Determination of transient temperature distribution inside a wellbore considering drill string assembly and casing program

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Abstract: Heat exchange efficiency between each region of the wellbore and formation systems is influenced by the high thermal conductivity of the drill string and casing, which further affects temperature distribution of the wellbore. Based on the energy conservation principle, the Modified Raymond), Simplified and Full-scale models were developed, which were solved by the fully implicit finite difference method. The results indicated that wellbore and formation temperatures were significantly influenced at the connection points between the drill collar and drill pipe, as well as the casing shoe. Apart from the near surface, little change was observed in temperature distribution in the cement section. In the open-hole section, the temperature rapidly decreased in the circulation stage and gradually increased in the shut-in stage. Most important, the simulated result from the full-scale model coincided with the measured field data better than the other numerical models. These findings not only confirm the effect of the drill string assembly and casing programs on the wellbore and formation temperature distribution, but also contribute to resource exploration, drilling safety and reduced drilling costs.

Keywords: thermophysical properties; heat transfer mechanism; thermal loss; thermal recovery; circulation and shut-in stages

1 Introduction

With the development of human society, world energy consumption will continue to increase in the coming decades. Consumption is expected to reach 20,679 million tons of oil equivalent (Mtoe) in 2040,

including 28% from oil, 27% from coal, 23% from natural gas, 15% from renewables, and 8% from nuclear energy sources[1-5]. As a result, hydrocarbon resources play a significant role in supplying the global energy market through 2040. Geothermal energy, a renewable and sustainable energy, is transferred from the interior to the surface of the Earth with approximately 592,638 TJ/year harnessed in 82 countries [6, 7]. It is well known that in order to obtain this resource, geothermal wells are drilled at selected locations to establish a channel from the bottom-hole to the surface. Globally, increasing amounts of hydrocarbon and geothermal resources are being found in fields that have very high thermal gradients, resulting in a series of technical challenges in the high temperature environments of drilling and production processes [8].

As the total vertical depth increases, there is an increase in the bottom-hole temperature, as well as the hydrostatic fluid column pressure. These two factors have opposing effects on fluid density. The increased hydrostatic fluid column pressure causes an increase in the fluid density due to compression. Conversely, the increase in temperature causes a decrease in the fluid density due to thermal expansion [9]. It is inferred that the two effects cancel each other out in shallow well drilling, which could ensure drilling safety using the formation temperature instead of the wellbore temperature. In deep formation drilling, however, wellbore temperature has a more significant influence than pressure on the properties and density of the wellbore fluid. Karstad et al. investigated the change of fluid density under different conditions, assuming a surface density of 2.04 g/cm^3 . The effective density at geothermal conditions is 1.98 g/cm^3 . Immediately following the onset of circulation, it increases to 1.99 g/cm^3 and continues to increase to 2.06 g/cm^3 after 12 h of circulation [10]. Madu et al. presented effective density change of high temperature wells in the Niger Delta. It was observed that drilling fluid density decreased with depth, especially at the high temperatures of the hole section, which could give rise to well control issues and stuck pipe incidents [11]. Ataga et al. emphasized the pressure profile in a circulating well, taking into account the temperature profile, to estimate frictional pressure loss,

effective density and bottom-hole pressure [12]. If the impact of temperature on effective density is disregarded, there may be disastrous effects. Kick and blow-outs due to under-balanced pressure or lost circulation and formation damage may occur as a result of over-balanced pressure when drilling through formations with a small gap between pore pressure and fracture pressure [9].

Determining the transient temperature distribution in and around a well accurately, under circulation and shut-in conditions, is a complex engineering task because of the dynamic thermal circulation (cooling) and shut-in (heating) processes [13, 14]. Many uncertain factors impact the heat exchange between the wellbore and formation, including well geometry, surface temperature, geothermal gradient, fluid inlet temperature, fluid flow rate, fluid properties, continuous circulation of drilling fluid and drilling stoppages [15-17]. To increase the depth of the borehole, high thermal conductivity drill string and casing were used, affecting the heat transfer efficiency in each region of the well. To ensure safe drilling and production, the borehole wall was sealed by casing and cement slurry, which varied the thermal resistance and further impacted heat transfer from formation to wellbore. To facilitate understanding of transient temperature behavior along a wellbore and to inform quick decision making, a reliable and accurate evaluation of temperature distribution requires a complete dynamic thermal study of fluid circulation and stopping circulation in and around the wellbore[18].

Because predicting transient temperature behavior is important, the study of temperature distribution has attracted significant interest. Two approaches have emerged to estimate wellbore temperature: analytical and numerical [19]. The analytical method is derived mainly from constant linear and cylindrical heat source models. Linear heat source models typically consider the analysis of conductive heat flow under radial, cylindrical, or spherical conditions [20]. These methods are widely used to infer the initial formation temperature in the shut-in stage, using the simple solving method. Kutasov et al. estimated geothermal

gradients with the approximate method from the log data and identified that duration of the shut-in time was required for more precise evaluation [21]. Verma et al. attempted to predict static formation temperature using the error propagation theory [22]. García-Gutierrez et al. calculated effective thermal conductivity of the cement sheath through experimental comparison of the line source method and the Jaeger method. Experimental thermal conductivity uncertainties were 4% and 11.8% for the line source and Jaeger methods, respectively [23]. Purely conductive heat transfer in the drilling fluid after circulation is assumed in these methods, ignoring convection effects.

Heat transfer between the wellbore and formation is a dynamic process, covering both thermal conduction and thermal convection. The complex conductive–convective heat flow process is investigated, using cylindrical heat source models. Heat transfer, the temperature difference between the borehole wall (cement/formation interface) and the undisturbed formation temperature, was considered as a pseudo-steady state expressed as a transient heat-conduction time function, $f(t)$ by adapting Carslaw and Jaeger’s solution [24]. Kabir et al. estimated the flow profiles and production rate with the unsteady-state heat transfer method in gas wells. The model results paralleled with the measured data [25]. Dong et al. developed the heat transfer model of multi-thermal fluid in perforated horizontal wellbores of heavy oil reservoirs. They found that oil viscosity influenced the heating length of horizontal wellbores and flowing characteristics of multi-thermal fluids [26]. With early injection, the wellbore’s heat capacity has a substantial impact on the heat transfer between the wellbore and the formation. A novel analytical transient heat-conduction time function was developed to calculate wellbore heat losses [27]. Gu et al. constructed a comprehensive mathematical model to estimate wellbore heat efficiency and to analyze steam injection performance reflected both in wellbores and in reservoirs. They concluded that high wellhead injection rate and wellhead steam quality can improve wellbore heat efficiency [28]. Cheng et al. considered the stochastic

approximation method with the well-log temperature data to calculate both the thermal conductivity and volumetric heat capacity [27]. They also predicted the distribution of oil saturation in reservoirs by a semi-empirical oil saturation model using inversion thermal properties distributions [29]. Wu et al. presented a semi-analytical model to estimate the temperature transient behavior after the well was shut-in [30]. Although these classic analytical methods provide solid bases for estimating the initial formation temperature, they cannot be used as a general tool for calculating the thermal properties of wellbore/reservoir systems due to the following errors [15]: i) simple assuming constant linear heat source are usually considered as wellbore/formation interface heat transfer; ii) because the thermal conductivity of the tubing and casing walls is substantially higher than other components in the wellbore, thermal resistance of the wellbore and formation are not taken into account; iii) heat exchange between the fluid inside and outside the drill pipe is not considered.

