Study on the influence of thermal characteristics of rolling bearings and spindle
resulted in condition of improper assembly

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Abstract: Thermal analysis of spindle-bearing system with improper assembly is crucial but insufficient. Aimed at this, spindle system of ball bearings with tilting outer ring was proposed. Firstly, based on quasi-static analysis, the bearing friction heat was analyzed and calculated, and the effect of tilting bearing outer ring on bearing's contact angle, contact force and friction heat was investigated. Secondly, steady thermal simulation analysis on spindle in consideration of tilting outer ring was completed. Afterwards, for validation purpose, outer spacers with different non-parallelism were respectively assembled on the spindle to simulate bearing outer ring's different tilt statuses. The results indicated that, bigger tilt angle of bearing outer ring results in greater bearing's heat generation power and higher steady temperature of bearing outer ring. Simultaneously, in comparison with non-tilting outer ring, outer ring tilted to a certain extend will lead to greatly increase in friction heat of the certain rolling element. Experimental results verified the simulation analysis as that increasing non-parallelism of outer spacer leads to decreasing temperature difference between front and rear bearings.

Keywords: Spindle-bearing system; improper assembly; thermal characteristics; non-parallelism

As an actuator, spindle-bearing system is the key component of a machine tool, and it is fairly crucial for machine tool's machining accuracy, reliability and service lifespan [1]. During the manufacturing process of a spindle, the assembly condition is one of the critical elements to determine its performance [2]. Since increasingly high demands on spindle service performances, such as accuracy and rotating speed, were raised by numerical control machine tools, strict

requirements on spindle assembly were also proposed at the same time. Currently, although a growing number of instruments and methods on precision measurement for spindle assembly were used, dependence on skilled assembly workers was still heavy during the assembly process. Nowadays, key parts with high quality and precision like rolling bearing, electric motor and lubrication system were applied on the spindle manufacturing process in many spindle production corporations, but spindles with excellent performance were just produced by few several enterprises. Obviously, assembly process is the main reason which profoundly influenced the spindle performance. Given this, it is extremely indispensable to study the impact on spindle performance caused by assembly quality. In the future, less reliance on skilled assembly workers by studying spindle assembly technology will become an effective way to ensure the assembly quality of spindles.

Since spindle assembly seriously affects the spindle performance, many scholars and research institutions have deeply studied this subject. Jang [3] summarized that misaligned journal bearing may be resulted from an individual or combination of the following factors: asymmetric bearing loading, elastic deflection and thermal distortion of the shaft; distortions caused by bearing housing supports; manufacturing tolerances and errors due to installation and assembly defects. To theoretically analyze the effect on system caused by assembly quality, Zhang [4] studied the whole vibration of system resulted from misalignment faults by establishing the dynamic model of rotor-ball bearing system under misalignment situation. Similarly, a model based technique for fault diagnosis of rotor-bearing system was proposed [5]. Besides, through experimental testing and rotor dynamic computer simulations, the radial stiffness of an angular contact ball bearing with inner race static misalignment and outer race dynamic misalignment under various axial loads were investigated by Ertas [6]. Different with the above researches, Yi [7] and Li [8] proposed new methods to reduce the effect of misalignment, namely a proper amount of misalignment which is beneficial for the dynamic characteristics for a misaligned matched rotor-bearing system and non-uniform preloads properly distributed which could decrease the heat generation power of bearing. On the issue about thermal characteristics of rotor-bearing system, Chen [9], Takabi [10] and Yan [11-12] established different thermal models to analyze the thermo-mechanical dynamic behaviors and thermal deformation of spindle by simulations and

experiments. Even more important, the impact on bearing performance resulted from outer spacer non-parallelism was numerically analyzed by Zhang H. [13]. In addition, some bearing enterprises provided many beneficial instructions on spindle assembly process in their handbooks [14]. However, all these were focusing on model of rotor-bearing system or thermal characteristics of bearing. When analyzing the thermal characteristics of spindle, influence on bearing caused by the quality of spindle assembly was not taken into account. What's more, researches on effect of thermal characteristics of bearings and spindle caused by improper assembly of spindle were extremely insufficient.

In fact, during the manufacturing process of a spindle, the case that there exists misalignment between the inner and outer ring of spindle bearing caused by improper assembly occurred frequently [3,13-14]. As the structural schematic diagram of a spindle shown in Fig.1, parts such as spacers and locknut were in direct contact with the bearing end face, so factors including non-parallelism of contact surface, improper way of screwing up a locknut and torque inequality when tightening end cover bolts even its improper sequence of tightening bolts would result in relative tilt between bearing outer ring and inner ring. Consequently, abnormal bearing rotating would take place, accompanying with spindle's temperature rise and vibration increase. Furthermore, bending deflection would appear on the spindle if the misalignment becomes more serious, which highly influenced the spindle rotating precision.

