Thermal design of large plate-fin heat exchanger for cryogenic air separation unit based on multiple dynamic equilibriums

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Abstract: This paper presents a thermal design method of large plate-fin heat exchanger (PFHE) for cryogenic air separation unit (ASU) based on multiple dynamic equilibriums (MDE). The enlarging of PFHE brings cascading thermal design difficulties. Renault number of multi-streams in long passages fluctuates in wide range which generates various working conditions. The fin type of each passage and the Re for flowing streams are pre-set firstly. Using single-banking with top priority, the Recursive Prior Plug-In (RPP) method is proposed to complete permutation of hot streams and cold streams. The preferable passage arrangement is selected and determined according to the variance of the cumulative heat load. The initial temperature of each ordered stream in all the passages is then obtained. The temperature difference sequence between the one-sided adjacent streams is built. The large heat transfer area in plane perpendicular to the flowing direction is divided into MDE according to the temperature difference sequence. For each equilibrium, the heat balance equations between hot quantity and cold quantity are built by Fourier's law of heat conduction and heat convection. The final equilibrium temperature of each ordered stream in all the passages can be obtained by iterative solving the simultaneous differential equations for all MDE. Each long stream is also divided into MDE along flowing direction and modelled by energy conservation principle in enthalpy change to obtain the actual phase. The heat transfer process about Re is divided into MDE according to the dynamic equilibriums between heat transfer and frictional resistance to determine the optimal fins under different working conditions. The heat transfer experiment is completed to verify the proposed method. The MDE is applicable for various stacking patterns of numerous long hot streams and cold streams under variable working conditions. The proposed method is especially useful for thermal design of large energy-efficient PFHE for ASU.

Key words: Cryogenic air separation unit (ASU); Large plate fin heat exchanger (PFHE); Thermal Design; Multiple Dynamic Equilibriums (MDE); Stacking Pattern; Working Condition

1 Introduction

The cryogenic air separation unit (ASU) separates atmospheric air into oxygen, nitrogen, argon and rarely other noble inert gases. Stimulated by the increasing needs for ASU throughout the industry, the trend of ASU is high productivity with remarkable energy-efficiency^{[1][2]}. The plate fin heat exchanger (PFHE) transfers heat by condensation, evaporation and liquefaction for ASU. The thermal efficiency of PFHE has remarkable effects on energy-efficiency of ASU^[3].

In recent years, many papers^{[4][5][6]} focus on heat transfer modeling, thermal efficiency, heat transfer experiment, simulation and visualization, optimization design.

Among them, in stacking pattern research ^[7], Prasad ^{[8][9]} researched the multistream PFHE based on stacking pattern of stream passages. Ghosh et al. ^[10] proposed optimum stacking pattern for multi-stream PFHE through a genetic algorithm. Zhao et al. ^[11] studied layer pattern optimization model for multi-stream plate-fin heat exchanger using genetic algorithm.

In thermal modeling, simulation and visualization research, Ref.[12] built multidimensional numerical model of heat exchanger. Hasan et al.^[13] researched operational modeling of multistream heat exchangers with phase changes. Hajabdollahi et al.^[14] proposed CFD modeling and multi-objective optimization method of compact heat exchanger using CAN method. Ref.^[15] modeled and simulated rectangular plain fin, strip offset fin, perforated fin, and wavy fin. Speetjens et al.^[16] studied visualisation of heat transfer in 3D unsteady flows.

In thermal experimental research, Ref.^[17] studied configuration parameters design and optimization for PFHE with serrated fin by multi-objective genetic algorithm. Khan et al.^[18] introduced experimental investigation of single phase convective heat transfer coefficient in a corrugated plate heat exchanger for multiple plate configurations. Gherasim et al.^[19] studied heat transfer and fluid flow experiment. Kotcioglu et al.^[20] researched experimental investigation for optimization of design parameters in a rectangular duct with plate-fins heat exchanger by Taguchi method.

In thermal optimization design, Ghosh et al.^[21] studied alternate algorithm for the analysis of multi-stream. Reneaume et al.^[22] proposed simulated annealing optimization method for PFHE. Ref.[23] introduced neural networks method cooperated with genetic algorithms optimization. Najafi et al.^[24] proposed multi-objective optimization of a plate and frame heat exchanger via genetic algorithm. Yousefi et al.^{[25][26]} proposed an imperialist competitive algorithm and an improved harmony search algorithm for optimal design of PFHE. Zarea et al. ^[27] studied optimal design of PFHE by a Bees Algorithm. Hajmohammadi et al.^[28] studied optimal design of unequal heat flux elements for optimized heat transfer inside a rectangular duct. Montanez-Morantes et al. ^[29] simulated single-phase multistreams in PFHE. Turgut ^[30] proposed hybrid chaotic quantum behaved particle swarm optimization algorithm for thermal design of PFHE.

In Ref. [31], the mean square error of cumulative heat load is reduced. Ref. [32] studied serrated fin by multi-objective genetic algorithm.

In conclusion, the above papers promote the research of PFHE in either theory or experiment. Most published studies are based on the assumption that there exist thermal equilibrium between the two adjacent streams. The assumption is feasible for the single-banking of few streams. However, the enlarging of PFHE brings cascading thermal design difficulties. Renault number of multi-streams in long passages fluctuates in wide range which generates various working conditions. Various stacking patterns including multiple-banking and hybrid-banking are inevitably used to complete passage arrangement. It is difficult to complete design of large PFHE by simply using the current methods. Therefore, a thermal design method of large plate-fin heat exchanger (PFHE) for cryogenic air separation unit (ASU) based on multiple dynamic equilibriums (MDE) is proposed and verified by experiment.

2 Design difficulties caused by enlarging of PFHE for cryogenic ASU

2.1 Cryogenic air separation unit and the heat exchange sub-system

Cryogenic ASU use the continuous process of stream compression, cooling, liquefaction and distillation. The cryogenic ASU mainly includes submachines which are air compressor (Label 2 in Fig.1), turbo expander (Labels 8, 9), heat exchanger (Labels 11,12,14), distillation packing tower (Labels 13,15,16,17). The ASU process mainly contains adsorption subsystem, rectification subsystem and heat exchange subsystem. The adsorption subsystem is used to remove impurities such as water, carbon dioxide, hydrocarbons, acetylene. The adsorption of CO₂ content should be less than 1 ppm (part per million, 1ppm=1e-6). The rectification subsystem is used to extract the different fractions of air according to the different boiling points.

The heat exchange subsystem functions as condensation, evaporation and liquefaction of hot and cold multi-streams. Heat exchange sub-system of ASU includes main heat exchanger (Label 11), subcooler (Label 12) and condenser evaporator (Label 14). In main heat exchanger, hot streams top-down also called forwards flow and cold

streams bottom up also called backwards flow exchange heat via countercurrent heat transfer. In subcooler, liquid air and liquid nitrogen are further cooled below the saturation temperature by gaseous nitrogen and waste nitrogen via countercurrent flow. In condenser evaporator, liquid oxygen falling from upper column evaporates and gaseous nitrogen rising from lower column condensates via phase change heat transfer.

1. self-cleaning air filter; 2. air compressor; 3. water pump; 4. air cooling tower; 5. water-cooling tower; 6. molecular sieve absorber; 7. electric heater; 8. Supercharger; 9. turbo-expander; 10. silencer; 11. main heat exchanger; 12. Subcooler; 13. lower column; 14. main condenser evaporator; 15. upper column; 16. crude argon column I ; 17. crude argon column II ; 18. liquid argon pump; 19. liquid argon; 20. gaseous nitrogen.

