Spatio-temporal Disruption of Thermocline by Successive Laminar Vortex Pairs in a Single Tank Thermal Energy Storage

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Abstract

The stratification efficiency of single tank thermal energy storage is affected by the internal mixing processes, especially in the thermocline region due to disturbances of different kinds. To study the mixing dynamics at the interface, we have conducted detailed numerical and supporting experimental studies for different Atwood numbers (stratification levels). Numerical experiments were conducted with two successive vortex pairs with three different time-lags (short, medium and long). For the short time-lag case, the preliminary vortex pair merges with the ensuing vortex pair. The merged single vortex pair peels back the thermocline layer causing mixing of the hot and cold fluids. The thermocline thickness increases as a result of the entrainment of the cold fluid into the hot fluid. The peeling process continues until buoyant forces lead to plume like structures that penetrate into the lighter fluid. For the medium and large time-lag cases, such merging of vortices was not observed. The vortex pair interacts separately with the thermocline region. The plume structure created by the first vortex pair interacts with the ensuing vortex pair. The altered interface (thermocline) thickness strongly depends on the nature of the vortex - thermocline interaction mechanisms. The thermocline effectiveness decreases consequent to such interactions and have been quantified in details in the current work.

Keywords: Computational Fluid Dynamics, Solar Thermal Storage, Efficiency of thermal energy storage, Vortex - Thermocline Interaction, Vortex merging

Nomenclature

a	Distance between vortex centres, m
A	Atwood number
d	Diameter of the tank, m
g	Acceleration due to gravity, m/s^2
h	Height of the tank, m
Re_v	Vortex Reynolds number
L_p	Penetration length, m
T	Absolute temperature, K
u	Radial velocity, m/s
v	Axial velocity, m/s
x	Radial direction, m
y	Axial direction, m
δ	Thermocline thickness, m
Γ	Circulation, m^2/s
μ	Viscosity, Ns/m^2
ν	Kinematic viscosity, m^2/s
ρ	Density, kg/m^3
τ	Flow time, s
ω	Vorticity, $1/s$
N	Brunt- Väisälä frequency, Hz
p	Time period, s
k	Thermal conductivity, W/mK
c_p	Specific heat, KJ/kgK
Ri	Richardson number
T_h	Hot fluid temperature, K
T_c	Cold fluid temperature, K
ω_c	Characteristics frequency of the vortex, $1/s$
δ_s	Disruption Index
$f1, f2$	Primary and secondary peak frequency for all simulation cases, Hz
t^*	Non dimensionalised time
u^*	Non dimensionalised velocity
ω_{max}	Core vorticity, $1/s$
τ_1	Time lag, s
T^*	Non dimesionalised temperature profile along the axial length of TES

1. Introduction

In concentrated solar power (CSP) plants, parabolic troughs in conjunction with thermal energy storage (TES) produce the power required to operate the generator. TES, an important subsystem of the CSP provides the required stability in power by regulating the discrepancy between the supply and demand. Out of the many identified storage technologies, a single tank TES offers the most economical storage option for CSP applications.

In a single tank TES, the hot and cold heat transfer fluids (HTF) are stored in an inherently stable stratified configuration. The hot and cold fluids are separated by a region of large density gradient called “thermocline” of thickness δ (Fig. 1a). The stability of the thermocline is one of the crucial factors that govern the efficiency of a single tank TES [1, 2]. A stable TES can significantly enhance the efficiency of the CSP plant. Instabilities inevitably cause undesirable mixing inside the tank. Mixing broadens thermocline thickness and reduces thermal storage performance.

Viscous fingering is a commonly observed phenomenon in dual media (porous filled) tanks and Kelvin- Helmholtz instability (shear instability) can generate vortices inside the TES. These vortices evolve and disrupt the thermocline region [3, 4]. The resultant undesirable mixing of fluids disturb the thermal stratification inside the tank [5]. These types of interface-collapses are observed during the charging cycle when the temperature of the incoming fluid is lower than that of the stored hot-fluid temperature [3, 4]. Slight variations in density can also cause Rayleigh-Taylor instability, leading to formation of vortices. Hence, it is highly desirable to predict the structure and path of vortices as well as the physics behind the interaction dynamics.

