A Lattice Boltzmann Algorithm for Simulating Conjugate Heat Transfer through Virtual Heat Capacity Correction

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Abstract:

A fundamental problem for lattice Boltzmann method (LBM) to simulate conjugate heat transfer between solid and fluid is that lattice Boltzmann equation (LBE) can only retrieve standard energy equation when solid has the same heat capacity (ρc) with that of fluid. When their heat capacities are different, the continuity of heat flux cannot be kept at the interface for conventional LBM. In present work, a lattice Boltzmann algorithm for simulating conjugate heat transfer through virtual heat capacity correction method is proposed to solve this problem. This algorithm, at first, assume that the solid has the same heat capacity (ρc) with that of fluid to keep the continuity of heat flux at the interface between solid and fluid, after that modify the temperature field and the heat flux distribution function according to real data of heat capacity of fluids or solids at the end of each time step. This algorithm also fits to conjugate heat conduction problems between solid and solid as long as we take one of the solids as a kind of fluid that does not flow. Several test cases including both steady and transient conjugate heat transfer with flat or curved interface are calculated to validate the present method. The results show that the proposed method has a very good performance in simulating solid-fluid and solid-solid conjugate heat transfer, which is easy in implementation, capable for solving both steady and transient heat transfer problems. Also, the present method can deal with curved interface easily without a necessary to find the normal direction of interface. However, this method suffers a stability problem while the heat capacity (ρc) of fluid is larger than that of solid, which will be eliminated in future work.

Key words:

Conjugate Heat Transfer, Lattice Boltzmann Method, Virtual Heat Capacity Correction, Numerical simulation

1. Introduction

Heat transfer across the interface between solid and fluid or solid and solid is one of fundamental issues in multiphase heat transfer. It has numerous applications in engineering problems such as cooling of electronic devices [1-2], lab-on-a-chip [3], micro-TAS [4], micro fuel cells [5], and microchannels [6] etc.

Depending on the numerical methods for solving the governing N-S and energy equations, various approaches have been proposed to keep temperature and energy (e.g. heat) flux continuity at the interface between the two domains of different solids, solid and fluid, or different fluids [7,8]. In the category of conventional numerical methods, Finite Volume Method (FVM), Finite Difference Method (FDM), Finite Element Method (FEM) have been successfully applied to simulate conjugate heat transfer problems. For example, Fiebig et al [9] made a numerical investigation on conjugate heat transfer in a finned-tube element with FVM. Ha et al [10] simulated three-dimensional natural convection and conduction in a differentially heated cubic enclosure with a heat-generating cubic conducting body with FVM. By taking FDM, Korichi et al [11] made a numerical simulation on conjugate heat transfer in a rectangular channel with mounted obstacles on upper and lower walls. More systematic introduction can be found in the review given by Dorfman and Renner [12] and the references therein.

Based on the particle distribution function of discrete velocity, lattice Boltzmann method (LBM) has been developed rapidly in recent years. As proved by some previous researchers, LBM is an effective and very simple method in simulating the solid-fluid flow systems [13-17]. Up to date, there are some researchers tried to spread the use of LBM to solid-fluid conjugate heat transfer problems. In the category of LBM, some improved SIMPLE-like algorithms are adopted to simulate conjugate heat transfer problem [18-20]. These methods assume that the solid has the same heat capacity ρc (ρ and c are, respectively, the density and the specific heat capacity) with that of the fluid, therefore, can only recover the steady energy equation. Also several difference methods have been proposed [21-23] to deal with conjugate heat transfer problems, in which both an equal temperature and a continuous heat flux at the interface of solid and fluid can be well guaranteed. However, the raised examples in those methods are only for solids with a square shape at steady state. Recently, Li et al. [24, 25] proposed a new difference method to ensure temperature and energy flux are continuous at a curved interface, but the difference formula is so complex that not easy to implement, especially for the cases when the normal direction of interface is not easy to find. Karani and Huber [26] suggested using an additional energy source term which introduces the jump conditions at the interface between two different phases to recover the energy equation approximately with first-order accuracy, this method can also be

