of separation of variable used in Ref. [38, 40]. The solution is

$$U(x, t) = u(x)v(t) = \sum_{n=1}^{\infty} [C_n \sin(\beta_n x) + D_n \cos(\beta_n x)] \exp\left(-\frac{k_{eq}}{(\rho c)_{eq}} \beta_n^2 t\right) \quad (B1)$$

where β_n are unknown constants. To satisfy the temperature boundary conditions ($x = -L, U(-L, t) = 0$ and $x = L, U(L, t) = 0$), there must be

$$[D_n \cos(\beta_n L) - C_n \sin(\beta_n L)] \exp\left(-\frac{k_{eq}}{(\rho c)_{eq}} \beta_n^2 t\right) = 0 \quad (B2)$$

$$[D_n \cos(\beta_n L) + C_n \sin(\beta_n L)] \exp\left(-\frac{k_{eq}}{(\rho c)_{eq}} \beta_n^2 t\right) = 0 \quad (B3)$$

If $D_n=0$, then $U(x,t) = \sum_{n=1}^{\infty} C_n \sin(\beta_n x) \exp\left(-k_{eq} / (\rho c)_{eq} \beta_n^2 t\right)$. $U(x,t)$ will vanish if we set $x=0$,

and it is obviously unreasonable. Thus there must be $C_n=0$ and $\cos(\beta_n L)=0$ ($\beta_n = (2n-1)/(2L), n=1,2,3\dots$). Under the initial condition ($t=0, U(x,0)=T_0-W(x)$), Eq. (B1) can be rewritten as

$$\sum_{n=1}^{\infty} D_n \cos(\beta_n x) = \frac{J^2}{2k_{eq}\sigma} (x^2 - L^2) - \frac{\Delta T}{2} \left(\frac{x}{L} + 1 \right) \quad (B4)$$

Multiplying both sides of Eq. (B4) by $\cos(\beta_n x)$ and performing integration along the interval $[-L, L]$, we can obtain the constant

$$D_n = \frac{(-1)^n}{L} \left[\frac{J^2}{k_{eq}\sigma\beta_n^3} + \frac{\Delta T}{\beta_n} \right] \quad (B5)$$

To this end, the transient part of the temperature field is obtained.

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Fig. 6 The transient interlaminar shear stress at the interface between N-type layer and insulating layer

Fig. 7 The transient interlaminar peeling stress at the interface between N-type layer and insulating layer

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