The interaction of vortices with the thermocline region is a very complex problem with enormous technological interest. Depending on the boundary and operating conditions, the vortex pairs can either penetrate or bounce from the thermocline region [6]. This collision is baroclinically enhanced and can generate strong vortical mixing inside the tank. Depending on the flow conditions, rich and diverse flow structures like ‘plume’, ‘billow’ and ‘internal gravity waves’ were observed in the vicinity of the thermocline. Similar vortex-thermocline interactions can be observed in many natural phenomena and have myriad applications across numerous fields of science and technology. The interaction of wake-vortices created by ships/submarines with thermocline in the ocean, the propagation of trailing-vortices generated by airplanes in the atmosphere and the discharge of buoyant jets from power plants are few examples.

Linden **had** analysed the collision and the subsequent mixing induced by a turbulent vortex on a sharp-density interface [7]. Owing to the impact of this collision, both the interface and the vortices **were** distorted. The buoyancy forces cause the interface to recoil and splash the dense fluid into the lighter counterpart. Linden **had** put forward the concept of connecting the penetration length of vortices and entrainment rate with Froude number (Fr). In a similar study, [6] **had shown** that in the Boussinesq limit, the interaction dynamics of laminar vortices (of size a and strength Γ) with a thin interface **was** essentially governed by the parameter AR , where A was the Atwood number across the interface and $R = a^3 g / \Gamma^2$. This parameter AR physically represents the ratio of buoyancy force and inertia force (Richardson number). For low Richardson number cases, the vortices penetrated the interface. During this process, the vortices shrunk and some of the dense fluid was ejected back into the lighter fluid. At large Richardson number cases (high AR), the vortex-density interface interaction bores resemblance to vortex-wall interaction. The effects of Froude number, Atwood number and Reynolds number on a weak stratified interface had been

studied by Marcus and Bell (1994) [8]. At high Reynolds numbers, they had observed the formation of secondary and tertiary vortices.

Formation of internal gravity waves were seen during the propagation of vortices through density stratified interface. Orlandi et al. had conducted experimental and numerical simulations of the impact of a vortex pair on a linearly stratified interface [9]. The objective of their study was to find out the criteria for gravity wave generation by a descending vortex. Orlandi et al. had posited that gravity waves can be created only when $\omega / N < 1$, where ω is the characteristic frequency of the vortex and N is the Brunt- Väisälä frequency respectively. They had observed a linear dependence of maximum penetration length with Froude number (U/NL), where U and L were characteristic velocity and length scales of the vortex ring. Recoil motion of the vortex with spiral instability was observed in the proximity of the interface.

In a recent study, Olsthoorn and Dalziel had compared the mixing efficiency of a sharp interface, perturbed by periodic vortex pairs [10,11]. They found that under certain flow conditions, mixing efficiency was independent of Richardson number. This novel result demonstrated the importance of periodic vortex structure in mixing dynamics. However, further analyses are required to find the flow conditions for which these findings hold.

Though the collision and the subsequent mixing induced by the vortex pair had been explored in many studies, their relation to the mixing efficiency had not been analyzed in any detail. In real-life situations, the vortices can form in a random way and there is a high possibility of multiple vortices, leading to disruption of the thermocline. In such cases, the criteria applicable to single vortex pairs may not be applicable for dual vortex pairs. Hence, a qualitative and quantitative comparison of the two scenarios demands further attention.

The present work shows how successive vortex pairs of fixed strength introduced (at certain phase lags) into a stable TES affect the thermocline performance. Flow physics behind the mixing mechanism have been analysed for different flow conditions. In an effort to qualitatively validate the numerical predictions, experiments at similar flow conditions have been performed. Detailed analysis involving disruption of temperature profiles have been presented, followed by quantification of disruption (δ_s) and thickness of thermocline (δ_T) due to vortex interactions.