Fig.1. PFHE functions as condensation, evaporation and liquefaction for ASU

2.2 Large Plate-fin heat exchanger and the design challenges

Large PFHE which is also called Braze Aluminum Plate-Fin Heat Exchanger (BAHX) or Braze plate heat exchangers (BPHE) consists of a stack of alternate flat plates called “parting sheets” and fin corrugations, brazed together as a block shown in

Fig.2. The streams flow along the passages made by the corrugations between the parting sheets. The corrugations serve as both secondary heat transfer surfaces and mechanical supports for the internal pressures between layers. The direction of width, height and length of PFHE is generally defined via the finned passage of core.

1. End header; 2.Layer or passage of heat transfer fins; 3.Parting sheet; 4. Side cap sheet; 5.Side header; 6. Flow equalizing wing; 7. Block (core); 8.Intermediate header; 9.Nozzles; 10.Side bars; 11.Distributor fins

Fig.2. Multi-stream vertical brazed aluminum plate-fin heat exchanger (PFHE)

The enlarging of PFHE causes cascading design difficulties. For example, width (W)(x) $\times$ height (H)(y) $\times$ length (L)(z) may be more than 6.0m $\times$ 3.0m $\times$ 10.0m; the number of passage may be more than 100; the pressure may be more than 9MPa (1MPa=10bar $\approx$ 145psi); permissible temperature differences between adjacent streams should be less than 10 K; surface area density, area/volume may be 1500 m²/m³. The

long flowing length (z) of layer may cause phase change for the stream. Multiple banking of passage arrangement will reduce thermal efficiency. How to solve these design challenges and obtain optimal equilibrium is an important research hotspot.

2.3 Surface efficiency and effective heat transfer area of different fins

Heat is energy in transit. The basic heat transfer equation

$$q = \frac{dQ}{dt} = Aq_A = \int_0^A K\Delta T dA \approx K\Delta T_m A \quad (1)$$

where Q is heat transferred (J); q is heat transfer rate (W); A is heat surface area (m^2); q_A is heat flux ($\text{W}\cdot\text{m}^{-2}$); K is heat transfer coefficient ($\text{W}\cdot\text{m}^{-2}\cdot\text{K}^{-1}$); ΔT is temperature difference, (K); ΔT_m is mean temperature difference (MTD), which depends upon the flow configurations of the streams.

Heat is transferred from or into a stream within a finned passage. Fins are the basic components of PFHE. The fin types include plain (straight), wavy (corrugated) fin, serrated (offset, OSF) fin, perforated fin, louvered fin, etc. The fins can also be further subdivided into rectangular fin, triangular fin, pin fin, herringbone fin, etc. The heat transfer surface area per meter in Z direction can be classified into primary heat transfer surface A_1 and secondary heat transfer surface A_2 . A_1 is provided by the bare parting sheet and the fin base directly brazed to the parting sheet. A_2 is provided by the fins. The ratio $A_2/(A_1 + A_2)$ of A_2 to the total $A_1 + A_2$ represents the ratio of heat transferred along direction of fin height to the total heat transfer surface area

$$A_1 = 2(1 - d_f t_f) \quad (2)$$

$$A_2 = 2d_f(h_f - t_f) \quad (3)$$

$$\frac{A_2}{A_1 + A_2} = \frac{d_f(h_f - t_f)}{(1 - d_f t_f) + d_f(h_f - t_f)} \quad (4)$$

where h_f is the fin height (mm); t_f is the fin thickness (mm); p_f is fin

pitch/spacing (mm) ; d_f is the fin density $d_f = p_f^{-1}$ (m^{-1}), i.e. number of fins per meter in X direction.

For different types of fins shown in Fig.3, l_p is wave length for wavy fins or tooth length for OSF fins. The fin type determines the cross-section of the passage which can be rectangle, circle, annulus, etc. The equivalent diameter d_e can be obtained by cross section area A and section perimeter l , for rectangle without fillet

$$A = d_f(h_f - t_f)(p_f - t_f) \quad (5)$$

$$l = d_f(2(h_f - t_f) + 2(p_f - t_f)) \quad (6)$$

$$d_e = 4 \frac{A}{l} = 4 \frac{d_f(h_f - t_f)(p_f - t_f)}{d_f(2(h_f - t_f) + 2(p_f - t_f))} = \frac{2(h_f - t_f)(p_f - t_f)}{(h_f - t_f) + (p_f - t_f)} \quad (7)$$

$$l * Z = A_1 + A_2 \quad (8)$$

The Renault number Re for flowing stream is defined by

$$Re = \frac{ud_e}{\nu} = \frac{\rho ud_e}{\mu} = \frac{\dot{V} \rho d_e}{A \mu} = \frac{\dot{m} d_e}{A \mu} \quad (9)$$

where u is fluid velocity($m \cdot s^{-1}$); ν is kinematic viscosity ($m^2 \cdot s^{-1}$); μ is dynamic viscosity (Pa·s); ρ is mass density in standard state ($kg \cdot m^{-3}$); $\dot{V}$ is the volume flow rate of stream in standard normal state ($Nm^3 \cdot s^{-1}$); $\dot{m}$ is mass flow rate in standard state ($kg \cdot s^{-1}$).

Obviously, Re is the core parameter used to determine the working condition of convective heat transfer between multi-streams.

Fin efficiency η_f is defined as the ratio of actual transferred heat to ideal transferred heat. The η_f is not connected with flow configurations. Using basic heat transfer Eq.(1),

$$\eta_f = \frac{Q_{actual}}{Q_{ideal}} = \frac{\int_0^A K \Delta T_{fin} dA}{\int_0^A K \Delta T_{sheet} dA} \approx \frac{\overline{T_{fin}} - T_{stream}}{\overline{T_{sheet}} - T_{stream}} \in (0,1) \quad (10)$$

where $\overline{T_{fin}}$ is mean temperature of the fins (K) ; $\overline{T_{sheet}}$ is mean temperature of sheet along the flowing direction (K).

The temperature $T_{fin}(x)$ depends on the type and shape of the fin. For circular fin with equal cross-section, $T_{fin}(x)$ obeys Bessel's differential equation. For straight fin with equal cross-section, $T_{fin}(x)$ obeys hyperbolic function “tanh”

$$T_{fin}(x) \sim \tanh(x) = \frac{\sinh(x)}{\cosh(x)} = \frac{e^x - e^{-x}}{e^x + e^{-x}} \in (-1, 1) \quad x \in \mathbb{R} \quad (11)$$

Surface efficiency η_s is defined as the comprehensive efficiency of A_1 and A_2 . The total transferred heat Q equals to the primary transferred heat Q_1 and secondary transferred heat Q_2 .

$$Q = Q_1 + Q_2 \quad (12)$$

$$K(A_1 + A_2)\eta_s \Delta T = K A_1 \Delta T + K A_2 \eta_f \Delta T \quad (13)$$

$$\eta_s = 1 - \frac{A_2}{A_1 + A_2} (1 - \eta_f) \quad (14)$$

Surface efficiency η_s can be used to obtain effective heat transfer area A_e .

$$A_e = A \eta_s \quad (15)$$

Fig.3. Geometric parameters of plain fin, serrated fin, wavy fin and perforated fin

