Optimization design of built-up thermal protection system based on validation of corrugated core homogenization

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ABSTRACT

An optimization procedure aiming to design a built-up thermal protection system is presented in this paper. The built-up thermal protection system comprises of classical Integrated Thermal Protection System (ITPS) and additional layer of insulation material which is attached to the cold surface of ITPS. In this paper, corrugated core sandwich panel which is fabricated using C/SiC composite material is considered as an ITPS when the core void is filled with insulation material. It is found in this paper that for the built-up TPS the temperature calculated based on corrugated core homogenization shows significant deviation from the exact solution. The significant advantage of the built-up TPS in insulation ability compared to the traditional ITPS is validated. An interesting phenomenon is also found

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that with the increasing insulation ability of the attached insulation, the temperature on the cold surface of the sandwich panel would converge to a stable value. Based on this finding, a two-step design procedure with high accuracy and high efficiency is exploited for the built-up thermal protection system. At the first step, the stable value of temperature on the cold surface of the sandwich panel is obtained utilizing the finite element technique, and is used as the boundary condition needed in the second step. At the second step, the two-dimensional transient heat transfer problem is transformed into a one-dimensional problem based on which the optimal thicknesses of the multiple insulation layers are obtained utilizing the finite difference formulation. Numerical results show that the thickness and the areal density of the built-up TPS can be significantly reduced by considering multilayer design.

Keywords: Thermal protection system; C/SiC composites; Corrugated core sandwich panel; Homogenization; Transient heat conduction

Nomenclature

t_T the thickness of the top face sheet of the sandwich panel (m)

t_W the thickness of the bottom face sheet of the sandwich panel (m)

t_B the thickness of the webs (m)

d the height of the sandwich panel (m)

θ the angle of the inclined webs (rad)

T temperature ($^{\circ}\text{C}$)

k thermal conductivity ($\text{W/m } ^{\circ}\text{C}$)

ρ density (kg/m^3)

c specific heat (J/kg °C)

V volume (m³)

t time (s)

h the height of the insulation layer (m)

i the i th time point

n the n th space point

q heat flux ((W/m²))

ε the emissivity of the surface

σ the Stefan-Boltzmann constant (W/m² K⁴)

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35 1. Introduction

36 Hypersonic aircrafts have to withstand extremely high aerodynamic heating during the
37 ascent and reentry stages. In order to ensure the safety and effectiveness of the high speed
38 flying aircrafts, the primary task is to maintain the underlying structure within acceptable
39 temperature. Thermal Protection Systems (TPS) are necessary in order to protect the internal
40 structure of the aircrafts from the high heat fluxes occurring on the external surfaces [1-4].

41 The current state-of-the-art TPS covers the outer surface of an aircraft with a layer
42 that does not carry significant loads but slows the high heat flow transferring into the internal
43 structure and prevents the underlying loadbearing structure from exceeding acceptable
44 temperature limits [5]. For this kind of traditional thermal protection system, the safety would
45 be easily destroyed and the reliability would be significantly reduced by thermal bowing
46 and pressure loads and other unpredictable external impact actions such as meteorite, rain, hail,

and dust, due to the insufficient mechanical property of the traditional TPS[6, 7]. Therefore, designing a structure to function simultaneously as a thermal insulator and an efficient aerodynamic load-bearing structure is actually urgently needed. A directly generated idea is to develop ceramic heat insulation material with better mechanical properties. However, there has not much work can be done with respect to develop new materials, and there is good scope for developing new structures[6]. An attractive alternative for thermal protection is integral thermal protection system[5,6, 8-10]. Bapanapalli et al. [6] were the first to use corrugated core sandwich panel to design thermal protection system, and proposed an ITPS concept and a preliminary design process in 2006. In that work they detailed expressed the advantages of corrugated core sandwich structure. Corrugated core sandwich panel can provide high in-plane stiffness and out of plane stiffness, and hence can withstand high aerodynamic pressure load and in-plane load that traditional TPS cannot afford. Meanwhile, the empty space in the corrugated core can be filled with non-load-bearing insulation material, and this makes ITPS has both thermal protection as well as load bearing capabilities. Besides, corrugated core sandwich structures are easier to manufacture when compared to truss-core sandwich structures. For honeycomb sandwich structure, it is not so easily achievable to pack insulation material in vacant spaces between the webs compared to corrugated core sandwich structure. In addition, considering the constant pursuit of a lightweight aircraft, the significant potential of reducing weight further highlights the advantages of ITPS[11]. As a comparison, research has demonstrated that no significant weight savings can be achieved by using a functionally graded metal foam-core sandwich structures[12].