2. Numerical Method

TES is modelled as a rectangular 2D domain of dimensions $100mm \times 500mm$. The configuration mimics the lab scale experimental rig used for validating the computational data. The aspect ratio (h/d) for the lab scale geometry used is 5, which is comparable with that used in industrial applications. The schematic of the computational domain and the coordinate systems are shown in Fig. 1. Here, x and y co-ordinates correspond to radial and axial directions. Velocities in radial, axial directions are represented by u and v respectively. The net resultant velocity is represented by V . (Fig. 1a). The properties of hot and cold fluids are denoted by subscripts h and c respectively (Fig. 1b). T_h and T_c denote the temperatures of the hot and cold fluid respectively. Fig. 1c shows the non-dimensional temperature profile (T^*) along the axial length of the TES where, $T^* = \frac{(T-T_c)}{(T_h-T_c)}$.

2.1. Governing Equations and numerical scheme

In a thermally stratified storage tank, the mixing dynamics is governed by continuity, momentum and energy equations. The governing equations are solved using *ANSYS CFX* [12].

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho U) = 0 \quad (1)$$

$$\frac{\partial(\rho U)}{\partial t} + \nabla \cdot (\rho U \otimes U) = -\nabla p + \nabla \cdot \tau + S_M \quad (2)$$

$$\frac{\partial(\rho e)}{\partial t} + \nabla \cdot (\rho U e) = +\nabla \cdot (\lambda \nabla T) - P \nabla \cdot U + \tau : \nabla U + S_E \quad (3)$$

$$\text{where, } \tau = \mu[\nabla U + ((\nabla U)^T) - \frac{2}{3} \delta \nabla \cdot U] \quad (4)$$

$$e = h - \frac{p}{\rho}$$

Here, high-resolution advection and second order upwind schemes were used for solving the governing equations [13]. Upper limit for the *RMS* values of residuals was set to 10^{-5} . The Courant number for the simulations is 0.5.

2.2. Flow and boundary conditions

For initializing the temperature in the domain, the following sigmoid temperature profile was used [3, 4]. **This configuration and spatial distribution of temperature at the end of the charging process [3, 4] can be closely represented by the sigmoid profile without any loss of generality.**

$$T = T_c + \frac{T_h - T_c}{1 + e^{-m(y-n)}}; \quad (5)$$

Here, m represents the shape parameter, which determines the thermocline thickness and n designate the axial location of the thermocline. In the current simulations, the values of m and n are 300 and $0.25m$ respectively. For the given profile, the thermocline thickness (δ_T) is 25 mm. The thickness of thermocline is calculated using the expression $\delta_{Th} - \delta_{Tc}$, where δ_{Th} is the lowest y value at which the temperature attains a value of $0.99 T_h$ and δ_{Tc} is the largest y value at which the temperature reaches a value of $1.01 T_c$. The operating temperatures for different cases are specified in Table 1. Side walls and the bottom of the domain were specified with

no slip adiabatic wall conditions (Refer Fig. 1b). Section A-A (Fig. 1a) is open to atmosphere.

Solar salt HITEC salt with the following thermo-physical properties was used as the heat transfer fluid

$$\rho(T) = 2090 - 0.636T (^{\circ}C) \quad (6)$$

$$C_p(T) = 1443 - 0.172T (^{\circ}C) \quad (7)$$

$$k(T) = 0.443 + 0.00019T(^{\circ}C) \quad (8)$$

$$\mu(T) = [22.714 - 0.12T + 2.281 \times 10^{-4} T^2 - 1.474 \times 10^{-7} T^3] \times 10^{-3} (^{\circ}C) \quad (9)$$

Here T is instantaneous temperature measured in degree Celsius. Numerical simulations are conducted for two Atwood numbers ($A1=0.034$, $A2=0.071$). Atwood number is a **dimensionless** number defined as $A = \frac{\rho_c - \rho_h}{\rho_c + \rho_h}$, where ρ_h and ρ_c are density of hot and cold fluid respectively. Atwood number designates the extent of thermal stratification across the thermocline.

Change in Atwood number is achieved by changing the value of T_h . The selected values of temperature and Atwood numbers are tabulated in table 1. Here t_l is the time lag maintained between the vortices. $A1t0$ and $A2t0$ represents the single vortex cases for the Atwood numbers $A1$ and $A2$ respectively.

