**DEVELOPMENT AND VALIDATION OF A THERMODYNAMIC MODEL FOR THE PERFORMANCE
ANALYSIS OF A GAMMA STIRLING ENGINE PROTOTYPE**

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Abstract

This work presents the development and validation of a numerical model that represent the performance of a gamma Stirling engine prototype. The model follows a modular approach considering ideal adiabatic working spaces; limited internal and external heat transfer through the heat exchangers; and mechanical and thermal losses during the cycle. In addition, it includes the calculation of the mechanical efficiency taking into account the crank mechanism effectiveness and the forced work during the cycle. Consequently, the model aims to predict the work that can be effectively taken from the shaft. The model was compared with experimental data obtained in an experimental rig built for the engine prototype. The results showed an acceptable degree of accuracy when comparing with the experimental data, with errors ranging from $\pm 1\%$ -8% for the temperature in the heater side, less than $\pm 1\%$ error for the cooler temperatures, and $\pm 1\%$ -8% for the brake power calculations. Therefore, the model was probed adequate for study the prototype performance. In addition, the results of the simulation reflected the limited performance obtained during the prototype experiments, and a first analysis of the results attributed this to the forced work during the cycle. The implemented model is the basis for a subsequent parametric analysis that will complement the results presented.

Keywords: Stirling engine; simulation and modelling; thermodynamic analysis; energy technology

Nomenclature

A	area (m ²)
A _o	external wet area of the tube (m ²)
C _f	non-dimensional friction coefficient
C _{fd}	form drag coefficient
C _{sf}	skin friction coefficient
C _p	constant pressure specific heat (J/kg K)
C _{pwater}	constant pressure specific heat for inlet water (J/kg K)
C _v	constant volume specific heat (J/kg K)
d	diameter (m)
d _{hy}	hydraulic diameter (m)
E	crank mechanism effectiveness
Err	error tolerance
Error1	absolute error calculated for T _c and T _e
Error2	absolute error calculated for T _k and T _h
Error3	absolute error calculated for T _{wk} and T _{wh}
f	friction factor coefficient
freq	engine frequency (Hz)
F _R	view factor
h	convective heat transfer coefficient (W/m ² K)
h _r	radiation heat transfer coefficient (W/m ² K)
h _{water}	water film heat transfer coefficient (W/m ² K)
k	thermal conductivity (W/mK)
K	piston to displacer swept volume ratio
l	length (m)
m	mass (kg)
n	number of flow resistance layers
m _{water}	mass flow of the inlet water (kg/s)
M	total mass of the working gas (kg)
NTU	Number of transfer units
P	pressure level (Pa)
P _{ch}	engine charging pressure (bar)
P _{br}	engine brake power (W)
Q	heat transfer rate (W)
Q _{hc}	heater heat transfer rate by cycle (J/cycle)
Q _{kc}	cooler heat transfer rate by cycle (J/cycle)
Q _{rc}	regenerator heat transfer rate by cycle (J/cycle)
Q _{ht}	total heating requirement for the engine (W)
Q _{kt}	total cooling requirement for the engine (W)
Q _{lossr}	heat loss due to imperfect regenerator (W)
Q _{lk}	heat loss due to internal conduction (W)
Q _{lsh}	heat loss due to shuttle conduction (W)
R	gas constant (J/kg K)
R _{ci}	conductive thermal resistance for tubes wall(K/W)
R _{fi}	fouling thermal resistance inside the tubes (K/W)
R _{fo}	fouling thermal resistance outside the tubes (K/W)
R _{hi}	convective thermal resistance inside the tubes (K/W)
t	time (s)
T	temperature (K)
T _{ad}	adiabatic flame temperature of the fuel (K)
T _{fM}	measured flame temperature (K)
T _{ratio}	cold to heat temperature ratio
T _{wi}	temperature at the internal wall of the tubes (K)
T _{wo}	temperature at the outer wall of the tubes (K)
T _{water_in}	inlet temperature of the water (K)
v	mean velocity (m/s)
V	volume (m ³)
V _{de}	total dead volume (m ³)
V _{swe}	expansion space swept volume (m ³)
V _{swc}	compression space swept volume(m ³)
W	work flow per cycle (J/cycle)
W _i	engine indicated work (J/cycle)

W _s	engine shaft work (J/cycle)
W _{ploss}	energy loss due to pressure drop (J/cycle)
W-	engine forced work (J/cycle)
X	dead volume ratio

Acronyms

ACM	Aspen Custom Modeller
CHP	Combined Heat and Power
SE	Stirling Engine

Subscripts

b	buffer space
c	compression space
d	displacer
e	expansion space
f	final value
h	heater space
hous	regenerator housing space
i	inside section
in	inlet flow
k	cooler space
M	measured values
o	outside section
out	outlet flow
r	regenerator space
w	wall
wh	heater wall
wk	cooler wall
0	initial value

Superscripts

+	positive variation
-	negative variation

Greek symbols

α	phase shift angle(rad)
α _s	surface absorptivity
γ	adiabatic constant
η _b	brake efficiency
η _b	mechanical efficiency
η _b	thermal efficiency
σ	Stefan-Boltzmann constant (W/m ² K ⁴)
ε	Regenerator effectiveness
ρ	fluid density (kg/m ³)
φ	Crank rotational angle (rad)
μ	viscosity (kg/m s)

1. Introduction.

Actual energy demand and environmental problems require intensive research for the development of efficient and sustainable energy solutions. In this scenario, the Stirling engine technology appears as a renewed solution [1], with the potential to meet the requirements at small-scale [2] thanks to its known theoretical capabilities. However actual designs are far from meeting the efficiency requirements needed to be commercially viable as shown by Thomas [3], Dong [4], and Gonzales-Pino [5]. This heightened the need for engineering tools, like numerical simulation, that could assess design improvements together with test measurements in order to optimize the engine performance before implementing them in the engine.

