Four-stage loop-type cascade traveling-wave thermoacoustic engine

M. Senga,¹⁾ S. Hasegawa^{2,a)}

¹ *Course of Mechanical Engineering, Tokai University, Hiratsuka,
Kanagawa 259-1292, Japan*

² *Department of Prime Mover Engineering, Tokai University, Hiratsuka
259-1292, Japan*

Abstract

We designed and built a prototype of a loop-type cascade traveling-wave thermoacoustic engine with four regenerators. The objective with this prototype engine was to achieve both high acoustic impedance at the regenerator position and a traveling-wave field in ducts, and to amplify the acoustic power by a factor equal to the number of regenerators. To achieve this objective, we enlarged the regenerator cross-sectional area and incremented the duct diameter. We measured and analyzed the acoustic field and verified that its characteristics met the objectives.

1. Introduction

Based on the phase relation between oscillatory pressure p and oscillatory velocity v for a traveling wave being equivalent to the phase relation for a gas in a Stirling engine [1], in 1979 Ceperley proposed a traveling wave engine [2] depicted in Fig. 1. This

a) Corresponding author. *E-mail address:* s.hasegawa@tokai-u.jp

traveling-wave thermoacoustic engine amplifies acoustic power when a traveling wave passes through the bundle of narrow channels (“regenerator”) with axial temperature gradient. Its gain is determined identically by the ratio of the absolute temperatures at the ends of the regenerator (that is, the ratio of the temperature T_H at the high-temperature end to the temperature T_C at the cold-temperature end). This proposal has led researchers to develop various traveling-wave thermoacoustic engines [3–5] and refrigerators [6–8]. Assuming a traveling-wave phase and negligibly small viscous loss, gain G for a thermoacoustic engine of power W is given [2] by

$$G = \frac{T_H}{T_C} = \frac{W_H}{W_C}. \quad (1)$$

where W_H and W_C are the acoustic power of the regenerator outlet and inlet, respectively.

With A denoting the cross-sectional area, the acoustic power is

$$W = \frac{A}{2} |p| |v| \cos \varphi. \quad (2)$$

Ceperley also proposed a cascade amplifier connecting n regenerators in series to increase the gain,

$$G = \left(\frac{T_H}{T_C} \right)^n. \quad (3)$$

Fig. 1 Traveling-wave thermoacoustic amplifier

Acoustic power amplification in this cascade traveling-wave thermoacoustic engine

was demonstrated by Biwa et al. in 2011[9]. However, connecting multiple regenerators causes deviations from a traveling wave phase. This occurs because a reflected wave arises in the regenerators. The number of regenerators n_r that can be inserted is then limited for that reason. Furthermore, effects of heat transport [10] from oscillatory velocity were large, and the thermal efficiency was low in the regenerator because the specific acoustic impedance $z = \rho_m c$, (where ρ_m is the mean density and c is the adiabatic sound speed) was low. Here, the specific acoustic impedance z is expressed as the ratio of oscillatory pressure to oscillatory velocity ($= p/v$). The relationship between the thermal efficiency and specific acoustic impedance z in a regenerator was also calculated by Ceperley in 1985[11]. His calculation results showed that regenerator thermal efficiency η_{Reg} is limited to around 10% of the Carnot efficiency η_c when z of the regenerator is equivalent to the characteristic acoustic impedance of a traveling wave in free space. Here, assuming a thermal input Q_H into the hot end of the regenerator, η_{Reg} and η_c are given as follows:

$$\eta_{\text{Reg}} = \frac{W_H - W_C}{Q_H} \quad (4)$$

$$\eta_c = 1 - \frac{T_C}{T_H}. \quad (5)$$

Ceperley furthermore showed that if z can reach $10\rho_m c$, then the thermal efficiency reaches 79% of the Carnot efficiency. According to his calculation, a high z is necessary to increase the thermal efficiency of the regenerator. Backhaus et al. built a $\lambda/4$ (λ : wavelength of the sound wave) mode traveling thermoacoustic engine [4] by attaching a branch resonator to a