A variety of parameters plays significant roles in heat transfer behavior between the wellbore and formation. Numerical modeling has been widely recognized as an effective and less expensive method to investigate heat exchange processes in each region of a well. To obtain complete transient solutions for such cases, numerical approaches, coupled with a superior mathematical algorithm, have been extensively used to predict the wellbore temperature and estimate the initial formation temperature to apply different operating conditions [19, 31-37]. Emami-Meybodi et al. using Laplace transform and finite Fourier cosine transform, developed a two-dimensional transient heat conduction model in the near wellbore region during the steam circulation stage. Steam injection was not suited to extremely low injectivity of the reservoir; to reduce the viscosity of heavy oil, the formation required preheating before steam injection [33]. Espinosa-Paredes et al. presented a feedback-based method to predict the unperturbed formation temperatures in the shut-in process [34]. García-Valladares et al. developed a numerical heat exchange model to estimate the production

characteristics in the geothermal well [35]. Wu et al. investigated the evolution of fluid and rock temperature during the circulation stage using a semi-analytic method. They identified the impact of the fluid injection rate on the change of the down-hole temperature [19]. Li et al. developed semi-infinite mathematical models to describe the dissociation process of natural gas hydrates, considering the Stefan moving boundary problem. Higher injection water temperature resulted in the increased participation of areal hydrates in dissociation [36]. Espinosa-Paredes et al. using an air–water mixture as drilling fluid, analyzed the thermal behavior of geothermal wells. The temperature reduction of the surrounding rock using the air–water mixture was less than that of liquid [37]. Few studies have focused on the effect of the drill string assembly and casing programs on heat transfer in each region of the wellbore and formation during fluid circulation and shut-in stages. Drill string assembly and casing programs, used to increase well depth and seal formation, influence changes in wellbore and formation temperature during the drilling and production processes. Investigating how the thermal properties of drill string assembly and casing programs affect the wellbore and surrounding formation temperature distribution is essential to design drilling and completion programs, optimize fluid properties, ensure drill safety and decrease the cost of well construction.

This study develops mathematical models to investigate the effect of drill string assembly and casing programs on the temperature distribution of a well during circulation and shut-in periods. The study incorporates the Modified Raymond model (which considers viscous dissipation and bit rotation and neglects the drill string assembly and casing program), a simplified model (considering neither the casing string nor the cement sheath) and a full-scale model (a full well composition). All were solved by the fully implicit finite difference method. The various temperature differences between the annulus and inside the drill strings, as well as between the surrounding formation and annulus under different operating conditions further analyzed the impact of the drill string assembly and casing programs on the complicated heat transfer

mechanism. Drawing from comparisons of the calculated results with the field data, the high accuracy of the transient heat transfer model was recommended.

2 Physical model

The circulation process of fluid in a wellbore is similar to a heat exchange system. The schematic diagram of the physical well drilling system and process is illustrated in Figure 1. Based on the transient heat transfer mechanism for the physical regions of the wellbore, four regions were considered for the heat transfer analysis. In region 1 (inside the drill string), drilling fluid enters the drill string with a specified temperature (T_{in}) at the surface and passes down the drill string in the z-direction. Its temperature (T_1) is dependent on the rate of heat convection down the drill string and the radial heat transfer with the inside wall of the drill string. In region 2 (drill string wall), the drill string wall temperature (T_2) is determined by the rate of heat exchange between the wall and flow down the drill string and up in the annulus, as well as the axial heat conduction in the drill string wall. In region 3 (annulus), fluid exits the drill string through the bit, enters the annulus at the bottom, then passes up the annulus and finally exits the annulus (T_{out}) at the surface [31, 34]. The annulus temperature (T_3) is determined by the rate of heat convection up the annulus, the rate of heat conduction between the annulus fluid and the outside wall of the drill string, and the rate of heat conduction between the wall of the well and annulus fluid. The temperature in region 4 (surrounding the wellbore/formation) corresponds to the heat conduction in the formation or the casing/cement sheath in the radial direction.

3 Mathematical model

3.1 Three-dimensional mathematical model

The heat exchange among each region of the wellbore and formation is described as a three-dimensional

transient temperature field $T(z, r, \theta, t)$. The three-dimensional governing equation of each region of the wellbore and formation systems for any direction under heat conduction and heat convection conditions is expressed as:

$$\rho C_p \frac{DT}{Dt} = \nabla \cdot (\lambda \nabla T) + \rho \nabla \cdot \mathbf{V} + q_s \quad (1)$$

To ensure that the discrete equation developed possesses the energy conservation data for each wellbore and formation region, Equation (1) defined the form of energy conservation as follows:

$$\frac{\partial}{\partial t}(\rho C_p T) + \nabla \cdot (\rho C_p T \mathbf{V}) = \nabla \cdot \left[\frac{\lambda}{C_p} \nabla \cdot (C_p T) \right] + S_T \quad (2)$$

$$S_T = \rho T \frac{DC_p}{Dt} - \nabla \cdot \left(\frac{\lambda}{C_p} T \nabla C_p \right) + \rho \nabla \cdot \mathbf{V} + q_s \quad (3)$$

The expansion of Equations (2) - (3) included a single assumption: the parametric variations including density, velocity and deviator stress tensor of fluid in the r and θ directions were neglected. Equations (2) - (3) could be expressed as a one-dimensional form[38]:

$$\frac{\partial(\rho C_p T)}{\partial t} + \frac{\partial}{\partial z}(\rho C_p v_z T) = S_T \quad (4)$$

$$S_T = \rho T \frac{\partial C_p}{\partial t} + \rho T v_z \frac{\partial C_p}{\partial z} + q_f + q_s + q_{in} \quad (5)$$

3.2 Modified Raymond model

Based on an analysis of Equations (4) and (5), Raymond made one of the best attempts at predicting the wellbore fluid temperature during the circulation stage. The temperature profiles in the drill strings, annulus and formation could be obtained by solving Equations 6 through 8. The heat generation by viscous dissipation and bit rotation, however, was neglected in the Raymond model [31]. To investigate the effect of the drill string assembly on the heat transfer mechanism of the wellbore, the Modified Raymond model considered the additional heat generated by the flow pressure drop and mechanical energy, resulting in the

principle developed as follows.

The drilling fluid entered the drill string at a specified temperature (T_{in}), then passed down the drill string.