2.3. Vortex generation

Vortices in the tank were created by ejecting a small slug of fluid through a nozzle (diameter d) for a short time period (p) at a temperature T_h (same as hot fluid temperature). This assures that the vortex advection through the hot fluid region occurs due to inertia and not affected by buoyancy forces. The vortices are introduced at an axial distance L ($0.22m$) above the centre of the thermocline (Fig. 1b).

The impulsive flow from the nozzle is modelled using the following expressions

$$v = \begin{cases} V_{max}.f(t)\frac{\tanh(d(1-|r|))}{\tanh(d)} & \text{if } |r| \leq 1; \\ 0 & \text{if } |r| > 1 \end{cases} \quad (10)$$

Where,

$$f(t) = \begin{cases} e^{-\left(\frac{t-t_0}{\sigma}\right)^2} & \text{if } t \leq 0.1 \\ 0 & \text{if } t > 0.1 \end{cases} \quad (11)$$

Where, $r = \frac{(z-0.05)}{0.0075}$, mean valve opening time $t_0 = 0.05$ s, $\sigma = 0.022$ and the term d is a constant taken as 5.5. The equations set to create the vortex pairs are similar to the one used in the study of Orlandi et al [9]. This comprise of spatial distribution of velocity coupled with a valve opening time function $f(t)$. Fig. 2 shows the variation in inlet velocities with flow time. Here, the term $f(t)$ is defined as a Gaussian distribution with maximum value of 1 (Fig. 2b). This profile accounts for gradual opening and closing of valve along with realistic velocity field inside the nozzle. The values of diameter d and time period p are 15 mm and 0.1s respectively.

Initially, simulations were performed for single vortex pair (cases $A1t0$ and $A2t0$, as shown in Table 1 and Fig. 2b). Subsequently, the second vortices are introduced at a predetermined time lag τ_1 with respect to the first pair of vortices (Fig. 2). The values of the time lag τ_1 are chosen as 1 s (short: cases $A1t1$ and $A2t1$), 3.8 s (medium: cases $A1t2$ and

A2t2) and 5 s (long: cases A1t3 and A2t3) respectively (Table 1). Schematic of rolled up vortices (of circulation strength Γ_0 and size a) and the thermocline region are shown in Fig. 1a. The velocity and time scales are made dimensionless using Γ_0 and a , where a is the distance between vortex centres (Fig. 1a) and Γ_0 is the circulation strength of the vortex at $t = 1$ s. Accordingly, $t^* = t \Gamma_0 / a^2$ and $u^* = ua / \Gamma_0$. Other relevant non-dimensionalized terms are $T^* = \frac{(T-T_c)}{(T_h-T_c)}$, Vortex Reynolds number ($Re_v = \frac{\Gamma}{2\pi\nu}$) and Richardson number ($Ri = a^3 Ag / \Gamma^2$) where ν is the kinematic viscosity.

2.4. Grid independence study

A mesh independence study is conducted to determine the optimum grid required to capture the flow physics. Three types of meshes of sizes 200×1000 (fine grid), 100×500 (adapted grid) and 40×200 (coarse grid) were considered for the simulations. Fig. 3 shows the effect of grid resolution on vorticity and temperature at $t^* = 24.3$ and $t^* = 32.5$. The differences between fine grid and adapted grid are negligible. Hence, to reduce the computational time, grid of 100×500 (adapted grid) is considered for further simulations. We have compared our single vortex interaction part of our work with the numerical results of Manu et al [3, 4] which in turn shows qualitative similarities with the experimental work of Orlandi [9] and Dahm et al [6]. It should be noted that the ratio of successive grid sizes is not uniform. This is however due to the nozzle diameter of 15mm, which mandates usage of mesh sizes that are perfect factors of the nozzle diameter. For example a grid size of 50×250 translates into element size of 2mm x 2mm, which will result in improper resolution of flow near the nozzle exit and thus may affect the inlet flow conditions which is undesirable. Balance between accuracy and computational power further substantiate the selection of the above optimized mesh size of 1mm x 1mm.