Different prototypes have been developed guided by simulation analysis. The simulation studies varied in complexity from simulation based on first order [6]; second order analysis as reported by Cheng [7], Mehdizadeh [8], Parlak [9], Strauss [10] and Tlili [11]; and computer fluid dynamics (CFD) analysis that include the works of Mahkamov [12], Ibrahim [13], and Wilson [14]. Among these methods, first order methods are simple and limited to estimate the power output and engine efficiency under ideal assumptions. On the other extreme, CFD analyses are very complex and require intensive computing resources[15]. Therefore, second order analyses have been preferred for first design and optimization studies of the engine considering a compromise between prediction accuracy and computational requirements. These second order methods include the mass and energy balances through the different spaces of the engine and also evaluate the friction and thermal losses using a decoupled approach.

Different studies have guided the development of Stirling engines prototypes. These include, novel configurations in the regenerator [16], the heat exchangers [17], the crank mechanism [18] and optimization studies [10]. However, there is still a need to develop improved engines that should present higher efficiency levels, fuel flexibility, and should also be easy to integrate within combined heat and power systems (CHP). It is especially important the mentioned integration capability, because of the great potential that combined heat and power systems presents as decentralized solutions based on renewable energy [19]. Some works that explored this integrations include Pålsson and Carlsen [20], Nishiyama [21], and Sato [22].

In this sense, the objective of this paper is the development of a thermodynamic-numerical model of a Stirling engine that should represent the performance of a new 1 kW gamma engine prototype built by GENOA Stirling Company in Italy. This model aims to assess through numerical simulation analysis the performance improvement of the GENOA engine prototype, and it is centred on a second order thermodynamic analysis implemented in Aspen Custom Modeller®. The numerical model is based on Urielli approach [23], it considers ideal adiabatic working spaces; limited internal and external heat transfer through the heat exchangers; and mechanical and thermal losses during the cycle. In addition, it includes the numerical evaluation of the mechanical efficiency taking into account the crank mechanism effectiveness and the forced work during the cycle, according to Senft methodology [24]. Therefore, the model combines Urielli and Senft approaches into a restructured numerical analysis that computes the work that can be effectively taken from the shaft. The model was validated with data obtained from an experimental rig built for the engine. The details about the methods used for the measurements are reported in Cardozo et al [25].

2. Mathematical model

A mathematical model for the simulation of Stirling engine systems was developed in a previous work [26]. This consisted on four main modules named ideal adiabatic, internal heat transfer, external heat transfer, and energy losses. This paper improves the model by adding the evaluation of the mechanical efficiency of the system, thus the improved model contains 5 modules. The first module corresponds to an ideal Stirling engine adiabatic model, which assumes ideal adiabatic compression and expansion spaces to estimate the main engine variables. The derivation of the equations that govern this system are explained in Urielli [23]. The outputs of this module are coupled to the internal heat transfer module, which through appropriate correlations evaluates the heat transfer, the temperature, and the thermodynamic properties of the working fluid inside the heat exchangers. The variation of the thermodynamic properties with the temperature is considered at every time step of the system. The next module, external heat transfer module, couples the heat transfer between the external walls at the hot and cold side of the engine. This is done through energy balances and heat transfer correlations, described in detail in Araoz et al [26]. The following module, energy losses module, evaluates the losses due to pressure drop, axial conduction, shuttle heat transfer, and imperfect regeneration once the cyclic steady state conditions were reached. Finally, the mechanical efficiency module permits to estimate the effect of forced work during the cycle and the effect that the design for the crank mechanism have on the performance of the engine.

The main variables that connect the modules are described below.

- **External heat transfer module.** This module considers the adiabatic flame temperature and the inlet temperature of the cooling fluid on the hot and cold side respectively. Therefore, the heat source (Q_h) and the heat sink (Q_k) are used to estimate the wall temperatures (T_{woh} , T_{wok}). This approach is proposed to couple the Stirling engine within the external heat and cooling sources respectively.
- **Internal Heat Transfer module.** The internal working gas temperatures (T_h , T_k) in the heater and cooler respectively are calculated using heat transfer correlations for steady state internal forced convective flow [27]. On the other hand, the regenerator analysis proposes the use of cyclic flow heat transfer correlations, which are more suitable for the flow conditions on this space [28]. Therefore, with these correlations the effect of limited heat transfer inside the engine is introduced in the model.
- **Ideal Adiabatic module.** The main operative variables such as net shaft work (W_s), heat and cooling demands (Q_h , Q_k), are calculated considering the internal working fluid, temperature distribution, and the engine geometric characteristics following Urielli's [23] approach.
- **Energy losses module.** The losses inside the engine are estimated to correct the ideal adiabatic outputs. This module considers the losses due to pressure drop, axial conduction, shuttle heat transfer, and imperfect regeneration.
- **Mechanical efficiency module.** The losses due to forced compression and expansion are evaluated, considering the buffer pressure (P_b), the shape of the cycle and the crank mechanism effectiveness (E)

The relationships between the modules are shown in Figure 1. The loops represent the iterative calculations to achieve the steady state cyclic conditions. The detailed report of the first four

modules can be found in Araoz et al [26], and the detailed description of the new mechanical efficiency module, is presented in the next section.