loop so as to reduce viscous loss in the regenerator. This engine achieved $z \gg \rho_m c$ in the regenerator, and its efficiency relative to the Carnot efficiency $\eta_{\text{Loop}} / \eta_c$ reached 42%. Here, if W flowing out into the branch ducts is denoted by W_B , then the loop efficiency η_{Loop} is

$$\eta_{\text{Loop}} = \frac{W_B}{Q_H}. \quad (6)$$

The apparatus of Backhaus et al. achieves $z \gg \rho_m c$ in the regenerator, but the attenuation of W caused by viscous loss [12, 13] is large because of the standing wave acoustic field in the duct^[4]. In 2010, Ueda et al. investigated the relationship between η_{Reg} and dimensionless dissipation Γ in the duct,

$$\Gamma = \frac{(W_H - W_C) - W_B}{W_H - W_C}, \quad (7)$$

by numerical calculation [14]. They investigated the relationship between η_{Reg} and Γ by varying the position, length, and flow-channel radius of a regenerator in a refrigerator using the $\lambda/4$ mode. According to this result, η_{Reg} and Γ affect the coefficient of performance (COP) of the refrigerator and establish a trade-off. For this reason, to maximize COP for the entire refrigerator, it is necessary to have optimal values of both η_{Reg} and Γ .

If this trade-off can be resolved and high η_{Reg} and low Γ can coexist, then it is possible to improve the thermal efficiency of the thermoacoustic engine. It is difficult to reduce Γ in the $\lambda/4$ mode thermoacoustic engine proposed by Backhaus et al. because a standing wave is established in the duct. Nonetheless, the thermoacoustic device with λ as the looped tube length proposed by Yazaki et al. [3] can reduce Γ because a traveling wave is in the duct.

77 Accordingly, if $z \gg \rho_m c$ can be satisfied locally at the position of the regenerator in the
 78 thermoacoustic device with looped tube length of λ , then it is possible that high η_{Reg} and
 79 low Γ coexist. Ceperley proposed enlarging locally the cross-sectional area of the
 80 regenerator section in comparison with the cross-sectional area of the duct as a means to
 81 satisfy $z \gg \rho_m c$ locally at the regenerator [11, 15]. The four-stage traveling-wave
 82 thermoacoustic power generator [16] by de Blok is an example of achieving $z \gg \rho_m c$ by
 83 changing the cross-sectional area. Through self-matching conditions, he produced a
 84 traveling wave with $z \approx \rho_m c$ by symmetrically placing a regenerator and an acoustic load
 85 inside a looped tube of length λ at every $\lambda/4$. In this device, the regenerator is placed
 86 centrally with respect to the acoustic load. Γ becomes large over half the total duct length
 87 relative to locations upstream of the regenerators because $z < \rho_m c$ downstream of the
 88 regenerators. Moreover, acoustic power amplification by factor n_r as proposed by Biwa et
 89 al.[9] cannot be obtained because an acoustic load is applied to every regenerator.
 90 Furthermore, self-matching conditions exist that must be satisfied, such as all regenerator
 91 temperatures being equal in principle and the natural frequencies of the electric generators
 92 being the same [16].

93 In this study, we designed and built a prototype engine that enables acoustic power
 94 amplification by factor n_r while satisfying self-matching conditions without using multiple
 95 generators. To accomplish this, we use the variation of duct cross-sectional area in a

traveling-wave thermoacoustic engine with four regenerators in a looped tube. We furthermore evaluated experimentally whether both $z \gg \rho_m c$ in the regenerator and small Γ in the ducts is possible.

2. Apparatus design

A. Design concept

In this section, we present the design concepts for an apparatus in which it is possible to connect four regenerators in series in a loop while satisfying both $z \gg \rho_m c$ in the regenerators and small Γ in the ducts. First, we consider the conceptual model shown in Fig.