3. Results

3.1. Mixing induced by the vortex pairs

Variation of vortex Reynolds number and Richardson number with time is shown in Fig. 4. The circulation of the vortex is estimated using $\Gamma = \int \omega_z dA$ where, $\omega_z = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$. Since circulation depends strongly on the integration area shape (enclosed by contour C, Fig. 1a), a threshold limit of $\omega_{max}/\sqrt{2}$ (where ω_{max} is the core vorticity) is used for the calculation. The plotted circulations are calculated by the contour integration of vorticity which lies above the threshold limit. The circulation for A2 case is higher than that of A1 case. This is presumably due to the high viscosity of hot fluid at low Atwood number case (A1: $\mu_{hA1} > \mu_{hA2}$). The non-dimensional form of circulation (Vortex Reynolds number Re_v) is nearly constant in A1t0 case (Fig. 4 a), whereas Re_v increases with time in A2t0. The distance between core of the vortex pairs are identical ($a=22 \text{ mm}$) in both the cases. Richardson number based on the vortex strength and vortex size is calculated using the expression $Ri = a^3 Ag / \Gamma^2$ [5]. As expected, the Ri values are high for high Atwood number cases (strong stratification).

3.1.1 Single vortex pair

The advecting vortices are symmetric up to the interface region. The mixing initialized by single vortex pairs on a thermocline region is shown in Fig. 5. The contours of vorticity ($\omega_z = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$) superposed on the temperature contours at various t^* are shown. These snapshots are reminiscent of the experiments of Orlandi and Dahm [9, 6]. We have observed that splashing occurs for low value of Richardson number. For high Richardson number, the vortex thermocline interactions mimics a vortex wall interaction. When the splashing mechanism is active, the kinetic energy of vortices is used to expunge the cold fluid out of the thermocline region (Fig. 5).

At low Atwood number case, the deformation of the thermocline region is substantial. The vortex pair penetrates the surface and create a crater at the interface. This process upsets the thermal stratification in the tank. As the vortices penetrates the thermocline region, it loses the momentum and shrinks in size due to the opposite sign of the vorticity generated by baroclinicity (Fig. 5a, $t^* = 34$). The shrinkage of vortex at the interface is an inviscid process. The vortex continuously peels off the interface fluid at the sides of the crater. This process continues until buoyant forces increase (due to mixing) and strips off the vortical structure in the form of rising plumes. The entrained fluid rolls up (with opposite sign to the corresponding parent vortices) with the rising plume and develops into *spade - like* structures (Fig. 5a, $t^* = 41.3$). This results in significant entrainment of cold fluid into the hot counterpart leading to the broadening of the thermocline region. The *spade - like* vortices ultimately diffuse. At $t^* = 977$, one can observe a steady but relatively thicker thermocline (Fig. 5a).

For high Richardson number case ($A2t0$), thermal stratification is relatively strong. Penetration and peeling processes are less prominent (Fig. 5b). As the vortices approach the interface, the thermocline region get distorted. Formation of secondary and tertiary vortices are observed in the simulations (Fig. 5b, $t^* = 32.2$). Subsequently offspring vortices orbit around the parent vortices as shown in Fig. 5b ($t^* = 34.9$). These vortices propagate upstream, spread laterally and eventually diffuses in the cold fluid (Fig. 5b, $t^* = 37, 40.3$ and 45.7). At $t^* = 1074.4$, the thermocline becomes steady with relatively thicker δ compared to the initial conditions.

3.1.2 Double vortex pair

Mixing dynamics of the thermocline in the case of two pairs of laminar vortices for $A1t1$ case is shown in Fig. 6. In this short time lag case (Fig. 6a), an interesting ‘vortex-

merging' mechanism is observed prior to the interaction with the thermocline. The ensuing vortex pair accelerates and merges with the preceding vortex pair as shown in Fig. 6a at $t^*=31.6$ [14]. The merged vortices interact with the thermocline region, create a crater and subsequently bounce back (at $t^*=34, 38.8$) into the hotter fluid. During the process of bouncing, colder fluid is sucked along with the vortex structure. This vortex subsequently dissociates into smaller eddies ($t^*=43.7$). By $t^*=971$, the thermocline completely settles down and an increase in its thickness is observed. The mechanism behind such merging has not been investigated in details here. Similar merging process has been observed in higher Atwood number case with short time lag ($A2t1$).

