**The influence of thermal contact resistance on the relative performance of heat pipe-fin
array systems**

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Highlights

- Impact of contact resistances on finned-heat pipes (HPs) is investigated.
- Two configurations: (i) HP-heat sink and (ii) HP-fins are considered.
- Influence of contact resistance on heat transfer within the HP is shown.
- Tradeoff between (i) low overall system resistance and (ii) range of operation is quantified.

Abstract

Externally-finned heat pipes (HPs) using individual fins (a HP-fin configuration) or a finned heat sink (HP-heat sink) are subject to thermal contact resistances at the HP-fin or HP-heat sink interfaces. A detailed HP model is used to quantify the influence of the contact resistances on the heat transfer and fluid flow within the HP, as well as on the overall heat transfer rate. For similar area-averaged contact resistance values, the HP-heat sink typically provides a lower overall thermal resistance than the HP-fin configuration. Under off-design operating conditions, however, the HP-heat sink option can underperform the HP-fin geometry. A thermal resistance model is also developed and its predictions compare qualitatively to those of the HP model. It is shown that the overall resistance of the HP-fin configuration is proportional to the number of fins in the array that are subject to a contact resistance. Using the detailed model it is also demonstrated that the order in which the fins (fins without versus fins with contact resistances) are placed within the array has little effect on the overall thermal resistance of the HP system.

Keywords

Heat pipe; fin arrays, contact resistance, design tradeoff

1. Introduction

Heat pipes (HPs) are devices that transfer heat more effectively than high thermal conductivity media of the same physical dimensions [1-2]. Heat transfer within HPs is characterized by various thermal resistances, including the small thermal-resistance processes of evaporation (within the HP evaporator section) and condensation (in the HP condenser section) at the vapor-liquid interfaces of an internal working fluid. Applications of HPs have been reviewed [3-15], and in many practical situations the limiting system-level thermal resistances are associated with the flow of single-phase fluids *external* to the HP's condenser or evaporator sections. The larger external thermal resistance will typically be on the condenser end of the HP if, for example, air is used as the coolant and a flowing liquid heats the HP evaporator section. External arrays of fins, foils, and porous metallic foams have been added to HPs to reduce the overall system thermal resistances [16-29].

Two- and 3D models of HPs that account for the complexity of transient, conjugate heat transfer phenomena, including phase change of the HP working fluid, have been developed [27, 28, 30-35]. Recently, Stark et al. [36] presented a detailed, experimentally-verified model capable of predicting the performance of a HP system involving fins that are attached to the condenser (cold) section of the HP. The 3D, transient mixed convection of air through the fin array was predicted by both direct simulation and use of a shear stress turbulence model, and the predictions of external convection were coupled to those of a 2D model that accounted for the pertinent heat transfer phenomena within the HP. It was shown that 3D effects within the HP did not significantly influence the performance of a HP-fin array system similar to that of interest in this study, justifying use of the 2D HP model.

In practice, thermal contact resistances will exist at the interface between the exterior surface of a HP and the roots of individually-mounted, external fins (or the base of a finned heat sink). The magnitudes of these resistances depend on the manner in which the HP and fins (or the finned heat sink) are bonded. A wide range of thermal contact resistance values have been reported for finned-tube heat exchangers [37-41], a geometrical configuration similar to the finned-HP of interest here.

The specific objective of this study is to quantify the effect of thermal contact resistances on the performance of a vertical HP system using two approaches: (*i*) a detailed 2D numerical HP model that accounts for the effects of fin conduction and external convection (the fins are

populated along the HP condenser section) and (ii) a thermal resistance network analysis. As will be shown and as expected, the presence of contact resistances decreases the performance of either the HP-fin or HP-heat sink configurations. However, it will also be demonstrated that the preferred system configuration (HP-fin versus HP-heat sink) is influenced by the presence of thermal contact resistances under certain operating conditions. This study represents, to the authors' knowledge, the first in depth investigation of how contact resistances modify heat transfer phenomena within the HP of such systems, and how they impact the range of conditions over which a HP-fin array system may successfully operate.

2. Numerical Models

Two numerical models, a detailed HP model and a thermal resistance model, will be used to investigate the influence of thermal contact resistances on the performance of an externally-finned HP. Again, the fins are either directly attached to the HP (HP-fin) or incorporated in a heat sink whose base is in contact with the HP (HP-heat sink).