Figure 1: Block diagram for the Stirling model

2.1 Governing equations

The equations included in the model are based in the mass, energy balances, and the equation of state for the working gas. These balances were applied to the control volumes shown in Figure 2.

Figure 2: Control volumes for Stirling engine , based on Urielli [23]

The mass balance is expressed as:

$$m_{in} - m_{out} = \frac{dm}{d\varphi} \quad (1)$$

The energy balance neglecting the energy kinetic terms:

$$\frac{dQ}{d\varphi} + c_{pin}T_{in}m_{in} - c_{pout}T_{out}m_{out} = \frac{dW}{d\varphi} + c_v \frac{d(mT)}{d\varphi} \quad (2)$$

The equation of state for the gas in the control volume:

$$PV = mRT \quad (3)$$

The balances were applied to each control volume to obtain a set of algebraic differential equations. This set was complemented with correlations for the heat transfer in the heat exchangers, and the losses of the engine. The details of the model development is presented in Araoz[26]. However, a summary of the equations is presented in Appendix B.

2.2 Mechanical efficiency and shaft work

The mechanical efficiency of an engine measures the amount of the work produced by the thermodynamic cycle (indicated work W_i), that can be effectively taken from the shaft, shaft work (W_s) [24].

$$\eta_m = \frac{W_s}{W_i} \quad (4)$$

The mechanical efficiency is evaluated with the fundamental efficiency theorem, considering a constant mechanism effectiveness (E) as developed by Senft [24] .

$$\eta_m = E - \left(\frac{1}{E} - E \right) \frac{W^-}{W_i} \quad (5)$$

Where, W- represents the forced work. This is the work that the crank mechanism must deliver to the piston to make it move in opposition to the pressure difference across it [24]. For example during the expansion process, when the pressure of the gas inside the working space is lower than the opposite buffer pressure, then the expansion process is forced. In a similar way, during the compression process, when the pressure inside the working space is higher than the opposite buffer pressure, then the compression is forced. Therefore, this forced work depends mainly on the cycle shape, and the buffer pressure level (P_b) and its calculated with the following expression [24].

$$W^- = \oint (P - P_b)^+ dV^- + \oint (P - P_b)^- dV^+ \quad (6)$$

The superscripts difference the two types of forced work, the first one during the compression (dV^-) when the buffer pressure is below the working space pressure $(P - P_b)^+$, and the second during the expansion (dV^+) when the buffer pressure is above the working space pressure $(P - P_b)^-$.

The modified model includes a numerical integration of Equation 6, and the evaluation of both: The mechanical efficiency from Equation 5, and the shaft work from Equation 4.

2.3 Brake thermal efficiency

The overall efficiency or brake thermal efficiency is defined as the ratio of the shaft work, W_s , and the net heat input of the engine, Q_{hc} . This can be calculated by the product of the thermal efficiency and the mechanical efficiency as shown in Equation 7. The additional module includes the estimation of the mechanical efficiency and the brake efficiency.

$$\eta_b = \frac{W_s}{Q_{hc}} = \frac{W_i}{Q_{hc}} \frac{W_s}{W_i} = \eta_t \eta_m \quad (7)$$

3. Simulation of the Genoa engine

3.1. System description

The Genoa Stirling is a two cylinder gamma type engine built as a prototype for research studies by GENOA Stirling S.R.L. company from Italy [29]. According to its specifications, it is capable to produce up to 1 kW electrical output with air as working fluid at 600 rpm rotational speed and with the heater temperature around 750 °C [29]. The main components of the engine such as the crankcase, the crank mechanism with the balancing flywheel, the heat exchangers and the generator of the engine are shown in Figure 3.

Figure 3: Genoa Stirling scheme

Additional pictures for the heater cooler and regenerator heat exchangers are shown in Figure 4.

a) Heater head

b) Cooler heat exchanger

c) Regenerator heat exchanger

Figure 4: Heat exchangers of the engine prototype

The gamma Stirling engine consists of two identical piston-displacer cylinders, connected to a common shaft, under similar operational conditions. Therefore, it is assumed that both cylinders present similar thermodynamic cycles and consequently the double cylinder thermodynamic analysis is simplified to one cylinder analysis. The validity of the similarity on both cylinders is a common approach on Stirling simulation studies [11,30–33]. In addition, the model assumes adiabatic expansion and compression spaces, and that the steady state cyclic conditions are reached.

The Stirling engine was used in an experimental rig, built at the Energy department, Royal Institute of Technology (KTH), Stockholm, Sweden. This rig consisted on the engine coupled to a pellet burner in order to produce heat and power simultaneously as shown in Figure 7.a. This configuration had technical limitations that are still being studied in order to improve both power and thermal outputs. But, despite of these limitations experimental results were obtained and these were compared with the model.