Therefore, it is essential to research the influence on bearings and spindle performances caused by the spindle assembly quality, particularly the influence on thermal characteristics of bearings and spindle. In this paper, quasi-static analysis on ball bearing with tilting outer ring was carried out firstly. Then the friction heat was analyzed and calculated, and the effect of tilting bearing outer ring on bearing's contact angle, contact force and friction heat was studied. Based on this, steady thermal simulation analysis on spindle with consideration of tilting outer ring was conducted. Afterwards, in order to research the influence on thermal characteristics of bearings and spindle resulted from spindle assembly quality via experiment, outer spacers with different non-parallelism were respectively assembled on the spindle to simulate bearing outer ring's different tilt statuses. Finally, with experiments, temperature changes of front/rear bearings and thermal displacements with the increase of outer spacer's non-parallelism were discussed.

1. Bearing force analysis with tilting outer ring

During the process of assembling a bearing, factors like improper assembly and non-uniform preloads would lead to relative tilt between bearing inner and outer ring. In comparison with a properly assembled bearing, contact parameters like contact angle and contact force between balls and bearing inner/outer ring would change when a bearing was improperly assembled. Generally, the clearance fit was selected between bearing housing and bearing outer ring, and the interference fit was designed between shaft and bearing inner ring. As a result, the outer ring would tilt if there exist non-uniform loads acting on the bearing outer ring. Thus, force analysis on the bearing with tilting outer ring was given in this section.

1.1 Quasi-static analysis

Assuming that the bearing inner ring was fixed after assembling, in condition of axial and radial loads, axial displacement δ_a and radial displacement δ_r occur on the outer ring. As shown in Fig.2, the tilt angle of outer ring was supposed as θ . Before and after the tilting of outer ring, distance between inner and outer raceway curvature center would change, as shown in Fig.3. For any angular orientation φ_j (about φ_j , see Fig.4) of balls, distance between inner and outer raceway curvature center [15] could be expressed as:

$$S_j = [(A_0 \sin \alpha_0 + \delta_a + \theta R_o \cos \varphi_j)^2 + (A_0 \cos \alpha_0 - \delta_r \cos \varphi_j)^2]^{1/2} \quad (1)$$

where

$$R_o = \frac{d_m}{2} - (f_o - 0.5)D_b \cos \alpha_0 \quad (2)$$

According to the relationship between load and displacement of ball-raceway contact, normal load Q_j of the j^{th} ball is given as:

$$Q_j = K_n A_0^{1.5} \left\{ \left[\left(\sin \alpha_0 + \bar{\delta}_a + R_o \bar{\theta} \cos \varphi_j \right)^2 + \left(\cos \alpha_0 - \bar{\delta}_r \cos \varphi_j \right)^2 \right]^{1/2} - 1 \right\}^{1.5} \quad (3)$$

where, $\bar{\delta}_a = \delta_a / A_0$, $\bar{\delta}_r = \delta_r / A_0$, $\bar{\theta} = \theta / A_0$.

For any angular orientation φ_j , contact angle α_j under static loads meets the following condition.

$$\sin \alpha_j = \frac{\sin \alpha_0 + \bar{\delta}_a + R_o \bar{\theta} \cos \varphi_j}{\left[\left(\sin \alpha_0 + \bar{\delta}_a + R_o \bar{\theta} \cos \varphi_j \right)^2 + \left(\cos \alpha_0 - \bar{\delta}_r \cos \varphi_j \right)^2 \right]^{1/2}} \quad (4)$$

Load Q_j , along the direction of contact angle, is normal load which acting on the raceway, and its axial and radial components are in static equilibrium with the axial load and radial load respectively. The equilibrium equation can be described as:

$$\begin{cases} F_a = \sum_{j=0}^Z Q_j \sin \alpha_j \\ F_r = \sum_{j=0}^Z Q_j \cos \varphi_j \cos \alpha_j \end{cases} \quad (5)$$

In the above nonlinear equations, δ_a and δ_r are unknowns, which can be solved by iteration method. After being solved, parameters like contact force, contact angle and displacements of bearing under static loads can be provided as the initial iterative values of quasi-static analysis.

Under static loads, distance A_0 between inner and outer raceway curvature center would increase to $(A_0 + \delta_i + \delta_o)$, but the line through inner/outer raceway curvature center and ball center is still collinear. However, centrifugal force would act on balls when the bearing rotating at high speed. As a consequence, contact angle of inner raceway with balls and outer raceway with balls are not equal, thus the line through inner/outer raceway curvature center and ball center is not collinear, as shown in Fig.5.