used to simulate conjugate heat transfer with curved interface approximately. This idea was developed by Rihab et al [27], in their paper, enthalpy rather than temperature is solved directly to make the expression of additional source term is easy to calculate. However, their method is only applicable for conjugate heat conduction. Another promising approach was proposed by Mohamad et al [28], in their method, the continuity of heat flux at the interface is ensured by coupling temperature difference with a new scaling law of energy distribution function. It is suitable for steady and unsteady conjugate heat transfer between two different immiscible media, what confines this method is that it requires the interface must locate at computational nodes.

Considering the complex geometry, and the difference of thermal properties between the two sides of an interface, the numerical simulations for conjugate heat transfer are generally not easy to be implemented with LBM. A more accurate, reliable, and simpler lattice Boltzmann algorithm is still desired. In this paper, a novel lattice Boltzmann algorithm called by virtual heat capacity correction method for simulating solid-fluid or solid-solid conjugate heat transfer is introduced, which is very easy to be implemented, and capable for solving both steady and transient heat transfer problems. Also, the present method can deal with curved interface easily, which does not need to find the normal direction of an interface. The remaining part of this paper is organized as follows. In Section 2, the conventional LBM is briefly reviewed and the virtual heat capacity correction lattice Boltzmann method is introduced in detail. To validate the present method, seven test cases are given in Section 3. The one-dimensional heat conduction problem is firstly considered in Section 3.1; followed by two unsteady heat conduction problems in a three-layer stratified medium in Section 3.2; thirdly, the heat conduction in a two-layered annulus is calculated in Section 3.3; the next is unsteady heat conduction in a square medium containing discrete circular media in Section 3.4; and finally, two natural convection problems in a square cavity with uniformly distributed solid blocks are simulated in Section 3.5 and Section 3.6 separately. In Section 4, summary and conclusions are addressed.

2. Numerical Method

In this paper, the single-relaxation-time(BGK) D2Q9 lattice Boltzmann model is used, the two lattice Boltzmann equations for simulating fluid flow and heat transfer, respectively, can be written as follows [29]:

$$f_i(\vec{x} + \vec{e}_i \Delta t, t + \Delta t) - f_i(\vec{x}, t) = -\frac{1}{\tau_f} [f_i(\vec{x}, t) - f_i^{(eq)}(\vec{x}, t)] + F_i(\vec{x}, t) \Delta t \quad (1)$$

$$g_i(\vec{x} + \vec{e}_i \Delta t, t + \Delta t) - g_i(\vec{x}, t) = -\frac{1}{\tau_g} [g_i(\vec{x}, t) - g_i^{(eq)}(\vec{x}, t)] \quad (2)$$

where f_i and g_i are the distribution functions of density and energy respectively, $f_i^{(eq)}$ and $g_i^{(eq)}$ are their corresponding distribution functions at equilibrium state, respectively. F_i is the discrete force term, τ_f and τ_g are the relaxation times related to, respectively, kinematic viscosity and thermal diffusivity, t is the time in simulation. The corresponding equilibrium distribution functions of $f_i^{(eq)}$ and $g_i^{(eq)}$ are expressed as follows.