3.2 Inputs for the model

The main inputs for the engine simulation are shown in Table 1. Supplementary inputs that include the design and operational characteristics of the engine are presented in Appendix A.

Table 1: Main parameters for the engine simulation

Parameter	Value	Definition	Description
freq	5 Hz		Frequency of the engine
X	1.3353	V_{de}/V_{swe}	dead volume ratio
K	0.3684	V_{swc}/V_{swe}	piston to displacer swept volume ratio
T_{ratio}	0.23	T_{ad}/T_{water_in}	cold to heat temperature ratio
P_{ch}	12.5 bar	----	engine charging pressure

The model also needs to consider the relation of the crank mechanism and the variation of the volumes inside the working spaces. Therefore, considering that the engine has gamma type configuration, the following relations for the expansion and compression spaces were included [24]:

$$V_e = V_{cle} + \frac{V_{swe}}{2}(1 + \cos(\varphi + \alpha)) \quad (8)$$

$$V_c = V_{clc} + (V_{swe} - V_e) + \frac{V_{swc}}{2}(1 + \cos(\varphi)) \quad (9)$$

Furthermore, the following volume derivatives were evaluated.

$$dV_e = -\frac{V_{swe}}{2} * \sin(\varphi + \alpha) \quad (10)$$

$$dV_c = -dV_e - \frac{V_{swc}}{2} * \sin(\varphi) \quad (11)$$

3.3 Numerical solution

The system consists of a set of algebraic differential equations, which are shown in Appendix B. These consider as boundary conditions that the temperatures of the working gas at the end of the cycle must be equal to the temperatures at the beginning of the cycle, once cyclic steady state conditions are reached. Therefore, an iterative shooting method [34], using a fourth order Runge Kutta scheme for the time discretization, was implemented for the numerical solution. The iteration process was done until cyclic steady state conditions, which is numerically reached when the difference between the assumed initial values and the values calculated at the end of the cycle are lower than a defined error. After the cyclic steady state solution was reached, the energy losses and the forced work were evaluated. The forced work was calculated using the classical Simpson 3/8 numerical integration rule [35]. The scheme in Figure 5 summarizes the iterative steps for the solution.

Figure 5: Calculation scheme for the Stirling engine model

The numerical solution was implemented in Aspen Custom Modeller® (ACM)[36], which is a product from Aspen Plus® that permits the elaboration of customized models [37]. This software has its own modelling language and can also be coupled with C++ procedures. The Layout of the model in ACM is shown in Figure 6. The blocks were programmed with the equations shown in the appendix B and then the solution of the system was obtained with the algorithm previously described.

Figure 6: Layout Stirling engine model in ACM

The descriptions of the blocks are shown in Table 2. Additional details of the block inputs are given in Appendix A.

Table 2: Description of the blocks for the ACM model

Block Name	Description
Comp-Exp	The block contains the data that describes the volume variation inside the engine. The swept, dead volumes, crank mechanism and the characteristics of the pistons.
Cooler	The block contains the geometrical data for the cooler heat exchanger.
Heater	The block contains the geometrical data for the heat exchanger.
Regenerator	The block contains the geometrical data for the regenerator and the details of the matrix porosity and material.
Ext-heat	The characteristics of the external heat source are contained in this block.
Mech_Efficiency	The block contains the parameters for the calculation of the engine mechanical efficiency.
CoolingFluid	The characteristics of the external cooling fluid are contained in the block.
WorkingGAS	The block contains the parameters for the calculation of the properties for the working gas inside the engine.
Stirling	This is the main block, and contains the main equations that describe the thermodynamic analysis of the engine.

4. Model validation

The geometrical and operational characteristics for the Genoa engine are described in Table 1 and appendix A. The engine was mounted in the experimental rig shown in Figure 7a. In addition, the temperatures of the working gas were measured at the different points of the engine shown in Figure 7b.

Figure 7: a) Schematic set-up of the Stirling engine integrated with a combustion chamber and a boiler [28]
b) Temperature measurement points for the working gas in the Stirling engine T2: Hot side; T10: Cold side; T11, T12: Hot and cold side of the regenerator [25]

The experimental rig used wood pellets as fuel. Additional temperatures measured for the validation were: The temperature close to the flame (T1), the water inlet temperature (T8), the water outlet temperature (T9). Other measurements are also as shown in Figure 8.

The temperature T1 was measured using a type K $\varnothing$ 1.5 mm Inconel 600 thermocouple. The additional temperatures shown in Figure 8 were measured using type K $\varnothing$ 1.0 mm thermocouples. Considering the type of thermocouples the expanded uncertainty was ± 3.2 °C with a coverage factor of 2. The speed of the engine crankshaft was monitored by a pulse sensor and a frequency to analog converter (OMRON E2A, and Red Lion IFMA) with an uncertainty $\pm 0.2\%$. The pressure inside the engine was measured with a pressure transducer (RS type 46) with analog signal and an uncertainty of ± 0.1 bar. All the measurements were recorded from the beginning to the end of the test using a data logger. Additional details of the measurements are reported in Cardozo et al [25].

neglected by the model, since it assumes steady state heat transfer conditions. On the other hand, the prediction of the mean temperature in the regenerator space (T_r), presents higher differences. This is analysed with the Figure 9 below, which shows the variations of the temperatures inside the heat exchangers assumed by the model.