2.1. HP model

Figure 1 shows a vertical HP-heat sink assembly that includes N annular fins incorporated into a heat sink of base thickness t_b . A second configuration (HP-fin, not shown) involves annular fins that are attached directly to the exterior surface of the HP at $r = r_{hp}$ (i.e., $t_b = 0$). In either configuration, the length of the HP, $L_{hp} = L_e + L_a + L_c$, includes an evaporator section (of length L_e), an adiabatic section (of length L_a) and a condenser section (of length L_c). Radially, the HP includes a vapor region ($0 \leq z \leq L_{hp}$; $0 \leq r \leq r_v$), a porous wick ($0 \leq z \leq L_{hp}$; $r_v < r \leq r_w$), and a solid wall ($0 \leq z \leq L_{hp}$; $r_w < r \leq r_{hp}$). The computational domain includes the entire HP, as well as the base of the heat sink (for $t_b = 1$ mm). The influence of convection from the un-finned regions of the heat sink (or from the HP between individually-mounted fins), as well as conduction within and convection from the fins, is described by application of appropriate boundary conditions applied at $r = r_b$ (or $r = r_{hp}$) as will be discussed shortly.

Fig. 1. Computational domain and fin array (HP-heat sink configuration).

Axisymmetric, 2D flow of the HP working fluid and axisymmetric heat transfer within the HP and fins is assumed, while thermal radiation is assumed to be negligible. The HP wick is taken to be saturated with the liquid phase of the HP working fluid, and advection effects within the wick are not included in the governing energy equations [42]. Buoyancy forces are accounted for in the vapor phase of the HP working fluid. Until the HP capillary limit (calculated using the technique in [1]) is reached, the wick is assumed to provide sufficient capillary pressure, in conjunction with gravitational forces, to transport the HP working fluid from the condenser to the evaporator section of the HP. A detailed description of the lengthy governing equations of the HP model, including conservation of mass, momentum and energy, is available elsewhere [33].

A uniform and constant temperature, T_e , is applied to the exterior of the HP ($r = r_{hp}$) along its evaporator section. In addition to the adiabatic section of the HP, the end caps ($z = 0$, L_{hp}) and the bottom (and top) horizontal surfaces of the heat sink base are assumed to be adiabatic. Convection occurs at the un-finned regions of the heat sink (or from the exposed HP

for the HP-fin configuration). For the HP-heat sink configuration the convection losses are described by

$$-k_w \frac{\partial T}{\partial r} \Big|_{z, r=r_b} = \overline{h_w} [T(z, r=r_b) - T_\infty] = \overline{h_w} \theta(z, r=r_b) \quad (1a)$$

where $\overline{h_w}$ is the area-averaged convection heat transfer coefficient on the base of the heat sink. The heat sink base is also cooled by radial conduction to the roots of each of the N annular fins of thickness t_f and outer radius r_f . Fin analysis [43] is used to generate an effective heat transfer coefficient, that is applied at $r = r_b$, so that

$$\begin{aligned} -k_w \frac{\partial T}{\partial r} \Big|_{z, r=r_b} &= \overline{h_{\text{eff}}} \theta(z, r=r_b) = \frac{\theta(z, r=r_b)}{R_f''} \\ &= \overline{h_f} \left[\frac{2}{t_f m} \times \frac{K_1(mr_b)I_1(mr_{fc}) - I_1(mr_b)K_1(mr_{fc})}{K_0(mr_b)I_1(mr_{fc}) + I_0(mr_b)K_1(mr_{fc})} \right] \theta(z, r=r_b) \end{aligned} \quad (1b)$$

where $r_{fc} = r_f + t_f / 2$ is used to account for convection from all exposed fin surfaces, and $m = \sqrt{2\overline{h_f} / k_f t_f}$. The parameter $\overline{h_f}$ is the area-averaged convection heat transfer coefficient associated with the top, bottom and tips of the fins, while I_0 (K_0) and I_1 (K_1) are modified, zero-order and first-order Bessel functions of the first (second) kind. Potential thermal contact resistance is considered at the HP-heat sink interface. The preceding analysis may be modified slightly and applied to the HP-fin configuration, but with Eqs. (1a) and (1b) applied at $r = r_{hp}$ and thermal contact resistances $R_{t,c}''$ specified at the roots of each fin which are added in series with the fin resistances R_f'' defined in Eq. (1b).