3 Divide large heat transfer area in various directions into MDE for PFHE

3.1 Arranging various kinds and types of streams into passages using RPP

Large PFHE has n passages, namely $n + 1$ sheets, ascending ordered from bottom to top. According to cryogenic process of ASU shown in Fig.1, there are two kinds and N_{type} types of streams in the n passages. The PFHE transfers heat between hot streams and cold streams which involves at least two streams and

passages, $n \geq N_{type} \geq 2$. The total number N_j of passages for j -th type stream is given by

$$N_j = \frac{\dot{m}_j}{q_{mj}A_j} = \frac{\dot{V}_j\rho_j}{q_{mj}A_j} \quad N_j \geq 1 \quad (16)$$

$$\sum_{j=1}^{N_{type}} N_j = n \quad (17)$$

$$q_m = \frac{\dot{m}}{A} = \frac{\rho\dot{V}}{A} = \frac{\rho \cdot uA}{A} = \rho u \quad (18)$$

where $\dot{m}_j$ is total mass flow rate for j -th type stream in standard state ($\text{kg}\cdot\text{s}^{-1}$); q_{mj} is the mass flux per unit area ($\text{kg}\cdot\text{m}^{-2}\cdot\text{s}^{-1}$); A_j is section area of each passage which depends on the fin parameters (m^2); $\dot{V}_j$ is the total volume flow rate for j -th type stream in standard normal state ($\text{Nm}^3\cdot\text{s}^{-1}$);.

The initial total heat load of hot streams q_h and cold streams q_c

$$N_h = \sum N_i \quad q_h = \sum N_i \cdot q_i \quad i \in \text{hot stream} \quad (19)$$

$$N_c = \sum N_j \quad q_c = \sum N_j \cdot q_j \quad j \in \text{cold stream} \quad (20)$$

$$n = N_h + N_c \quad (21)$$

The initial mean unbalance of heat load for each passage can be obtained using q_h and q_c

$$\bar{q} = \frac{1}{n} \sum_{i=1}^n q^{(i)} = \frac{q_h + q_c}{n} \quad (22)$$

For convenience of calculations of summation Eq.(22),

$$q_i > 0 \quad i \in \text{hot stream} \quad (23)$$

$$q_j < 0 \quad j \in \text{cold stream} \quad (24)$$

The cumulative heat load $\sigma_q^{(i)}$ of i -th passage can be calculated using recursion:

$$\sigma_{q^{(i)}} = \sigma_{q^{(i-1)}} + q^{(i)} \quad i = 1, \dots, n \quad (25)$$

$$\sigma_{q^{(0)}} = 0 \quad (26)$$

$$\sigma_{q^{(n)}} = q_h + q_c = \sum_{i=1}^n q^{(i)} = n\bar{q} \quad (27)$$

The passage arrangement means arranging all $N_j \times j \ j \in [1, N_{type}]$ streams into n passages to according to the unbiased variance of the cumulative heat load $\sigma_q^{(i)}$

$$\sigma_{\sigma_q} = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (\sigma_{q^{(i)}} - \bar{\sigma}_q)^2} \quad i \in [1, n] \quad (28)$$

$$\bar{\sigma}_q = \frac{1}{n} \sum_{i=1}^n \sigma_{q^{(i)}} \quad (29)$$

Single banking means there is no same kinds of streams between the adjacent streams. Multiple banking or hybrid-banking means there exists same kinds of streams between the adjacent streams. The number of N_h, N_c determines the stacking pattern of passage arrangement

$$N_{min} = \min (N_h, N_c) \quad (30)$$

$$N_{max} = \max (N_h, N_c) \quad (31)$$

Passage arrangement is a permutation problem which has numerous arrangements. Passage arrangement can be realized by enumeration method, exclusion method, heuristics method, etc. Recursive Prior Plug-In (RPP) method is proposed to obtain permutation number P_{total} to overcome the disadvantage of time-consuming and low efficiency.

If $N_h = N_c$

$$P_{total} = 2 \cdot A_{N_h}^{N_h} \cdot A_{N_c}^{N_c} = 2 \cdot N_h! \cdot N_c! \quad (32)$$

If $|N_h - N_c| = 1$

$$P_{total} = N_h! \cdot N_c! \quad (33)$$

If $2 \leq |N_h - N_c| \leq N_{min} + 2$

$$P_{total} = A_{N_{min}}^{N_{min}} \cdot A_{N_{max}}^{N_{min}+1} \cdot A_{N_{min}+1}^{N_{max}-N_{min}-1} \quad (34)$$

If $N_{min} + 2 \leq |N_h - N_c| \leq 2N_{min} + 3$

$$\begin{aligned} P_{total} &= A_{N_{min}}^{N_{min}} \cdot A_{N_{max}}^{N_{min}+1} \cdot A_{N_{max}-N_{min}-1}^{N_{min}+1} \cdot A_{N_{min}+1}^{N_{max}-2N_{min}-2} \\ &= A_{N_{min}}^{N_{min}} \cdot A_{N_{max}}^{2N_{min}+2} \cdot A_{N_{min}+1}^{N_{max}-2N_{min}-2} \end{aligned} \quad (35)$$

If $(s-1)N_{min} + s \leq |N_h - N_c| \leq sN_{min} + s + 1$

$$\begin{aligned} P_{total} &= A_{N_{min}}^{N_{min}} \cdot A_{N_{max}}^{N_{min}+1} \cdot A_{N_{max}-N_{min}-1}^{N_{min}+1} \cdot \dots \cdot A_{N_{max}-(s-1)N_{min}-s+1}^{N_{min}+1} \cdot A_{N_{min}+1}^{N_{max}-sN_{min}-s} \\ &= A_{N_{min}}^{N_{min}} \cdot A_{N_{max}}^{sN_{min}+s} \cdot A_{N_{min}+1}^{N_{max}-sN_{min}-s} \end{aligned} \quad (36)$$

$$s = \text{ceil}\left(\frac{N_{max}}{N_{min}+1}\right) \quad s \in Z^+ \quad (37)$$

The Eq.(36) is the general case with $s + 1$ times plug-in when $N_h \neq N_c$ obtained by mathematical inducting of Eq.(33)-(35). RPP combines two basic types of layer arrangement: single banking with top priority and multiple banking with lower priority.

The effective secondary heat transfer surface A_e can be obtained by the fin efficiency η_f . For single banking, the effective heat transfer surface area A_e for a passage can be estimated:

$$A_e = A_1 + \eta_{f,single} \phi_f A_2 \quad (38)$$

$$\eta_{f,single} = \frac{\tanh(\beta)}{\beta} \quad (39)$$

$$\beta = h_e \sqrt{\frac{2K_\alpha}{\lambda_f t_f}} \quad (40)$$

where K_α is the effective heat transfer coefficient of a stream ($\text{W} \cdot \text{m}^{-2} \cdot \text{K}^{-1}$); λ_f is the thermal conductivity of fin material ($\text{W} \cdot \text{m}^{-1} \cdot \text{K}^{-1}$); h_e is effective heat transfer distance (m).