However, the effects of heat shorts of the webs of corrugated core are so significant that much longer heat transfer path hence much thicker filled insulation are frequently needed which finally leads to a low structural efficiency ITPS. Much work has been done to address this shortcoming of the corrugated core sandwich ITPS [13-15]. But the too complicated manufacturing process involved makes these design schemes very difficult to accomplish.

The restriction of the fabrication makes the current design of ITPS difficult to effectively avoid the heat short. Actually, excellent heat insulation can be obtained by attaching insulation layer to the classical ITPS externally. In contrast to the traditional TPS which is tiled on the outer surface of the aircraft, in a built-up thermal protection system, the insulation layer is attached to the cold surface of the sandwich panel or the ITPS. In this design framework, the brittle insulation layer is protected from environmental load such as heat blowing and aerodynamic pressure, which makes the insulation material with significant thermal insulation capabilities but weaker mechanical property as an expense can be fully utilized, such as aerogel [16]. Besides, multilayer design can be implemented to the attached insulation layer to provide more excellent heat insulation [17]. Although this type of built-up TPS is no longer classical ITPS or TPS, however, it can be an effective alternative for the traditional TPS and ITPS before the fabrication being significantly improved.

No matter for traditional TPS or ITPS, cumbersome optimization calculation is needed to obtain an optimum design. Martinez et al. [5,8,9] developed a micromechanical method to homogenize the sandwich panel and calculated the equivalent thermal forces and the stresses in an ITPS subjected to mechanical and thermal loads. Gogu et al. [10] implemented a material selection study for an ITPS to seek material combinations together with the corresponding

optimal panel geometry for low-mass designs. Xie et al. [18] provided an optimization procedure accounting for the mass minimization in the case of the multilayer insulation of thermal protection system. And they also utilized the globally convergent method of moving asymptotes (GCMMA) for heat transfer analysis and thermal-mechanical analysis [11]. In order to implement more efficient optimization calculations, for the design of traditional multilayer TPS, various analytical, semi-analytical and numerical methods have been developed recently [19-23]. Sheikholeslami and Ganji [24] provide a brief review of researches on heat transfer via semi analytical and numerical methods. For the design of built-up thermal protection system, optimization calculations are also needed. In the preliminary design process of the ITPS, the sandwich core is frequently homogenized to reduce the cumbersome calculation generated in the optimization design [6]. For the ITPS, the result calculated based on core homogenization shows good agreement with the exact result [5]. However, for the built-up TPS, the effectiveness of core homogenization of the sandwich panel has never been validated in the existing literature according to the authors' knowledge, and this will be addressed in this paper.

Besides, hypersonic aircrafts suffer high surface temperature frequently up to 1500 °C during their high-Mach flight. However, the melt temperature of the ordinary aluminum and titanium alloy is about 200 °C to 300 °C, and even the nickel-base superalloy can generally only endure 800 °C to 1000 °C. Due to the attractive properties such as superior specific mechanical properties under elevated thermal oxidative environments, carbon fiber reinforced silicon carbide matrix composites C/SiC has been studied extensively [25-26]. However, there is few literature concerning with the thermal protection system design based on C/SiC

composite sandwich structure [27-30].

In summary, firstly, it is difficult to apply the traditional ITPS in practice at the present stage due to the significant heat short effect. In view of this point, the present paper provides a concept of built-up thermal protection system which is composed of C/SiC composite corrugated core sandwich panel and insulation layer attached to the cold surface of the sandwich panel. Secondly, for the built-up TPS, the effectiveness of core homogenization of the sandwich panel is not validated till now and the design procedure based on the 2-D finite element model incurs cumbersome computational time. Therefore, the present paper examines the effectiveness of core homogenization of the built-up TPS next. At last, a design procedure based on multilayer design for the attached insulation material is developed to obtain the optimal structural configuration.