The fluid's temperature is determined by the rates at which ① heat is transported down the drill string, ②

heat is exchanged between the drill string and the annulus fluid, ③ heat is generated by fluid friction, ④

the temperature of the drill string fluid changes with time. Consequently, the equation that describes the fluid

temperature inside the drill string as a function of time, t , and depth, z is[31]:

$$Q_m - \rho_m q c_m \frac{\partial T_1(z, t)}{\partial z} - 2\pi r_1 U [T_1(z, t) - T_3(z, t)] = \pi \rho_m c_m r_1^2 \frac{\partial T_1(z, t)}{\partial t} \quad (6)$$

As the fluid exits the drill string through the bit and flows up the annulus, its temperature is determined by

the rates at which ① heat is convected up the annulus, ② heat is exchanged between the annulus fluid and

the drill string fluid, ③ heat is exchanged between the formation borehole wall and the annulus fluid, ④

heat is generated by the fluid friction and drill string rotation, ⑤ the temperature of the annulus fluid

changes with time. Therefore, the equation that describes the temperature of fluid in the annulus as a function

of t and z is:

$$\rho_m q c_m \frac{\partial T_3(z, t)}{\partial z} + 2\pi r_1 U [T_1(z, t) - T_3(z, t)] - 2\pi r_3 h_3 [T_3(z, t) - T_4(z, t)] + Q_a = \pi \rho_m c_m (r_3^2 - r_2^2) \frac{\partial T_3(z, t)}{\partial t} \quad (7)$$

The heat transfer in the formation can be simplified as a two-dimensional conduction problem that

assumes radial symmetry around the wellbore. When the well is divided into many small segments, the

vertical heat diffusion is negligible due to a relatively small temperature difference. The conductive heat

transfer in the adjacent reservoir becomes a one-dimensional problem, where the formation temperature is

regarded as a function of injection time and radial distance [27]. An energy balance in the formation leads to

the governing equation of heat transfer in the formation in cylindrical coordinates, described as:

$$\frac{\lambda_f}{\rho_f c_f} \frac{1}{r} \frac{\partial}{\partial r} \left[r \frac{\partial T_f(r, Z, t)}{\partial r} \right] = \frac{\partial T_f(r, Z, t)}{\partial t} \quad (8)$$

As derived from the Modified Raymond model, Equations (6) and (7) represent the heat transfer models from the inside drill string and annulus, respectively. No heat transfer model of the drill string wall exists. The heat transfer regions of the wellbore consisted of the fluid inside the drill string, the drill string and the annulus fluid. The wellbore temperature distribution was determined by heat exchange among these regions. The drill string's temperature distribution was calculated by the vertical heat conduction in the drill string and the convective heat exchange between the drill string and the fluid columns surrounding it (fluid inside the drill string and annulus, respectively). The Modified Raymond model used an over-all heat transfer coefficient (U) between the drill string and annulus to express the heat exchange quantity[27, 28, 39], which could have a significant influence on the wellbore temperature distribution, and the formation was comprised predominately of rock, rather than casing or cement sheath. Thus, the Modified Raymond model was considered as having no effect on the drill string assembly and casing programs, relative to the heat transfer between the wellbore and formation.

3.3 Simplified model

The simplified model considered the heat exchange between the inner wall of the drill string and drilling fluid inside the drill string, as well as the outer wall of the drill string and annulus fluid. The effect of the casing and cement sheath was neglected [13, 32]. The transient heat transfer models of the wellbore and formation were developed as follows.

Based on the development principle of Equation (6), the fluid temperature model of the inside drill string included the rate of heat convection between the fluid and inside wall of the drill string in the r direction. The energy balance equation of the inside drill string is described as[32]:

$$Q_m - \rho_m q c_m \frac{\partial T_1(z, t)}{\partial z} - 2\pi r_1 h_1 [T_1(z, t) - T_2(z, t)] = \rho_m c_m \pi r_1^2 \frac{\partial T_1(z, t)}{\partial t} \quad (9)$$

The wall temperature of the drill string is determined by the fluid flowing velocity inside the drill string and annulus. Its energy term includes three parts: ① the first term on the left-hand side of Equation (10) accounts for the vertical heat conduction in the drill string, ② the radial heat convection exchange between the drill string and the fluid inside and outside the strings, ③ the heat accumulation in an element of the drill

string wall. The energy balance of the drill string wall is:

$$\lambda_w \frac{\partial^2 T_2(z, t)}{\partial z^2} + \frac{2r_1 h_1}{(r_2^2 - r_1^2)} [T_1(z, t) - T_2(z, t)] - \frac{2r_2 h_2}{(r_2^2 - r_1^2)} [T_2(z, t) - T_3(z, t)] = \rho_w c_w \frac{\partial T_2(z, t)}{\partial t} \quad (10)$$

Based on an analysis of Equation (7), the factors influencing the extra annulus temperature included the rate of heat convection between the annulus fluid and the drill string wall, as well as the annulus fluid and wellbore face. The energy balance in the annular is expressed as:

$$\rho_m q c_m \frac{\partial T_3(z, t)}{\partial z} + 2\pi r_2 h_2 [T_2(z, t) - T_3(z, t)] - 2\pi r_3 h_3 [T_3(z, t) - T_4(z, t)] + Q_a = \rho_m c_m \pi (r_3^2 - r_2^2) \frac{\partial T_3(z, t)}{\partial t} \quad (11)$$

where T_3 and T_4 are the temperatures of the annular fluid and borehole wall, respectively.

In the simplified model, the transient heat transfer considered only the effect of the drill string. The variation in the casing program (including the depth of casing, cement sheath, static drilling fluid and rock) surrounding the formation was not considered, whereas the wellbore wall (comprised of rock) was considered. The transient heat transfer model of the wellbore surrounding formation is the same as Equation (8).

3.3 Full-scale model

As the well depth increased, the heat exchanges and resulting temperature distributions among each region of the wellbore and formation were influenced, due to the greater complexity of the drill string assembly and casing programs. The transient heat transfer models, under actual drilling conditions, need to be developed to accurately obtain the wellbore and formation temperatures. The full-scale model incorporated the drill string in the wellbore, the casing and the cement sheath in the formation. As shown in Equations (9) - (11), the transient heat transfer models of the wellbore and the heat exchange models of the wellbore wall and surrounding formation were given as follows:

Based on the energy balance of the volume element for the wellbore wall, its quantity heat was described in four parts: ① the vertical heat conduction in the wellbore wall, ② the heat convection exchange with the annulus fluid, ③ the radial of heat conduction between the wellbore wall and cement sheath, ④ the

energy accumulation in a volume element. The mathematical model is:

$$\lambda_4 \frac{\partial^2 T_4(z, t)}{\partial z^2} + \frac{2r_3 h_3 [T_3(z, t) - T_4(z, t)]}{(r_4^2 - r_3^2)} - \frac{2\lambda_5 [T_4(z, t) - T_5(z, t)]}{(r_4^2 - r_3^2)} = \rho_4 c_4 \frac{\partial T_4(z, t)}{\partial t} \quad (12)$$

In each region of the heat transfer medium surrounding the wellbore, the heat flow is considered to be conduction only in the vertical and radial directions. Their heat transfer equations were written as:

$$\frac{\partial^2 T_i(z, t)}{\partial r_i^2} + \frac{1}{r_i} \frac{\partial T_i(z, t)}{\partial r_i} + \frac{\partial^2 T_i(z, t)}{\partial z^2} = \frac{\rho_i c_i}{\lambda_i} \frac{\partial T_i(z, t)}{\partial t} \quad (13)$$

The convective heat transfer coefficients of the computed models were obtained from literatures [18, 40].