$$f_i^{(eq)} = w_i \rho \left[1 + 3(\vec{e}_i \cdot \vec{u}) + \frac{9}{2}(\vec{e}_i \cdot \vec{u})^2 - \frac{3}{2}\vec{u} \cdot \vec{u} \right] \quad (3)$$

$$g_i^{(eq)} = w_i T \left[1 + 3(\vec{e}_i \cdot \vec{u}) + \frac{9}{2}(\vec{e}_i \cdot \vec{u})^2 - \frac{3}{2}\vec{u} \cdot \vec{u} \right] \quad (4)$$

and the discrete forcing terms of $F_i^{(eq)}$ is:

$$F_i(\vec{x}, t) = \omega_i \left(1 - \frac{1}{2\tau_f} \right) \left[\frac{\vec{e}_i - \vec{u}(\vec{x}, t)}{c_s^2} + \frac{\vec{e}_i \cdot \vec{u}(\vec{x}, t)}{c_s^2} \vec{e}_i \right] \cdot \vec{F}(\vec{x}, t) \quad (5)$$

where $c_s = 1/\sqrt{3}$ is the lattice sound speed. The bulk properties of density, velocity and temperature are obtained by:

$$\rho = \sum_i f_i \quad (6)$$

$$\rho \vec{u} = \sum_i f_i \vec{e}_i + \frac{\vec{F}}{2} \Delta t \quad (7)$$

$$T = \sum_i g_i \quad (8)$$

The relaxation times τ_f and τ_g related to kinematic viscosity and thermal diffusivity

of fluid, respectively, can be given by the following equations:

$$\nu = (\tau_f - 0.5)c_s^2\Delta t \quad (9)$$

$$\alpha = (\tau_g - 0.5)c_s^2\Delta t \quad (10)$$

By using Chapman-Enskog expansion [30], this lattice Boltzmann model can recover the following energy equation:

$$\frac{\partial T}{\partial t} = \nabla \cdot (\alpha \nabla T) - \nabla \cdot (\vec{u} T) \quad (11)$$

Where α is the thermal diffusivity, $\vec{u}$ is the velocity. For solid, there is no convection term $\nabla \cdot (\vec{u} T)$ and the model can recover the energy equation of transient heat conduction as follows:

$$\frac{\partial T}{\partial t} = \nabla \cdot (\alpha \nabla T) \quad (12a)$$

The above equation can also be written as:

$$\rho c \frac{\partial T}{\partial t} = \nabla \cdot (k \nabla T) + \rho c \cdot \nabla \cdot \left(\frac{1}{\rho c} \right) \cdot (k \nabla T) \quad (12b)$$

When ρc do not change with position, the energy equation for solid is in a form of:

$$\rho c \frac{\partial T}{\partial t} = \nabla \cdot (k \nabla T) \quad (13)$$

Therefore, it is only when ρc does not change with position ($\nabla \cdot \left(\frac{1}{\rho c} \right) = 0$) that the equation (12a) is equivalent to equation (13). However, in general, the conjugate heat transfer problems of solid-fluid or solid-solid may have different ρc so that the energy equation (13) cannot be recovered if the LBM scheme is not modified probably. Karani and Huber [26] gave a detailed illustration about the difference between equations (12a) and (13), and they proved that the boundary conditions at the interface can be satisfied automatically if the solid-fluid system or solid-solid system have the same constant heat capacities in all areas

Now, a novel lattice Boltzmann algorithm for solid-fluid conjugate heat transfer with different heat capacities will be introduced. Firstly, assume that the solid has the

same ρc with that of fluid to avoid the second term in the right side of equation (12b). Then, calculate equations (1)-(8) through the conventional steps. For the solid area, these steps are equivalent to solving the following equation:

$$(\rho c)_f \frac{\partial T}{\partial t} = \nabla \cdot (k_s \nabla T) \quad (14)$$

It is worthy to point out that the right side $\nabla \cdot (k_s \nabla T)$ does not relate to ρc , neither the heat flux $\vec{q} = k_s \nabla T$, so that the calculated $\vec{q}$ in the solid area is correct after using virtual heat capacity. To make a unit time step integration for equation (14) from t^n to t^{n+1} , we can get:

$$(\rho c)_f (T^{n+1} - T^n) = \int_{t^n}^{t^{n+1}} \nabla \cdot (k_s \nabla T) dt \quad (15)$$

However, the correct integral energy equation in the solid area should be:

$$(\rho c)_s (T_c^{n+1} - T^n) = \int_{t^n}^{t^{n+1}} \nabla \cdot (k_s \nabla T) dt \quad (16)$$

Combining equations (15) and (16), the correct temperature field T_c^{n+1} in the solid area can be obtained directly. Then we can get the right heat flux field $\vec{q}$ and temperature field T_c^{n+1} of the solid area at the end of every time step.