For s multiple banking, the parting sheet between the two layers becomes a secondary surface and the fin efficiency η_f is reduced. The effective heat transfer surface area A_e

$$A_e = \frac{s-1+\eta_{f,multiple}}{s} A_1 + \eta_{f,multiple} A_2 \quad s \geq 2 \quad (41)$$

$$\eta_{f,multiple} = \left(\frac{1}{\beta+\gamma} \right) \left(\frac{B-1}{B+1} \right) \quad (42)$$

$$\gamma = \frac{\beta}{h_e} \left(\frac{1}{d_f} - t_f \right) \quad (43)$$

$$B = \left(\frac{1+\gamma}{1-\gamma} \right) e^{2\beta} \quad (44)$$

3.2 Multiple Dynamic Equilibrium in plane perpendicular to the flowing direction

There are three modes of heat transfer: conduction, convection, and radiation. In general a body may involve specified heat flux, convection and radiation simultaneously. Thermal efficiency ε is generally defined as the ratio of real transferred heat quantity and maximum transferred heat quantity.

$$\varepsilon = \frac{Q_{min}}{Q_{max}} \in (0,1) \quad (45)$$

$$Q_{min} = \min (Q_h, |Q_c|) \quad (46)$$

$$Q_{max} = \max (Q_h, |Q_c|) \quad (47)$$

If ε approaches 1, $\varepsilon \rightarrow 1$, it means $1 - \varepsilon \rightarrow 0$, it can be considered as a thermal equilibrium state with relative thermal equilibrium error $1 - \varepsilon$ between hot and cold streams.

The energy balance equation means heat transferred to the body in all modes equals to the heat transferred from the body in all modes. The general form of the heat transfer differential equation in the Cartesian coordinate system using Fourier's law of heat conduction

$$\rho C_P \frac{\partial T}{\partial t} = \nabla(\lambda \nabla T) + \dot{Q} \quad (48)$$

$$\rho C_P \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left(\lambda \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(\lambda \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left(\lambda \frac{\partial T}{\partial z} \right) + \dot{Q} \quad (49)$$

where ∇ is Laplace operator, $\nabla T = \text{grad } T$; ρ is substance density ($\text{kg} \cdot \text{m}^{-3}$); the mass

specific heat capacity C_p at constant pressure P is the amount of energy needed to raise the temperature of 1 kg of a substance by 1 K; λ is the thermal conductivity ($\text{W}\cdot\text{m}^{-1}\cdot\text{K}^{-1}$).

Fig.4. Multiple Dynamic Equilibrium in plane perpendicular to the flowing direction

Temperature gradient ∇T points to the direction of temperature increase

$$\nabla T = \lim_{\Delta h \rightarrow 0} \frac{\Delta T}{\Delta h} \vec{h} = \frac{\partial T}{\partial h} \vec{h} \quad (50)$$

Heat flux q_A ($\text{W} \cdot \text{m}^{-2}$)

$$q_A = q/A = -\lambda \nabla T \quad (51)$$

The initial temperature of each section of i -th passage stream can be expressed as

$$T = \{T^{(1)}, \dots, T^{(i)}, \dots, T^{(n)}\} \quad i = 1, \dots, n \quad (52)$$

The temperature difference $\Delta T^{(i)}$ between the i -th passage and the one-sided adjacent $(i + 1)$ -th streams

$$\Delta T = \{\Delta T^{(1)}, \dots, \Delta T^{(i)}, \dots, \Delta T^{(n-1)}\} \quad (53)$$

$$\Delta T^{(i)} = T^{(i+1)} - T^{(i)} \quad i = 1, \dots, n - 1 \quad (54)$$

The large heat transfer area in plane perpendicular to the flowing direction is divided into multiple dynamic equilibriums (MDE) according to the temperature difference sequence.

$$MDE = \{E^{(1)}, \dots, E^{(i)}, \dots, E^{(k)}\} \quad k \in \mathbb{Z}^+ \& k \leq \frac{n-1}{2} \quad (55)$$

For the i -th equilibrium $E^{(i)}$, the number of streams $N_{E^{(i)}}$ can be determined by thermal equilibrium criterion according to $\Delta T^{(i)}$

$$\prod_{j=1}^{N_{E^{(i)}}} \Delta T^{(j)} < 0 \quad i = 1, \dots, k \quad (56)$$

The number of streams in each equilibrium,

$$N_E = \{N_{E^{(1)}}, \dots, N_{E^{(i)}}, \dots, N_{E^{(k)}}\} \quad (57)$$

$$n = \sum_{i=1}^k N_{E^{(i)}} \quad N_{E^{(i)}} \geq 2 \quad (58)$$

Eq.(58) proves that there exist thermal equilibrium among more than two streams. Therefore, Eq.(58) is available for various stacking patterns of hot and cold streams.

The effective heat transfer distance h_e is the Euclidean distance between position $y|_{T=\max(|T|)}$ where $T = \max(|T|)$ and the position $y|_{\nabla T \rightarrow 0}$ where $\nabla T \rightarrow 0$. Each

equilibrium state has its corresponding h_e

$$h_e = \{h_e^{(1)}, \dots, h_e^{(i)}, \dots, h_e^{(k)}\} \quad (59)$$

$$h_e^{(i)} = |y|_{T=\max(|T|)} - y|_{\nabla T \rightarrow 0} = N_{E^{(i)}} \frac{h_f}{2} \quad (60)$$

Eq.(60) proves that h_e will increase when multiple-banking is employed during passage arrangement.

Temperature gradient in y direction of all the streams

$$\nabla T = \{\nabla T^{(1)}, \dots, \nabla T^{(i)}, \dots, \nabla T^{(k)}\} \quad (61)$$

$$\nabla T^{(i)} = \frac{1}{h_e^{(i)}} \int_0^{h_e^{(i)}} \frac{\Delta T}{dy} dy \quad (62)$$

Heat flux of all the ordered streams using Eq.(51)

$$q_A = \{q_A^{(1)}, \dots, q_A^{(i)}, \dots, q_A^{(k)}\} \quad (63)$$

For each thermal equilibrium, there exist a dynamic thermal equilibrium state. For the i -th equilibrium, $i = 1, \dots, k$, using wall conduction thermal, convective thermal, the thermal balance equations can be built

$$Q_w^{(i-1)} + Q_\alpha^{(i-1)} = Q_w^{(i)} + Q_\alpha^{(i)} \quad i = 1, \dots, k \quad (64)$$

$$Q_w^{(i-1)} = q_A^{(i-1)} d_f^{(i-1)} A_1^{(i-1)} = -\lambda_f^{(i-1)} \nabla T^{(i-1)} d_f^{(i-1)} A_1^{(i-1)} \quad (65)$$

$$Q_\alpha^{(i)} = K_\alpha^{(i)} A_2^{(i)} \Delta T^{(i)} \quad (66)$$

where $K_\alpha^{(i)}$ is convective heat transfer coefficient ($\text{W} \cdot \text{m}^{-2} \cdot \text{K}^{-1}$).

On the basis of the k equilibriums, the mean temperature $\overline{T_{sheet}^{(i)}}$ ($i = 1, \dots, n + 1$) of each sheet whose position is $y = (i - 1)h_f$ can be further obtained. The final equilibrium temperature of each ordered stream in all the passages can be obtained by iterative solving the simultaneous differential equations for all MDE. The temperature inhomogeneity of streams can be obtained by unbiased variance

$$\sigma_{T^{(i)}} = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (T^{(i)})^2} \quad (67)$$

3.3 Multiple Dynamic Equilibrium of long stream along flowing direction

Each long stream is also divided into MDE along flowing direction and modelled by energy conservation principle in enthalpy change to obtain the actual phase of stream.