2. Geometry and heat description of C/SiC corrugated core sandwich panel

For the C/SiC composite corrugated core sandwich panel, uniform heat boundary condition is assumed on the top surface of the top face sheet in this paper. By considering that the geometry and material of the sandwich panel is uniform along the y direction, the heat conduction problem of the actual structure can be analyzed based on a two-dimensional model in the xz plane, as shown in Fig. 1(a). In addition, the two-dimensional model can be further simplified to a unit cell model by utilizing the periodic property of the sandwich panel in the x direction. Without loss of generality, the geometry properties of the corrugated core sandwich structure are chosen as: $t_T = t_W = t_B = 2 \text{ mm}$, $\theta = \pi/3$, $d = 14 \text{ mm}$, $p = 16.93 \text{ mm}$. The density and specific heat of C/SiC are 2100 kg/m^3 and $1420 \text{ J/kg}^\circ\text{C}$, respectively. The thermal conductivity of C/SiC is given in Table 1[30].

Table 1

Temperature dependent thermal conductivity of C/SiC [30]

T (°C)	k_s (W/m °C)	T (°C)	k_s (W/m °C)
20	9.3	700	5.0
100	8.2	800	4.9
200	7.2	900	4.8
300	6.4	1000	4.7
400	5.9	1100	4.6
500	5.5	1200	4.5
600	5.2		

Uniform heat flux is assumed injecting on the top surface of the corrugated core sandwich panel, and part of the heat returnsto the outer environment through radiation and convection occurringon the top surface.Forintegrated thermal protection system, insulation material **such as Mullite thermal insulationblanket**is packed into the void of the core.Therefore, the remainder heat flows from the top face sheet along the webs and insulations and arrives at the bottom face sheet.The left and right sides of the unit cell are adiabaticdue to the symmetry and periodic on these two sides. The bottom surface is assumed adiabatic here. Therefore, conservative result is obtained in the design process.**Fig.1(b)** shows the schematic of heat transfer in a unit cell. The top surface of hypersonic aircrafts withstands extremely high temperature during high speed flight. Since the top face sheet is commonly thin, the filled insulation would endure high temperature which makes hightemperature resistance very important for the insulations. Besides, by considering the convenience requirement for the insulation padding, thermalblanket is assumed as the insulations which are filled into the sandwich core void in this paper, and its property parameters are given in **Table A.1** in Appendix A.It should be pointed out that this paper is focus on the design

procedure, not on the material selection as being investigated in [10].

Without loss of generality, the thermal boundary condition is given in terms of temperature on the top surface of the sandwich structure, and the value is shown in Fig. 2. The heat conduction problem addressed here is nonlinear and transient, and considers the temperature dependence of the material property. The finite element procedure pack ABAQUS [31] and the finite differential formula are used to solve the heat transfer problem. The finite element method has been widely used in the heat conduction problem due to its strong versatility for complex geometry and material [32, 33].

3. Validation of corrugated core homogenization

3.1 Formulas of the homogenization

In the existing literatures, the core of ITPS is frequently homogenized as a homogeneous material to reduce the computation load. And during homogenizing, the horizontal webs are no longer considered which means that the inclined webs are connected to the face sheets directly. Compared to the original model shown in Fig. 1(a), the simplified and homogenized model is shown in Fig. 3. The homogenized properties of the core are calculated by the rule of mixtures formulae. The formulae below show the homogenized properties for density, specific heat and thermal conductivity [6]:

effective density

$$\rho_C = \frac{\rho_W V_W + \rho_I V_I}{V_C} = \frac{\rho_W d_W + \rho_I (p \sin \theta - d_W)}{p \sin \theta} \quad (1)$$

effective specific heat

$$c_C = \frac{c_W \rho_W V_W + c_I \rho_I V_I}{\rho_C V_C} = \frac{\rho_W d_W c_W + \rho_I c_I (p \sin \theta - d_W)}{\rho_W d_W + \rho_I (p \sin \theta - d_W)} \quad (2)$$

effective conductivity

$$k_C = \frac{k_W A_W + k_I A_I}{A_C} = \frac{k_W d_W + k_I (p \sin \theta - d_W)}{p \sin \theta} \quad (3)$$

where, subscripts W, I, and C indicate web, insulation, and homogenized material, respectively.

