The mechanism of heat transfer enhancement using longitudinal vortex generators in a laminar
channel flow with uniform wall temperature

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Abstract

Heat transfer enhancements are used extensively to improve the performance of heat exchangers and to reduce their running cost. Longitudinal vortices can enhance heat transfer greatly. To enrich the mechanism of heat transfer enhancement produced by longitudinal vortices, this paper addresses the mechanisms of heat transfer enhancement enforced by longitudinal vortices generated by vortex generators (VGs) from another view point. The view point is based on the convective transport equation of heat flux, a description of convective heat transfer process in the process parameter of heat flux $\mathbf{q}$. Based on this view point, the heat transfer enhancement mechanism is explained by the contributions of the combinations of flux with velocity gradient and velocity with flux gradient to the heat transfer. To find these contributions the heat transfer in a channel with and without VGs is studied numerically, and the differences of these contributions under conditions with and without VGs are compared in the frame of the convective transport equation of heat flux. The results show that longitudinal vortices greatly enhance the local convection contribution terms that promote the local transport of heat flux in span direction, which enforces the transports of heat flux along the main flow direction and heat flux along the normal direction by enlargements of the values of the corresponding convection contribution terms. The later results in the increase of the heat flux per unit temperature difference on the top and the bottom surfaces, and hence the heat transfer enhancement is enforced by longitudinal vortices. The local terms intensify the local transport of heat flux in span direction, but do not do so in an average measure in the presently studied case.

Keyword: convection; heat transfer enhancement; longitudinal vortices; mechanism; vortex generator.

Nomenclature

a	height of rectangular winglet, m
A	area of heat transfer, m^2
b	base length of winglet, m
B	width of the channel, m
c_p	specific heat capacity, kJ/kgK
d_e	hydraulic diameter of flow channel, m
f	friction factor: $f=\Delta p d_e/(L_x \rho u_{\max}^2/2)$
g	height of vortex generator
h	convective heat transfer coefficient, W/m^2K
H	height of channel, m
J	heat transfer enhancement factor
l	span length of the rectangular winglet, m
L	length of the channel, m
n	normal direction
Nu	Nusselt number: $Nu=hd_h/\lambda$
p	static pressure, Pa
q	component of heat flux, W/m^2
$\mathbf{q}$	heat flux vector, W/m^2
Re	Reynolds number: $Re=\rho \cdot u_{\max} \cdot d_e/\mu$
S	distance between the tips of rectangular winglet, m
SW	span average convection contribution term, W/m^2s
T	temperature, K
t	time, s
u, v, w	components of velocity vector, m/s
W	convection contribution term, W/m^2s
X_v	axial distance from the entrance to the wing tip, m
x, y, z	coordinates, m
Special characters	
ρ	density, kg/m^3

μ	Viscosity, kg/(ms)
λ	thermal conductivity, W/mK
Δ	difference value
β	attack angle of vortex generator, °
ξ, η, ζ	coordinates, m
Subscripts	
a	air
bulk	cross-section averaged value
$c-x, c-y, c-z$	related to velocity
$e-x, e-y, e-z$	related to velocity gradient
f	fluid
in	inlet
local	local value
m	average value
plain	channel of without VGs
s	span average value
w	wall
x, y, z	along the coordinate directions

1. Introduction

Heat exchangers are widely used in many fields. It is very important to reduce their energy and material consumption, and to improve their efficiency. To design heat exchangers with good performance, many methods of heat transfer enhancement are used. Thus, the investigations on heat transfer enhancement have been intensified. Heat transfer enhancement with small energy consumption may be achieved by generating structured secondary flow via longitudinal vortices [1-45]. This method has received more attention because it introduces relatively small friction loss of fluid flow with enhanced heat transfer [1-45]. The reason is that the wall friction loss depends mainly on the wall normal gradient of the velocity component along the main flow direction, and does not depend on the gradient of the velocity components formed by secondary flow, so this method has been attributed to the heat transfer enhancement methods of the third generation [11, 12 and 16].