Based on relative axial displacement δ_a , relative radial displacement δ_r and relative angular displacement θ , the axial and radial distances between inner and outer raceway curvature center are given as follows [15] respectively.

$$A_{1j} = A_0 \sin \alpha_0 + \delta_a + R_o \theta \cos \varphi_j \quad (6)$$

$$A_{2j} = A_0 \cos \alpha_0 - \delta_r \cos \varphi_j \quad (7)$$

By introducing variables X_1 and X_2 , the following equations can be deduced from Fig.5.

$$\begin{cases} \cos \alpha_{ij} = \frac{X_{2j}}{(f_i - 0.5)D_b + \delta_{ij}} \\ \sin \alpha_{ij} = \frac{X_{1j}}{(f_i - 0.5)D_b + \delta_{ij}} \\ \cos \alpha_{oj} = \frac{A_{2j} - X_{2j}}{(f_o - 0.5)D_b + \delta_{oj}} \\ \sin \alpha_{oj} = \frac{A_{1j} - X_{1j}}{(f_o - 0.5)D_b + \delta_{oj}} \end{cases} \quad (8)$$

Taking into account the geometric relationship, the following formulas could also be obtained from Fig.5.

$$\begin{cases} (A_{1j} - X_{1j})^2 + (A_{2j} - X_{2j})^2 - [(f_o - 0.5)D_b + \delta_{oj}]^2 = 0 \\ X_{1j}^2 + X_{2j}^2 - [(f_i - 0.5)D_b + \delta_{ij}]^2 = 0 \end{cases} \quad (9)$$

When the spindle rotating at high speed, loads including centrifugal force F_c , gyroscopic moment M_g , contact forces Q_i and Q_o would act on the balls, as shown in Fig.6.

For each rolling element, it should meet force balance on the horizontal and vertical directions:

$$\begin{cases} Q_{ij} \sin \alpha_{ij} - Q_{oj} \sin \alpha_{oj} - \frac{M_{gj}}{D_b} (\lambda_{ij} \cos \alpha_{ij} - \lambda_{oj} \cos \alpha_{oj}) = 0 \\ Q_{ij} \cos \alpha_{ij} - Q_{oj} \cos \alpha_{oj} + \frac{M_{gj}}{D_b} (\lambda_{ij} \sin \alpha_{ij} - \lambda_{oj} \sin \alpha_{oj}) + F_{cj} = 0 \end{cases} \quad (10)$$

where, for outer raceway control, $\lambda_{ij} = 0$ and $\lambda_{oj} = 2$.

Centrifugal force and gyroscopic moment of the j^{th} ball can be expressed as:

$$F_{cj} = \frac{1}{2} m d_m \omega^2 \left(\frac{\omega_m}{\omega} \right)_j^2 \quad (11)$$

$$M_{gj} = J \omega^2 \left(\frac{\omega_b}{\omega} \right)_j \left(\frac{\omega_m}{\omega} \right)_j \sin \beta \quad (12)$$

here, refer to Appendix to get ω_b , ω_m and β .

In order to get δ_a and δ_r , force equilibrium equations on bearing outer ring are established as follows:

$$\begin{cases} F_a - \sum_j \left(Q_{oj} \sin \alpha_{oj} - \lambda_{oj} \frac{M_{gj}}{D_b} \cos \alpha_{oj} \right) = 0 \\ F_r - \sum_j \left(Q_{oj} \cos \alpha_{oj} + \lambda_{oj} \frac{M_{gj}}{D_b} \sin \alpha_{oj} \right) \cos \varphi_j = 0 \end{cases} \quad (13)$$

The type of bearings is 7014C produced by NSK, some structural parameters are shown in Table 1.

According to the structural parameters of bearing and the initial value δ_a and δ_r solved by Eq. (5), combining with Eq. (6)~(13), a Matlab program based on Newton-Raphson method has been developed. The corresponding calculation flow chart is shown in Fig.7, whose main processes are as follows: a) Assuming that the initial axial displacement and initial radial displacement of bearing outer ring are $[\delta_{a0}, \delta_{r0}]$, after calculating $X_{1j}, X_{2j}, \delta_{ij}$ and δ_{oj} from Eq. (5), one can get $[\delta_a, \delta_r]$ after repeated calculation through Eq. (6)~(13). b) If $[\Delta\delta_a, \Delta\delta_r]$ between two adjacent calculations meets the error requirement, solution is done and contact parameters like $Q_{oj}, Q_{ij}, \alpha_{oj}, \alpha_{ij}$ can be obtained; otherwise, change $[\delta_{a0}, \delta_{r0}]$ and calculate again until $[\Delta\delta_a, \Delta\delta_r]$ between two adjacent calculations meets the error requirement. Before the calculation, bearing loads F_r and F_a in Eq. (13) need to be determined firstly. Here F_a is chosen as 1400N according to the recommended value from bearing handbook [16]. Besides, it needs to be pointed out that the experimental spindle used in this paper is driving by a motor through the driving belt, as shown in Fig.8. Because the driving belt force F_b acting on the spindle was tested as 400N, and according to a simple force analysis, the radial forces acting on every front and rear bearing are different, denoted as $F_{rf}=290\text{N}$ and $F_{rr}=685\text{N}$, respectively. In addition, since the tilting bearing outer ring and radial force have the same circumferential direction, the value of tilt angle in Fig.2 can specify the tilt direction uniquely.

1.2 Results of quasi-static analysis

According to bearing handbook, the allowable tilt angle between inner and outer ring of rolling bearing should be generally smaller than 0.069° . To prevent relatively large tilt angle from damaging bearings and facilitate the following experiments, select the tilt angles as $0^\circ, 0.0052^\circ, 0.0104^\circ, 0.0156^\circ$ and 0.0208° . The selection process is detailed in Section 3.1.