The values of $\overline{h_w}$ and $\overline{h_f}$ have been obtained for a geometrically-similar configuration with the same operating conditions considered here through use of an experimentally-verified, (3D, elliptic, including buoyancy effects) shear stress model of the air flow within the fin array [36]. Values of the area-averaged contact resistances for either the HP-heat sink or HP-fin configurations, $R_{t,c}''$, are based on data from the literature, as discussed later.

Equations (1) through (12) of [33], specification of an initial, uniform temperature of 27°C and zero velocities everywhere, along with the preceding boundary conditions complete the 2D HP model description. Note that, although the problem is steady in the mean, initial conditions are specified because the HP code accounts for the transient response before a steady state is reached, providing a unique pressure distributions within the HP [33]. The finite volume approach of Patankar [44] is used to discretize the governing equations. Convergence is assumed when residual mean square values less than 10^{-6} are achieved across all of the dependent variables for heat transfer and flow inside the HP. The solutions are deemed to be grid independent. Specifically, a grid of 190×76 ($z \times r$) yields predictions of overall heat transfer rates within 0.5% of those attained with a grid of 240×96 ($z \times r$) and the coarser grid was used here. Note that 12, 8, and 4 control volumes were included through the thickness of each fin in the grid sensitivity study. Predicted local temperatures using 8 and 12 control volumes in each fin are nearly identical, and 8 control volumes were used here. A typical computation time required for one simulation is 1 hr on a 3.5 GHz, 16 GB Ram computer. Additional details are available in [36].

2.2. Thermal resistance model, HP-fin configuration

To quickly sort design options, it is desirable to develop thermal resistance models (TRMs) for HP systems, and the TRM developed here is similar to those described in [5, 45, 46]. However, the current model accounts for N individual fins and potential contact resistances at the N HP-fin interfaces. A similar model could be developed for the HP-heat sink configuration.

The thermal resistance network for a finned HP with N individual fins is shown in Fig. 2. To consider the potential for variability in thermal contact resistances from fin-to-fin, which could result due to damage during operation, for example, $\tilde{N}$ of the N fins are specified to have no thermal contact resistance at their roots. The remaining $N - \tilde{N}$ fins are subject to a thermal contact resistance, $R_{t,c}$.

As shown in Fig. 2, four individual thermal resistances are applied to the exterior of the HP condenser section to account for the presence of all N fins. The bottom exterior resistance shown, R_w , is associated with convection from the exposed surface of the HP to the ambient fluid (air). For fins having no contact resistance at their base, a parallel path for heat loss from the HP

condenser section is through the $\tilde{N}$ fins, represented by a single thermal resistance $R_f/\tilde{N}$ where R_f is the thermal resistance of one fin. Heat loss through the $N - \tilde{N}$ fins subject to a contact resistance and is represented by the top heat flow path consisting of the contact and fin resistances in series.

Fig. 2. Thermal resistance network (HP-fin configuration).

The thermal resistance associated with the HP working fluid vapor, R_v , is due to evaporation occurring at the HP wick-vapor interface, the pressure drop and associated temperature change of the vapor as it flows upward in the HP, and condensation at the vapor-wick interface [1, 5, 45, 46]. Flow of the HP working fluid vapor also redistributes the temperature distribution within the HP vapor, as will be discussed in conjunction of Fig. 3. The value of R_v is typically small and, in the limiting case, is equal to zero.

3. Results and discussion

3.1. Geometrical parameters and operating conditions

Results are presented for two HP-fin configurations involving $N = 6$ annular fins, each of thickness $t_f = 0.97$ mm and pitch $S_f = 10.8$ mm. As noted previously, the fins are either (i) individually attached to the HP condenser wall of length L_c , or (ii) are part of a cylindrical heat sink of base thickness $t_b = r_b - r_{hp} = 1$ mm and length L_c . The mass of the two configurations is specified to be identical, with $r_f = 53.42$ mm (51.67 mm) for the individual fin (heat sink) case.

The copper-water HP (modeled after an Enertron HP-HD06DI17500BA) and aluminum fins, as well as pertinent thermophysical properties and HP dimensions, are taken from [36] and are listed in Table 1. As in the reference, the upstream velocity of air in crossflow through the fin array is $V = 1.5$ m/s, and the air temperature is $T_\infty = 25^\circ\text{C}$. The heat transfer coefficient for the bare (un-finned) portion of the exposed HP condenser wall (at $r = r_{hp}$) is $\overline{h}_w = 51.5$ W/m²·K, while that of the exposed surfaces of the fins is $\overline{h}_f = 19.5$ W/m²·K. Both values are taken from [36].