Figure 9: Temperature variation along the heat exchangers and regenerator temperature assumed by the model (T_r)

From Figure 9, it can be seen that the model assumes that the temperatures at the interfaces heater-regenerator and cooler-regenerator were equal to the temperatures at the cooler (T_k) and heater (T_h) spaces respectively. Therefore, the average temperature at the regenerator (T_r) was calculated with these values. This assumption neglects the axial temperature variation along the heater and cooler, which is reflected on the measurements taken at the exact interfaces positions T_{11} and T_{12} . This explains the difference between the average regenerator temperature calculated with the measured temperatures (T_{rM}) and the calculated with the estimations of the model T_r as it is shown in Table 3. However, considering that the model was capable to calculate within a good degree of accuracy the power output measured during the experimental runs, it can be inferred that the error for the regenerator temperature estimation have little influence on the brake power calculation. This is shown in Table 4, where the values for the measured and calculated brake power are compared at different operating conditions. The percentage error ranges from $\pm 1.31\%$ to $\pm 7.94\%$, which is an acceptable approximation for first design calculations.

Table 4: Measured and predicted brake power

Time (s)	T_{IM} (K)	Measured Frequency (Hz)	Measured Pressure (bar)	Brake Power (W) Experimental	Brake Power (W) Calculated	Error %
3780-3900	1387.8	5.17	12.50	54.72	53.59	2.06
3900-4020	1382.9	5.26	12.50	55.39	52.08	5.97
4020-4140	1393.0	5.27	12.50	55.61	53.49	3.81
4140-4200	1377.8	5.33	12.50	46.35	50.03	7.94
4200-4380	1383.5	5.28	12.50	53.59	51.97	3.02
4380-4560	1377.7	5.36	12.50	50.91	50.33	1.14
4560-4680	1385.7	5.29	12.50	50.96	51.63	1.31
4680-4800	1384.3	5.34	12.50	55.9	51.53	7.82
4800-4980	1366.9	5.56	12.54	47.13	46.13	2.12

5. Results and discussion

This section presents additionally results for the simulation of the engine under the experimental conditions described before. This aims to completely describe the thermodynamic performance of the engine and thus identify the main limitations that the engine presents.

5.1 Temperature variation

Figure 10 shows the temperature variation in the different spaces of the engine cylinder, once the cyclic steady state conditions are reached. This figure displays the sinusoidal variation of the temperatures inside the compression (T_c) and expansion (T_e) spaces. It can also be seen that the expansion space presents periods with elevated temperatures, which results into a high thermal stress for the material and therefore further engine deterioration. In addition, the figure also shows that the mean temperatures for the working fluid inside the heater (T_h) and cooler (T_k) are close to the heat exchangers wall's temperature (T_{wk} , T_{whe}). This indicates a good heat transfer rate on both heat exchangers, and consequently a good thermal performance based on the model assumptions. However, it is important to notice that this performance will decrease with the time due to the fouling on the heat exchangers which is not accounted for in the engine model.

Figure 10: Temperature variation along the engine, flame temperature $T_{ad}=1388$ K

5.2. Mass distribution and volumes variation

The mass distribution and volumes variation for the engine during a complete cycle are shown in Figure 11 and Figure 12 respectively. These variations permit to analyse the engine dynamics during the compression and expansion processes.

Figure 11: Volumes variation during the engine cycle

Figure 12: Mass variation inside the engine spaces during a complete cycle

Figure 11, permits to identify the following processes: The compression, characterized by the decrease in the total volume, from the time around $t=0.01$ to $t=0.04$; the heating process, when the total volume variation is not pronounced and the temperatures increase, around $t=0.04$ to $t=0.06$; the expansion process when the total volume increases around $t=0.06$ to $t=0.09$; and the cooling process when the volume stays almost constant and the temperatures decrease, at the times around $t=0.09$ to $t=0.10$ and $t=0$ to $t=0.01$.

The compression period starts with the increment of the mass in the compression space, and a decrease of the mass in the expansion space as shown in Figure 12. The decreasing mass in the expansion space indicates a good dynamic for the compression process, because it is desirable to keep low the hotter portion of the mass during this period. However, the mass on the compression space is too high, which is not desirable, since this will be reflected in a large negative compression work. In addition, the expansion process also presents a reduced performance due to the low values for the mass in the expansion space during the expansion process. This represents an expansion with

low hotter mass and thus a low working output to the shaft. Furthermore, the low mass in the expansion space during the heating period might be the main cause for the high temperatures reached. Therefore, the volumes and mass flow dynamics of the reference case should be improved to reach higher work outputs and avoid the overheating of the expansion chamber.

Figure 12, also shows that the mass in the heater and cooler are really small compared with the mass in the regenerator during the complete cycle. This reflects the high importance of the regenerator efficiency for the engine performance.

5.3. Work flow

Table 5, shows the simulation results for the compression and expansion work during a single cycle. This table also presents the different work losses estimated for the system.