A different collapse sequence is observed for medium and large phase lag cases ($A1t2$ and $A1t3$) in comparison to the short phase lag case ($A1t1$). The preceding vortex pair creates a rising *spade-like* structure (Fig. 6b, $t^*=34$), exactly similar to the single vortex pair simulations (Fig. 5a, $t^*=41.3$). The second vortex directly collides with the plume structure of the primary vortex before reaching the thermocline. Due to the collision, the second vortex advects downstream with reduced strength and subsequently interact with the thermocline region (Fig. 6b). A series of smaller offspring vortices are formed during the course of the interactions. The final thermocline thickness is the resultant of the net effect of the vortex-vortex, vortex-wall and vortex-thermocline interactions. The thermocline stabilizes by $t^*=971$ (Fig. 6b).

An experimental corroboration of the present simulation was conducted for the same flow conditions. The test section was a glass chamber of dimension $100mm \times 100mm \times 500mm$. Analogous conditions were created with water at top and brine solution ($\rho_1 = 1050 \text{ kg/m}^3$ and $\rho_2 = 1000 \text{ kg/m}^3$) at bottom, separated by a sharp density gradient. A vortex generator, located at a distance 0.22 m from the centre of the interface was used to create the vortices. Vortices were created by ejecting a small slug of fluid from a 10 gallon pressure

tank, through a pipe (1.41 *cm* diameter) for a short time period (0.5 *s*), controlled by a solenoid valve. Axi-symmetric laminar vortices are formed and propagate towards the interface. The second vortex is generated after a finite time lag (1 *s*). The primary vortex pair penetrates through the interface at $t=1.7$ *s* (Fig. 7). Following the penetration, buoyancy induces partial stripping of the vortex to form a rising plume type structure. This plume collides with the second vortex at $t=2.2$ *s*. Subsequently, the two coherent structures ($t=2.36$ *s*) merges and forms a toroidal shape ($t=2.43$ *s*) which expands laterally (Fig. 7). **The experimental pictures have been provided to corroborate this qualitatively only and is not meant for a one to one quantitative validation.**

3.2. Penetration length and stratification efficiency

Variation of penetration length of the vortex pair is tracked for all the combinations of Atwood numbers and time lags (Fig. 8 and Fig. 9). Penetration length is calculated by tracking the trajectory of the vortex core. The penetration length is non-dimensionalized by L . Initially, penetration length varies linearly with flow time marked as ‘FV’ in Figs. 8 and 9. As the vortex interacts with the thermocline, buoyancy becomes dominant leading to plume type structure which rises upwards. The reverse penetration or evolution of the *plume* structure is extracted by tracking the local temperature (path RP in Figs. 8-9). This is achieved by measuring the spatial location in hot fluid regime (above the thermocline region) in which the local temperature drops to 99% of T_h . The trajectory of ensuing (second) vortices (at a time lag τ_l with the primary vortex) are similarly obtained by tracking the core of the vortex pair (‘region ‘EV’). Interaction point of the upward moving *plume* with the ensuing vortices is marked as P in Fig. 8 and 9.

Figs. 8a and 9a designate the single vortex trajectory at two different Atwood numbers ($A1$ and $A2$). It can be seen that the penetration length as the vortex approaches the thermocline is

linear (path FV). At point “O”, the vortex starts interacting with the thermocline interface leading to crater formation (as explained earlier). The interaction leads to deviation of the penetration length from its linear trajectory (TP in Figs. 8a and 8a). Subsequently, the plume forms (due to buoyancy effects) and rises in the reverse (upwards) direction. The path “RP” tracks the penetration of the plume. The plume generally follows a linear trajectory (RP; except in the early developing stages) with constant velocity. However due to gravitational effects, it starts to dissipate towards the end and exhibits a flow reversal at point M (Fig. 8 and 9). Though both Atwood numbers show qualitatively similar trends, there are subtle differences. As an example, for the higher Atwood number, the plume contains less energy and hence rises upto a lower height (by 29 %; Figs. 8a and 9a).