Table 1. Thermophysical properties and pertinent dimensions.

	HP working fluid	HP wall	HP wick (sintered powder of porosity 0.4 [47])	Heat sink
Material	Water [1]	Copper [43]	Copper saturated with water (effective properties) [1]	AL 3003-h14 [48]
Density, ρ (kg/m ³)	996.6 (liquid)	8933	5758.44	2730
Thermal conductivity, k (W/m K)	0.6132 (liquid) 0.01932 (vapor)	401	200.8	159
Specific heat, c_p (J/kg K)	4181 (liquid) 1882 (vapor)	385	647.78	893
Viscosity, μ (Pa s)	8614×10^{-7} (liquid) 91.74×10^{-7} (vapor)			
Latent heat, h_{fg} (kJ/kg)	2434.9			
Vapor pressure, p_{sat} (Pa)	4352			
Dimensions (mm)		$L_e = 50.8$ $L_a = 49.2$ $L_c = 75$ $r_v = 2.35$ $r_{hp}-r_w = 0.3$ [47]	$r_w-r_v = 0.35$ [47]	In text

For cases involving a base thickness of $t_b = 1$ mm the value of $\overline{h_w}$ is determined by correcting the $t_b = 0$ value through application of the Hilpert correlation [43] to account for the larger radius, $r = r_b > r_{hp}$. After implementing the correction, the convection coefficient associated with the exposed portions of the heat sink base is $\overline{h_w} = 44.3$ W/m²·K. Since the exposed areas of the fins for the $t_b = 0$ and $t_b = 1$ mm configurations are nearly identical, the value of $\overline{h_f} = 19.5$ W/m²·K is applied to both the HP-heat sink and HP-fin configurations.

Finned tubes are typically formed by expanding the tube radius (e.g. by introducing high pressure oil within the tubes) to form a snug fit between the tube and individual, intermittently-spaced fins. Various fabrication methods lead to thermal contact resistances per unit area ($R''_{t,c}$) ranging from 0 to 10^{-4} m² K/W [37-41]. Here, $R''_{t,c}$ values of 0, 10^{-5} and 10^{-4} m² K/W are specified for both (i) individual fins that are attached directly to the HP and (ii) the heat sink base-HP interface. Since the heat sink base-HP contact area is larger than the fin base-HP contact area, the contact resistances (K/W) for the $t_b = 1$ mm configuration are smaller than those associated with the $t_b = 0$ configuration. Specifically, for $R''_{t,c} = 10^{-5}$ m² K/W, $R_{t,c} = 0.007$, 0.547 and 0.091 K/W for (i) the heat sink base-HP interface, (ii) the interface between a single fin and the HP, and (iii) $N = 6$ fins acting in parallel, respectively. For $R''_{t,c} = 10^{-4}$ m² K/W the corresponding resistances are $R_{t,c} = 0.070$, 5.47 and 0.911 K/W.

3.2. Detailed HP model predictions

Steady-state temperature distributions within the HP (and in the $t_b = 1$ mm thick base of the heat sink) are shown in Fig. 3. The predictions are associated with a uniform HP evaporator temperature of $T_e = 60^\circ\text{C}$ and contact resistances of 0, 10^{-5} and 10^{-4} m² K/W. Isotherms are drawn at intervals of 0.1 K over the range $330.5 \text{ K} \leq T \leq 333 \text{ K}$. For either the $t_b = 0$ or $t_b = 1$ mm configurations, the fins (not shown) impact the temperature distributions within the HP wall, wick, and working fluid. Specifically, the fins depress local temperatures within the HP wall, wick, and working fluid (and in the heat sink base). As evident from inspection of the isotherm distributions within the HP vapor, overall temperature differences along the HP are reduced as thermal contact resistances are increased, as expected.

(a)

(b)

Fig. 3. Temperature distributions for $t_b = 1$ mm (left), $t_b = 0$ (right) with $T_e = 60^\circ\text{C}$: (a) $R''_{t,c} = 0$, (b) $R''_{t,c} = 10^{-5} \text{ m}^2 \text{ K/W}$, (c) $R''_{t,c} = 10^{-4} \text{ m}^2 \text{ K/W}$.