The effects of a single longitudinal vortex embedded in one side of the wall in the turbulent boundary layer on heat transfer were experimentally studied by Eibeck and Eaton [3, 8]. Pauley and Eaton [7] studied the effects of a longitudinal vortex pair embedded in one side of the wall in a turbulent boundary layer on heat transfer characteristics. The results show that the vortex pair decays slower in a longer distance than a single vortex does, which has a great impact on heat transfer. Russell [4] carried out an experimental study of a pair of embedded vortex generators (VGs) on the channel and compared their heat transfer and pressure drop characteristics. It is found that a pair of embedded vortex generators is a superior method to enhance heat transfer. The capacity of heat transfer enhancement of VGs depends on their shape [10, 19, 22, 31, 34, 37, 44] and other parameters such as the angle of attack [1-20, 23, 24, 27, 28, 32, 33, 36, 39, 40], the height of VGs [1-20, 23, 24, 27, 28, 32, 36, 39, 40], the arrangement of VGs [21, 25, 35, 38], the

ratio of the size of VGs to the size of the channel [26] and the manufacturing type of VGs (VGs are mounted or punched) [26, 27]. According to the reports of Tiggelbeck et al. [10], the best performance is given by the delta winglets, closely followed by the rectangular winglets. For the flat tube bank fin heat exchanger, the optimum attack angle of VGs is 40° [45], and the optimum height of VGs is 0.8 times the net fin spacing [20]. Didarul [25] found that heat transfer with rectangular VGs of jagged arrangement is better than that of the same angle arrangement. Wang et al. [26] found that heat transfer is enhanced significantly when the longitudinal vortex generators are mounted in the narrow channel, Wu et al. [27] studied heat transfer enhancement by punching rectangular winglets and triangular winglets of rectangular channel in laminar fluid flow, the results show that strong heat transfer enhancement and low coefficient of flow friction in the zone near the winglet is affected by punching holes in the wall. Wang et al. [29] found that mounted VGs have no obvious difference with punched VGs.

The longitudinal vortices can be classified as secondary flow, which is the flow on the cross section normal to the main flow. Recently, our group found that the intensity of secondary flow determines the heat transfer characteristics of the fin side of tube bank fin heat exchanger having VGs on the fin surface [30, 41-44]. A dimensionless parameter of the intensity of secondary flow is given by Song et al. [42]. This literature indicates that in many cases of using VGs as the measure to enhance heat transfer, the intensity of secondary flow determines the heat transfer characteristics.

The above mentioned literature is only a small part of the literature that addresses the heat transfer enhancement using VGs. This literature mainly presents the characteristics of the heat transfer enhancement enforced by VGs, and VGs' parametric effect on heat transfer enhancement. Above mentioned literature demonstrates that longitudinal vortices can enhance the momentum

and heat exchanges between vortices and the boundary layer, produce strong flow instability, and also increase the mixture of fluid in different temperatures. These are the presently acknowledged heat transfer enhancement mechanisms produced by longitudinal vortices. Unfortunately, these acknowledged mechanisms are too vague to be discovered in detail. Until now, they have not been strictly proven by some suitable terms in available equations describing convective heat transfer. Motivated by this fact, the present paper tries from another view point to address the mechanisms of heat transfer enhancement enforced by longitudinal vortices. The view point is based on the recent works done by our group [46-51], which have reported a description of the convective heat transfer process in the process parameter of $\mathbf{q}$: convective transport equation of heat flux. This equation shows that the velocity gradient and the velocity have different contributions to convective heat transfer, and also these contributions are dependent on the heat flux transferred and its gradient itself. To find these contributions, the heat transfer in a channel with and without VGs is studied numerically in this paper, and the differences of these contributions under conditions with and without VGs are compared in the frame of the convective transport equation of heat flux. Based on these differences the mechanism of heat transfer enhancement enforced by longitudinal vortices is disclosed.

2. Physical model and mathematic formulation

Regarding the motivation of this paper, it does not try to repeat the studies performed by available literature, such as studying the parameter effects of VGs on heat transfer characteristics. Instead, this paper tries to compare the differences of heat transfer under conditions with and without VGs in the frame of the convective transport equation of heat flux. With this consideration, in order not to bring other uncertainties (vortices produced by factors other than VGs), we choose the structure according to Ref. [10]. The structure of four vortex generators is experimentally studied in Ref. [10]. According to the viewpoint of Ref. [10], the results show that

winglets perform better than wings. The studied model consists of the two plates and many rectangular winglet pairs located at one plate as shown in Fig. 1. The computational model configurations are $H=4$ mm, $B=20$ mm, and $L=120$ mm. The leading point of VGs is located at a distance $X_v = H$ from the inlet of the channel, the distance between the VGs is $S=0.2H$, the base length of VGs is $l=3H$, the height of VGs is $h=H/2$, the angle of attack β is 30 deg. The inlet air temperature is $T_a=42^\circ\text{C}$, the wall temperature of the channel is $T_w=30.8^\circ\text{C}$.