3.4 Initial and boundary conditions

Specifying the initial and boundary conditions of different computed models is a necessity. Equation (14) describes the initial temperature profiles for each region of a well as being the geothermal gradient.

$$\begin{cases} T_1(z, t=0) = T_1(z=0, t) + G_f z \\ T_2(z, t=0) = T_2(z=0, t) + G_f z \\ T_3(z, t=0) = T_3(z=0, t) + G_f z \\ T_4(z, t=0) = T_s + G_f z \\ \text{L L} \end{cases} \quad (14)$$

The inlet fluid temperature is obtained from drilling fluid logging on site. The initial conditions of the inlet temperature can be given as:

$$T_1(z=0, t) = T_i \quad (15)$$

The fluid temperature at the exit of the drill string is the same as the fluid temperature at the entrance of the annulus, resulting in similar temperatures at the bottom-hole [32].

$$T_1(z=H, t) = T_2(z=H, t) = T_3(z=H, t) \quad (17)$$

At the interface between the wellbore wall (cement/rock) and the annular region for the fluid return, the heat quantity of flux out for the formation is the same as the flux in to the annulus that is expressed in Equation (18)[41].

$$\lambda_4 \left[\frac{\partial T_4(r, z, t)}{\partial r} \right]_{r=r_3} = \left[h_3 + \frac{2\pi\lambda_m\lambda_4}{\lambda_4 \ln(r_3/r_2) + \lambda_m \ln(r_4/r_3)} \right] [T_4(r_4, z, t) - T_3(r_3, z, t)] \quad (18)$$

The formation surface and outer radial boundary conditions are considered to have no cross-boundary heat flows, which are described in Equations (19) and (20), respectively.

$$\left. \frac{\partial T_f(r, z, t)}{\partial z} \right|_{z=0} = 0, \text{ or } T_f(r, z=0, t) = T_s \quad (19)$$

$$\left. \frac{\partial T_f(r, z, t)}{\partial r} \right|_{r \rightarrow \infty} = 0, \text{ or } T_f(r \rightarrow \infty, z, t) = T_s + G_f z \quad (20)$$

4 Numerical solutions

4.1 Computational grid formulation

Designing equations to obtain temperature distributions over time is complicated. Developed models incorporating solution methods are based on the fully implicit finite difference techniques. The wellbore and the adjacent formation are represented by a two-dimensional mesh grid, including a number of radial elements. This is due to the casing program and a variable number of vertical elements, depending on well depth. Each radial element corresponds to a different portion of the wellbore cross-section from the inside drill string into the formation [41-44]. Five regions, or components, as shown in Figure 2, are identified as necessary in the mesh grid: (1) the inside drill string ($0 \leq r \leq r_1$), (2) the drill string wall ($r_1 < r \leq r_2$), (3) the annular fluid ($r_2 < r < r_3$), (4) the interface between the wellbore wall (casing/rock) and the annular fluid ($r = r_3$), (5) the cement sheath/ stagnant fluid/ rock ($r > r_3$).

4.2 Discrete model

Based on the above computational grid unit, a set of partial differential equations can be presented as a finite difference form using the finite differences technique for each element of the grid to describe the transient heat exchange for an implicit form. A set of non-linear algebraic equations are then solved using an

iterative method to achieve convergence of the simulated temperature to the measured data. Equation (8) of finite difference can be defined as follows.

The heat transfer model of the cylindrical wall is given as:

$$\frac{\lambda}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) = \frac{\lambda}{r_i} \left[\frac{1}{\Delta r} \left(r_{i-0.5} \frac{\partial T_{i-0.5}}{\partial r} - r_{i+0.5} \frac{\partial T_{i+0.5}}{\partial r} \right) \right] \quad (21)$$

$$\text{where, } \frac{\partial T_{i-0.5}}{\partial r} = \frac{1}{\Delta r} (T_{i-1} - T_i), \quad \frac{\partial T_{i+0.5}}{\partial r} = \frac{1}{\Delta r} (T_i - T_{i+1})$$

Based on Fourier's Law, the heat through a unit of area is expressed as:

$$\phi = 2\pi r \lambda \frac{\partial T}{\partial r} = 2\pi r_{i+0.5} \lambda \frac{T_{i+1} - T_i}{r_{i+1} - r_i} \quad (22)$$

The heat of the cylindrical wall is also defined as:

$$\phi = \frac{2\pi\lambda(t_1 - t_2)}{\ln(r_2'/r_1')} = \frac{2\pi\lambda(T_{i+1} - T_i)}{\ln(r_2'/r_1')} \quad (23)$$

Equation (22) is equivalent to Equation (23), thus the following equation appears as:

$$r_{i+0.5} = \frac{r_{i+1} - r_i}{\ln(r_{i+1}/r_i)} ; \quad r_{i-0.5} = \frac{r_i - r_{i-1}}{\ln(r_i/r_{i-1})} \quad (24)$$

Based on Equation (24), Equation (21) is also expressed as:

$$\frac{\lambda}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) = \frac{\lambda}{r_i \Delta r_i} \left[\frac{1}{\ln(r_i/r_{i-1})} (T_{i-1} - T_i) - \frac{1}{\ln(r_{i+1}/r_i)} (T_i - T_{i+1}) \right] \quad (25)$$

The time discretization of the right side of Equation (8) is the discretization equation represented as:

$$\frac{\lambda}{r_i \Delta r_i} \left[\frac{1}{\ln(r_i/r_{i-1})} (T_{i-1} - T_i) - \frac{1}{\ln(r_{i+1}/r_i)} (T_i - T_{i+1}) \right] = \frac{T_f(r_w, Z, t)_{i,j}^{n+1} - T_f(r_w, Z, t)_{i,j}^n}{\Delta t} \quad (26)$$

Application of the above definitions allows the equation for each node to be written in a single, generalized vector form:

$$\alpha_{ij} T_{i-1,j}^{n+1} + \beta_{ij} T_{i,j}^{n+1} + \gamma_{ij} T_{i+1,j}^{n+1} + \delta_{ij} T_{i,j-1}^{n+1} + \lambda_{ij} T_{i,j+1}^{n+1} = C_{ij} \quad (27)$$

The matrix form of the finite difference for each node is given:

$$a\bar{T}^{n+1} = \bar{C} \quad (28)$$

The finite difference equations are solved by the fast successive over-relaxation (*SOR*) iteration method.

The following general form for each node is expressed as:

$$T_{i,j}^{v+1} = \frac{\omega}{B_{i,j}} \left[F_{i,j} T_{i,j}^N + A_{i,j} T_{i-1,j}^{v+1} + C_{i,j} T_{i+1,j}^v + D_{i,j} T_{i,j-1}^{v+1} + E_{i,j} T_{i,j+1}^v \right] + (1-\omega) T_{i,j}^{v,N+1} \quad (29)$$

The calculation accuracy depends on the meshing elements and the size of the interval values. In general, it is observed that the vertical element size has to be less than 3% of the total well depth to ensure that the annulus temperature profile remains independent of the vertical element size [32]. The radial element size is increased as the exponential relationship.