Table 5: Engine work flow per cycle

Model Output per cycle	Aspen Custom Modeller(ACM)
Expansion Work (W_e , J/cycle)	52.62
Compression Work (W_c , J/cycle)	-23.39
Pressure drop lost heater(J/cycle)	0.21
Pressure drop lost cooler(J/cycle)	0.07
Pressure drop lost regenerator (J/cycle)	0.28
Total lost due to Pressure drop (J/cycle)	0.56
Net Indicated Work (W_i , J/cycle)	28.67
Forced Work (W_- , J/cycle)	23.49
Brake Work Output (W_{br} , J/cycle)	5.18

The temperatures measured and the temperatures calculated show a good thermal performance of the engine. But, the measured brake power was very low. Different problems on the engine design, and operational conditions may explain these very low result. However, additional experimental instrumentation is needed for a detailed design study. For this reason, the present analysis considers a theoretical approach that may be later complemented with experimental studies. This theoretical approach considers Equation 7. From this equation, and considering that the thermal performance was found acceptable, the main losses should correspond to a low mechanical efficiency of the prototype. This mechanical efficiency is reduced by the presence of forced work during the cycle, and mechanical friction on the crank mechanism.

Figure 13, presents the evaluation of the forced work in a pressure volume diagram for the gas cycle inside the gamma prototype. From this it can be seen that the forced work (W_-) is mainly due to the forced expansion process. This means that at the experimental conditions, large part of the cyclic work may have been used to complete the forced expansion process and thus the real engine output is smaller than expected.

Figure 13: Pressure volume diagram and forced work during the cycle

The results discussed above, are complemented with the variation of the compression (W_c), expansion (W_e), and net indicated work (W_i) during the cycle shown in Figure 14.

Figure 14: Work flow during the engine cycle

Figure 14 shows that during the first part of the cycle, from $t=0$ to $t=0.045$, the compression and expansion spaces present exchanged roles. This means that an increment of the volume is presented in the compression space and a decrement of the volume is present in the expansion one. This reduced the engine performance, but it cannot be avoided since the gas needs to pass from one space to another. Regarding the second part of the cycle, from $t=0.045$ to $t=0.095$, the expansion and compression are shaped as expected and thus indicate a better dynamic during this period. However,

considering that large part of the expansion process is forced, the net brake work is low as reported in Table 5.

From the previous analysis it can be concluded that a detailed evaluation of the volumes dynamics, the cranks mechanism effectiveness, and the forced work during the cycle, must be considered in order to re-design the engine for a better performance. This will be covered on a detailed parametric study to be reported on a next article.

5.4. Heat Flow

Table 6, presents the results for the heat flow and corresponding heat losses through the heat exchangers calculated at the end of a single cycle [26]. As it can be seen, the total heat requirements are almost three times the requirements calculated without considering the losses. It can also be seen that the shuttle conduction losses represent the main heat loss during the cycle. These correspond to the losses due to the oscillation of the hot displacer across the temperature gradient in the working spaces of the engine.

Table 6: Heat flow and heat losses during the cycle

Heat Exchanger Space	Heat Flow (J/cycle)
Heater flow (Q_{hc} , J/cycle)	52.82
Cooler flow (Q_{kc} , J/cycle)	-23.56
Regenerator flow (Q_{rc} , J/cycle)	0.05
Heat Losses	
Internal conduction losses (Q_{ikc} , J/cycle)	26.98
Shuttle conduction losses (Q_{shc} , J/cycle)	80.04
Regenerator losses during heating (Q_{lossrc} , J/cycle)	18.62
Regenerator losses during cooling (Q_{lossrc} , J/cycle)	-18.62
Total Heat Requirements	
Heating Requirements (Q_{htc} , J/cycle)	178.47
Cooling Requirements (Q_{ktc} , J/cycle)	42.18

The cyclic variation for the heat flow is additionally shown in Figure 15. The heat requirements for the heater and cooler present slight variations during the entire cycle. On the other hand, the regenerator presents high variations, managing large quantities of heat. This confirms the large importance of this heat exchanger on the engine performance.

Figure 15: Heat flow variation during the engine cycle

5.5 Brake Power and Brake Efficiency

The engine brake power is defined as the net brake work per cycle (W_s), times the engine frequency (freq).

$$P_{br} = W_s \times \text{freq} \quad (12)$$

The net brake work, and the total heat requirement, presented on Table 5 and Table 6 respectively are doubled considering the double cylinder engine. These values are reported on Table 7, which also reports the thermal and brake efficiencies for the engine.

Table 7: Power output and efficiency of the engine

Brake Power (W)	Heat Requirement (W)	Thermal Efficiency (%)	Mechanical Efficiency (%)	Brake Efficiency (%)
53.58	1845.35	16.10	18.10	2.90

The results reflect the low performance of the engine under the experimental conditions. This was mainly attributed to the forced work and the mechanical efficiency as it was analysed in the previous section. In addition, complementary works will broaden this analysis with the aim of propose improvements on the engine design and operational parameters.

6. Conclusions

In the present work a thermodynamic model for a Stirling engine was improved, by including the numerical evaluation of the forced work and the mechanical efficiency, then validated against experimental data, and finally implemented for the simulation of an engine prototype. The numerical

model developed considered the analytic approach proposed by Senft [24], but extended its application for the case of the more realistic adiabatic working spaces assumptions. Consequently, the effective work taken from the shaft is better estimated and thus used for a more complete analysis of the thermal and mechanical performance of an engine. For this article, the analysis considered a novel gamma engine prototype, under the experimental conditions of a micro scale combined heat and power system fuelled by wood pellets.