Figure 1 Configurations of the studied channel: (a) Channel with array of VGs; (b) computational domain.

To build up the mathematical model, the following assumptions are made. (1) The fluid has constant properties. (2) Fluid flow is a steady laminar flow. (3) Viscous dissipation and the body force are not considered. Based on these assumptions the compact forms of the governing equations in physical space are as follows.

The continuity equation

$$\frac{\partial}{\partial x_i}(\rho u_i) = 0 \quad (1)$$

The momentum equation

$$\frac{\partial}{\partial x_i}(\rho u_i u_k) = \frac{\partial}{\partial x_i}(\mu \frac{\partial u_k}{\partial x_i}) - \frac{\partial p}{\partial x_k} \quad (k = 1, 2, 3) \quad (2)$$

The energy equation

$$\frac{\partial}{\partial x_i}(\rho c_p u_i T) = \frac{\partial}{\partial x_i}(\lambda \frac{\partial T}{\partial x_i}) \quad (3)$$

On the inlet cross section of fluid flow:

$$u(x, y, z) = u_{in}, v(x, y, z) = 0, w(x, y, z) = 0, T(x, y, z) = T_{in} \quad (4)$$

On the outlet cross section of fluid flow:

$$\frac{\partial u(x, y, z)}{\partial x} = 0, \frac{\partial v(x, y, z)}{\partial x} = 0, \frac{\partial w(x, y, z)}{\partial x} = 0, \frac{\partial T(x, y, z)}{\partial x} = 0 \quad (5)$$

On the symmetric surfaces:

$$\frac{\partial u(x, y, z)}{\partial n} = 0, v(x, y, z) = 0, \frac{\partial w(x, y, z)}{\partial n} = 0, \frac{\partial T(x, y, z)}{\partial n} = 0 \quad (6)$$

On the fin surface:

$$u(x, y, z) = 0, v(x, y, z) = 0, w(x, y, z) = 0, T = T_w \quad (7)$$

The characteristic length is

$$d_e = 2H \quad (8)$$

The Reynolds number and the friction factor are defined as

$$Re = \rho u_{in} d_e / \mu \quad (9)$$

$$f = \Delta p d_e / (L \rho u_{in}^2 / 2) \quad (10)$$

The local convective heat transfer coefficient is determined by

$$h_{local} = -\lambda \frac{\partial T}{\partial n} / (T_{in} - T_w) \quad (11)$$

where T_c is the characteristic temperature of the fin side fluid, and it is defined as

$$T_c = T_{in} / \left(\int_0^1 \int_0^1 (T(x, y) - T_w) dx dy / \int_0^1 \int_0^1 dx dy \right) \quad (12)$$

The span average heat transfer coefficient is obtained through

$$h_s = \int_B h_{local} dy / B \quad (13)$$

The average heat transfer coefficient is obtained through

$$h_m = \iint_A h_{local} dA / A \quad (14)$$

where A is the total area of the heat transfer surface.

The local and average Nu are determined by

$$Nu_{local} = h_{local} d_e / \lambda, \quad Nu_s = h_s d_e / \lambda, \quad Nu_m = h_m d_e / \lambda \quad (15)$$

3. Numerical method

The simulation domain in the physical space (x - y - z coordinates) is transformed into a rectangular parallelepiped in the computational space (ξ - η - ζ coordinates) using the transfinite interpolation algebraic method. In the computational space, the numerically solved equation is shown in Eq. (A1). The governing equations in the form of Eq. (A1) are discretized on the collocation grid system based on using the finite volume method [52]. The power scheme is used in discretizing the convective terms, and the second order central difference scheme is used in discretizing the diffusion terms. SIMPLE algorithm [52] is used to couple the velocity and pressure. The treatment of the vortex generators and the detailed verification of the numerical method are given by Lin [53], which will not be repeated here.