The refill procedure consists of the following steps:

- a) Assume the solid has the same ρc with that of fluid, calculate f_i and g_i .
- b) Calculate density ρ , velocity $\vec{u}$ and temperature T
- c) Calculate $f_i^{(eq)}$ and $g_i^{(eq)}$.
- d) Correct temperature T_c^n at the solid area using the following equation:

$$(\rho c)_s (T_c^{n+1} - T^n) = (\rho c)_f (T^{n+1} - T^n) = \int_{t^n}^{t^{n+1}} \nabla \cdot (k \nabla T) dt \quad (17)$$

e) Calculate $g_i^{(eq,new)}$

f) Find Correct g_i^{new} by keeping the heat flux $\bar{q}$ in the solid area from changing, $\bar{q}$ for the solid area can be calculated by [31]:

$$\bar{q} = (\rho c)_f \cdot \left(1 - \frac{1}{2\tau_g}\right) \sum_i (g_i - g_i^{(eq)}) \bar{e}_i \quad (18)$$

Since the term $g_i - g_i^{(eq)}$ can also be replaced by $g_i^{new} - g_i^{(eq,new)}$.

g) Back to step a).

For the cases of solid-solid conjugate heat transfer, we can take one of the solids as a kind of fluid that does not flow. The calculation procedures are the same with those of a solid-fluid problem. However, in order to ensure equation (14) is convergent during the iterations, we have to add a confinement $(\rho c)_s / (\rho c)_f \geq 1$, which can be regarded as the application limitation of this method. **For the situation of $(\rho c)_s / (\rho c)_f < 1$, the present scheme is always divergent, the fundamental reason needs to be analyzed deeply in numerical stability.** In other words, the proposed method can only simulate a problem of solid-fluid conjugate heat transfer where the ρc of solid is equal or bigger than that of fluid. Fortunately, for a solid-solid conjugate heat transfer problem, the confinement can be easily eliminated as long as we take the solid having a smaller ρc as the fluid.

3. Results and discussion

In this section, seven test examples are investigated to validate the present lattice Boltzmann algorithm, which include (1) 1D steady heat conduction in two-layered composite material, (2) two unsteady heat conduction problems in three-layered stratified medium, (3) steady heat conduction in two-layered annulus, (4) unsteady heat conduction in a square medium containing discrete circular media, (5) natural convection in a 2D square cavity with 16 scatted small square solid blocks, (6) natural

convection in a square cavity containing 16 circular solid blocks.

3.1 One-dimensional conjugate heat conduction

Fig.1 Schematic of two-layered stratified medium

At first, a one-dimensional heat conduction problem in a two-layered stratified system is considered. As shown in Fig. 1, the two materials have different thermophysical properties, for instance, $k_1/k_2=4$ and $\alpha_1/\alpha_2=4$. The whole media have a uniform initial temperature T_c at time $t=0$. At time $t>0$, a hot temperature T_h ($T_h>T_c$) is imposed at the upper boundary while the lower boundary is kept at T_c . As an illustration, we solve the temperature field by using two methods, i.e. the conventional method that solves equation (12a) directly without considering the difference of ρc , and the present method that solves equation (13) through the steps we proposed above. **The distribution of dimensionless temperature $\theta=(T-T_c)/(T_h-T_c)$ is compared with its analytical solution, as shown in Fig 2.**

Fig.2 The comparisons between numerical results of the present method and the conventional method and the analytical solution

Figure 2 shows that the data obtained by the proposed LBM match very well with the analytical solution, on the other hand, the conventional LBM taking $(\rho c)_1 = (\rho c)_2$ and $k_1 : k_2 = \alpha_1 : \alpha_2$ by default cannot get the right answer.