For i -th equilibrium of each stream, the transferred heat $q^{[i]}$ equals to sensible heat $q_{sensible}^{[i]}$ and latent heat $q_{latent}^{[i]}$

$$q^{[i]} = q_{sensible}^{[i]} + q_{latent}^{[i]} \quad (68)$$

$$q_{sensible}^{[i]} = \rho^{[i]} A^{[i]} u^{[i]} C_p^{[i]} \Delta T^{[i]} = \dot{m}^{[i]} C_p^{[i]} \Delta T^{[i]} \quad (69)$$

$$q_{latent}^{[i]} = \dot{m}^{[i]} \gamma^{[i]} \quad i = 1, 2, 3, \dots \quad (70)$$

where $q_{sensible}^{[i]}$ is sensible heat transfer rate in standard state without phase change (W); $\rho^{[i]}$ is mass density in standard state ($\text{kg} \cdot \text{m}^{-3}$); $A^{[i]}$ is heat transfer area (m^2); $u^{[i]}$ is fluid velocity in standard state ($\text{m} \cdot \text{s}^{-1}$); $C_p^{[i]}$ is mass specific heat capacity at constant pressure ($\text{J} \cdot \text{kg}^{-1} \cdot \text{K}^{-1}$); $\Delta T^{[i]}$ denotes temperature change of i -th section (K); $q_{latent}^{[i]}$ is latent heat transfer rate with phase change (W); $\dot{m}^{[i]}$ is mass flow rate in standard state ($\text{kg} \cdot \text{s}^{-1}$); $\gamma^{[i]}$ is latent heat of phase change ($\text{J} \cdot \text{kg}^{-1}$).

$q_{latent}^{[i]}$ is associated with phase change which is greater than $q_{sensible}^{[i]}$. Temperature change cause phase change and enthalpy drop or rise. Enthalpy H is the total energy a fluid contains including both internal energy and the fluid-mechanical potential energy. When phase changes, heat quantity should be revised as:

$$Q^{[i]} = (q_{sensible}^{[i]} + q_{latent}^{[i]})t = \Delta H^{[i]} = \dot{m}^{[i]} \Delta \dot{H}^{[i]} \quad (71)$$

where $\Delta H^{[i]}$ is the enthalpy change in the substance (J); $\Delta \dot{H}^{[i]}$ is mass specific enthalpy change ($\text{J} \cdot \text{kg}^{-1}$).

The $\Delta \dot{H}$ from T_{ref} to T can be got by polynomial definite integration

$$\Delta \dot{H}^{[i]}(T) = \Delta \dot{H}^{[i]}(T_{ref}) + \int_{T_{ref}}^T C_P(T) dT \quad (72)$$

The constant-pressure specific heat C_P can be related to the constant-volume specific heat C_V which is easy to measure in a calorimeter

$$C_P = \frac{\partial^2 H}{\partial m \partial T} = \frac{d\dot{H}}{dT} = C_V + R \quad (73)$$

The temperature differences $\Delta T_{end1}^{[i]}, \Delta T_{end2}^{[i]}$ between two streams at end1 and end2 of i -th section can be obtained for various flow configuration such as counterflow, co-current flow, crossflow and cross counterflow. For counterflow, for the two streams at the end1 and end2

$$\Delta T_{end1}^{[i]} = T_{h,o}^{[i]} - T_{c,i}^{[i]} \quad (74)$$

$$\Delta T_{end2}^{[i]} = T_{h,i}^{[i]} - T_{c,o}^{[i]} \quad (75)$$

When multi-streams undergo phase changes (evaporation or condensation), $\Delta T_m^{[i]}$ should be approximated by integral mean temperature difference IMTD

$$\Delta T_m^{[i]} = IMTD^{[i]} = \frac{\left| \int_0^{z^{[i]}} \Delta T dz \right|}{\int_0^{z^{[i]}} dz} = \frac{\left| \int_0^{z^{[i]}} (T_h^{[i]}(z) - T_c^{[i]}(z)) dz \right|}{\int_0^{z^{[i]}} dz} = \frac{\sum_{i=1}^n AMTD^{[i]}}{n} \quad (76)$$

For i -th sections, the required effective surface area $A_{e,h}^{[i]}, A_{e,c}^{[i]}$ can be obtained from mean temperature difference $\Delta T_m^{[i]}$, or from the heat transfer coefficient $K_{\alpha,h}, K_{\alpha,c}$ of hot and cold streams:

$$K^{[i]} A_e^{[i]} = \sum \frac{Q^{[i]}}{\Delta T_m^{[i]}} = \frac{(\sum K_{\alpha,h}^{[i]} A_{e,h}^{[i]}) \cdot (\sum K_{\alpha,c}^{[i]} A_{e,c}^{[i]})}{\sum K_{\alpha,h}^{[i]} A_{e,h}^{[i]} + \sum K_{\alpha,c}^{[i]} A_{e,c}^{[i]}} \quad i = 1, 2, 3, \dots \quad (77)$$

Where $K^{[i]}$ is heat transfer coefficient of streams in i -th sections.

4 Divide heat transfer process with various Re into MDE for large PFHE

4.1 Multiple Dynamic Equilibrium of various working conditions

For each passage, there exists a dynamic equilibrium, where the heat thermal and cold thermal keep a dynamic equilibrium synchronously. The heat transfer efficiency and pressure drop are two factors which depended and conditioned upon each other in the dynamic equilibrium of the heat transfer process.

The heat transfer factor j (j -factor) of the fin is defined

$$j = StPr^{2/3} \quad (78)$$

$$St = \frac{K_\alpha}{u\rho C_P} = \frac{K_\alpha}{q_m C_P} = \frac{Nu}{RePr} \quad (79)$$

$$Pr = \frac{C_P \mu}{\lambda} \quad (80)$$

$$Nu = \frac{K_\alpha d_e}{\lambda} = \frac{q_A d_e}{\lambda \Delta T} = \frac{q_A d_e}{\lambda \left(\frac{T_i + T_o}{2} - \bar{T}_{sheet} \right)} \quad (81)$$

where St denotes Stanton number; Pr is Prandtl number; Nu is Nusselt number; K_α is convective heat transfer coefficient ($\text{W} \cdot \text{m}^{-2} \cdot \text{K}^{-1}$), denoted respectively as $K_{\alpha,h}$, $K_{\alpha,c}$ for hot and cold stream; q_m is the mass flux per unit area of stream ($\text{kg} \cdot \text{m}^{-2} \cdot \text{s}^{-1}$); q_A is heat flux ($\text{W} \cdot \text{m}^{-2}$).

Using Eq.(81), the outlet temperature T_o can be expressed as

$$T_o = \frac{2q_A d_e}{\lambda N_u} + 2\bar{T}_{sheet} - T_i \quad (82)$$

When the j changed into j' with ratio $\varepsilon_j = (j' - j)/j$,

$$\frac{\Delta N_u}{N_u} = \frac{j \varepsilon_j Re \times \left(\frac{C_P \mu}{\lambda} \right)^{1/3}}{j Re \times \left(\frac{C_P \mu}{\lambda} \right)^{1/3}} = \varepsilon_j \quad (83)$$

The changed outlet temperature T'_o of the stream can be obtained

$$\begin{aligned} T'_o &= \frac{2q_A d_e}{\lambda N_u (1 + \varepsilon_j)} + 2\bar{T}_{sheet} - T_i = \frac{T_o + T_i - 2\bar{T}_{sheet}}{1 + \varepsilon_j} + 2\bar{T}_{sheet} - T_i \\ &= \frac{T_o + 2\varepsilon_j \bar{T}_{sheet} - \varepsilon_j T_i}{1 + \varepsilon_j} \end{aligned} \quad (84)$$