Axial HP wall temperature distributions for the $t_b = 0$ and $t_b = 1$ mm configurations at $0 \leq L \leq L_{hp}$, $r = r_{hp}$ are presented in Fig. 4. For either the $t_b = 0$ or 1 mm configuration the fins induce appreciable thermal depressions in the condenser section of the HP wall in a manner similar to that described in [36] for a HP-fin configuration with no contact resistance. Because of thermal buffering attributed to the heat sink base, temperature depressions are always larger for the individual fin configuration ($t_b = 0$) relative to those associated with the based heat sink ($t_b = 1$ mm). As expected, magnitudes of the local depressions are reduced as $R''_{t,c}$ is increased. As suggested in the discussion of Fig. 3, the average temperature drop along the HP wall ($\Delta T = T_e - \overline{T_c}$) decreases as the thermal contact resistances are increased. Specifically, $\Delta T = 3.19$, 3.17 and 3.07 K for the heat sink ($t_b = 1$ mm) case with no, low, and high thermal resistances. For the individual fin ($t_b = 0$) configuration, $\Delta T = 3.04$, 2.84 and 1.94 K for no, low, and high thermal resistances.

Fig. 4. Heat pipe wall temperatures: (a) $R''_{t,c} = 0$, (b) $R''_{t,c} = 10^{-5} \text{ m}^2 \text{ K/W}$, (c) $R''_{t,c} = 10^{-4} \text{ m}^2 \text{ K/W}$.

Mass fluxes of the liquid phase HP working fluid (water) are reported in Fig. 5; local *radial* mass fluxes at the vapor-wick interface (m''_r , $r = r_v$; LHS of Fig. 5) and local *axial* fluxes averaged over the wick cross-sectional area (m''_a , $r_v \leq r \leq r_w$; RHS of Fig. 5) are shown.

Inspection of the radial fluxes in the HP evaporator section (negative radial mass fluxes), shows

that overall mass transfer rates (and as will be shown, overall heat rates) are reduced as thermal contact resistances are increased. Variations in the radial mass fluxes within the condenser section (positive mass fluxes) are more sensitive to the thermal contact resistance for the HP-fin configuration than for the HP-heat sink configuration, as expected from consideration of the thermal depressions shown in Fig. 4.

Fig. 5. Local mass flux along the HP at the vapor-wick interface (left) and within the wick (right): (a) $R''_{t,c} = 0$, (b) $R''_{t,c} = 10^{-5} \text{ m}^2 \text{K/W}$, (c) $R''_{t,c} = 10^{-4} \text{ m}^2 \text{K/W}$.

Axial mass fluxes within the HP wick (RHS of Fig. 5) are zero at $z = 0$ and L_{hp} , as required. Downward mass fluxes are increased from the top of the HP until the HP adiabatic section is reached, reflecting the condensation of the HP working fluid in the overlaying sections of the HP. The waviness evident in the axial mass flux distributions in the condenser section is associated with the high local radial mass fluxes in the vicinity of each fin root. The axial mass flux is reduced from its maximum value in the HP adiabatic section to zero at $z = 0$ as water evaporates radially inward in the HP evaporator section. The influence of the thermal contact resistance on the axial mass flux is more pronounced for the individual fin ($t_b = 0$) cases, as expected from the preceding discussion. Axial vapor velocities along the centerline of the HP are reported in Fig. 6. Peak vapor velocities are small relative to the sonic limits for HPs [1], and are reduced with increasing contact resistances.

Although the HP sonic limit is far from being reached, other limitations are important. For example, the capillary limit ($\sim 44 \text{ W}$) can be calculated for the HP considered here using procedures described in [1]. Since the same HP is used in either system, this capillary limit applies to both the HP-heat sink and the HP-fin configurations. The maximum axial mass flux associated with the capillary limit is identified by a vertical line in Fig. 5 (RHS). As evident, the HPs operate just below the capillary limit for all of the cases considered so far, but may reach the limit as different operating conditions (e.g. increased evaporator temperatures) are encountered.

Fig. 6. Heat pipe centerline vapor velocities: (a) $R''_{t,c} = 0$, (b) $R''_{t,c} = 10^{-5} \text{ m}^2 \text{ K/W}$, (c) $R''_{t,c} = 10^{-4} \text{ m}^2 \text{ K/W}$.

Overall heat transfer rates are shown in Fig. 7 (LHS). As evident in Fig. 7a, minor differences exist between the heat rates for the two physical configurations ($t_b = 0$ and 1 mm) when contact resistances are non-existent. Both the HP-heat sink and HP-fin configurations exhibit reduced heat rates as the thermal contact resistance is increased (LHS of Figs. 7a through 7c). As expected, the HP-heat sink ($t_b = 1$ mm) performance is not as sensitive to $R''_{t,c}$ as that of the individual fin configuration ($t_b = 0$).