A mesh size dependence test is conducted in order to check the accuracy of the present method. The numbers of meshes in y axis are given by 50, 100, 200 and 400, respectively. **Note that the interface in simulation consists of the overlapped edges of two adjacent meshes which belong to different media by default.**

Table 1

The errors of temperature fields with different relaxation times and meshes

<i>Error</i>			
$N=50$	$N=100$	$N=200$	$N=400$

$\tau_{g2} = 0.8$	1.43×10^{-13}	6.01×10^{-13}	2.42×10^{-12}	9.90×10^{-12}
$\tau_{g2} = 1.1$	6.13×10^{-14}	2.59×10^{-13}	1.05×10^{-12}	4.21×10^{-12}
$\tau_{g2} = 1.7$	2.84×10^{-14}	1.53×10^{-13}	4.62×10^{-13}	1.89×10^{-12}
$\tau_{g2} = 2.9$	1.22×10^{-14}	4.52×10^{-14}	1.78×10^{-13}	7.11×10^{-13}

To analyze the difference between present results and the analytical results quantitatively, the errors by using different relaxation times and meshes are presented in Table 1. Only τ_{g2} , the relaxation times of medium 2, are displayed here because the virtual heat capacities are taken for medium 1 so that the relaxation times are virtual as well. The error definition is:

$$Error = \frac{\sum_{j=1}^N |T_j^a - T_j^c|}{N} \quad (19)$$

Where T_j^a is the analytical temperature at point j , while T_j^c is the calculated temperature, and N is the number of grids in y axis. As seen from Table 1, the errors are too small to be distinguished from rounding errors induced by computer. Based on these small errors, it can prove that present method can get good results very close to the analytical solution for steady heat conduction with straight interface.

3.2 Unsteady heat conduction in a three-layer stratified medium

This case aims to test the ability of the present method in dealing with transient conjugate heat transfer. The physical model is a three-layered composite medium as shown in Fig. 3. The thermophysical parameters are defined as $k_1 = k_3 = 1.0$, $k_2 = 0.1$, $\alpha_1 = \alpha_3 = 1.0$ and $\alpha_2 = 3.0$. The three layers have a uniform initial temperature T_c at time $t = 0$. At time $t > 0$, the lower boundary is fixed at a constant temperature of

$T_h (T_h > T_c)$ while the upper is fixed at T_c , and the two vertical boundaries are adiabatic.

Fig.3 Schematic of three-layer stratified media

Defining a dimensionless temperature $\theta = (T - T_c) / (T_h - T_c)$, then the analytical solutions can be obtained [32]. To confirm the rightness of the present method, the unsteady conjugate heat transfer problem is simulated, the dimensionless temperature fields at transient time $t = 0.1$, $t = 0.5$, $t = 1$, $t = 2$ and steady state are obtained, and these curves together with those of analytical solutions are shown in Fig. 4(a). It can be seen that the results calculated by the present method agree very well with the analytical solutions in both unsteady and steady states, the method we proposed above has a good performance in dealing with transient conjugate heat conduction problems. On the contrary, the results (Fig. 4(b)) for conventional LBM without considering the different heat capacities of the three layers are mismatching with the analytical ones.

Fig.4 Comparisons between the analytical solutions and the results calculated by (a) the present method and (b) the conventional LBM for transient heat conduction in a three-layered medium.

Furthermore, we investigate this problem for another group of thermophysical properties. Fig. 5 displays comparisons of temperature distribution at different time between the results calculated by present method and previous numerical results [27] calculated by Control Volume Method (CVM). The thermophysical properties are set the same as those of reference [27], which are: $k_1 = k_3 = 1$, $k_2 = 0.1$,

$$(\rho c)_1 = (\rho c)_3 = 5, \quad (\rho c)_2 = 1.$$

An overview of Fig. 5(a) shows that the results simulated by present method agree very well with that of CVM solution at transient period. For results at steady state, our numerical results are very close to analytical solutions while the results of CVM have a little obvious deviation. Similarly, for conventional LBM, the results (Fig. 5(b)) deviate with right solutions in a large scale at both transient and steady state.