2. Here, a single unit comprises an upstream duct, a cold heat exchanger CHX (temperature T_C), a regenerator REG, a hot heat exchanger HHX (temperature T_H), and a downstream duct. We assume a traveling wave with specific acoustic impedance $\rho_m c$ enters the regenerator from the upstream duct. To achieve $z \gg \rho_m c$, the cross-sectional area of the regenerator is enlarged as proposed by Ceperley and de Blok [11, 16]. Furthermore, the apparatus is designed such that in the duct $z \approx \rho_m c$ to establish a small Γ . When there is no viscous dissipation as stated by Ceperley, oscillatory pressure does not vary at either end of a regenerator that is sufficiently short in comparison with the wavelength, and ideally the oscillatory velocity is amplified by the ratio of the absolute temperatures at each end. To maintain $z \approx \rho_m c$ in the downstream duct after the regenerator, the cross-sectional area ratio

115 A_b (downstream duct cross-sectional area) relative to the upstream duct cross-sectional area
116 A_a (upstream duct cross-sectional area) should be set to the absolute temperature ratio,
117 $A_b/A_a = T_H/T_C$. Fig. 3 shows the model designed based on the design concept, Table I shows
118 its various specifications, and Table II shows specifications for each heat exchanger and
119 regenerator. The regenerators used were general-purpose honeycomb ceramics. The flow
120 path diameter was determined as shown in Fig. 3.

121 The loop length of the experimental apparatus in this study was set to 4.8 m. We
122 experimentally verified that the objective $z \gg \rho_m c$ is realized both at the regenerators and
123 for the traveling wave in the ducts in this experimental apparatus (referred to as the
124 “proposed model (Model A)” for which the regenerator area and duct diameter have been
125 changed). For comparison and verifying the effect of locally enlarging the regenerator
126 cross-sectional area and the effect of incremental duct enlargement, a reference experiment
127 was performed using an apparatus with regenerators that all have diameter 32 mm and
128 internal duct diameters of 14 mm (referred to as the “comparison model (Model B)”). All
129 specifications for Model B other than diameter were set to be the same as those for Model
130 A. Furthermore, as shown in Fig. 3, three pressure transducers were attached to each duct
131 section, and the pressure oscillation of the apparatus was measured using an FFT analyzer.
132 From the measured pressure oscillation, we used the Two-sensor method to measure the
133 acoustic field, and calculated the acoustic power gain from the acoustic power distribution.

134

135

136

Fig. 2 Conceptual model

137

138

139

Fig. 3 Proposed model (Model A)

140

Table I System specifications (Model A)

	Unit 1		Unit 2		Unit 3		Unit 4	
	Diameter [mm]	Length [m]	Diameter [mm]	Length [m]	Diameter [mm]	length [m]	Diameter [mm]	Length [m]
duct	10	0.5	12	0.5	14	0.5	17	0.5
CHX	23	0.03	28	0.03	32	0.03	40	0.03
REG	23	0.03	28	0.03	32	0.03	40	0.03
HHX	23	0.03	28	0.03	32	0.03	40	0.03
duct	12	0.5	14	0.5	17	0.5	22	0.5

Table II Specifications of heat exchanger and regenerator

	Flow path [mm]	Porosity [%]	Temperature [K]
CHX	2	72	300
REG	0.6	80	
HHX	2	72	623