3.2 Validation of core homogenization of ITPS

It has been shown in reference [6] that for corrugated core sandwich ITPS, the temperature on the bottom surface of the classical ITPS calculated based on core homogenization agrees well with the result calculated based on the exact model. However, that work did not discuss the effect of web thickness when comparing the results from the two models. Besides, significant difference in material property can be found between the C/SiC composites used in this paper and the metal alloy used in that work. In view of these points, validation of core homogenization of C/SiC composite ITPS considering various values of web thickness is first given below.

The comparison of the results from original model, model without horizontal webs and homogenized model is implemented. By taking the ITPS with 2 mm thick webs as example, Fig. 4 gives the finite element meshes of the model without horizontal webs and the original model. The two models have 1879 and 1890 quadrangular heat transfer elements DC2D4, respectively. We are concerned with the temperature on node 1 and node 2 shown in Fig. 4. For the homogenized model, one-dimensional transient heat conduction finite element analysis can be implemented which involves 24 linear heat transfer elements DC1D2. Besides, two boundary conditions are assumed at the interface of two adjacent regions, namely, continuity of temperature, which implies perfect thermal contact, and continuity in heat flux.

Fig. 5 shows the comparison of results from the three models with the thickness of the

webs being 1 mm, 2 mm, 3 mm and 4 mm. In all the calculations in this paper, the initial temperature of the structure is chosen as 20 °C. As shown in Fig.5, the results from the three models show small discrepancy during the entire time except that the temperature on node 1 of the models with 1 mm web shows a little obvious error. And with the increasing of the web thickness, the difference of the temperature on node 1 calculated from different models significantly decreases. This is because that the effect of heat short substantially increases with the web thickness and promotes uniform temperature on the bottom surface of the ITPS. It can be noted that with the increasing of web thickness the temperature at node 2 from different models converges to the same value more rapidly than that at node 1. This is because that node 2 stays away from the webs and hence is weakly affected by the heat short effect. Therefore, the accuracy of the homogenization of sandwich core would be enhanced with the increasing web thickness.

Meanwhile, the mesh sensibility analysis should be conducted to insure the reliability of the conclusions. Taking the calculation of the temperature at node 1 and node 2 of the original model with 2 mm webs as an example, Fig. 6 shows the absolute value of relative error of the result from the model with 1890 elements and 11915 elements with respect to the result from the model with 47636 elements, respectively. It can be noted that the maximum error is about 1 %, and for most time range, the error is less than 0.02 %. Therefore, the results from model with 1890 elements can be considered as accurate.

Fig. 7 shows the temperature on the top and bottom surfaces of the model with 1 mm webs. As can be seen, the temperature on the bottom surface stays high level due to the effect of heat short. In order to decrease the heat short effect, methods such as reducing the web

thickness and increasing the core height or combining the two methods may be adopted. However, these methods would lead to insufficient mechanical ability and over-height or over-weight. Another simple but effective method is to attach additional insulation to the bottom surface of the sandwich panel. Compared to the traditional TPS and ITPS, this concept produces a built-up thermal protection system. Although the effectiveness of the core homogenization of the traditional ITPS has been validated as shown in Fig. 5, the accuracy of the homogenization has not and should be examined for the built-up TPS.

3.3 Validation of homogenization of built-up thermal protection system

Without loss of generality, the validation of core homogenization is implemented considering a built-up TPS with 1 mm webs. As shown in Fig. 8, insulated tile is attached to a traditional ITPS on the bottom surface, and the material parameters of the tile are given in Table A.2 in Appendix A.

Fig. 9 gives the comparison of results from three models with different tile thickness, like 5 mm, 10 mm, 30 mm and 40 mm. As can be seen, obvious discrepancy between the result from the homogenized model and that from the other two models can be found for all tile thicknesses. It can be noted that the difference between the result from the model without horizontal webs and that from the original model significantly decreases with the increasing tile thickness. This is because the increasing insulation ability of the tile layer ensures enough time for the uniformization of the heat flux in the tile layer.

Fig. 10 compares the insulation performance of the classical ITPS and the built-up TPS with the same sandwich structure and tile layer with various thicknesses. The temperature on the innermost surface is shown. As can be seen, the tile layer can provide considerable

insulation which validates the effectiveness of the concept of built-up TPS.