The change ratio of outlet temperature T_o of the stream can be obtained

$$\frac{\Delta T_o}{T_o} = \frac{T_o' - T_o}{T_o} = \frac{\frac{2q_A d_e}{\lambda Nu} \left(\frac{-\varepsilon_j}{1+\varepsilon_j} \right)}{T_o} = \frac{T_o + T_i - 2\bar{T}_{sheet}}{T_o} \left(\frac{-\varepsilon_j}{1+\varepsilon_j} \right) = \left(\frac{2\bar{T}_{sheet} - T_i}{T_o} - 1 \right) \left(\frac{\varepsilon_j}{1+\varepsilon_j} \right) \quad (85)$$

The change ratio of outlet temperature T_o can be expressed by piecewise function according to the ε_j :

$$\varepsilon_j \in (0, +\infty) \quad \frac{\Delta T_o}{T_o} \in \left[\left(\frac{2\bar{T}_{sheet} - T_i}{T_{o,max}} - 1 \right) \left(\frac{\varepsilon_j}{1+\varepsilon_j} \right), \left(\frac{2\bar{T}_{sheet} - T_i}{T_{o,min}} - 1 \right) \left(\frac{\varepsilon_j}{1+\varepsilon_j} \right) \right] \quad (86)$$

$$\varepsilon_j \in (-1, 0) \quad \frac{\Delta T_o}{T_o} \in \left[\left(\frac{2\bar{T}_{sheet} - T_i}{T_{o,min}} - 1 \right) \left(\frac{\varepsilon_j}{1+\varepsilon_j} \right), \left(\frac{2\bar{T}_{sheet} - T_i}{T_{o,max}} - 1 \right) \left(\frac{\varepsilon_j}{1+\varepsilon_j} \right) \right] \quad (87)$$

The fanning friction factor f (f -factor) can be defined by the frictional pressure loss ΔP across a plate-fin passage and at any associated entry, exit for single-phase stream

$$f = \Delta P \frac{d_e}{2L\rho u^2} = \Delta P \frac{\rho d_e}{2Lq_m^2} = \Delta P \frac{d_e^2}{2LRe\mu u} \quad (88)$$

Where ΔP is pressure drop (Pa); L is the passage length (m).

Using Eq.(88), the pressure drop ΔP can be expressed as

$$\Delta P = \frac{2fLq_m^2}{\rho d_e} \quad (89)$$

When the f changed with ratio $\varepsilon_f = (f' - f)/f$, the changed pressure drop $\Delta P'$ of the stream can be obtained

$$\Delta P' = \frac{2fLq_m^2}{\rho d_e} (1 + \varepsilon_f) \quad (90)$$

The changed ratio of pressure drop ΔP of the stream can be obtained

$$\frac{\Delta(\Delta P)}{\Delta P} = \frac{\Delta P' - \Delta P}{\Delta P} = \frac{\frac{2fLq_m^2 \varepsilon_f}{\rho d_e}}{\frac{2fLq_m^2}{\rho d_e}} = \varepsilon_f \quad (91)$$

From Eq.(84) and Eq.(90), improving the j -factor generally improves the f -factor. Therefore, the heat transfer factor and friction factor are regarded as two inter-constraint measures.

$$\varepsilon_j \cdot \varepsilon_f \leq 0 \quad \varepsilon_j, \varepsilon_f \in (-1, 1) \quad (92)$$

The equilibrium degree between two inter-constraint measures can characterize the equilibrium between heat transfer factor and friction factor. Therefore, the heat transfer process can be pre-divided into MDE according to the dynamic equilibriums between two inter-constraint measures using the value of $\frac{dj}{df}$. The interval integral of $\frac{dj}{df}$ can be used to improve the MDE by iteration. Renault number of multi-streams fluctuates in wide range which generates various working conditions. The MDE help to determine the optimal fins under various working conditions.

4.2 Validation and Verification of the Multiple Dynamic Equilibrium

Using cryogenic ASU process, the multi-streams: $N_{type} = 6$. The cold streams B, C, D, F are oxygen gas (O_2), nitrogen gas (N_2), wasted N_2 , pressure N_2 . The hot streams A, E are Air, pressure air. The total mass flow rate $\dot{m}$ of A, B, C, D, E, F stream is respectively $11.26 \text{ kg}\cdot\text{s}^{-1}$, $3.56 \text{ kg}\cdot\text{s}^{-1}$, $10.27 \text{ kg}\cdot\text{s}^{-1}$, $9.93 \text{ kg}\cdot\text{s}^{-1}$, $4.78 \text{ kg}\cdot\text{s}^{-1}$, $6.58 \text{ kg}\cdot\text{s}^{-1}$. The heat load $q_A:q_B:q_C:q_D:q_E:q_F=1.4165:-1:-1.0635:-1.0522:1.2976:-0.8100$.

Fig.5. Heat load $q^{(i)}$, cumulative heat load $\sigma_q^{(i)}$ and mean temperature $\overline{T^{(i)}}$ of stream in i -th passage, where (a)(c)(e) are about the passage arrangement without using RPP method, (b)(d)(f) are about the passage arrangement using the proposed RPP method.

In Fig.5(a), there exists three-banking among 4,5,6 layers. In Fig.5(b), the passage

arrangement is improved by using RPP method. In Fig.(c), all layers are divided into 22 MDE using Eq.(56), $N_E = \{2,2,3,4,2,2,3,2,2,2,2,2,3,2,2,2,2,2,3,2,2,2\}$, in Fig.(d), all layers are divided into 24 MDE, $N_E = \{2,2,3,2,2,2,2,2,2,3,2,2,2,2,3,2,2,2,2,3,2,2,2\}$. In Fig5.(e), unbiased variance of each stream $\sigma_{T(i)}=72.5299$, unbiased variance of temperature difference $\sigma_{\Delta T(i)}=125.6709$. In Fig5.(f), unbiased variance of each stream $\sigma_{T(i)}=72.5299$, unbiased variance of temperature difference $\sigma_{\Delta T(i)}=130.2573$.

Fig.6. Mean temperature $\overline{T^{(i)}}$ of stream in i -th passage before and after using MDE method, where (a) (c) (e) are about before using MDE method under low, moderate and full working conditions ; (b) (d) (f) are about after using MDE method under low, moderate and full working conditions.

The original temperature of Fig.6 is shown in Fig.5(f). In Fig6.(a), unbiased

variance of each stream $\sigma_T=37.1914$ K. In Fig5.(b), unbiased variance of each stream $\sigma_T=25.1633$ K. Passage temperature inhomogeneity is decreased from 37.1914 to 25.1633 K with ratio of -32.34% using Eq.(67). From Fig5.(c)(d), passage temperature inhomogeneity is decreased from 12.1447 K to 8.4561 K with ratio of -30.37%. From Fig5.(e)(f), passage temperature inhomogeneity is decreased from 9.5889 K to 6.7512 K with ratio of -29.59%. After using MDE method, the temperature differences $\Delta T_{end1}, \Delta T_{end2}$ between two streams at end1 and end2 are also reduced, which improves the heat transfer coefficient of large PFHE using Eq.(77).

5 Heat transfer experiment under various working conditions

5.1 Experimental schematic of heat transfer for different types of fins

The heat transfer experiment is about different types of cores and fins under various working conditions shown in Fig.7. In physical experiment, temperature accuracy is 0.1K, differential pressure accuracy is 1Pa, volumetric flux accuracy is 0.1ml. An atmospheric pressure (atm) is 0.1013 Mpa. Actual pressure is the sum of gage pressure and atm, which is automatically controlled by the electromagnetic solenoid valve. The outlet temperature of multi streams can be obtained by thermocouple sensors placed in XOY plane. When $\varepsilon \rightarrow 1$ using Eq.(45), it can be considered as dynamic equilibrium between hot streams and cold streams.