B. Determination of apparatus frequency

For the proposed model with incremental enlargement of ducts (Model A) and the comparison model without incremental enlargement of ducts (Model B), we compared the acoustic field using the frequency at which acoustic power gain reaches its maximum. To do this, the frequency at which the gain becomes a maximum was determined for Model A and for Model B, using loudspeakers and measuring the gain in the experimental apparatus. To give extraneous oscillation, a loudspeaker was attached to the end of the branch duct. Three pressure transducers were attached in each duct section to measure the oscillatory pressure for both models. The temperature of each hot heat exchanger was maintained by

an electrical heater and the temperature of each cold heat exchanger was maintained by circulating coolant around the heat exchanger. The inside of the apparatus was filled with air at atmospheric pressure as the working fluid. The acoustic power distribution in the ducts was calculated using the measured oscillatory pressure obtained by the Two-sensor method [17, 18], and varying the frequency. The gain G_{all} for the entire loop was calculated from

$$G_{\text{all}} = \frac{W_{\text{out}}}{W_{\text{in}}}. \quad (8)$$

Here, W_{in} and W_{out} are the acoustic power at the start ($x = 0$ m) and end ($x = 4.8$ m) of the loop, respectively. Fig. 4 shows the results of the experiment using loudspeaker excitation. With f on the horizontal axis and G_{all} on the vertical axis, the circles and triangles indicate the results for Model A and Model B, respectively; the black circle $\bullet$ and black triangle $\blacktriangle$ indicate the maximum gain point for each model. In the figure, the maximum G_{all} was 4.57 at $f = 57$ Hz for Model A and 3.37 at $f = 54$ Hz for Model B.

Fig. 4 Amplification gain G_{all} vs. frequency f

174

175 From this, it was found that Model A has a higher gain over the entire loop. To drive
176 the apparatus at a frequency that gives a maximum gain for both Model A and Model B, the
177 acoustic impedance distribution at the branch duct of the apparatus was measured by the
178 two-sensor method. Then, a variable load was determined such that an equivalent
179 impedance [19–21] to each optimized frequency is obtained for Model A and Model B. The
180 load was then attached to the branch duct.

181

182 **3. Experimental results**

183 We performed self-oscillation experiments on the apparatus constructed as described in
184 Section 2-B and compared Model A and Model B. The experiments were performed by
185 attaching an acoustic load to the end of the branch duct, with an equivalent impedance to
186 the frequency at the maximum gain point determined in Section 2-B for Model A and
187 Model B, respectively. A stacked screen mesh was used as an acoustic load. As mentioned
188 above, the frequency was 57 Hz for Model A, and 54 Hz for Model B. The other apparatus
189 configuration and cold / hot heat exchanger temperature settings were the same as the
190 conditions shown in Section 2-A.

191 Pressure transducers were attached to each duct in the same way as described in
192 Section 2-B and the acoustic field in the ducts was calculated from the measured oscillatory

193 pressure by the two-sensor method. Figs. 5–7 show the normalized acoustic power W/W_0
 194 distribution, the phase difference between pressure and velocity φ distribution, and the
 195 normalized acoustic impedance $|z|/\rho_m c$ distribution. In each figure, the solid line indicates
 196 results for Model A, the dotted line indicates results for Model B, and black dots indicate
 197 measurement points. The curves in these figures were extrapolated from the values
 198 measured by the Two-sensor method (measurement points indicated by circles and triangles
 199 in Fig. 4), by calculating the acoustic field in the duct by analytical solution [Ref. 18,
 200 Eqs. (2) and (3)]. This method can be applied to parts where there is no change in
 201 cross-sectional area of the duct, and no change in temperature. In this study, measurements
 202 could not be performed at the positions of the heat exchanger and regenerator (indicated by
 203 hatch mark) as these position. In Fig. 5, acoustic power is normalized by the acoustic
 204 power W_0 of the initial point of the loop ($x = 0$); in Fig. 7, acoustic impedance is
 205 normalized by the characteristic acoustic impedance $\rho_m c$. Furthermore, as the acoustic
 206 impedance is normalized by $\rho_m c$, the value 1 indicates that the acoustic field is equivalent
 207 to a traveling-wave field in free space. From Fig. 5, it is found that Model A has small
 208 viscous dissipation across the entire apparatus. In contrast, the gain of the entire loop
 209 becomes small for Model B because sections exist where viscous dissipation is large. G_{all}
 210 with self-oscillation was 4.5 for Model A and 3.2 for Model B. Next, Fig 6 shows that
 211 while φ changes in the range of -45° to 30° for Model A, this range is about -70° to 70° for