At rotating speed 8000rpm, contact angle between balls and outer/inner ring under different tilt angle of bearing outer ring are shown in Fig.9 and Fig.10 respectively. It can be observed that the contact angle varies relatively slightly when the tilt angle of bearing outer ring is 0° . At this time, the maximal and minimal contact angle between balls and outer ring is 14.43° and 12.96° , with a

margin of 1.47° . The maximal and minimal contact angle between balls and inner ring is 20.91° and 19.20° , with a margin of 1.71° . However, with the increase of tilt angle, the contact angle curve fluctuates with larger amplitude, along with saltation in Ball 6 and Ball 16. When the tilt angle reaches to 0.0208° , the maximal and minimal contact angle between balls and outer ring turn into 15.74° and 10.93° respectively, with a greater margin of 4.81° . The maximal and minimal contact angle between balls and inner ring become to 21.07° and 15.98° , with a larger margin of 5.09° . It indicates that the value and distribution of balls' contact angle are highly affected by the bearing outer ring's tilt angle.

At the same rotating speed, contact force between balls and outer/inner ring under different tilt angle are displayed in Fig.11 and Fig.12 respectively. Obviously, the contact force changes gently when the tilt angle is 0° . At present, the maximal and minimal contact force between balls and outer ring is 176.2N and 115.0N, with a margin of 61.2N. The maximal and minimal contact force between balls and inner ring is 136.1N and 74.4N, with a margin of 61.7N. While with the increase of tilt angle, the contact force curve demonstrates a larger fluctuates. When the tilt angle grows to 0.0208° , the maximal and minimal contact force between balls and outer ring change to 235.5N and 90.8N, with a larger margin of 144.7N. The maximal and minimal contact force between balls and inner ring turn to 195.5N and 49.7N, with a bigger margin of 145.8N. Similarly, this reveals that the value and distribution of balls' contact force are greatly influenced by the tilt angle.

2. Steady thermal analysis on spindle

Bearing heating would result in change of fit conditions and lubrication statuses as well as thermal deformation of spindle, which highly affects the machining performance of a spindle. Contact parameters calculated above lay a basis for spindle's steady thermal analysis. In this section, bearing heat generation power under tilting outer ring is calculated firstly, then it is regarded as heat source to conduct steady thermal simulation on the spindle.

2.1 Bearing heat generation

For rolling bearing with grease lubrication, the friction heat generation power are mainly composed of heat H_d generated from differential sliding on contact zones between balls and

raceways, heat H_s generated from spinning sliding on contact zones between balls and raceways, heat H_g generated from balls' gyroscopic rotation, heat H_v generated from lubricating grease's viscosity and heat H_c generated from sliding between cages and balls [17]. In reality, gyroscopic rotation has been reduced to the largest extent owing to the physical design of modern rolling bearings, so ignoring the friction heat H_g is feasible. In addition, since the acting forces between balls and cage is very small in most of the time, heat H_c could also be neglected [15]. Therefore, heat H_d , H_s and H_v are mainly considered and analyzed in this section.

1) Heat H_d generated from differential sliding

Along the major axis direction of the contact ellipse on contact zones between balls and raceways, heat generation power can be calculated as follows:

$$H_{nxj} = \frac{1}{J} \int \tau_{nxj} v_{nxj} dA_{nj} = \frac{a_{nj} b_{nj}}{J} \int_{-1}^1 \int_{-\sqrt{1-q^2}}^{\sqrt{1-q^2}} \tau_{nxj} v_{nxj} dt dq \quad (14)$$

Similarly, along the minor axis direction of the contact ellipse on contact zones between balls and raceways, heat generation power gives:

$$H_{nyj} = \frac{1}{J} \int \tau_{nyj} v_{nyj} dA_{nj} = \frac{a_{nj} b_{nj}}{J} \int_{-1}^1 \int_{-\sqrt{1-q^2}}^{\sqrt{1-q^2}} \tau_{nyj} v_{nyj} dt dq \quad (15)$$

where, $n=i/o$ refers to the outer ring and inner ring respectively; J is the conversion constant from unit $N \cdot m/s$ to W ; $\tau_{nxj}, \tau_{nyj}, v_{nxj}, v_{nyj}$ are the friction shearing stress and sliding velocity on contact surface respectively. A_{nj} is the contact area and a_{nj}, b_{nj} are the semi-major and semi-minor axis of contact ellipse.

The friction shearing stress at any point (x, y) on contact area is expressed as:

$$\tau = \frac{3\mu Q}{2\pi ab} \left[1 - \left(\frac{x}{a} \right)^2 - \left(\frac{y}{b} \right)^2 \right]^{1/2} \quad (16)$$

where, μ is the average sliding friction coefficient.