Figure 2 Schematic view of the grid system used in simulation

A grid system of $154 \times 94 \times 40$ is adopted in the computation domain. Moreover, a serious verification for the grid independence of the numerical solution has been made to ensure the accuracy of the numerical results. As indicated in Table 1, the influence of increasing the grid size or reducing the grid size on the numerical results is very small. The maximum difference of Nu and f between the three grid systems is less than 1.0%.

Table 1 The result dependence on the grid size

Grid size($x \times y \times z$)	Nu	f
$107 \times 80 \times 32$	8.108	0.4686
$154 \times 94 \times 40$	8.108	0.4997
$161 \times 104 \times 44$	8.108	0.4964

In addition, we carefully checked the accuracy of our code to ensure its correctness. The manuscript model is simulated using Fluent software (ansys15.0). The results are shown in Table 2, Nu number is 8.11 calculated by our own code, and Nu number is 8.22 calculated by Fluent software (ansys15.0), with an error rate of 1.4%. At the same time, the model of Ref. [53] is also simulated using Fluent software (ansys15.0), the results show that the error is 0.8%. These indicate that our code is accurate.

Table 2 Comparison of Nu numbers obtained by our code and Fluent software (ansys15.0)

model	Nu	our code	Fluent software	error
manuscript	Nu	8.11	8.22	1.4%
Ref. [53]	Nu	15.31	15.43	0.8%

4. The convective transport equation of heat flux

Using the model described by Eqs. (1)-(7) the fluid flow field and the temperature field can be obtained. Then the convective heat transfer characteristics can also be obtained. The main aim of this paper is not to obtain the convective heat transfer characteristics, but to find explanations for the behavior of the obtained convective heat transfer characteristics. To achieve this aim further analysis should be carried out. A deeper analysis of convective heat transfer should use the convective transport equation of heat flux, which is discussed in [46-51]. Here, we repeated it again.

The convective transfer equation of the heat flux can be written as follows.

$$\frac{\partial \mathbf{q}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{q} + (\nabla \mathbf{v}) \cdot \mathbf{q} = \alpha \nabla^2 \mathbf{q} \quad (16)$$

This can be written as

$$\frac{\partial \mathbf{q}}{\partial t} + \mathbf{W}_c + \mathbf{W}_e = \alpha \nabla^2 \mathbf{q} \quad (17)$$

For a steady state case

$$\mathbf{W}_c + \mathbf{W}_e = \alpha \nabla^2 \mathbf{q} \quad (18)$$

where

$$\mathbf{W}_c = (\mathbf{v} \cdot \nabla) \mathbf{q}, \quad \mathbf{W}_e = (\nabla \mathbf{v}) \cdot \mathbf{q} \quad (19)$$

$$S\mathbf{q} / \Delta T = \int_0^B \frac{\mathbf{q}}{B\Delta T} dy, \quad S\mathbf{W}_e / \Delta T = \int_0^B \frac{(\nabla \mathbf{v}) \cdot \mathbf{q}}{B\Delta T} dy, \quad S\mathbf{W}_c / \Delta T = \int_0^B \frac{(\mathbf{v} \cdot \nabla) \mathbf{q}}{B\Delta T} dy \quad (20)$$

$$S(\mathbf{W}_c + \mathbf{W}_e) / \Delta T = (S\mathbf{W}_c + S\mathbf{W}_e) / \Delta T \quad (21)$$

The component forms of Eqs.(16-21) are shown in the Appendix. W_{c-x} , W_{c-y} and W_{c-z} are the contribution of the velocity to the transport of q_x , q_y and q_z , respectively, and W_{e-x} , W_{e-y} and W_{e-z} are the contribution of the velocity gradient to the transport of q_x , q_y and q_z , respectively. To compare the two convective heat transfer cases in the same frame, it is necessary to normalize W_{c-x} , W_{c-y} , W_{c-z} , W_{e-x} , W_{e-y} , W_{e-z} , q_x , q_y and q_z using temperature difference, $\Delta T = T_w - T_c$, thus, these parameters will be $W_{c-x}/\Delta T$, $W_{c-y}/\Delta T$, $W_{c-z}/\Delta T$, $W_{e-x}/\Delta T$, $W_{e-y}/\Delta T$, $W_{e-z}/\Delta T$, $q_x/\Delta T$, $q_y/\Delta T$ and $q_z/\Delta T$. For these local parameters and their y-direction average values, $SW_{c-x}/\Delta T$, $SW_{c-y}/\Delta T$, $SW_{c-z}/\Delta T$, $SW_{e-x}/\Delta T$, $SW_{e-y}/\Delta T$, $SW_{e-z}/\Delta T$, $Sq_x/\Delta T$, $Sq_y/\Delta T$ and $Sq_z/\Delta T$ are defined in the Appendix.