Model B. This indicates that the main traveling-wave field is maintained by the incremental enlargement of the duct diameter in Model A, in contrast to the field shifting to a main standing-wave field in Model B. Also, as seen in Fig. 7, the change in $|z|/\rho_m c$ increases with each engine unit in Model B, the change being in the range 0.2–1.6. In Model A, the change is small even when passing an engine unit and is in the range 0.4–1.2. This also indicates that the traveling wave is maintained in the ducts by incremental enlargement of duct diameter. Also, concerning z at the position of the regenerator, from the continuity of volumetric flow rate, using the upstream duct cross-sectional area and regenerator cross-sectional area, and the specific impedance at the upstream duct end point z_A , the specific acoustic impedance at the regenerator position can be expressed as $z_{reg} = (A_{reg}/A_a) z_A$. In Model A, A_{reg}/A_a is about 4.0–5.5, so from the relation above, the specific impedance at the regenerator position is predicted to increase.

Furthermore, in the experiment, the acoustic power flowing out into the branch duct W_B and total heat input Q were $W_B = 0.28$ W, $Q = 800$ W for Model A, and $W_B = 0.2$ W, $Q = 990$ W for Model B. Because the working gas was air at atmospheric pressure, the acoustic power output was low, and most of the input heat is thought to have been released into the atmosphere or simply lost through thermal conduction. However, in comparing Model A with Model B, a traveling wave was achieved, the phase of the regenerator position was closer to a traveling wave, and the acoustic power gain was improved, and

hence has higher thermal efficiency than Model B [11]. Therefore, it is believed possible to improve the thermal efficiency in an actual machine significantly using pressurized gas as the working gas in Model A, and by insulating the heat input part.

Fig. 5 Normalized acoustic power distribution

Fig. 6 Phase between pressure amplitude and velocity amplitude distribution

Fig. 7 Normalized acoustic impedance distribution

4. Conclusion

We designed and built a prototype of a loop-type cascade traveling-wave thermoacoustic engine with four regenerators. This engine enables acoustic power amplification by a factor n_r while satisfying both $z \gg \rho_m c$ in the regenerators and small Γ in the resonators by enlarging their cross-sectional area and varying the duct cross-sectional area. We measured the acoustic field and verified that $z \gg \rho_m c$ at the regenerators and small Γ in the ducts were satisfied simultaneously by varying the duct cross-sectional area and enlarging the regenerator cross-sectional area. When the regenerators were enlarged and the ducts were incrementally enlarged, comparisons with an unmodified apparatus showed that the traveling wave was maintained in the duct and the traveling wave phase was achieved in the regenerator, hence the acoustic power gain in the whole loop was improved by a factor of 1.3.

256

257 **Acknowledgements**

258 This study was financially supported by the Advanced Low Carbon Technology
259 Research and Development Program (ALCA, Grant No.13414425) of Japan Science and
260 Technology Agency.

261

262 **References**

263 [1] I. Urieli and D. M. Berchowitz, Stirling Cycle Engine Analysis (Hilger, Bristol, UK,
264 1984).

265 [2] P. H. Ceperley, “A pistonless Stirling engine—The traveling wave heat engine”, J.
266 Acoust. Soc. Am., **66**, 1508–1513 (1979).

267 [3] T. Yazaki, A. Iwata, T. Maekawa, and A. Tominaga, “Traveling Wave Thermoacoustic
268 Engine in a Looped Tube”, Phys. Rev. Lett. 81, 3128–3131 (1998).

269 [4] S. Backhaus and G. W. Swift, “A thermoacoustic Stirling heat engine”, Nature
270 (London) 399, 335–338 (1999).

271 [5] M. E. H. Tijani and S. Spoelstra, “A high performance thermoacoustic engine”, J. Appl.
272 Phys. 110, 093519 (2011).