(a)

(b)

Fig. 7. Heat rates (left) and heat rate ratios (right): (a) $R''_{t,c} = 0$, (b) $R''_{t,c} = 10^{-5} \text{ m}^2 \text{ K/W}$, (c) $R''_{t,c} = 10^{-4} \text{ m}^2 \text{ K/W}$.

In all cases reported in Fig. 7 (LHS), Q increases as the evaporator temperature is increased until a capillary limit is reached, corresponding to a critical evaporator temperature. Prior to reaching its capillary limit, the HP-heat sink configuration out-performs the HP-fin geometry because of its smaller overall contact resistance (K/W). When the capillary limit is reached, phase change within the HP is curtailed, and it transitions to operation in a conduction mode. Therefore, heat rates at evaporator temperatures exceeding critical values are predicted based on the assumption of stagnant HP gas and liquid phases. These heat rates are small relative to rates associated with the properly-operating HP. A similar effect is evident for the $t_b = 0$ configuration.

Although no HP would be designed to operate beyond its capillary limit, the HP-fin configuration ($t_b = 0$) could potentially provide superior performance relative to the HP-heat sink geometry in situations for which there might be, for example, an excursion in HP evaporator temperatures beyond expected or design values. In other words, a tradeoff exists between (i) achieving a low HP-fin array system thermal resistance and (ii) broadening the range of conditions for which the HP (and, therefore, the HP-fin array system) can operate. This tradeoff ultimately depends on the magnitude of thermal contact resistances between the HP and the fin array and is quantified in terms of a dimensionless heat rate ratio involving the heat sink ($t_b = 1 \text{ mm}$) and individual fin array ($t_b = 0$) configurations

$$Q_r = Q(t_b = 1 \text{ mm})/Q(t_b = 0) \quad (2)$$

Consider an excursion in the HP evaporator temperature from its design value. The dependence of Q_r on the evaporator temperature is shown on the RHS of Fig. 7. Except for a narrow window at $T_e \approx 68^\circ\text{C}$, $Q_r \approx 1$ for $R_{t,c}'' = 0$ (Fig. 7a). In contrast, for the largest contact resistance considered here (Fig. 7c), the dimensionless heat ratio is large ($Q_r \approx 2$) until the capillary limit is reached for the HP-heat sink configuration ($t_b = 1$ mm). When the critical evaporator temperature is reached for the $t_b = 1$ mm configuration, the heat rate ratio suddenly decreases to a small value ($Q_r \approx 0.06$) until a second capillary limit is exceeded for the HP-fin geometry ($T_e \approx 100^\circ\text{C}$ in Fig. 7c) after which Q_r increases to a value of approximately unity as both HP configurations operate in the conduction mode. Hence, the HP-heat sink ($t_b = 1$ mm case) offers superior performance relative to the HP-fin configuration ($t_b = 0$) under expected operating conditions (or when the HPs of both configurations are operating beyond their respective critical evaporator temperatures). However, the HP-fin configuration is superior to the HP-heat sink option over a potentially large intermediate range of T_e , corresponding to operation between the two critical temperatures. Hence, thermal contact resistances affect not only the HP heat rates, but can influence the choice of whether to use a based heat sink or individual fins in finned HP devices.

3.3. TRM predictions

The individual resistances shown in Fig. 2 are defined and quantified in Table 2 or described in [5]. Unique to the finned HPs considered here, thermal contact resistances at the root of each fin are determined using $R_{t,c}'' = 0$ or 10^{-4} m² K/W. The thermal resistance associated with the HP working fluid vapor, R_v , has been estimated using a novel, iterative technique recently developed [36]; the thermal resistance associated with this particular HP under similar operating conditions is $R_v = 0.092$ K/W.

Table 2. Thermal resistance descriptions, formulae, and values.