In the end, by accumulating the local heat in each ball contact area, heat H_d for the whole bearing can be calculated as

$$H_d = \sum_{n=i}^{n=o} \sum_{j=1}^{j=Z} (H_{nxj} + H_{nyj}) \quad (17)$$

2) Heat H_s generated from spinning sliding

Spinning friction is another important factor that causes heat generation of a bearing. The spinning friction moment on contact area between balls and raceways are computed as:

$$\begin{cases} M_{si} = \frac{3u_{si}Q_i a_i \varepsilon_i}{8} \\ M_{so} = \frac{3u_{so}Q_o a_o \varepsilon_o}{8} \end{cases} \quad (18)$$

The total heat generated from spinning friction in bearing is given by

$$H_s = \omega_{si} \cdot M_{si} + \omega_{so} \cdot M_{so} \quad (19)$$

where, ω_{si} and ω_{so} denote spin angular velocity of balls with inner and outer ring contact zone respectively, which can be obtained by referring to Appendix.

3) Heat H_v generated from lubricating grease's viscosity

Palmgren [18] has deduced that friction moment M_v generated from lubricating grease's viscosity is as follows:

$$M_v = 10^{-10} f_0 (vn)^{2/3} d_m^3 \quad (20)$$

where, coefficient f_0 depends on bearing design and lubrication method. Parameter ν means kinematic viscosity of lubricant.

So heat H_v could be given as:

$$H_v = M_v \cdot \omega = \frac{\pi n}{30} \times 10^{-10} f_0 (vn)^{2/3} d_m^3 \quad (21)$$

By adding these three kinds of friction heat together, the total heat generation power of bearing can be expressed as:

$$H = H_d + H_s + H_v \quad (22)$$

2.2 Bearing heat generation analysis results

Refer to Fig.1, supposing that a certain kind of spindle assembly error brings about tilting outer ring of front bearings. Based on the above quasi-static solution results and bearing fiction heat analysis, heat generation power under different tilt angles and rotating speeds can be obtained.

At rotating speed 2000rpm, 4000rpm, 6000rpm and 8000rpm, heat generation power of front and rear bearings with tilt angle are shown in Fig.13. As can be seen, heat generation power of two front bearings get higher and higher with the increasing tilt angle of bearing outer ring, but heat

generation power of two rear bearings remain about the same because of no tilt on rear bearing's outer ring. Comparing that at 0° with 0.0208° of tilt angle, heat generation power of front bearing increases 1.7W, 2.9W, 3.1W and 2.8W respectively at rotating speed 2000rpm, 4000rpm, 6000rpm and 8000rpm, but the growth rate of heat generation power becomes more and more smaller with the increase of rotating speed. A conclusion could be drawn that the tilt angle of bearing outer ring has less impact on bearing's heat generation when it rotates at higher speed. When the tilt angle of outer ring is 0° , heat generation power of front and rear bearing with rotating speed are shown in Fig.14. It can be speculated that because of the driving belt force acting on the spindle, a larger radial load F_{rr} and smaller radial load F_{rf} make the heat generation power of rear bearing be slightly larger than that of front bearing.

At rotating speed 8000rpm, each ball's heat generation power of front bearing under different tilt angle is shown in Fig.15. Similar with the contact force curves in Fig.11, the curve changes relatively gentle when the tilt angle is 0° . However, with the increase of tilt angle, the curve displays a greater fluctuation. It reveals that the value and distribution of balls' heat generation power are greatly influenced by the tilt angle of bearing outer ring. From Fig.11, Fig.12 and Fig.15, it is obviously observed that Ball 1 of front bearing is the rolling element that manifests the most increase in contact force and heat generation power, which is caused by the reason that the tilting bearing outer ring and radial force have the same circumferential direction in the position of Ball 1. Besides, because of the joint effect of bearing radial force and tilt angle, Ball 6 and Ball 16 present saltation in contact parameters and heat generation power in Fig.9, Fig 10, Fig.11, Fig.12 and Fig.15, but other balls demonstrate a successional rule. Heat generation power and contact force of Ball 1 of front bearing are shown in Table 2. At rotating speed 2000rpm, 4000rpm, 6000rpm and 8000rpm, heat generation power of Ball 1 increases 68%, 63%, 54% and 44% respectively when the tilt angle grows from 0° to 0.0208° . And contact force of inner and outer ring with Ball 1 increases 44% and 34% respectively when the tilt angle goes up from 0° to 0.0208° . Apparently, overheat or overload locally like this would seriously increase bearing failure probability. Therefore, an analysis result can be made that bearing's local failure probability is highly affected by the tilt angle of bearing outer ring.

2.3 Spindle steady thermal analysis and results

Steady thermal analysis on spindle with Ansys Workbench software has been conducted on the base of heat generation power. The structural schematic diagram of spindle is shown in Fig.1. Heat generation power of four bearings serves as the heat input of spindle. For outer raceway control, generally heat generation power of outer raceway is very small, so heat generation power of bearing can be assigned equally to bearing inner ring and balls [19]. In addition, heat generation power is applied as surface load since friction heat mainly generates on contact surface.

At rotating speed 2000rpm, 4000rpm, 6000rpm and 8000rpm, steady temperature distribution under tilt angle of 0° , 0.0052° , 0.0104° , 0.0156° and 0.0208° can be obtained based on simulation analysis. Under different simulation conditions, steady temperature of bearing outer ring is shown in Table 3. It demonstrates that increasing tilt angle generally results in growing outer ring steady temperature of front bearings, but that of rear bearings has no substantial change.