4.3 Model solving process

The heat transfer model was solved using the iterative method as follows: ① input the physical parameters of the wellbore and formation, including the heat transfer parameters, fluid density, fluid temperature, et al, ② divide the mesh grids of the wellbore and formation in the vertical and radial directions, and discretize the mathematical models in space and time, ③ define the initial temperature distributions along the wellbore and formation that input the transient heat transfer models ④ predict the wellbore temperature ($T_{3,j}^{t_0+\Delta t}$) at $t = t_0 + \Delta t$ based on the initial data ($T_{3,j}^{t_0}$), ⑤ compare the calculated outlet temperature ($T_{3,0}^{t_0+\Delta t}$) with measured data, and estimate the computational accuracy when satisfied with tested data; return to calculate if not satisfied, ⑥ correct the calculation until the wellbore temperature accuracy is satisfied, namely $\|T_{3,j}^{t_0+\Delta t} - T_{3,j}^{t_0}\| \leq \varepsilon_T$. If not satisfied, return to ③ and correct the calculation, ⑦ the calculation process is over at $t = t_0 + \Delta t$. The temperature is calculated at the next time step.

5. Results and discussions

5.1 Basic data

The basic calculation data in this study, derived from literature [32], are presented as follows.

5.2 Results analysis of different numerical models

5.2.1 Contrastive analysis of results during circulation stage

The annulus temperature profiles, as a function of well depth under different circulation times, are calculated by three mathematical models - the Modified Raymond, Simplified and Full-scale models - and are presented in Figure 3 (a-c). The annulus temperatures slightly increased with the increase of circulation time beyond the intersection point between the annulus temperature and initial formation temperature. Conversely, the annulus temperatures decreased below the intersection point. The decrease of the annulus temperature at initial circulation is larger than at later circulation; namely, the drop scale from 0 to 5 h is much more than that from 5 h to 10 h. The rapid decrease of the annulus temperature occurred at the section of bottom-hole rather than at the top section. This is attributed to the annulus fluid constantly obtaining heat quantity from the formation due to the convection and conduction at the bottom-hole section, leading to an increase of the annulus fluid temperature and decrease of the formation temperature. At a certain depth, the annulus fluid temperature was larger than formation temperature when the fluid was flowing up in the upper annulus. Although the heat quantity of the annulus above the intersection point could transfer into the inside drill string and formation via heat exchange and heat conduction, when the annulus fluid flows up, more heat quantity of the annulus fluid was absorbed from the formation in the bottom-hole than the heat quantity transferring to the inside drill string and formation in the top section of the well. A higher annulus fluid temperature than formation temperature in the top section results

In addition, as seen from Figure 3 (a-c), when the section was over 3000 m, the annulus fluid temperature, calculated by the Modified Raymond method, gradually decreased with an increase in circulation time; the calculation results of the Simplified model indicated little change of annulus temperature as a variation of circulation time. The Full-scale model showed no obvious change in the annulus temperature as the circulation time increased. The consideration of varying factors in the models leads to differences in calculation results. The drill string assembly was neglected in the Modified Raymond model. It may be possible that the heat exchange occurred between the formation and the wellbore fluid, including the inside drill string and annulus, rather than only the annulus fluid. If so, this would lead to more heat quantity from the formation transferring into the annulus due to the increase of the fluid, resulting in a greater variation of annulus temperature. The drill string assembly's influence on temperature distribution of the annulus was considered in the Simplified model. As the heat conduction and convection of the annulus temperature occurred between the drill string and formation, the annulus temperature varied little, as compared to the results of the Modified Raymond model. In the Full-scale model, when the depth of the well was above 3000 m (Table 1), the formation was sealed by casing and cement. The heat conductivity coefficient of the casing and formation are 43.75 and 2.25 w/m.°C, respectively (Table.1), and the ratio of both values is 19.4. The heat exchange between the annulus and cement occurred rapidly, leading to an easily reached thermodynamic equilibrium state. Almost no change occurred in the annulus fluid temperature with the variation of circulation time in the cementing section.

The annulus fluid temperature profiles at 10 h circulation are compared using the Modified Raymond, Simplified and Full-scale mathematical models and are displayed in Figure 3 (d). The calculated changes of the annulus temperature appeared in the order: Modified Raymond > Full-scale > Simplified. The singularity phenomenon is explained as follows: the heat exchange between the whole wellbore fluid (inside drill string

and annulus) and formation was approximately considered in the Modified Raymond model, as the drill string assembly was neglected. This resulted in a greater heat quantity of formation carried from the bottom-hole to surface due to of the increase of fluid volume during 10 h circulation. A rapid drop of annulus temperature occurred. The findings were in accordance with the results of Figure 3 (a). The higher annulus temperature calculated by the Simplified model was obtained because the heat transfer of the wellbore structure was not considered, resulting in less heat in the formation carrying through the wall of the drill string to the well after a long circulation. The effect of the drill string assembly and casing program on the annulus temperature distribution was considered in the Full-scale model. When the annulus fluid flowed up, the heat exchange of the annulus fluid occurred between the fluid inside the drill string and the formation (casing, cement, static drilling fluid and rock), leading to the calculated data of the Full-scale model being distributed between the Modified Raymond and Simplified model.

To further understand the heat transfer mechanism, the temperature difference distribution profiles between the annulus and inside drill string fluids were investigated using different mathematical models during the circulation stage. Figure 4 (a-c) shows a great temperature difference between the annulus and drill string, arising from the heat exchange efficiency among each unit of the wellbore and surrounding formation. Under different circulation times, the annulus temperatures calculated by the models were all higher than the inside drill string temperature in the circulation process, aside from the calculated data obtained by the Full-scale model near the outlet. This phenomenon indicates that the annulus heat quantity transferred into the inside drill string through the wall of the drill string induces the heat exchange between them during the circulation stage. The temperature of the volume units at surface was set at 16 °C and the inlet temperature at 24 °C. As a result, the inlet temperature was higher than the outlet temperature of the annulus at surface; the negative difference between them, as the circulation time increased, is shown in

Figure4 (c). The heat transfer of the medium near surface are comprised of rock for the Raymond and Simplified models rather than that of the casing, leading to more heat quantity transferred into the formation and a decline in the annulus fluid temperature. These two factors, applied in the Full-scale model, induced a decrease in the annulus temperature.

Additionally, as shown in Figure 4 (a, b), the maximum temperature difference between the annulus and inside drill string occurred at a depth of 2300 m for the Modified Raymond and Simplified models. This may be due to the well's total depth of 4600 m. The fluid gained heat quantity from the formation below the section of annulus and transferred heat into the formation and inside drill string above that section. This leads to a lower heat loss at any depth point of the annulus, except for the depth of 2300 m where the maximum temperature difference occurred. In the Full-scale model, however, the maximum temperature difference data appeared at a depth of 3000 m, which was the joint point of the casing and rock. As seen from Table 1, the wellbore was comprised of rock, rather than casing, below the depth of 3000 m. The heat quantity of the annulus fluid absorbed from the formation in the open-hole section could not rapidly transfer to the upper formation and inside drill string. This resulted in an elevation in the annulus temperature and an increase in the temperature difference. The heat quantity could be a timely transfer to the cement section of the formation due to a higher coefficient of heat conduction of the casing, leading to a decline in the annulus temperature and a decrease in the temperature difference.