4. Design procedure of built-up TPS

As illustrated in the last section, the built-up TPS provides more significant insulation than the classical ITPS. Actually, the insulation performance can be further enhanced by implementing multilayer design for the attached insulation layer of the built-up TPS. Aerogel with excellent insulation performance only works below certain temperature, generally 800 °C-900 °C. It is required that the temperature of the bottom surface of the sandwich panel cannot exceed the upper limit of the workable temperature of aerogel material when aerogel is directly attached to the sandwich panel. However, this requirement is difficult to satisfy for aircrafts in high speed flight. Although high-temperature-resistant ceramic tile can endure much higher temperature, much thicker tile is needed to reduce the innermost surface temperature due to the much higher heat conductivity compared to aerogel material. An effective choice is to design the attached insulation in terms of multilayer. For the multilayer design, the restraint must be satisfied is that the temperature of each layer is lower than their respective upper limits of the workable temperature.

In order to make the optimization design process more time saving, the sandwich core is traditionally homogenized. However, as shown in the last section, homogenization of the sandwich core which is however effective for the classical ITPS is invalid for the built-up TPS. Therefore, in the design of the built-up TPS, the corrugated core sandwich panel must be exactly analyzed into a two-dimensional model. An interesting result is found in the numerical simulations that the temperature on the bottom surface of the sandwich panel converges to a stable level with the increasing insulation ability of the attached layer. Without loss of generality,

a built-up TPS composing of sandwich panel and two additional layers is taken as an example, as shown in Fig. 11. The material parameters of aerogel are given in Table A.3 in Appendix A. Considering that the temperature of each layer cannot exceed their upper limits, we are concerned with temperatures of the three nodes as shown in Fig. 11.

Temperature of node 1 associated with four kinds of aerogel thickness, like 5 mm, 10 mm, 20 mm and 30 mm and 10 mm tile, is shown in Fig. 12. As can be seen, temperature on the bottom surface of the sandwich panel converges to a stable level with the increasing of insulation ability of the attached insulation layer. This interesting phenomenon may be explained based on the analysis of the temperature gradient. Since adiabatic condition on the innermost surface of the built-up TPS is assumed in this paper, the less insulation ability of the insulation layer, the more rapidly the heat transferring to the adiabatic surface. And then the temperature gradient hence the temperature at the cold surface of the sandwich panel varies more easily. With the increasing of the insulation ability of the attached insulation layer, the influence of the heat reflection at the adiabatic surface on the temperature of the bottom surface of the sandwich panel is significantly reduced. Therefore, the temperature of the cold surface of the sandwich panel stays at a stable value due to the constant temperature gradient at the surface. Based on this finding, a useful two-step strategy can be developed for the optimization design of the built-up TPS. Firstly, the heat conduction of the built-up system with high insulation ability layers may be analyzed by using finite element program, and temperature on the cold surface of the sandwich panel can be extracted. And secondly, new model only including the insulation layers can be established. Since the attached insulation layers are homogeneous along the other two orthotropic coordinate directions, the original

2-D heat conduction problem can be transformed into a 1-D heat conduction problem along the stacking up direction. The temperature extracted in the first step is selected as the temperature boundary of the new model. For the new model, optimization based on one-dimensional heat conduction analysis can be implemented with high efficiency. The governing equation of the one-dimensional transient heat conduction problem can be expressed as

$$\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial z} \left(k \frac{\partial T}{\partial z} \right) \quad (4)$$

Explicit difference scheme is used to solve Eq. (4), where forward difference scheme for time derivative and backward difference scheme for the first order spatial derivative and central difference scheme for the second order spatial derivative. Detail derivations are given in Appendix B. The constraints of the optimization problem taking the thickness of each insulation layer as design variable can be expressed as

$$T_{1\max}(h_1, h_2) \leq 800 \text{ }^{\circ}\text{C} \quad (5)$$

$$T_{2\max}(h_1, h_2) \leq 180 \text{ }^{\circ}\text{C} \quad (6)$$

Eq. (5) and Eq. (6) indicate that the temperature of the aerogel layer and the innermost surface cannot exceed the design value. The optimization objective considered here is minimum weight of the system, i.e. seeking smallest areal density $m = m_0 + \rho_1 h_1 + \rho_2 h_2$, where m_0 is the areal density of the sandwich panel with filled insulations.

It should be mentioned here that the optimal thickness of the insulation layers are obtained here by using the graphic method rather than other optimization algorithm developed with more complex mathematical theory. The graphic method draws the feasible zone and the isogram of the objective function in the design space. And then the location which makes the

objection function minimum within the feasible zone can be easily found. The graphic method provides intuitive evolution of the objective function with the design variable which helps to perceive the essential mechanism of the problem.