To rationalize the comparison between simulation and experimental results and weaken the influence originated from the steady temperature of bearing 1 and 2, outer ring's average temperature of front bearing 1 and 2 was defined as temperature of front bearing set t_{front} and that of rear bearing 1 and 2 was defined as temperature of rear bearing set t_{rear} . Steady temperature of front and rear bearing set based on simulation analysis and the corresponding quadratic fit curves are shown in Fig.16. It can be observed that t_{rear} is generally higher than t_{front} . With the increase of tilt angle, t_{front} presents gradually increasing trend, but t_{rear} remains about the same and just fluctuates gently, which can be explained by Fig.13.

3 Experiments and discussion

3.1 The test rig

In order to simulate bearing outer ring's tilting statuses when improperly assembled, outer spacers with different non-parallelism were respectively assembled on the spindle.

When there exists non-parallelism on the outer spacer, the bearing outer ring in contact to which would tilt, as shown in Fig.17(a). Define the altitude difference between the highest and lowest point of outer spacer as non-parallelism Δh , as shown in Fig.17(b), and the tilt angle of outer spacer can be written as:

$$\theta = \arctan(\Delta h / D) \quad (23)$$

where, D denotes outer diameter of outer spacer.

As stated in Section 1.2, five groups of tilt angle 0° , 0.0052° , 0.0104° , 0.0156° and 0.0208° were set in the experiment. The selection process is shown in Table 4.

When an outer spacer with non-parallelism assembled on the spindle, the unparallel side is in contact with the end face of front bearing 2. Therefore, the non-parallelism of outer spacer would be reflected in the front bearings after spindle assembly finished. That is, the two front bearings would rotate with the same tilt angle as outer spacer.

To verify the simulation results, a test rig that can study the influence on spindle steady thermal characteristics, which resulted from the end face non-parallelism of outer spacer, was designed. The experiment was conducted in the United Laboratory of NSK and Xi'an Jiaotong University. Spindle used in the experiment is a precision spindle with rated speed 8000rpm, built-in broach and belt drive system. The schematic view of the spindle can be seen in Fig.1. The bearing type is 7014C, with DBB (Duplex Back to Back) arrangement and grease lubrication. In addition, driving motor, test platform and industrial control system are included in this test rig. The industrial control system can control the rotating speed of motor, real-time display and store data such as temperature of bearing outer ring and the circumstance.

MISUMI thermocouple temperature sensors with resolution of 0.1°C , which used to measure outer ring temperature of all bearings, are installed in the spindle. The temperature sensors are embedded in the spindle and very close to the bearing outer ring, which could ensure the accuracy of measured temperature to the greatest extent. Master-ball target (The maximum roundness is 50nm) produced by Lion Precision is installed in the spindle through a chunk. Aimed at the target are three mutually orthogonal (defined as X , $-Y$ and $-Z$ axis respectively to coincide with the coordinate system of Fig.3) high precision capacitance displacement sensors, with a range of $250\mu\text{m}$, linearity of $\pm 0.15\%$ FSD (Full Scale Output) and peak-peak resolution of $0.25\mu\text{m}$, which to measure the displacements of master-ball target in three directions. Mounting position schematic diagram of sensors and the relevant entities are shown in Fig.18. Here, X , $-Y$ and $-Z$ axis mean horizontal radial direction, vertical radial direction and axial direction of spindle, respectively.

End face non-parallelism of outer spacer used in the spindle is 0μm, 10μm, 20μm, 30μm and 40μm, respectively. The test rig locates in a laboratory with constant temperature of 24±0.5℃. For each experimental process, the spindle rotates at 2000rpm in the beginning and speeds up to 4000rpm after the bearings outer ring reaching thermal equilibrium, and so on until getting to thermal equilibrium at 8000rpm. Refer to ISO 13041-8-2004 [20], the definition of ‘*thermal equilibrium*’ can be expressed as follows: Monitor three intervals in succession with every $\Delta T=10\text{min}$, if the difference between the highest and lowest temperature is not more than 0.1℃ during every ΔT , then consider the spindle to come to thermal equilibrium.

3.2 Experimental results and analysis

3.2.1 Temperature change during the experiment

When the non-parallelism of outer spacer is 20μm, original data curve of outer ring temperature of front bearing 1 (To simplify and clarify the plot, outer ring temperature of the other three bearings have not been listed here and they have similar curve.) during the whole experiment test as well as the curve after averaging and smoothing are shown in Fig.19. To prevent the two curves from overlapping, vertical axes with different scales are drawn. Steady temperature and its error with confidence level of 95% are labeled in the figure. As can be seen, the spindle gets to thermal equilibrium at every rotation speed, and the steady temperature of experimental test is gently lower than that of simulation analysis in Table 3.