To gain insight into the heat transfer mechanism between the wellbore and formation during the circulation stage, the temperature difference profiles were studied at circulation 10 h using three mathematical models. As illustrated in Figure 5 (a-c), the temperature difference profiles between different radial distances of the formation temperature and annulus temperature were met at various points for each model. The intersection points of depth are 1000 m, 2000 m and 3000 m, corresponding to Modified

Raymond, Simplified and Full-scale models, respectively. Below the depth of the intersection point, the formation temperatures were higher than the annulus temperature. Conversely, above the depth of the intersection point, the formation temperatures were lower than the annulus temperature. Over all, the temperature difference between the distances of the surrounding formation and annulus, below the depth of the intersection point, was larger than that of above depth. Two reasons account for this. First, the temperature surrounding the borehole wall of the formation continuously declined. Second, the temperature difference increased when far distances existed between the formation and annulus. Here, the annulus fluid constantly absorbed the heat quantity of the formation at the bottom-hole during the circulation process. The temperature difference becomes small, however, when the heat quantity of the formation was carried from the bottom-hole to the surface, thereby heating the upper section of the formation. The variation tendency of the temperature difference, shown in Figure 5 (a-c), was in accordance with that of Figure 3 (a-c).

In addition, the least temperature difference, located in the upper section of the well, was calculated by the Full-scale model, due to a higher coefficient of heat transfer for the cement section. Based on these results, one could infer that the annulus temperature measured by the temperature measuring instrument in the cement section was determined to be the initial formation temperature. In the open-hole section, the initial formation temperature was obtained by the measured data and by mathematical algorithms, such as the integral proportional control principle.

5.2.2 Contrastive analysis of results during shut-in stage

The annulus temperature profiles, using different mathematical models during the shut-in stage, are presented in Figure 6 (a-c). The annulus temperature gradually recovered to its initial formation temperature and reached thermodynamic equilibrium state with the increase of shut-in time. The Modified Raymond method calculations revealed that the greatest variation of the annulus temperature occurred between the

surrounding formation and annulus during the circulation period. Above the intersection point of the annulus temperature and initial formation temperature, the annulus temperature slightly decreased with the increase of shut-in time. This occurred because the annulus temperature was higher than the formation temperature during the circulation stage, as shown in Figure 3 (a-c) and Figure 5 (a-c). The heat quantity of the annulus transferred to the formation by heat conduction, decreasing the annulus temperature. Below the intersection point, the annulus temperature elevated as the shut-in time increased. Here, the annulus temperature was lower than the formation temperature due to the heat quantity absorbed from the formation during the circulation stage. The heat quantity of the formation transferred to the annulus, increasing the annulus temperature.

Through observations comparing Figure 3 (c) with Figure 6 (c), the annulus temperature was nearly equal to the formation temperature in the cement section, aside from the outlet, due to the higher coefficient of heat conduction in the casing. The thermodynamic equilibrium state by heat conduction was easily reached in the shut-in stage. Figure 6 (d) shows the compared data calculated by different models at a shut-in of 10 h. The annulus temperature calculations are in the order: Modified Raymond > Simplified > Full-scale models, respectively, except for the temperature near the outlet. The distribution tendency of the data was consistent with that of the circulation stage, which verified the correction of the developed and solved models. Based on the performance of the Full-scale model from the results of Figure 3 (d) and Figure 6 (d), the annulus temperature would be undervalued if the Modified Raymond model were adopted. The Simplified model, however, obtained the opposite results.

Figure 7 (a-c) presents the temperature difference distributions between the annulus and inside drill string obtained during the shut-in period. An increased shut-in time leads to a smaller temperature difference as the wellbore temperature gradually recovered to the initial temperature. The recovery rate of temperature at the

1 initial shut-in was faster than that of the latter. In comparing Figure 4 (a-c) with Figure 7 (a-c), even though
2 the variation tendency of the temperature difference was the same as that of the circulation stage, the
3 temperature difference is smaller than that of the circulation stage (ranging from -1°C to 1°C). The smallest
4 temperature difference was calculated by the Modified Raymond model. The annulus and inside drill string
5 fluids, taken as a whole due to neglecting the heat conduction of the drill string, resulted in the decrease of
6 the temperature difference. Because the drill string assembly consisted of drill pipe (4000 m) and drill collar
7 (600 m) at their joint point, the fluctuation of the temperature difference was generated due to the differing
8 wall thicknesses of the components. The difference of the drill string assembly and the efficiency of heat
9 conduction were further affected. In addition, fluctuation of the temperature difference also occurred at the
10 casing shoe point for the Full-scale method, indicating that the casing program during the shut-in stage
11 influenced wellbore temperature distribution.

12 Comparing the results of Figure 5 (a-c) and Figure 8 (a-c) reveals that the temperature difference between
13 the surrounding formation and annulus in the shut-in stage was lower than that of the circulation stage. As
14 the annulus fluid was in static state, no heat quantity was carried from the annulus to the surface. The heat
15 exchange between the formation and annulus, continuously generated by heat conduction, led to a smaller
16 temperature difference than that of the circulation stage. The smallest temperature differences occurred near
17 the depths of 1000 m, 2600 m and 3000 m for the Raymond, Simplified and Full-scale models, respectively.
18 Above the depth of 1000 m and 2600 m, the annulus temperature was higher than the formation temperature.
19 The heat quantity of the annulus transferred to the formation for the Raymond and Simplified models; below
20 those depths, the annulus temperature was lower than the formation temperature, resulting in the annulus
21 fluid absorbing the heat quantity from the formation. The above results were in accordance with Figure 5 (a,
22 b).

In the cement section, for the Full-scale model, the annulus and formation temperatures were almost equal, aside from the near outlet. During the shut-in stage, the measured annulus data from the down-hole temperature measuring instrument in the cement section may be determined as the initial formation temperature. In the open-hole section, two methods were used to obtain the initial formation temperature: ① accessing the heat exchange between the annulus and formation, which reaches the thermodynamic equilibrium state during a long shut-in, ② combining the measured data and developed transient heat transfer model to obtain the initial temperature. Based on the Full-scale mathematical model in the circulation and shut-in stages, the initial formation temperature could be extrapolated at any time in the drilling process.