273 [6] T. Yazaki, T. Biwa and A. Tominaga, “A pistonless Stirling cooler”, Appl. Phys. Lett.
274 80,157–159 (2002).

- 275 [7] B. Yu, E.C. Luo, S. F. Li, W. Dai and Z.H. Wu, “Experimental study of a
276 thermoacoustically-driven traveling wave thermoacoustic refrigerator”, *Cryogenics* 51,
277 49–54 (2011).
- 278 [8] L. M. Zhang, J. Y. Hu, Z. H. Wu, E. C. Luo, J. Y. Xu and T. J. Bi, “A 1 kW-class
279 multi-stage heat-driven thermoacoustic cryocooler system operating at liquefied
280 natural gas temperature range”, *Appl. Phys. Lett.* 107, 033905 (2015).
- 281 [9] T. Biwa, R. Komatsu and T. Yazaki, “Acoustical power amplification and damping by
282 temperature gradients”, *J. Acoust. Soc. Am.*, **129**, 132–137 (2011).
- 283 [10] S. Hasegawa, K. Kondo and Y. Oshinoya, “Experimental verification of heat transport
284 by acoustic wave”, *Appl. Therm. Eng.*, **78**, 551–555 (2015).
- 285 [11] P. H. Ceperley, “Gain and efficiency of a short traveling wave heat engine”, *J. Acoust.*
286 *Soc. Am.*, **77**, 1239–1244 (1985).
- 287 [12] G. W. Swift, *Thermoacoustics: A Unifying Perspective for Some Engines and*
288 *Refrigerators* (Acoustical Society of America through the American Institute of Physics,
289 Melville, New York, (2002).
- 290 [13] A. Tominaga, “Thermodynamic aspects of thermoacoustic theory”, *Cryogenics* 35,
291 427–440 (1995).
- 292 [14] Y. Ueda, B. M. Mehdi, K. Tsuji and A. Akisawa, “Optimization of the regenerator of a
293 traveling-wave thermoacoustic refrigerator”, *J. Appl. Phys.*, **107**, 034901 (2010).

294 [15]X. Zhang and J. Chang, “Onset and steady-operation features of low temperature
 295 differential multi-stage travelling wave thermoacoustic engines for low grade energy
 296 utilization”, *Energ. Convers. Manage.*, **105**, 810–816, (2015).

297 [16]K. de Blok, “Novel 4-stage traveling wave thermoacoustic power generator”,
 298 Proceedings of ASME 2010 3rd Joint US-European Fluids Engineering Summer
 299 Meeting and 8th International Conference on Nanochannels, Microchannels, and
 300 Minichannels, FEDSM2010-ICNMM2010, FEDSM-ICNMM2010-30527 (2010).

301 [17]A. M. Fusco, W. C. Ward and G. W. Swift, “Two-sensor power measurements in lossy
 302 ducts”, *J. Acoust. Soc. Am.*, **91**, 2229–2235 (1992).

303 [18]T. Biwa, Y. Tashiro, H. Nomura, Y. Ueda and T. Yazaki, “Experimental verification of a
 304 two-sensor acoustic intensity measurement in lossy ducts”, *J. Acoust. Soc. Am.*, **124**,
 305 1584–1590 (2008).

306 [19]H. Hatori, T. Biwa and T. Yazaki, “How to build a loaded thermoacoustic engine”, *J.*
 307 *Appl. Phys.*, **111**, 074905 (2012).

308 [20]W. Dai, E. Luo, X. Wang and Z. Wu, “Impedance match for Stirling type cryocoolers”,
 309 *Cryogenics*, **51**, 168–172 (2011).

310 [21]L. M. Zhang, Y. Y. Chen and E. C. Luo, "How to match an acoustical load to a
 311 thermoacoustic heat engine?", *AIP Conference Proceedings* 1573, 1445–1452 (2014).
 312