Equal-temperature curve of the top and bottom surfaces of the aerogel layer is shown in Fig. 13. As can be seen, in order to satisfy the constraint condition, the thickness of tile layer is at least 8 mm. With the increasing of the thickness of the tile, the thickness of the aerogel may be allowed to decrease. However, since the insulation ability of the tile is much lower than that of the aerogel, the total thickness and weight of the system would significantly increase with the increasing thickness of the tile layer. The same conclusion can also be observed from the equal-density curve shown in Fig. 13. Consequently, the optimum combination is tile layer 8 mm and aerogel layer 35 mm, and the total height and areal density of the built-up TPS is 59 mm and 2.75 g/cm^2 , respectively. Fig. 14 gives the temperature on the innermost surface when only tile layer is considered. The minimum thickness of the tile is 56 mm, and the total height and areal density of the system is 72 mm and 3.04 g/cm^2 respectively. Therefore, the multilayer design can significantly reduce the height and weight of the system. As also can be seen in Fig. 13, the temperature on the top surface of aerogel layer converges to a stable level with the increasing of insulation ability of the additional layer.

Fig. 15 gives the temperature calculated from the built-up TPS models with and without core homogenization, where the tile layer is 8 mm and the aerogel layer is 35 mm. As can be seen, the calculation based on core homogenization gives accurate result for the innermost surface. However, the temperature calculated from the homogenized model shows significant discrepancies for the top surface temperature of the aerogel layer, and this may fail the aerogel

and hence the system in actual flight environment. As a result, during the design of the built-up thermal protection system, the sandwich structure filled with insulations should be exactly modeled rather than be homogenized.

5. Conclusions

A type of built-up thermal protection system comprising of C/SiC composite corrugated core sandwich panel filled with insulation material and attached insulation layers is provided in this paper to compensate the insufficiency of traditional ITPS due to the significant heat short problem. Transient heat transfer analysis for the classical ITPS with different web thickness is first addressed by utilizing the finite element procedure ABAQUS and the effectiveness of core homogenization is validated. And then, the heat transfer of the built-up TPS is analyzed. Numerical result shows the significant advantage of the built-up TPS in insulation ability compared to the traditional ITPS. It is found that core homogenizing is not suitable for the built-up TPS. Therefore, during the design of the built-up TPS the sandwich structure should be exactly modeled. It is also found that the temperature on the cold surface of the sandwich panel of the built-up TPS converges to a stable level with the increasing insulation ability of the attached insulation layers. Based on this finding, a two-step design procedure is suggested for the built-up TPS to reduce the cumbersome computational time encountered by the computation based on the 2-D model. In the first step, the stable temperature of the cold surface of the sandwich panel is first obtained by using the finite element technique. And then, in the second step, the 2-D heat transfer model is transformed into a 1-D problem. Benefited from this key transformation, multilayer design for the attached insulation layer based on the finite difference formulation can be efficiently implemented. Numerical results confirm the

qualities of high accuracy and high efficiency of the two-step design procedure. The significant advantage of the built-up TPS in reducing the height and weight of the thermal protection system is also validated.

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Appendix A. Material parameters

Table A.1 Material parameters of Mullite thermal insulation blanket

T (°C)	k_s (W/m °C)	c (J/kg °C)	ρ (kg/m ³)
20	0.032	/	/
300	0.036	/	/
500	0.059	/	/
700	0.093	1000	100
900	0.136	/	/
1100	0.181	/	/
1300	0.241	/	/

Table A.2 Material parameters of ceramic tile

T (°C)	k_s (W/m °C)	c (J/kg °C)	ρ (kg/m ³)
20	0.05	/	/
300	0.06	/	/
500	0.07	1000	300
800	0.08	/	/
1100	0.09	/	/

Table A.3 Material parameters of aerogel

T (°C)	k_s (W/m °C)	c (J/kg °C)	ρ (kg/m ³)
20	0.023	700	/
100	0.023	745	/
200	0.024	733	/
300	0.025	757	/
400	0.027	806	337
500	0.029	768	/
600	0.030	817	/
700	0.032	743	/
800	0.034	920	/