After defining these parameters, the present study should compare the difference of these parameters for the case with VGs and without VGs, and then try to find the mechanisms of convective heat transfer enhancement enforced by VGs. In the following sections, the heat transfer enhancement by VGs is presented in the beginning, and then the reason why VGs can enhance convective heat transfer is discussed. The present study is not attempting to find out the parameter's effect on heat transfer enhancement. Thus, in the following sections, the

configuration and arrangement of VGs are not changed, and at the same time, the working condition is also the same, $Re=200$.

5. Results and discussions

5.1 The heat transfer characteristics of the cases with and without VGs

5.1.1 The Nu_{local} characteristics

Figure 3 shows the distribution of Nu_{local} on the wall surfaces of the channels with VGs and without VGs on the bottom wall. It is found from Fig 3(a) that Nu_{local} has no change along the y direction, Nu_{local} decreases greatly along the x direction, and the distributions of Nu_{local} on the top and bottom surfaces are the same for the case without VGs. As shown in Figs. 3(b), (c), on both surfaces, the distributions of Nu_{local} have been changed greatly along the y direction for the case with VGs. On the bottom surface there are two strips around VGs in which Nu_{local} is small. Nu_{local} is larger in a comparably large area that is located downstream of VGs and is enveloped partially by VGs. On the top surface, around the line $y=B/2$ there are two strips in which Nu_{local} is small, and near $y=0$ and $y=B$ there are two regions in triangular shape in which Nu_{local} is large.

Figure 3 The distributions of Nu_{local} on the top and the bottom surfaces: (b) the case with VGs on the bottom surface; (c) the case with VGs on the top surface.

On the top surface, as shown in Fig. 4(a), for the case with VGs, along the line of $y=B/6$, Nu_{local} is larger than that for the case without VGs in the region of $0.004\text{ m} < x < 0.08\text{ m}$. Along the

line $y=B/3$, Nu_{local} of the case with VGs is larger than that of the case without VGs in the region $0.004\text{ m} < x < 0.035\text{ m}$. As $x > 0.035\text{ m}$, Nu_{local} of the case with VGs is smaller than that of the case without VGs. Along the line $y=1/2B$, Nu_{local} of the case with VGs is smaller than that of the case without VGs as $0.006\text{ m} < x < 0.028\text{ m}$, but it is larger than Nu_{local} of the case without VGs as $x \geq 0.028\text{ m}$.

On the bottom surface, as shown in Fig. 4(b), Nu_{local} of the case with VGs along the line of $y=B/6$ is smaller than that of the case without VGs in the region of $x < 0.07\text{ m}$, in the other region, Nu_{local} is nearly the same for both cases. Along the line of $y=B/3$, Nu_{local} of the case with VGs is smaller than that of the case without VGs in the region of $0.003\text{ m} < x < 0.02\text{ m}$, but Nu_{local} of the case with VGs is slightly smaller than that of the case without VGs in the region of $0.02\text{ m} < x < 0.11\text{ m}$. Along the line of $y=B/2$, Nu_{local} of the case with VGs is much larger than that of the case without VGs in the region of $x > 0.003\text{ m}$.

To clearly indicate the heat transfer enhancement caused by VGs, Figs. 4 (c) and (d) present the distributions of $Nu_{local}/Nu_{local, plain}$ on lines of $y=B/6$, $y=B/3$ and $y=B/2$ on the top and bottom surfaces. Where, $Nu_{local, plain}$ represents the local Nusselt number of the case without VGs.

On the top surface, as shown in Fig.4 (c), along the line of $y=B/6$, $Nu_{local}/Nu_{local, plain}$ increases initially to the maximum value of about 1.48 (at $x=0.018\text{ m}$) and then decreases along the main stream direction, it is greater than 1.0. Along the line $y=B/3$, $Nu_{local}/Nu_{local, plain}$ increases firstly to the maximum value of about 1.53 (at $x=0.015\text{ m}$) and then decreases along the main stream direction. It is less than 1 as $x > 0.04\text{ m}$. Along the line $y=B/2$, $Nu_{local}/Nu_{local, plain}$ increases at first to the maximum value of about 1.03 (at $x=0.004\text{ m}$) and then decreases, it reaches its minimal value of 0.81 (at $x=0.017$) , and then increases along the main stream. It is greater than 1 as $x > 0.035\text{ m}$.