3.2.2 Temperature difference between front and rear bearing set

Since the steady temperature of bearing outer ring, when the spindle reaching thermal equilibrium, is influenced by the circumstance temperature, and the data of two tests are different, so define temperature difference between front and rear bearing set as follows

$$\Delta t = t_{rear} - t_{front} \quad (24)$$

to reflect the effect on steady thermal characteristics of front and rear bearings resulted from non-parallelism of outer spacer. Temperature difference Δt with non-parallelism of outer spacer in simulation analysis and experimental test are shown in Fig.20. It can be observed that increasing non-parallelism of outer spacer (from 0μm to 40μm) leads to decreasing temperature difference Δt both in simulation analysis and experimental test. This is because that t_{rear} is

generally higher than t_{front} , and with the increase of tilt angle, t_{front} gets higher, but t_{rear} remains about the same, as shown in Fig.16. Meanwhile, Δt are different because that factors, including the ignorance of details in modeling, inaccuracy of thermal parameters and assembly error resulted from repeated installation, could have brought about this difference between the simulation analysis and experimental test. That the simulation analysis is verified by experimental tests indicates the correctness of theoretical analysis in this paper.

3.2.3 Thermal displacements of master-ball target

Preliminary research on thermal displacements of master-ball target in X , $-Y$ and $-Z$ axis are shown in Fig.21. Take the displacements at the very beginning of the experimental test as $0\mu\text{m}$, and positive or negative value in vertical scale means decrease or increase of the measured displacements, respectively. On the one hand, it can be seen that under every kind of non-parallelism, the measured displacement decreases in $-Z$ axis, increases in $-Y$ axis, and basically remains unchanged in X axis with the growing rotating speed. This is because the increase in temperature rise of bearings and spindle, resulted from the growth of spindle rotating speed, leads to thermal extension in $-Z$ axis and ‘*rising of spindle*’ in $-Y$ axis. On the other hand, at a certain kind of rotating speed, the displacement of thermal extension in $-Z$ axis and ‘*rising of spindle*’ in $-Y$ axis become more serious with the increase of non-parallelism of outer spacer. It manifests that thermal displacements in X , $-Y$ and $-Z$ axis would be influenced by the non-parallelism of outer spacer.

At the end, it should be noted that the assembly orientation of outer spacer may exert an effect on the thermal displacements, so further study on the influence of thermal displacements resulted from different assembly orientation’s outer spacer would be conducted in the future work.

4. Conclusion

In this paper, to investigate the influence on thermal characteristics of bearings and spindle resulted from improper assembly, a spindle-bearing system of ball bearings with tilting outer ring was proposed. Quasi-static analysis on bearings and steady thermal simulation on spindle in consideration of tilting outer ring were conducted. And the experiments verified the simulation analysis. The conclusions of this paper can be drawn as follows:

(1) The value and distribution of balls’ contact force/angle are highly influenced by the tilt angle

of bearing outer ring. Bigger tilt angle leads to bigger maximal contact force/angle, smaller minimal contact force/angle and more non-uniform contact force/angle distribution.

(2) Bearing's heat generation power gets higher with the increasing tilt angle of bearing outer ring.

What's more, heat generation power and contact force of a certain ball are highly influenced by the tilt angle of bearing outer ring.

(3) In this spindle, increasing non-parallelism of outer spacer (from 0 μ m to 40 μ m) leads to decreasing temperature difference between front and rear bearing set.

(4) Preliminary research shows that temperature rise of bearings and spindle leads to thermal extension in -Z axis and '*rising of spindle*' in -Y axis. Increasing non-parallelism of outer ring leads to the growth of thermal extension displacement.

Acknowledgments

The authors would like to express their gratitude to the United Laboratory of NSK and Xi'an Jiaotong University. Also, the authors gratefully acknowledge the supports of National Science Foundation of China under Grant No. 51575434 and the Fundamental Research Funds for the Central Universities and National Science and Technology Major Project of China under Grant No. 2015ZX04014021.

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Nomenclature

		Greek letters	
A	Distance between inner and outer ring raceway curvature center /mm		
A_1	Introduced variable /mm	α	Contact angle /rad
A_2	Introduced variable /mm	β	Angle between ball velocity vector and pitch circle /rad
a	Semi-major axis of Hertz contact ellipse on contact zone /mm	δ	Displacement or deflection /mm
b	Semi-minor axis of Hertz contact ellipse on contact zone /mm	ε	Elliptic integral of the second kind on contact zone
D_b	Ball diameter /mm	θ	Angular displacement /rad
d_m	Bearing pitch diameter /mm	λ	Raceway control coefficient
F_a	Axial load /N	μ	Friction coefficient
F_c	Centrifugal force /N	ν	Kinematic viscosity / $\text{mm}^2 \cdot \text{s}^{-1}$
F_r	Radial load /N	φ	Balls' angular orientation /rad
f	Curvature coefficient of bearing race	ω	Rotating speed / $\text{rad} \cdot \text{s}^{-1}$
H	Heat generation power /W	Subscript	
J	Moment of inertia / $\text{Kg} \cdot \text{mm}^2$	0	Original
K	Load-displacement coefficient along normal direction / $\text{N} \cdot \text{mm}^{-1.5}$	i	Inner ring