Appendix B. Finite difference formulation for the 1-D transient heat conduction

B.1 Governing equations

Since the thermal conductivity of the material considered in this paper is temperature dependent as shown in Appendix A, the one-dimensional transient heat conduction given in Eq. (4) can be expressed as

$$\rho c \frac{\partial T}{\partial t} = \frac{\partial k}{\partial z} \frac{\partial T}{\partial z} + k \frac{\partial^2 T}{\partial z^2} \quad (\text{B.1})$$

Explicit difference scheme is used to solve Eq. (B.1), where forward difference scheme for time derivative and backward difference scheme for the first order spatial derivative and central difference scheme for the second order spatial derivative. Then there has

$$\begin{aligned} \frac{\partial T}{\partial t} &= \frac{T(z_n, t_{i+1}) - T(z_n, t_i)}{\Delta t} \\ \frac{\partial T}{\partial z} &= \frac{T(z_n, t_i) - T(z_{n-1}, t_i)}{\Delta z} \\ \frac{\partial^2 T}{\partial z^2} &= \frac{T(z_{n-\Delta z}, t_i) + T(z_{n+\Delta z}, t_i) - 2T(z_n, t_i)}{(\Delta z)^2} \end{aligned} \quad (\text{B.2})$$

By substituting Eq. (B.2) into Eq. (B.1), we have

$$T_n^{i+1} = T_n^i + G_\Delta (T_n^i - T_{n-1}^i)^2 + F_\Delta (T_{n-1}^i + T_{n+1}^i - 2T_n^i) \quad (\text{B.3})$$

where, $G_\Delta = \frac{\Delta t}{\rho c (\Delta z)^2} \frac{\partial k}{\partial T}$, $F_\Delta = \frac{k \Delta t}{\rho c (\Delta z)^2}$, and for convenience, k can be obtained using polynomial curve fitting based on the data given in Appendix A.

B.2 Continuum at the interface

Two boundary conditions are assumed at the interface of two adjacent materials, namely, continuity of temperature, which implies perfect thermal contact, and continuity in heat flux. Taking two adjacent material p and l as an example, as shown in Fig. B.1, continuity of temperature at the interface requires $T_{n,p}^i = T_{1,l}^i$, $T_{n,p}^{i+1} = T_{1,l}^{i+1}$. Consider the energy equilibrium at the boundary point n of material p . The input heat from the left point is

484 $k_p \frac{\partial T}{\partial z} = k_p \frac{T_{n-1,p}^i - T_{n,p}^i}{\Delta z_p}$. The heat from the material l is q_{lp} and the heat variation with time at
 485 point n is $\frac{\Delta z_p}{2} \rho_p c_p \left(\frac{T_{n,p}^{i+1} - T_{n,p}^i}{\Delta t} \right)$. Therefore, the energy equilibrium at point n can be
 486 expressed as

$$487 \quad k_p \frac{T_{n-1,p}^i - T_{n,p}^i}{\Delta z_p} + q_{lp} = \frac{\Delta z_p}{2} \rho_p c_p \left(\frac{T_{n,p}^{i+1} - T_{n,p}^i}{\Delta t} \right) \quad (\text{B.4})$$

488 Similarly, considering the energy equilibrium at the boundary point 1 of material l has

$$489 \quad k_l \frac{T_{2,l}^i - T_{1,l}^i}{\Delta z_l} + q_{pl} = \frac{\Delta z_l}{2} \rho_l c_l \left(\frac{T_{1,l}^{i+1} - T_{1,l}^i}{\Delta t} \right) \quad (\text{B.5})$$

490 From Eq. (4) and Eq. (5) and $q_{lp} = -q_{pl}$, there has

$$491 \quad T_{n,p}^{i+1} = T_{1,l}^{i+1} = T_{n,p}^i + \frac{2}{\frac{1}{F_{\Delta,p}} + \frac{1}{F_{\Delta,pl}}} (T_{n,p}^i - T_{n,p}^i) + \frac{2}{\frac{1}{F_{\Delta,lp}} + \frac{1}{F_{\Delta,l}}} (T_{2,l}^i - T_{1,l}^i) \quad (\text{B.6})$$

$$492 \quad \text{where, } F_{\Delta,p} = \frac{k_p}{\rho_p c_p} \frac{\Delta t}{(\Delta z_p)^2}, F_{\Delta,pl} = \frac{k_p}{\rho_l c_l} \frac{\Delta t}{\Delta z_l \Delta z_p}, F_{\Delta,lp} = \frac{k_l}{\rho_p c_p} \frac{\Delta t}{\Delta z_p \Delta z_l}.$$