Fig. 5 Comparisons between CVM and (a) the present method and (b) the conventional LBM (b) for transient heat conduction in a three layered medium.

3.3 Steady heat conduction in a two-layered annulus

Fig.6 Schematic of two-layered annulus

This case is designed to test our method against the curved interface. The model is a two-layered annulus with different thermophysical properties as shown in Fig. 7. Define r as the distance from the point to the center point, the area where $r \leq R_1$ belongs to medium 1, and its conductivity and heat capacity, respectively, are $k_1 = 0.2$

and $(\rho c)_1 = 5.0$. The area where $R_1 < r \leq R_2$ ($R_2 = 2R_1$) belongs to medium 2, which has a conductivity and a heat capacity, respectively, of $k_2 = 0.2$ and $(\rho c)_2 = 1.0$. The boundary of outer annulus is set as the following Dirichlet condition:

$$T(r = R_2, \varphi) = \cos 2\varphi \quad (20)$$

A direct-forcing immersed boundary method is used [\[33\] here for this boundary](#). For the inner circular interface between the two different media, the geometry can not be expressed exactly with square meshes. An approximation is used here by giving a definition that all of the nodes where $r \leq R_1$ have the thermophysical parameters of medium 1. It corresponds a staircase approximation for curved interface in geometry.

This problem is solved by using both the present and the conventional LBM. The results, including the radial distribution of temperature and heat flux in both horizontal ($\varphi = 0$) and vertical ($\varphi = \pi/2$) directions are compared with the analytical solutions [\[25\]](#). It is noted that the heat flux is calculated by equation (18).

As it is shown in Fig. 7, the present method can give good outputs that agree very well with the analytical solutions, however, the conventional method cannot get the right results. The comparisons indicate that our proposed method is competent for conjugate heat conduction with curved interface as well.

Fig.7 Radial distributions of temperature (a) and heat flux (b) in a two-layered annulus

3.4 Unsteady heat conduction in a square medium containing discrete circular media

To investigate the present method against transient heat conduction with a complex curved interface, heat conduction in a square medium containing 25 circular intrusions (Fig.8) with different thermal properties is calculated in this part.

Fig. 8 Square medium containing 25 discrete circular intrusions

The side length of the square is H , all of the circulars inside the square have the same radiuses $r = 0.08H$, the distance from a wall to the center of the nearest circular medium is $0.1H$, and the distance between the centers of two adjacent circles is $0.2H$. The thermophysical properties adopted in the simulation are $k_1=1, k_2=10$ and $(\rho c)_1=40, (\rho c)_2=10$ for the square and the circles, respectively. Here, the left wall is imposed with constant hot temperature T_h while the rest three walls are imposed with constant cold temperature T_c . The whole computational area is set at a uniform temperature T_c initially.

(a1) $t = 0.1$, present

(a2) $t = 0.1$, reference [27]

(b1) steady, present

(b2) steady, reference[27]

Fig.9 Comparisons between the present LBM scheme and reference [27] (solid line for LBM and dotted line for CVM) at transient period $t = 0.1$ (a1, a2) and steady state (b1, b2)

This problem also has been simulated by Rihab et al [27] with another LBM scheme and CVM. In Fig. 9, the dimensionless temperature field $\theta = (T - T_c) / (T_h - T_c)$ at a transient state of $t = 0.1$ and the steady state are displayed and compared with the results of Rihab et al [27]. It can be seen that the distribution and the shape of isotherms show good agreements.