5.2.3 Analysis of temperature distribution for critical depth point during the circulation and shut-in stages

The outlet, the middle of the annulus and the bottom-hole temperature distributions, with the variation times of circulation and shut-in, were investigated by three models, with results presented in Figure 9 (a-c). The outlet temperatures increased in the circulation stage and in the shut-in stage for the three calculated models. The reasonable explanations are that the initial outlet temperature (16 °C) was lower than the inlet temperature (24°C) during the circulation stage. Near the surface, the heat quantity of the inside drill string was transferred to the annulus by convection and conduction. Concurrently, the heat quantity in the bottom-hole was carried to the surface due to the fluid flow up, resulting in an increase in the outlet temperature. From the results in Figure 7 (a-c), it is easy to discern that the temperatures surrounding the formation were also lower than the annulus temperature during the shut-in period. At this point, the heat quantity of the annulus was transferred into the formation and the annulus temperature at the outlet decreased.

In addition, as shown in Figure 9 (a), in the circulation and shut-in stages, the sequence of calculated outlet temperatures from high to low are Simplified > Modified Raymond > Full-scale. The reasons are: ① the heat conductivity coefficient of the casing was larger than that of the formation, as the casing program was neglected in the Simplified model. This lead to less of a heat exchange occurring between the annulus and the formation. In turn, the outlet temperature increased, as shown in Figure 3 (b, d). ② As the casing program and drill string assembly were not considered in the Modified Raymond model, it is surmised that the entirety of the fluids in the inside drill string and annulus generated a heat exchange with the formation, resulting in more heat quantity absorbed from the formation than that of the Simplified model. Subsequently, the outlet temperature of the Modified Raymond model was lower than that of the Simplified model. ③ As the drill string assembly and casing program were analyzed during the development and solving of the Full-scale model process, much more of the heat quantity of the annulus was transferred into the formation, as shown in Figures 5 (c) and 8 (c). The lowest outlet temperature was calculated by the Full-scale model.

As shown in Figure 9 (b), in the middle depth of 2300 m, the largest variation tendency was obtained using the Modified Raymond model during the circulation and shut-in stages. The slow variation tendencies were acquired using the Simplified and Full-scale models at different times. Based on the analysis of results from Figures 3, 5, 6, and 8 (a-c), the temperature difference is a result of the drill string assembly and casing programs.

Figure 9 (a, b) compared with Figure 9 (c) shows that the variation tendency of the temperature at the bottom-hole was larger than that of the middle and outlet. Figure 9 (c) reveals that the variation tendency of the temperature for the Modified Raymond model was much larger than that of the Simplified and Full-scale models; similar variation tendencies of these two models were obtained. This is because the wall of the wellbore from 3000 m to 4600 m (the depth of the third casing was 3000m) was comprised of rock for the

Simplified and Full-scale models. Thus, the heat exchange in that section was nearly equivalent for the two models, resulting in the same temperature at the bottom-hole. In addition, the shut-in time was much longer than the circulation time. **If the annulus temperature at the bottom-hole was recovered to the initial formation temperature, the heat exchange between the wellbore and formation in the circulation stage was greater than that in the shut-in stage, generated by convection and conduction.**

6. Field application

The OM-41C geothermal well in Kenya was used as a practical example to investigate the field application of the three transient heat transfer models. The characteristics of OW-41C are: 1900 m above sea level; from the surface to target stratum, the well consists predominately of rhyolite, trachyte, basalt and a small quantity of tuff; total well depth is 3000 m. The basic data of this well is given as follows.

Figure 10 (a) shows that during the different shut-in periods, the temperature profiles in the OM-41C geothermal well were measured at 1 h, 10 h, 3 days, 11 days, 31 days and 60 days. The wellbore temperature gradually recovered with an increase of shut-in time, and then reached the thermodynamic equilibrium state between the wellbore fluid and formation. Based on evaluation of the temperature log spanning the shut-in of 60 days, **it is assumed that the temperature approached its initial formation temperature due to the long thermal recovery period[45, 46].** The amplitude of temperature variations in the upper section, from surface to 750 m, were lower than that of the third casing section (750 – 3000m); **the number of layers of casing for the upper section was greater than that of the bottom-hole section, which resulted in an increase of heat transfer.** As presented in Figure 10, **temperature differences at unit time during the initial shut-in stage (1 – 10 h) was higher than that of later times (3 – 61 days).** In addition, the circulation time (10 h) can be much shorter than that of the shut-in time (60 days) if, after circulation, the annulus temperature recovered to the

initial formation temperature. The practical measured data was in solid agreement with the analyzed results in Section 5, validating the correctness of the numerical model.

The simulated results using the Modified Raymond, Simplified and Full-scale numerical models are illustrated in Figure 10 (b). Although the variation tendencies exhibited the same characteristics for predicted and measured data, some differences in calculated accuracy appear. To investigate the precision of various models in more depth, the comparison of the predicted results and measured data were presented as a percentage deviation. As seen from Figure 10 (b), the comparison results of the temperature differences between the Full-scale model calculation and logged data (60 days) indicated agreement, with the average error at 0.95%. This offered the best temperature approximation to the field measured data among the three numerical models. The average errors of the Simplified and Modified Raymond models were 1.52% and 2.15%, respectively. A comparison of the three models' results prove that the drill string assembly and casing programs had a significant influence on heat transfer among each region of the wellbore and formation systems. The accuracy of the predicted results would be influenced if the thermal conductive components were neglected during the development and solution processes of the model. Each region of the wellbore and formation, as a system, needs to be considered in the heat transfer models. While this adds more complexity to the task, it also improves precision in reflecting the real transfer heat processes inside and outside a well.

7 Conclusions

(1) In the casing cement section, the annulus and formation temperatures underwent minimal change with variations in circulation and shut-in times, aside from near the surface. In contrast, in the open-hole section, the temperatures rapidly decreased during the circulation stage and gradually increased during the shut-in stage.

(2) In the circulation and shut-in stages, the temperature difference distributions between the annulus and

inside drill string were significantly influenced by the drill string assembly and casing programs. The temperature fluctuations occurred at connection points between the drill collar and drill pipe, as well as the casing shoe.

(3) The change variation in degrees of temperature calculated by the numerical models was in the order: Modified Raymond > Simplified > Full-scale models, respectively.

(4) Simulated results from the Full-scale model along the wellbore corresponds the best with the measured field data through comparison of the three numerical models. These results validate that the drill string assembly and casing programs play an important role in determining the accuracy and applicability of the developed transient heat transfer model.