493 **B.3 Boundary condition**

494 The heat boundary condition such as incident heat flux, convection, radiation rejection
 495 and adiabatic can be similarly addressed as the interfacial condition, for simplicity, only
 496 brief equations are given and the detail derivations are not shown here. The incident heat flux,
 497 heat radiation, heat convection and adiabatic condition can be separately expressed as

$$498 \quad T_1^{i+1} = T_1^i + 2F_{\Delta} (T_2^i - T_1^i) + \frac{2\Delta t}{\rho c \Delta z} q_w - \frac{2\Delta t}{\rho c \Delta z} \varepsilon \sigma \left((T_1^i)^4 - T_e^4 \right) \quad (\text{B.7})$$

$$499 \quad T_n^{i+1} = T_n^i + 2F_{\Delta} (T_{n-1}^i - T_n^i) - \frac{2\Delta t}{\rho c \Delta z} \varepsilon \sigma \left((T_n^i)^4 - T_e^4 \right) \quad (\text{B.8})$$

$$500 \quad T_n^{i+1} = T_n^i + 2F_{\Delta} (T_{n-1}^i - T_n^i) - \frac{2\Delta t}{\rho c \Delta z} h (T_n^i - T_e) \quad (\text{B.9})$$

$$501 \quad T_n^i = T_{n-1}^i \quad (\text{B.10})$$

502 where, q_w is the incident heat flux, ε is the emissivity of the surface, σ is the
 503 Stefan-Boltzmann constant, T_e is the ambient temperature, h is the free convection heat
 504 transfer coefficient and the temperature in Eq. (B.7) and Eq. (B.8) is Kelvin temperature.

505

Figure captions

Fig. 1(a) Geometry of unit cell of corrugated core sandwich panel

Fig. 1(b) Heat description of unit cell of an ITPS

Fig.2 Temperature boundary condition on the top surface of the sandwich panel

Fig. 3(a) Simplified ITPS model without horizontal webs

Fig. 3(b) Homogenized model of ITPS

Fig. 4(a) Finite element model of ITPS unit cell without horizontal webs

Fig. 4(b) Finite element model of ITPS unit cell

Fig. 5 Temperature of different modes with four kinds of web thickness

Fig. 6 Absolute value of relative error of the result from the model with 1890 elements and 11915 elements with respect to the result from the model with 47636 elements

Fig. 7 Temperature on the top and bottom surfaces of ITPS with 1 mm webs

Fig. 8 Finite element mesh of built-up TPS

Fig. 9 Comparison of the results from the three models with different tile thickness

Fig. 10 Temperature on the innermost surface of ITPS and built-up TPS with various values of tile thickness

Fig. 11 Finite element mesh of a built-up TPS

Fig. 12 Temperature of node 1 associated with different aerogel thickness

Fig. 13 Equal-temperature curve of the temperature on the top and bottom surfaces of aerogel layer and equal-area density curve of the whole system

Fig. 14 Temperature on the innermost surface when only tile is considered

Fig. 15 Temperature calculated from the models with and without corrugated core

528 homogenizing

529 Fig. B.1 Continuity condition at two adjacent material

530

531 **Table captions**

532 Table 1 Temperature dependent thermal conductivity of C/SiC [30]

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534 Table A.2 Material parameters of ceramic tile

535 Table A.3 Material parameters of aerogel

536

Figures

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Fig. 4(b) Finite element model of ITPS unit cell

(a) web thickness, 1 mm

(b) web thickness, 2 mm

(c) web thickness, 3 mm

(d) web thickness, 4 mm

Fig. 5 Temperature of different models with four kinds of web thickness

Fig. 6 Absolute value of relative error of the result from the model with 1890 elements and 11915 elements with respect to the result from the model with 47636 elements

Fig. 7 Temperature on the top and bottom surfaces of ITPS with 1 mm webs

Fig. 8 Finite element mesh of built-up TPS

(a) tile thickness: 5 mm

(b) tile thickness: 10 mm

(c) tile thickness: 30 mm

(d) tile thickness: 40 mm

Fig. 9 Comparison of the results from the three models with different tile thickness

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