3.5 Natural convection in a square cavity with 16 scatted solid blocks

The five cases given above belong to solid-solid conjugate heat transfer. The following cases are used to validate the present method for solid-fluid conjugate heat transfer. As shown in Fig. 10, in a large square cavity, there are 16 small square blocks uniformly distributed. The top and bottom walls of the cavity are thermally insulated, while the left and right walls are kept at constant temperatures of T_h and T_c ($T_h > T_c$) respectively. Therefore, natural convection will occur in the square cavity due to the buoyancy force. This model can be taken as a simplified model for natural convection in porous media. Here, the height of the square cavity is H , while the heights of those

small blocks are $0.15H$, the distance from a wall to the nearest block is $0.05H$, and the distance between two adjacent blocks is $0.1H$. No slip boundary condition is given for all solid walls of the cavity and the blocks.

In present work, a direct-forcing pressure-based LBM [17] is used to solve solid-fluid interaction flow problem, and the virtual heat capacity correction method is used to calculate solid-fluid conjugate heat transfer problem.

Fig.10 Nature convection in a square cavity filled with 16 solid blocks

The dimensionless parameter Rayleigh number is defined as $Ra = g\beta(T_h - T_c)H^3 / \nu\alpha_f$, where g and β are gravitational acceleration and volumetric thermal expansion coefficient of the fluid, respectively. ν and α_f are, respectively, the kinematic viscosity and thermal diffusivity of fluid phase. Prandtl number is defined as $Pr = \nu / \alpha_f$. The parameters of Ra and Pr are given by, respectively, 10^5 and 1.0 for all cases in this section. The variable parameters are k_s / k_f and $(\rho c)_s / (\rho c)_f$, and the value of $(\rho c)_s / (\rho c)_f$ is confined to be greater than or equal to 1.0 to ensure the numerical convergence.

To prove the correctness of the present method for solid-fluid conjugate heat transfer problem, the average Nusselt numbers of the left boundary ($\overline{Nu}$) are compared

with those of references . As shown in Table 2, the calculated $\overline{Nu}$ using the present method agree well with those of the previous numerical simulations [26,31,32]. However, the $\overline{Nu}$ obtained by the conventional LBM do not change with k_s/k_f when α_s/α_f is constant, which are obviously incorrect.

Table 2

Comparison of the average Nusselt number, $Ra = 10^5$, $Pr=1.0$, $\alpha_s/\alpha_f = 1.0$

k_s/k_f	[34]	[35]	[26]	Present LBM method	Conventional LBM method
1	1.233	1.193	1.185	1.2503	1.2503
10	2.030	2.066	2.031	2.0507	1.2503
100	2.313	2.394	2.4506	2.3357	1.2503

Table 3 shows the average Nusselt numbers $\overline{Nu}$ for different values of $(\rho c)_s/(\rho c)_f$ with the parameters of $Ra = 10^5$, $Pr = \nu/\alpha_f = 1$, and $k_s/k_f = 100$. The average Nusselt number $\overline{Nu}$ has little change with the value of $(\rho c)_s/(\rho c)_f$ increasing, which is believed reasonable because the stable temperature fields are only related to the change of k_s/k_f while Ra and Pr are constant. This regularity has also been observed by Karani and Huber [26].

Table 3

Comparison of the average Nusselt numbers at $Ra = 10^5$, $Pr = \nu/\alpha_f = 1$, and

$$k_s/k_f = 100$$

$(\rho c)_s/(\rho c)_f$	1	10	100
$\overline{Nu}$	2.3343	2.3352	2.3357

It can be concluded that the present method is also competent to simulate solid-fluid conjugate heat transfer with different ρc , however, a confinement condition of $(\rho c)_s/(\rho c)_f \geq 1$ is needed for the stability or the convergence of the output, which will be eliminated in the future work.

At last, to visualize the nature convection in cavity, streaming field and temperature field at a condition of $Ra = 10^5$, $Pr = 1.0$, $k_s/k_f = 100$ and $\alpha_s/\alpha_f = 1.0$ are displayed in Fig.11.