Thermal resistance	Description	Formula [43]	Value (K/W)
<i>Adiabatic section</i>			
$R_{w,a}$	Axial conduction in wall	$\frac{L_a}{k_w \pi (r_{hp}^2 - r_{wi}^2)}$	22.8
$R_{wi,a}$	Axial conduction in wick	$\frac{L_a}{k_{wi} \pi (r_{wi}^2 - r_v^2)}$	44.1
<i>Condenser section</i>			
$R_{w,c,a}$	Axial conduction in wall	$\frac{0.5L_c}{k_w \pi (r_{hp}^2 - r_{wi}^2)}$	17.4
$R_{w,c,r}$	Radial conduction in wall	$\frac{\ln(\frac{r_{hp}}{r_{wi}})}{2\pi k_w L_c}$	0.00056
R_w	Convection resistance between air and exposed condenser wall	$\frac{1}{\bar{h}_w 2\pi r_{hp} (L_c - N t_f)}$ ¹	14.9
R_f	Thermal resistance for 1 fin	Analytical annular fin analysis ²	4.76
$R_{t,c}$	Contact resistance for 1 fin	$\frac{R'_{t,c}}{2\pi r_{hp} t_f}$ ³	5.47
$R_{wi,c,a}$	Axial conduction in wick	$\frac{0.5L_c}{k_{wi} \pi (r_{wi}^2 - r_v^2)}$	33.6
$R_{wi,c,r}$	Radial conduction in wick	$\frac{\ln(\frac{r_{wi}}{r_v})}{2\pi k_{wi} L_c}$	0.00147
<i>Evaporator section</i>			
$R_{w,e,r}$	Radial conduction in wall	$\frac{\ln(\frac{r_{hp}}{r_{wi}})}{2\pi k_w L_e}$	0.00082
$R_{w,e,a}$	Axial conduction in wall	$\frac{0.5L_e}{k_w \pi (r_{hp}^2 - r_{wi}^2)}$	11.8
$R_{wi,e,r}$	Radial conduction in wick	$\frac{\ln(\frac{r_{wi}}{r_v})}{2\pi k_{wi} L_e}$	0.00216
$R_{wi,e,a}$	Axial conduction in wick	$\frac{0.5L_e}{k_{wi} \pi (r_{wi}^2 - r_v^2)}$	22.8
<i>Vapor region</i>			
R_v	Vapor thermal resistance	$\frac{0.5L_e + L_a + 0.5L_c}{k_v \pi r_v^2}$ ⁴	0.092

¹ $\bar{h}_w = 51.54 \frac{W}{m^2 K}$ [36]

² From Eqs. (3.96) and (3.97) [43], and $\bar{h}_f = 19.5 \frac{W}{m^2 K}$ [36].

$$^3 R_{t,c}'' = 10^{-4} \frac{m^2 K}{W}$$

$$^4 k_v = 70,000 \frac{W}{m K} [36]$$

Predicted overall heat rates are presented in Fig. 8 for $0 \leq \tilde{N} \leq N = 6$. Again, $\tilde{N}$ is the number of fins that are unaffected by a contact resistance. Note that for $1 \leq \tilde{N} \leq N - 1 = 5$, multiple overall heat rates exist for each value of $\tilde{N}$. For example, for $\tilde{N} = 1$, the single fin with no contact resistance could be placed at any of 6 different locations in the fin array, resulting in 6 unique heat rates (differences in the heat rates are expected, since each fin is exposed to a unique root temperature, as is evident upon consideration of Fig. 4.). However, it was found that the largest differences in the heat rates predicted by the HP model correspond to placement of all of the zero-contact resistance fins at the top of the array versus placement of the same fins at the bottom of the array. The upper bounds (HPUB) and lower bounds (HPLB) of heat rates predicted by the detailed HP model are shown in Fig. 8, along with predictions of the TRM.

As evident in Fig. 8, the heat rate decreases nearly linearly as the number of fins suffering from contact resistances ($N - \tilde{N}$) increases. Predictions of the HP model are confined to a narrow window defined by their upper and lower bounds. Predictions of the TRM are in qualitative agreement with those of the HP model. Quantitatively, the TRM predictions are within 18 ($\tilde{N} = 0$) to 25 percent ($\tilde{N} = 6$) of the HP model predictions when $R_v = 0$. When $R_v = 0.092$ K/W, the TRM predictions are within 12 percent of those of the HP model over the entire range of $\tilde{N}$. Specification of an arbitrary, yet reasonable value of $R_v = 0.22$ K/W results in TRM predictions (not shown in Fig. 8) that are within 3.5 percent of the detailed HP predictions over the entire range of $\tilde{N}$, with an average discrepancy of less than 1.9 percent.

Fig. 8. Overall heat rate dependence on the number of fins with no thermal contact resistance, as predicted by the HP and TRM models.