Fig.7. Experiment of heat transfer of different streams in the passages

The parameters of three kinds six types of fins are shown in Table 1.

Table 1 The parameters of three kinds six types of cores and fins

Fin parameters		h_f (mm)	t_f (mm)	p_f (mm)	d_e (mm)	A (m ²)	A_1+A_2 (m ²)	$A_2/(A_1+A_2)$
Kind1	Type1	3.98	0.51	4.21	3.5813	3.0496 e-3	3.4062	0.4840
	Type2	4.65	0.42	3.86	3.7943	3.7697 e-3	3.9741	0.5515
	Ratio ε	16.83%	-17.65%	-8.31%	5.95%	23.61%	16.67%	13.96%
Kind2	Type3	5.98	0.39	2.82	3.3875	4.8169 e-3	5.6879	0.6970
	Type4	6.72	0.35	2.67	3.4012	5.5350 e-3	6.5094	0.7330
	Ratio ε	12.37%	-10.26%	-5.32%	0.41%	14.91%	14.44%	5.17%
Kind3	Type5	7.58	0.36	2.23	2.9706	6.0544 e-3	8.1525	0.7943
	Type6	8.51	0.29	2.14	3.0203	7.1061 e-3	9.4112	0.8163
	Ratio ε	12.27%	-19.44%	-4.04%	1.67%	17.37%	15.44%	2.77%

5.2 Experimental results under variable working conditions

Using Eq. (78) and Eq. (88), the heat transfer factor (j -factor) and friction factor (f -factor) of the three kinds six types of fins are computed real-timely in experiment.

The heat transfer factor, friction factor, mean outlet temperature and pressure drop of six types of fins under three working conditions are shown in Fig.8.

Fig.8. The heat transfer factor, friction factor, mean outlet temperature and pressure drop of six types of fins under three working conditions where (a) is about Type1; (b) is about Type2; (c) is about Type3; (d) is about Type4; (e) is about Type5; (f) is about Type6.

The mean outlet temperature and pressure drop of different type6 fins under three working conditions are shown in Table 2.

Table 2 The outlet temperature and pressure drop of different fins

Parameters		Mean outlet temperature $\bar{T}_o$ (K)			Mean pressure drop $\bar{\Delta P}$ (Pa)		
		Working condition1	Working condition2	Working condition3	Working condition1	Working condition2	Working condition3
Kind1	Re	[656,1486]	[1486,3433]	[3433,4207]	[656,1486]	[1486,3433]	[3433,4207]
	Type1	186.7119	162.5515	158.0263	880.4486	3265.7732	6235.1779
	Type2	181.1402	158.2676	154.0037	1021.1443	3828.4982	7356.8864
	Ratio ε	-2.98%	-2.64%	-2.55%	15.98%	17.23%	17.99%
Kind2	Re	[610,1221]	[1221,3105]	[3105,4409]	[610,1221]	[1221,3105]	[3105,4409]
	Type3	194.3485	161.9545	154.7929	557.6148	2271.554	4866.9071
	Type4	189.3140	158.3833	151.5717	636.6846	2591.6161	5579.4223
	Ratio ε	-2.59%	-2.21%	-2.08%	14.18%	14.09%	14.64%
Kind3	Re	[651,1193]	[1193,2552]	[2552,3873]	[651,1193]	[1193,2552]	[2552,3873]
	Type5	201.7294	191.1424	177.6385	845.8369	2370.8716	6312.0556
	Type6	196.8891	187.1284	173.8211	958.5013	2704.2162	7193.8498
	Ratio ε	-2.40%	-2.10%	-2.15%	13.32%	14.06%	13.97%

From Table 2, under working condition for kind1 fins, from type1 to type2, the mean outlet temperature $\bar{T}_o$ decreases by the ratio of -2.98%, -2.64%, -2.55% and the pressure drop $\bar{\Delta P}$ increases by the ratio of 15.98%, 17.23%, 17.99%. Under working condition for kind2 fins, from type3 to type4, the $\bar{T}_o$ decreases by the ratio of -2.59%, -2.21%, -2.08% and the $\bar{\Delta P}$ increases by the ratio of 14.18%, 14.09%, 14.64%. Under working condition for kind3 fins, from type5 to type6, the $\bar{T}_o$ decreases by the ratio of -2.40%, -2.10%, -2.15% and the $\bar{\Delta P}$ increases by the ratio of 13.32%, 14.06%, 13.97%.

According to Eq.(92), under working condition1, the value of $|\varepsilon_{\bar{T}_o}|/\varepsilon_{\bar{\Delta P}}$ for each kind is 0.1865, 0.1827, 0.1802, therefore, the preferable fin is kind1>kind2>kind3.

Under working condition2, the value of $|\varepsilon_{\overline{T_o}}|/\varepsilon_{\overline{\Delta P}}$ is 0.1532, 0.1568, 0.1494, therefore, the preferable fin is kind2> kind1>kind3. Under working condition3, the value of $|\varepsilon_{\overline{T_o}}|/\varepsilon_{\overline{\Delta P}}$ is 0.1417, 0.1421, 0.1539 in therefore, the preferable fin is kind3>kind2>kind1. Based on the experimental results, the fin type can be determined in thermal design.

The outlet temperature of streams in different passages under variable working conditions is shown in Fig.9.

Fig.9. Outlet temperature of streams in different passages under variable working conditions where (a) is about Type1; (b) is about Type2; (c) is about Type3; (d) is about Type4; (e) is about Type5; (f) is about Type6.

In Fig.9(a), the mean unbiased variance of the temperature $mean(\sigma_{T_o}) = 3.0411$ K, the maximum unbiased variance of the temperature $max(\sigma_{T_o}) = 3.8237$ K at $Re = 4001$, the minimum unbiased variance of the temperature $min(\sigma_{T_o}) = 1.6967$ K at $Re = 658$. In Fig.9(b), $mean(\sigma_{T_o}) = 3.1664$ K, $max(\sigma_{T_o}) = 3.9810$ K at $Re = 4003$,

$\min(\sigma_{T_o}) = 1.7671$ K at $Re=660$. In Fig.9(c), $\text{mean}(\sigma_{T_o}) = 2.5842$ K, $\max(\sigma_{T_o}) = 4.1082$ K at $Re=1773$, $\min(\sigma_{T_o}) = 1.4104$ K at $Re=612$. In Fig.9(d), $\text{mean}(\sigma_{T_o}) = 2.6905$ K, $\max(\sigma_{T_o}) = 4.2778$ K at $Re=1775$, $\min(\sigma_{T_o}) = 1.4690$ K at $Re=619$. In Fig.9(e), $\text{mean}(\sigma_{T_o}) = 3.8567$ K, $\max(\sigma_{T_o}) = 5.6160$ K at $Re=1192$, $\min(\sigma_{T_o}) = 2.3836$ K at $Re=2552$. In Fig.9(f), $\text{mean}(\sigma_{T_o}) = 4.0152$ K, $\max(\sigma_{T_o}) = 5.6160$ K at $Re=1198$, $\min(\sigma_{T_o}) = 2.4809$ K at $Re=2559$.