The simulation results were compared with the experimental data measured during long time runs of the system. The model performance was very good for the prediction of the temperatures in the different spaces of the engine. In addition, the estimations for the net brake power also presented results similar to the measured values. However, additional experimental work should be performed to obtain data to validate the calculation of the different losses through the engine.

According to the results obtained, the thermal performance of the engine was found acceptable, and thus the low power output measured is preliminary attributed to a reduced mechanical efficiency of the system. The possible reasons for this low performance were further analysed with the different results for the temperatures variation, mass and volume variation, pressure drops, and the pressure volume diagrams obtained with the model. According to these analyses, the dynamics of the volumes variation and the crank mechanism may also be improved, in order to obtain higher net work during the cycle. In addition, it was found that the engine performance is very sensitive to the effect of the buffer pressure. These results will be extended with a sensitivity analysis for the system on a complementary work that aims to identify better the effect of the different parameters on the engine performance.

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APPENDIX A: DETAILED STIRLING ENGINE PARAMETERS

Table A1: Inputs for the Cooler in ACM

Variable	Value	Units	Description
d_o	0,005	m	tubes external diameter
d_i	0,003	m	tubes internal diameter
k_w	14,200	W/m K	material conductivity
L	0,032	m	tubes length
num	162	--	number of tubes
sl	0,005	m	space between tubes

Table A2: Inputs for the Heater in ACM

Variable	Value	Units	Description
d_e	0,005	M	tubes external diameter
d_i	0,0031	M	tubes internal diameter
k_w	14,2	W/m K	material conductivity
len	0,149	m	tubes length
num	36,0	--	number of tubes
sl	0,005	m	space between tubes

Table A3: Inputs for the Regenerator in ACM

Variable	Value	Units	Description
D_{in}	0,078	m	regenerator housing internal diameter
d_{out}	0,107	m	regenerator housing external diameter
d_{wire}	2,1e-004	m	wire diameter of the matrix
k_{wr}	27,0	W/m K	thermal conductivity of the matrix material
L_r	0,07	m	length of the regenerator housing
porosity	0,87		matrix porosity

Table A4: Inputs for the expansion- compression spaces and crank mechanism

Variable	Value	Units	Description
v_{clc}	4,4e-006	m^3	compression space clearance volume
v_{cle}	2,6e-005	m^3	expansion space clearance volume
v_{swc}	9,26e-005	m^3	compression space swept volume
v_{swe}	2,5134e-004	m^3	expansion space swept volume
$dispd$	0,062	m	displacer diameter
$displ$	0,07	m	displacer length
$effmek$	0,8	--	mechanism effectiveness
freq	5	Hz	Frequency
j_{gap}	0,006	M	gap between cylinder displacer and wall
k_{pist}	16,27	W/m K	piston conductivity

pbuff	1,2e+006	Pa	buffer pressure
phase	90,0	deg	phase angle advance
pmean	1,25e+006	Pa	mean operating pressure
strk	0,035	m	displacer stroke
dispd	0,062	m	displacer diameter

Table A5: Working and Cooling Fluid Inputs in ACM

Variable	Value	Units	Description
Working fluid	Air	---	working fluid inside the engine
Cooling Fluid	Water	-----	cooling fluid through the engine cooler
T _{cooling}	288	K	Inlet temperature of the cooling fluid

Table A6: Fouling factors and external combustion inputs in ACM

Variable	Value	Units	Description
Tad	1387	K	Flame temperature in the combustion chamber
absorp	0,70	---	absorptivity of the heater material

Appendix B

Main equations for the Stirling engine model

Stirling engine module

Mean Pressure

$$P = \frac{MR}{\left(\frac{V_c}{T_c} + \frac{V_k}{T_k} + \frac{V_r}{T_r} + \frac{V_h}{T_h} + \frac{V_e}{T_e} \right)}$$

Pressure variation

$$\frac{dP}{d\varphi} = - \frac{\gamma P \left(\frac{\left(\frac{\partial V_c}{\partial \varphi} \right)}{T_{ck}} + \frac{\left(\frac{\partial V_e}{\partial \varphi} \right)}{T_{he}} \right)}{\frac{V_c}{T_{ck}} + \gamma \left(\frac{V_k}{T_k} + \frac{V_r}{T_r} + \frac{V_h}{T_h} \right) + \frac{V_e}{T_{he}}}$$

Mass of the working gas in the different spaces

$$m_c = p \left(\frac{V_c}{RT_c} \right); m_k = p \left(\frac{V_k}{RT_k} \right); m_r = p \left(\frac{V_r}{RT_r} \right); m_h = p \left(\frac{V_h}{RT_h} \right); m_e = p \left(\frac{V_e}{RT_e} \right)$$

Mass accumulation

$$\frac{dm_k}{d\varphi} = \frac{m_k}{P} \left(\frac{\partial P}{\partial \varphi} \right); \frac{dm_h}{d\varphi} = \frac{m_h}{P} \left(\frac{\partial P}{\partial \varphi} \right); \frac{dm_r}{d\varphi} = \frac{m_r}{P} \left(\frac{\partial P}{\partial \varphi} \right)$$

$$\frac{dm_c}{d\varphi} = \frac{P \left(\frac{\partial V_c}{\partial \varphi} \right) + \frac{V_c}{\gamma} \left(\frac{\partial P}{\partial \varphi} \right)}{RT_{ck}}; \frac{dm_e}{d\varphi} = \frac{P \left(\frac{\partial V_e}{\partial \varphi} \right) + \frac{V_e}{\gamma} \left(\frac{\partial P}{\partial \varphi} \right)}{RT_{he}}$$