For dual vortices (Figs. 8b-d and 9 b-d), the first vortex pair follows a very similar trajectory as the single vortex counterpart. The plume structure is also similar. The ensuing or the second vortex also mimics the first one (except it is offset in time) although it shows minor fluctuations since it advects in the strained field of the first vortex. ‘P’, designates the point of collision of the second vortex with the plume of the first one at different time lags. In case of short time lag (Fig. 8b, 9b), the second vortex merges with the primary one and the resultant plume exhibits an oscillatory motion. In case of longer time lags (Fig. 8c-d, 9c-d), the second vortex interacts mainly with the rising plume of the primary vortex. The effect leads to substantial energy dissipation and the plume decays rapidly beyond the point ‘P’.

The axial temperature profiles $T(y)$ at different time instances for low and high Richardson numbers are shown in Figs. 10 and 11 respectively. During the mixing process, the axial temperature deviate significantly from the initially prescribed sigmoid profile ($t^* = 24.3$). Instantaneous $T(y)$ profiles are distorted exhibiting inflections due to vortical interactions (Figs. 5-6). Subsequently a broadened stable thermocline is developed inside the

TES after large time instance ($t^* = 971$). Thermocline thickness provide a relatively easy way of evaluating the performance of TES.

For the Atwood number $A1$ and $A2$ at $t^* = 24.3$ and $t^* = 26.9$ respectively, the thermocline profile shifts towards the left (Fig. 10, 11) in comparison to the initial sigmoid profile. This shows that the thermocline initially gets compressed by the vortex at the early instants of interaction (Fig. 10a, 11a). For lower Atwood number, the vortex-thermocline interactions ($t^* > 90$) lead to severe oscillations in the axial temperature profiles (Fig. 10b-d). The amplitude of the oscillations progressively amplify with increase in time lag. As an example, $T(y)$ exhibits the most severe perturbations for $A1t3$ (large time lag). This is because, $A1t3$ corresponds to multiple vortices impinging on the thermocline successively within a very short time span. For high Atwood numbers, the trends are relatively similar as can be seen in Fig. 11. However, the magnitude of temperature oscillations even for large time delay is comparatively lower than the low Atwood number counterpart. This is expected, since the high Atwood number is akin to vortex-wall type interaction and full penetration and mixing with the thermocline is not attained (Fig. 6).

Thermocline thickness is calculated from these temperature profiles at two different time instances, $t^*=0$ and $t^*=971$ (after complete dissipation of the vortices). These values are tabulated in table 2 for all Atwood numbers and phase lags. Distortion index is a useful parameter which measures the degree of stratification (pre and post vortical interactions)

$$\delta_S = \int_0^h \frac{|(T_{t^*=0} - T_{t^*=971})|}{h(T_h - T_c)} \quad (12)$$

Here, $T_{t^*=0}$ and $T_{t^*=971}$ correspond to the temperature profiles corresponding to pre (initial) and post (after steady state) vortical interactions along the axial height of the TES. Sample normalized temperature difference between initial and final $T(y)$ profiles is shown Fig. 12 (a) for a single vortex case ($A1$). The area enclosed between the two temperature profiles (Fig.

12a) can be used to calculate δ_S as shown in Fig. 12b. Similar procedure was followed for all cases ($A1t0-A1t3$, $A2t0-A2t3$) as analysed in this article. The calculated values are tabulated in Table 2. As can be expected, the value of δ_S is higher for lower Richardson number cases. For lower Richardson number cases, maximum distortion index is observed for short time lag case. Vortex merging phenomena could be the reason for this large increase in distortion index. In medium and long time lag cases the *spade - like* (plume) structure reduces the momentum of the ensuing vortex and the subsequent collisions are mild. Hence a reduction in deformation index is observed. However, the values of distortion index for higher Richardson numbers are anomalous. Maximum distortion index is observed in $A2t2$ case. A vortex-side wall interaction phenomena observed in this case. Presumably this could be the reason for this anomalous behaviour. Hence the aspect ratio of the tank plays an important role for determining the efficiency of the TES.