M_g	Gyroscopic moment /N·mm	o	Outer ring
n	Spindle speed /r·min ⁻¹	a	Axial direction
Q	Normal load /N	r	Radial direction
R	Trajectory radius of raceway curvature center /mm	j	The j^{th} ball at angular orientation
t_{front}	Average temperature of outer ring of front bearings /°C	s	Spinning motion
t_{rear}	Average temperature of outer ring of rear bearings /°C	b	Ball
X_1	Introduced variable /mm	m	Cage motion
X_2	Introduced variable /mm	$roll$	Rolling motion

Appendix

$$\gamma' = D_b / d_m \quad (\text{A-1})$$

$$\tan \beta = \frac{\sin \alpha_o}{\cos \alpha_o + \gamma'} \quad (\text{A-2})$$

$$\frac{\omega_b}{\omega} = \frac{-1}{\left(\frac{\cos \alpha_o + \tan \beta \sin \alpha_o}{1 + \gamma' \cos \alpha_o} + \frac{\cos \alpha_i + \tan \beta \sin \alpha_i}{1 - \gamma' \cos \alpha_i} \right) \gamma' \cos \beta} \quad (\text{A-3})$$

$$\frac{\omega_m}{\omega} = \frac{1 - \gamma' \cos \alpha_i}{1 + \cos(\alpha_i - \alpha_o)} \quad (\text{A-4})$$

$$\omega_o = -\omega_m \quad (\text{A-5})$$

$$\omega_i = \omega - \omega_m \quad (\text{A-6})$$

$$\omega_{si} = -\omega_b \sin(\alpha_i - \beta) + \omega_i \sin \alpha_i \quad (\text{A-7})$$

$$\omega_{so} = \omega_b \sin(\alpha_o - \beta) - \omega_o \sin \alpha_o \quad (\text{A-8})$$

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Table 1. Some structural parameters of bearing 7014C

Parameter	Value	Parameter	Value
Bearing bore diameter d (mm)	70	Bearing outside diameter (mm)	110
Number of balls Z	20	Bearing width B (mm)	20
Ball diameter D_b (mm)	11.906	Pitch diameter d_m (mm)	90.5
Coefficient of inner raceway curvature f_i	0.520	Coefficient of outer raceway curvature f_o	0.520
Elastic modulus of rings and balls (MPa)	2.08E5	Poisson's ratio of rings and balls	0.3
Initial contact angle ($^\circ$)	15	Density of rings and balls (kg/mm ³)	7.8E-6

Table 2. Heat generation power (HGP) and contact force of Ball 1 of front bearing (Unit: W, N)

Item	Tilt angle of bearing outer ring					Largest growth rate
	0°	0.0052°	0.0104°	0.0156°	0.0208°	
HGP:2000rpm	1.65	1.88	2.15	2.45	2.78	68%
HGP:4000rpm	3.58	4.05	4.58	5.17	5.82	63%
HGP:6000rpm	6.02	6.70	7.46	8.32	9.26	54%
HGP:8000rpm	9.12	9.94	10.88	11.95	13.12	44%
Contact force of inner ring	136.07	148.46	162.50	178.28	195.54	44%
Contact force of outer ring	176.25	188.58	202.57	218.29	235.51	34%

Table 3. Steady temperature of bearing outer ring based on steady thermal analysis (Unit: °C)

Rotating speed (rpm)	Bearing and column No.	Tilt angle of bearing outer ring									
		0°		0.0052°		0.0104°		0.0156°		0.0208°	
		1 ^①	2	1	2	1	2	1	2	1	2
2000	Front	28.6	28.4	28.6	28.5	28.6	28.6	28.7	28.6	28.7	28.6
	Rear	28.9	28.8	28.8	28.8	28.9	28.8	28.9	28.8	28.8	28.8
4000	Front	32.4	32.6	32.5	32.6	32.7	32.7	32.8	32.7	32.8	32.8
	Rear	33.6	33.0	33.5	33.0	33.6	33.1	33.5	33.1	33.5	33.1
6000	Front	36.5	36.7	36.6	36.7	36.6	36.8	36.6	36.9	36.7	36.9
	Rear	37.8	37.1	37.6	37.1	37.4	37.1	37.5	37.2	37.5	37.1
8000	Front	41.2	40.3	41.3	41.3	41.3	41.5	41.4	41.3	41.5	41.8
	Rear	42.1	41.3	41.9	41.3	41.9	41.3	42.0	41.1	42.1	41.2

Note①: Number ‘1’ or ‘2’ means column number of bearing and ‘1’ means the bearing closer to spindle nose.

‘Front’ and ‘Rear’ mean front bearings and rear bearings, respectively.

Table 4. The actual non-parallelism and tilt angle of outer spacers in the experiment

Group	Theoretical tilt angle ($^{\circ}$)	Theoretical non-parallelism (μm)	Non-parallelism Δh after rounding (μm)	Actual tilt angle θ ($^{\circ}$)
1	0	0	0	0
2	0.005	9.6	10	0.0052
3	0.01	19.2	20	0.0104
4	0.015	28.8	30	0.0156
5	0.02	38.4	40	0.0208

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