Mass Flow

$$m_{ck} = -dm_c; m_{he} = dm_e; m_{kr} = m_{ck} - dm_k; m_{rh} = m_{he} + dm_h$$

Conditional Temperatures

$$\begin{aligned} \text{If } m_{ck} > 0 \quad \text{then } T_{ck} &= T_c \quad \text{else } T_{ck} = T_k \\ \text{If } m_{he} > 0 \quad \text{then } T_{he} &= T_h \quad \text{else } T_{he} = T_e \end{aligned}$$

Temperatures

$$\frac{dT_c}{d\varphi} = T_c \left(\frac{\left(\frac{\partial P}{\partial \varphi} \right)}{P} + \frac{\left(\frac{\partial V_c}{\partial \varphi} \right)}{V_c} - \frac{\left(\frac{\partial m_c}{\partial \varphi} \right)}{m_c} \right); \frac{dT_e}{d\varphi} = T_e \left(\frac{\left(\frac{\partial P}{\partial \varphi} \right)}{P} + \frac{\left(\frac{\partial V_e}{\partial \varphi} \right)}{V_e} - \frac{\left(\frac{\partial m_e}{\partial \varphi} \right)}{m_e} \right)$$

Energy

$$\frac{dQ_k}{d\varphi} = \frac{V_k \left(\frac{\partial P}{\partial \varphi} \right) C_v}{R} - C_p (T_{ck} m_{ck} - T_{kr} m_{kr}); \frac{dQ_r}{d\varphi} = \frac{V_r \left(\frac{\partial P}{\partial \varphi} \right) C_v}{R} - C_p (T_{kr} m_{kr} - T_{rh} m_{rh})$$

$$\frac{dQ_h}{d\varphi} = \frac{V_h \left(\frac{\partial P}{\partial \varphi} \right) C_v}{R} - C_p (T_{rh} m_{rh} - T_{he} m_{he}); \frac{dW_c}{d\varphi} = P \left(\frac{\partial V_c}{\partial \varphi} \right); \frac{dW_e}{d\varphi} = P \left(\frac{\partial V_e}{\partial \varphi} \right)$$

Internal heat transfer module

Heat transfer from the heater wall to the working gas

$$Q_h = \frac{1}{R_{cih} + R_{hih} + R_{fih}} (T_{woh} - T_h)$$

Heat transfer from the cooler wall to the working gas

$$Q_k = \frac{1}{R_{cik} + R_{hik} + R_{fik}} (T_{wik} - T_k)$$

Heat loss during the regenerator process

$$Q_{lossr} = (1 - \varepsilon) \times Q_r$$

Regenerator effectiveness

$$\varepsilon = \frac{NTU}{1 + NTU}$$

External heat transfer module

Heat transfer from the flame to the external wall of the heater

$$Q_h = \frac{1}{\frac{1}{h_{rh} A_{oh}} + R_{foh}} (T_{ad} - T_{woh})$$

$$h_{rh} = \alpha_s \sigma A_{oh} F_R (T_{ad} + T_{woh}) (T_{ad}^2 + T_{woh}^2)$$

Estimation of the outlet temperature of the cooling fluid

$$T_{wok} = T_{water_in} + Q_k \left(\frac{1}{h_{ok} A_{ok}} + \frac{1}{2m_{water} C_{pwater}} \right)$$

$$h_{ok} = \frac{1}{\frac{1}{h_{water}} + R_{fok}}$$

Energy Losses

Pressure drop in the heat exchangers

$$\Delta P = \frac{f}{d_{hy}} \frac{1}{2} \rho v^2 l$$

Pressure drop in the regenerator: Based on the correlations of Thomas and Pittman [38]

$$\Delta P = C_f n \frac{\rho}{2} u^2$$

$$C_f = C_{fd} + \frac{C_{sf}}{Re}$$

$$\begin{array}{ll} C_{sf} = 68.556 & C_{fd} = 0.5274 \quad \text{wire screens} \\ C_{sf} = 70.035 & C_{fd} = 0.9307 \quad \text{metal felts} \end{array}$$

Total Pumping losses

$$W_{\text{ploss}} = \int_0^{2\pi} \left(\sum_{i=1}^{i=3} \Delta P_i \times \frac{dV_e}{d\theta} \right) d\theta$$

Energy losses due to shuttle conduction

$$Q_{\text{sh}} = 0.4 \frac{Z^2 K_{\text{pist}} D_d}{J L_d} (T_e - T_c)$$

Mechanical Efficiency module

Mechanical efficiency

$$\eta_m = \frac{W_s}{W_i}$$

Mechanical efficiency considering the mechanism effectiveness and forced work

$$\eta_m = E - \left(\frac{1}{E} - E \right) \frac{W -}{W_i}$$

Forced work

$$W = \oint (P - P_b)^+ dV^- + \oint (P - P_b)^- dV^+$$

Brake efficiency

$$\eta_b = \frac{W_s}{Q_{ht}} = \frac{W_i}{Q_{ht}} \frac{W_s}{W_i} = \eta_t \eta_m$$