Comparisons on outlet temperature between MDE method and other methods are shown in Table 3. It proves that the more accurate outlet temperature can be obtained by using the MDE method.

Table 3 Comparisons on outlet temperature between MDE method and other methods

Outlet Temperature (K)		Kind1		Kind2		Kind3	
		Type1	Type2	Type3	Type4	Type5	Type6
Experimental Results	$\text{mean}(\sigma_{T_o})$	3.0411	3.1664	2.5842	2.6905	3.8567	4.0152
	$\max(\sigma_{T_o})$	3.8237	3.9810	4.1082	4.2778	5.6160	5.6160
	$\min(\sigma_{T_o})$	1.6967	1.7671	1.4104	1.4690	2.3836	2.4809
Before using MDE method	$\text{mean}(\sigma_{T_o})$	3.0499	3.1752	2.5916	2.6985	3.8422	4.0345
	$\max(\sigma_{T_o})$	3.8353	3.9644	4.1213	4.2642	5.6339	5.6510
	$\min(\sigma_{T_o})$	1.6879	1.7775	1.4022	1.4777	2.3977	2.4956
After using MDE method	$\text{mean}(\sigma_{T_o})$	3.0396	3.1648	2.5829	2.6918	3.8547	4.0130
	$\max(\sigma_{T_o})$	3.8264	3.9838	4.1111	4.2808	5.6120	5.6201
	$\min(\sigma_{T_o})$	1.6956	1.7660	1.4095	1.4680	2.3852	2.4826

5.3 Simulation of fluid velocity and pressure under variable working conditions

Streams to and from the heat exchanger enter and leave by means of header and nozzle. The ends of header and inclination of nozzle have crucial influences on

uniform fluid distribution, especially for large PFHE. The fluid velocity and pressure inside header under three working conditions is shown in Fig.10.

Fig.10. Fluid velocity and pressure inside header of type6 fins under three working conditions where (a) is at $\text{Re}=1000$; (b) is at $\text{Re}=2027$; and (c) is at $\text{Re}=3070$.

As shown in Fig.10, the maximum fluid velocity u is increased with the increasing of Re under three working conditions. $u = 3.626 \text{ m} \cdot \text{s}^{-1}$ at Re=1000 is shown in (a); $u = 7.351 \text{ m} \cdot \text{s}^{-1}$ at Re=2027 is shown in (b); $u = 1.113\text{e}1 \text{ m} \cdot \text{s}^{-1}$ at Re=3070 is shown in (c). The pressure P is also increased with the increasing of Re under three working conditions. $P = 805.8 \text{ kPa}$ at Re=1000 is shown in (a); $P = 3.459\text{e}1 \text{ kPa}$ at Re=2027 is shown in (b); $P = 7.866\text{e}1 \text{ kPa}$ at Re=3070 is shown in (c). The simulation results are consistent with the experimental results.

The comparisons of the proposed MDE method with previously published studies are shown in Table 4.

Table 4 Comparisons of the methods with previously published studies

Comparative Items	Previously published studies	The proposed MDE method
Number of streams in each thermal equilibrium	2 ^[9]	More than two streams shown in Eq.(58)
Effective heat transfer distance	Half fin height ^[9]	More than half fin height shown in Eq.(60)
Phase state of multi streams	Single-phase ^[29]	Considering phase change shown in Eq.(72)
The total number of passages	14 ^[31]	More than 14 shown in Fig.5, Fig.6 using Eq.(36)
Number of multi streams	1 hot stream and 3 cold streams ^[31]	2 hot streams and 4 cold streams shown in Fig.5, Fig.6
Outlet temperature inhomogeneity	4.1% ^[31]	29.59% shown in Fig.6
Type of fin	Serrated fin ^[32]	Plain fin, serrated fin and wavy fin shown in Table.1
Number of working conditions	One ^[24]	More than one shown in Fig.8, Fig.9, Fig.10

The fin parameters are finally determined according to the adaptability to the working conditions by comparing six different fins and cores. According to Eq.(77),

the total heat transfer coefficient of large PFHE is increased from $2365 \text{ W}\cdot\text{m}^{-2}\cdot\text{K}^{-1}$ to $2879 \text{ W}\cdot\text{m}^{-2}\cdot\text{K}^{-1}$ with ratio of 21.73% using the proposed MDE method.

6 Conclusions

(1) Multiple Dynamic Equilibriums (MDE) is proposed to complete design of large PFHE. The large heat transfer area in plane perpendicular to the flowing direction is divided into MDE according to the temperature difference sequence between the one-sided adjacent streams. The MDE is not based on the assumption that there exist thermal equilibrium between the two adjacent streams. Therefore, MDE is applicable for various stacking patterns of hot and cold streams.

(2) The heat transfer process about Re is divided into MDE according to the dynamic equilibriums between heat transfer and frictional resistance to determine the optimal fins under different working conditions. It is especially useful for the condition that Renault number of multi-streams fluctuates in wide range which generates various working conditions.

(3) The experiment of heat transfer and flow resistance is completed to determine heat transfer factor and flow resistance factor under different Reynolds number. The proposed method is especially useful for thermal design of large energy-efficient PFHE for ASU.

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NOMENCLATURE

A = cross-section area, m^2

A_1 = primary heat surface area, m^2

A_2 = secondary heat surface area, m^2

A_e = effective heat surface area per meter, m^2

C_P = mass specific heat capacity at constant pressure, $J \cdot kg^{-1} \cdot K^{-1}$

d_e = equivalent diameter, mm

d_f = fin density, m^{-1}

f = fanning friction factor (f -factor)

h_e = effective heat transfer distance (m)

h_f = fin height/ inside height, mm

$\dot{H}$ = mass specific enthalpy, $J \cdot kg^{-1}$

j = heat transfer factor (j -factor)

λ = thermal conductivity, $W \cdot m^{-1} \cdot K^{-1}$

m = mass of substance, kg

N_E = number of streams in equilibrium

p_f = Fin pitch/spacing, m

P = pressure, Pa

q = heat transfer rate, W

q_A = heat flux, $W \cdot m^{-2}$

q_m = mass flux per unit area, $kg \cdot m^{-2} \cdot s^{-1}$

Q = heat transfer quantity, J

Re = Reynolds number

t_f = fin thickness, mm

T = temperature, K

$T_{c,i}, T_{c,o}$ = cold fluid terminal temperatures (inlet , outlet), K

$T_{h,i}, T_{h,o}$ = hot fluid terminal temperatures (inlet , outlet), K

K = heat transfer coefficient, $\text{W} \cdot \text{m}^{-2} \cdot \text{K}^{-1}$

K_{α} = is convective heat transfer coefficient, $\text{W} \cdot \text{m}^{-2} \cdot \text{K}^{-1}$

u = fluid velocity, $\text{m} \cdot \text{s}^{-1}$

ν = kinematic viscosity, $\text{m}^2 \cdot \text{s}^{-1}$

$\dot{V}$ = volume flow rate of stream ($\text{N m}^3 \cdot \text{s}^{-1}$)

ΔH = enthalpy change, $\text{J} \cdot \text{kg}^{-1}$

ΔP = pressure drop, Pa

ΔT = temperature change, K

ε = Thermal efficiency

η_f = Fin efficiency

η_s = Surface efficiency

μ = dynamic viscosity, $\text{Pa} \cdot \text{s}$

ρ = mass density of a stream, $\text{kg} \cdot \text{m}^{-3}$

SUBSCRIPTS

c cold

h hot

i inlet

o outlet

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