Fig.11 Streamlines (left) and isotherms (right) under a condition of $Ra = 10^5$, $Pr = 1.0$,

$$k_s/k_f = 100 \text{ and } \alpha_s/\alpha_f = 1.0$$

3.6 Natural convection in a square cavity containing 16 circular solid blocks

Another nature convection problem with solid-fluid conjugate heat transfer is investigated in this part. We consider a square cavity containing 16 circular solid blocks as depicted in Fig. 12. The bottom and top walls are adiabatic. The left wall is kept at a uniform hot temperature T_h while the right wall is kept at a uniform cold temperature

T_c ($T_h > T_c$). Inside the cavity, 16 circular solid blocks are distributed uniformly, the rest space in the cavity is filled up with fluid. Therefore, the porosity of the cavity ϕ is related to the diameter of every circular block (d_p) in a formula of $\phi = 1 - 16\pi d_p^2 / 4H^2$, and $\phi = 0.64$ is taken in this section. The two important parameters, Ra and Pr , are given by 10^5 and 0.7 , respectively, and the definitions are identical to that in section 3.5. Here, the thermal diffusivities of solid and fluid are confined at $\alpha_s = \alpha_f = 1.0$, the variable parameter is the thermal conductivity ratio of k_s/k_f only.

Fig.12 Nature convection in a square cavity containing 16 circular solid blocks

Similar data analysis to the last section, the average Nusselt numbers of the left wall ($\bar{Nu}$) are compared with the previous numerical results [36] obtained by LBM source formulation and finite volume method (FVM). The results calculated by the present method and Hu et al [36] are listed in Table 4, it can be seen obviously that $\bar{Nu}$ increases with thermal conductivity ratio k_s/k_f increasing, and the results show a good agreement with a maximum deviation less than 4.49%. However, the conventional LBM can get not right results when $(\rho c)_s \neq (\rho c)_f$ as well.

Finally, visualizations of streaming field and temperature field under the condition of $Ra = 10^5$, $Pr = 0.7$, $k_s/k_f = 100$, $\alpha_s/\alpha_f = 1.0$ are given in Fig.13.

Table 4

Comparison of the average Nusselt number, at $Ra = 10^5$, $Pr = 0.7$, $\alpha_s/\alpha_f = 1.0$

k_s/k_f	LBM by Hu et al [36]	FVM by Hu et al [36]	Present LBM method	Conventional LBM method
1	1.2614	1.2986	1.2403	1.2403
10	2.0429	2.0375	2.0153	1.2403
100	2.2957	2.2656	2.3357	1.2403

Fig.13 Streamlines (left) and isotherms (right) under a condition of $Ra = 10^5$, $Pr = 0.7$,

$$k_s/k_f = 100 \text{ and } \alpha_s/\alpha_f = 1.0$$

4. Conclusions

The virtual heat capacity correction of lattice Boltzmann algorithm is introduced in this paper to solve the solid-fluid and the solid-solid conjugate heat transfer problems. This method can simulate conjugate heat transfer correctly when the solid has different ρc with that of fluid while the conventional LBM method can not get the right results.

Seven test cases are raised and the results show that the present method has a very good performance in simulating solid-fluid and solid-solid conjugate heat transfer at both transient and steady states. The advantages of the present method can be concluded as follows: (1) this method is easy in implementation, the correction procedures after conventional calculating steps are very simple; (2) it can deal with the curved interface easily, and does not need to find the normal direction of an interface; (3) it can also be used for simulating transient conjugate problems; (4) it can be expanded to three-dimensional conjugate heat transfer naturally.

Unfortunately, the present method can only ensure the convergence at a condition of $(\rho c)_s/(\rho c)_f \geq 1$ for solid-fluid conjugate heat transfer problem that limits its application, which will be eliminated in the future work.

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