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Nomenclature

C_p Specific heat at constant pressure [J/kg.°C]
 c_f Specific heat capacity of the formation [J/kg.°C]
 c_m Specific heat capacity of the drilling fluid [J/kg.°C]

1	c_w	Specific heat capacity of the drill string [J/kg.°C]
2	c_4	Specific heat capacity of the borehole wall medium [J/kg.°C]
3	$\overset{\circ}{\underset{\circ}{\mathbb{P}}}$	Deviatoric stress tensor of the fluid flow in the wellbore
4	G_f	Geothermal gradient [°C/100m]
5	h_1	Convection coefficient of the inner wall of the drill string [W/(m ² .°C)]
6	h_2	Convection coefficient of the outer wall for the drill string [W/(m ² .°C)]
7	h_3	Convection coefficient of the borehole wall [W/(m ² .°C)]
8	i	Node number in the r direction
9	j	Node number in the z direction
10	n	Time node
11	Q_a	Energy source of unit length inside the annular [W·m ⁻¹]
12	Q_m	Energy source of unit length inside the drill string [W·m ⁻¹]
13	q_s	Heat source term [W·m ⁻¹]
14	q_f	Heat resulting from fluid friction losses [W·m ⁻¹]
15	q_{in}	Heat resulting from heat exchange among regions [W·m ⁻¹]
16	q	Flow rate of the drilling fluid [m ³ /s]
17	r_f	Formation radius [m]
18	r_1	Inside radius of the drill string [m]
19	r_2	Outer radius of the drill string [m]
20	r_3	Wellbore radius [m]
21	r_1', r_2'	Radical radius of formation [m]
22	T_1	Fluid temperatures inside the drill string [°C]
23	T_2	Wall temperature of the drill string [°C]
24	T_3	Annular fluid temperature [°C]
25	T_4	Borehole wall temperature [°C]
26	T_f	Formation temperature [°C]
27	T_i	Inlet temperature [°C]
28	T_o	Outlet temperature [°C]
29	T_s	Surface temperature [°C]
30	t	Time variable [s]
31	U	Overall heat transfer coefficient between the drill string and annulus [W·m ⁻² .°C ⁻¹]
32	$\overset{1}{\underset{1}{V}}$	Velocity vector of the fluid in the wellbore
33	z	Depth variable [m]
34		
35	Greek letters	
36	ρ	Medium density [kg/m ³]
37	ρ_f	Formation density [kg/m ³]
38	ρ_m	Density of the drilling fluid [kg/m ³]
39	ρ_w	Density of the drill string [kg/m ³]
40	ρ_4	Density of the borehole wall medium (casing/rock) [kg/m ³]
41	λ	Thermal conductivity [W·m ⁻¹ .°C ⁻¹]

- 1 λ_f Thermal conductivity of the formation [$\text{W}\cdot\text{m}^{-1}\cdot^\circ\text{C}^{-1}$]
2 λ_w Thermal conductivity of the drill string [$\text{W}\cdot\text{m}^{-1}\cdot^\circ\text{C}^{-1}$]
3 λ_4 Thermal conductivity of the borehole wall (casing/rock) [$\text{W}\cdot\text{m}^{-1}\cdot^\circ\text{C}^{-1}$]
4 λ_5 Thermal conductivities of cement sheath/rock [$\text{W}\cdot\text{m}^{-1}\cdot^\circ\text{C}^{-1}$]
5 Φ Heat through the unit area [J/m^2]
6 α_{ij} , β_{ij} , γ_{ij} , δ_{ij} , and λ_{ij} Matrices coefficients
7 *SOR* Gauss-Seidel iterative method if ω is equal to 1 in the equation (29); *SOR* is the over relaxation
8 method if ω is more than 1; *SOR* is the under relaxation method if ω is less than 1.
9
10 **Subscript**
11 1,2,3,4,5 inside drill string, wall of drill string, annulus, borehole, cement sheath/rock, respectively
12 *f* formation
13 *m* drill fluid

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Figures and Tables Captions

Figure 1. The schematic diagram of heat exchange between wellbore and formation

Figure 2. Schematic diagram of mesh grids of the wellbore and formation, (a) planform, (b) profile map.

Figure 3. The annulus temperature profiles for mathematical models under circulation condition.

Figure 4. The temperature difference profiles between annulus and inside drill string for different mathematical models during circulation stage.

Figure 5. The temperature difference profiles between formation and annulus for different mathematical models at circulation 10 h.

Figure 6 The annulus temperature profiles for three mathematical models during the shut-in stage

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Table 1 Basic calculation parameter of a well from the reference [32].

Table 2 Basic calculation parameter of The OM-41C geothermal well

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Modified Raymond model

(b) Simplified model

(c) Full-scale model

(d) Data comparison at circulation 10 h

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(a) Modified Raymond model

(b) Simplified model

(c) Full-scale model

Figure 4. The temperature difference profiles between annulus and inside drilling string for different mathematical models during circulation stage.

(a) Modified Raymond model

(b) Simplified model

(c) Full-scale model

Figure 5. The temperature difference profiles between formation and annulus for different mathematical models at circulation 10 h.

(a) Modified Raymond model

(b) Simplified model

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(d) Data comparison at shut-in 10 h

Figure 6. The annulus temperature profiles for three mathematical models during the shut-in stage

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Figure.8. The temperature difference profiles between formation and annulus for the different models at the shut-in 10 h

(a) Outlet temperature

(b) Temperature of middle annulus

(c) Bottom-hole temperature

Figure 9. The annulus temperatures at outlet, middle and bottom-hole during the circulation and shut-in stages

(a)

(b)

Figure 10. The logged and predicted temperature profiles of OM-41C geothermal well in shut-in stages. (a) The measured data in different shut-in periods; (b) Comparison of measured data and predicted data for different numerical models.

Table 1 Basic calculation parameter of a well from the reference [32]

Parameter style	Specific parameter	Inner diameter (mm)	Outer diameter (mm)	Depth (m)	Cementing section (m)	Heat cap (J/kg.°C)	Thermal cond (w/m.°C)		
Drill string assembly	Drill pipe	151	168	4000	\	400	43.75		
	Drill collar	70	171	\	\				
Casing program	First casing	486	508	600	0				
	Second casing	318	340	1500	300				
	Third casing	224	244	3000	1400				
Thermal conductivity of the drill fluid			1.75 w/m.°C		Thermal conductivity of the cement			0.7 w/m.°C	
Thermal conductivity of the formation			2.25 w/m.°C		Thermal capacity of the drill fluid			1600 J/kg.°C	
Thermal capacity of the cement			2000 J/kg.°C		Thermal capacity of the formation			800 J/kg.°C	
The total depth of well			4600 m		The diameter of open hole			213mm	
Flow rate of drill fluid			13.2 l/s		Surface temperature			16°C	
Geothermal gradient			2.23 °C/100		Inlet temperature			24°C	

Table 2 Basic calculation parameter of The OM-41C geothermal well

Parameter style	Specific parameter	Inner diameter (mm)	Outer diameter (mm)	Depth (m)	Heat cap (J/kg.°C)	Thermal cond (w/m.°C)	
Drill string assembly	Drill pipe	112	127	\	400	43.75	
	Weigh drill pipe	102	127	57.6			
	Drill collar	71.4	165.1	109.8			
Casing program	First casing	315.3	339.7	306			
	Second casing	220.6	244.5	750			
	Third casing	152.5	177.8	3000			
weight on bit			120-150 KN		Thermal conductivity of the drill fluid		0.85 w/m.°C
rotational speed			60 rpm		Thermal conductivity of the cement		0.7 w/m.°C
flow rate			22 L/s		Thermal conductivity of the formation		2.25 w/m.°C
fluid density			1050 kg/m³		Thermal capacity of the drill fluid		1200 J/kg.°C
fluid viscosity			0.052 Pa s		Thermal capacity of the cement		2000 J/kg.°C
The diameter of open hole			215mm		Thermal capacity of the formation		900 J/kg.°C
Circulation time			10 h		Surface temperature		31°C