4. Conclusions and recommendations

A detailed HP model has been used to quantify the influence of external contact resistances on the heat transfer and fluid flow within a finned HP, as well as on the overall heat transfer rate of the HP-fin array system. For the same area-averaged contact resistance values, a HP-heat sink configuration provides lower overall thermal resistance than a HP-individually finned configuration under design conditions. However, the HP-fin option can significantly outperform the HP-heat sink configuration if the HP capillary limit is accounted for during off-design operation. The thermal resistance model has quantified the magnitude of various resistances within the system, and has provided predictions that compare favorably to those of the HP model. The overall heat rate of the HP-fin configuration is proportional to the number of fins in the array that are not subject to a contact resistance at the HP-fin interface. It was also found that the order in which the fins (fins with no versus fins with contact resistances) are placed within the fin array has little effect on the overall thermal resistance of the HP system considered here.

The modeling approach used here neglects three-dimensional phenomena and possible variability of contact resistances that might occur on a fin-by-fin basis, such effects could also be included in future studies. Constriction resistances [43] within the HP wall and heat sink base, as

evidenced by the compacted isotherms in the vicinities of the fin roots, may be important to quantify and include in future refinements of the TRM model. The utilization of multiple HP-fin array configurations in large systems including the effects of contact resistances could also be considered in future investigations of finned HPs and HP systems.

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Nomenclature

c_p	specific heat (J/kg K)
g	gravitational acceleration (m/s ²)
h	convection coefficient (W/m ² K)
h_{fg}	latent heat (J/kg)
k	thermal conductivity (W/m K)
L	length (m)
m	fin constant (1/m)
m''	local mass flux (kg/s m ²)
N	number of fins
p_{sat}	vapor pressure (Pa)
Q	heat rate (W)
R	thermal resistance (K/W)
R''	thermal resistance per unit area (m ² K/W)
r, z	coordinate directions (m)
S	fin pitch (m)
T	temperature (K)
t	thickness (m)
U	heat pipe centerline vapor velocity (m/s)
V	air velocity (m/s)

Greek symbols

θ	reduced temperature (°C)
μ	dynamic viscosity (Pa s)
ρ	density (kg/m ³)

Superscripts

--	average
~	no contact resistance

Subscripts

a	adiabatic, axial
b	heat sink base
c	condenser, contact
e	evaporator
eff	effective
f	fin
fc	corrected fin radius
hp	heat pipe

r	radial, ratio
t	thermal
v	vapor
$v-wi$	vapor-wick interface
w	wall
wi	wick
$w-wi$	wall-wick interface
∞	ambient

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Table captions

Table 1. Thermophysical properties and pertinent dimensions.

Table 2. Thermal resistance descriptions, formulae, and values.

Figure captions

Fig. 1. Computational domain and fin array (HP-heat sink configuration).

Fig. 2. Thermal resistance network (HP-fin configuration).

Fig. 3. Temperature distributions for $t_b = 1$ mm (left), $t_b = 0$ (right) with $T_e = 60^\circ\text{C}$: (a) $R''_{t,c} = 0$, (b) $R''_{t,c} = 10^{-5} \text{ m}^2 \text{ K/W}$, (c) $R''_{t,c} = 10^{-4} \text{ m}^2 \text{ K/W}$.

Fig. 4. Heat pipe wall temperatures: (a) $R''_{t,c} = 0$, (b) $R''_{t,c} = 10^{-5} \text{ m}^2 \text{ K/W}$, (c) $R''_{t,c} = 10^{-4} \text{ m}^2 \text{ K/W}$.

Fig. 5. Local mass flux along the HP at the vapor-wick interface (left) and within the wick (right): (a) $R''_{t,c} = 0$, (b) $R''_{t,c} = 10^{-5} \text{ m}^2 \text{ K/W}$, (c) $R''_{t,c} = 10^{-4} \text{ m}^2 \text{ K/W}$.

Fig. 6. Heat pipe centerline vapor velocities: (a) $R''_{t,c} = 0$, (b) $R''_{t,c} = 10^{-5} \text{ m}^2 \text{ K/W}$, (c) $R''_{t,c} = 10^{-4} \text{ m}^2 \text{ K/W}$.

Fig. 7. Heat rates (left) and heat rate ratios (right): (a) $R''_{t,c} = 0$, (b) $R''_{t,c} = 10^{-5} \text{ m}^2 \text{ K/W}$, (c) $R''_{t,c} = 10^{-4} \text{ m}^2 \text{ K/W}$.

Fig. 8. Overall heat rate dependence on the number of fins with no thermal contact resistance, as predicted by the HP and TRM models.