3.3. Thermocline oscillations

Periodic oscillations of the thermocline region are observed in the simulations. In a thin thermocline (low δ_T), the disturbances propagate horizontally like surface waves. However, depending on the level of stratification, the wave can propagate vertically as well. The maximum permitted internal wave frequency in a stratified medium is given by Brunt-Väisälä frequency (N), i.e., the gravity waves can be created only when $\omega/N < 1$, where ω the characteristic frequency of the vortex [9].

$$N = \sqrt{-\frac{g}{\rho} \frac{d\rho}{dy}} \quad (13)$$

The normalized axial velocity $V/(\Gamma_0/a)$ at the center of the thermocline interface is shown in Fig.13. The amplitude of the velocity exhibits damping before reducing to zero with flow time. The system subsequently returns to an equilibrium state.

Fig.14 (a) and (b) shows the power spectral density ($e(f)$) for both Atwood number cases at different time lags. The period of oscillation is estimated by plotting the spectra of the velocity signal. It is seen that f_1 and f_2 frequencies show higher spectral power density for all the Atwood number cases as listed in Table 2. All the observed frequencies (for Atwood number 0.034 and 0.071) are of similar order but less than that of the buoyancy frequency N (1.2 Hz for A1, 1.4 Hz for A2).

4. Conclusions

Mixing dynamics initiated by successive laminar counter rotating vortices have been studied in a thermocline based thermal energy storage tank. Computations have shown that the vortices interact and results in spatio-temporal distortion of the stable thermocline. The vortices exhibit a wide range of dynamics like merging for short time lags. However in cases of medium and large time lags, the succeeding vortex interacts with the plume structure caused due to the interaction of the first vortex and the thermocline. These flow features were also observed in the experiments. Thermocline thickness and deformation index (parameters for measuring stratification efficiency) of the thermocline for cases were calculated. Stratification efficiency is higher for higher Richardson number. The impingement of vortices created oscillations in the thermocline region. The frequencies of the oscillations were always less than natural buoyant frequency.

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Table 1: Parameter space

Case	T_h (K)	T_c (K)	A	τ_1 (sec)	Γ_0 (m^2/s)	a (m)	Ri_o
<i>A1t0</i>	723	523	0.034		0.0047	0.022	0.16
<i>A1t1</i>	723	523	0.034	1	0.0047	0.022	0.16
<i>A1t2</i>	723	523	0.034	3.8	0.0047	0.022	0.16
<i>A1t3</i>	723	523	0.034	5	0.0047	0.022	0.16
<i>A2t0</i>	923	523	0.071		0.0052	0.022	0.27
<i>A2t1</i>	923	523	0.071	1	0.0052	0.022	0.27
<i>A2t2</i>	923	523	0.071	3.8	0.0052	0.022	0.27
<i>A2t3</i>	923	523	0.071	5	0.0052	0.022	0.27

Table 2: Thermocline thickness, distortion index, Brunt- Väisälä frequency, primary and secondary peak frequencies for all simulation cases

Case	δ_T ($t=0s$) (m)	δ_T ($t=100s$) (m)	δ_S	N (Hz)	f_1 (Hz)	f_2 (Hz)
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$A1t0$	0.025	0.1	3.6	1.2	0.56	0.265
$A1t1$	0.025	0.119	5.6	1.2	0.46	0.3
$A1t2$	0.025	0.117	5.2	1.2	0.26	0.5
$A1t3$	0.025	0.114	4.9	1.2	0.47	0.42
$A2t0$	0.025	0.073	1.8	1.4	0.4	0.32
$A2t1$	0.025	0.1	3.3	1.4	0.4	0.7
$A2t2$	0.025	0.106	3.5	1.4	0.68	0.4
$A2t3$	0.025	0.111	3.2	1.4	0.7	0.4

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