Figure 4 The distributions of Nu_{local} and $Nu_{local}/Nu_{local, plain}$ on the intersection lines by the planes of $y = B/6$, $y=B/3$, $y=B/2$ and the planes of $z=0$ and $z=H$: (a) Nu_{local} on the top surface; (b) Nu_{local} on the bottom surface; (c) $Nu_{local}/Nu_{local, plain}$ on the top surface; (d) $Nu_{local}/Nu_{local, plain}$ on the bottom surface.

On the bottom surface, Fig.4 (d) indicates that along the line of $y=B/6$, $Nu_{local}/Nu_{local, plain}$ decreases initially to the minimal value of about 0.70 (at $x=0.02$ m) and then increases to the maximum value along the main stream direction, it reaches 1 near the outlet of the channels. Along the line $y=B/3$, $Nu_{local}/Nu_{local, plain}$ initially decreases to the minimal value of about 0.6 (at $x=0.015$ m) and then increases to the maximum value of about 1.04 (at $x=0.02$ m). It further decreases along the main stream direction, reaching 0.96 near the outlet of the channel. Along the line $y=B/2$, $Nu_{local}/Nu_{local, plain}$ initially increases to the maximum value of about 1.58 (at $x=0.018$ m) and then decreases to the minimum value of about 1.04 (at $x=0.04$ m).

m) and then decreases along the main stream direction. Even at the outlet region, it still has a value of about 1.1.

The heat transfer enhancement by longitudinal vortices shown in Fig.3 and Fig.4 is consistent. Nusselt number is significantly changed by VGs, and Nusselt number on the bottom surface mounted VGs changes more greatly than on the top surface. Fig. 4 effectively reflects this change.

5.1.2 The difference of the span average Nusselt number

The span average Nusselt number, Nu_s , is shown in Fig.5 (a). It is found that VGs increase Nu_s on the top surface in the region of $x < 0.04$ m, and increase Nu_s on the bottom surface in the region of $0.007 \text{ m} < x < 0.05$ m. On the bottom surface in the region of $x < 0.007$ m Nu_s of the case with VGs is smaller than Nu_s of the case without VGs. In the region $0.015 \text{ m} < x < 0.05$ m, Nu_s on the bottom surface is larger than Nu_s on the top surface. Downstream Nu_s is nearly the same for the cases with and without VGs. Nu_s averaged over the top and the bottom surfaces is also shown in Fig.5, which lies between Nu_s of the top and the bottom surfaces.

Figure 5(b) presents the distributions of $Nu_s/Nu_{s,\text{plain}}$ averaged on the top surface, on the bottom surface, and on both surfaces. Where $Nu_{s,\text{plain}}$ is the span averaged Nusselt number of the case without VGs. $Nu_s/Nu_{s,\text{plain}}$ of the bottom surface has a slight increase near upstream VGs. near VGs, it decreases initially to its minimum value of about 0.93, then increases to its maximum value of about 1.23. It further decreases along the main stream direction, as $x > 0.07$ m, reaching 1. $Nu_s/Nu_{s,\text{plain}}$ of the top surface initially increases to its maximum value of 1.2 (at $x = 0.012$ m), then decreases to 1 at $x = 0.07$ m. $Nu_s/Nu_{s,\text{plain}}$ of both surfaces initially increases, then decreases slightly, it increases again to its maximum value of 1.19 (at $x = 0.013$ m), then further decreases to 1 at $x = 0.07$ m.

Figure 5 The distribution of Nu_s and $Nu_s/Nu_{s,plain}$ along the x direction: (a) Nu_s ; (b) $Nu_s/Nu_{s,plain}$.

5.1.3 Longitudinal vortices

The velocity fields on the cross section of the channel are presented in Fig. 6. These series of the velocity fields clearly give the developing of vortices in the channel. Figs. 6(a)-(d) show secondary flow on the cross section at $x=0.015$ m, 0.019 m, 0.024 m and 0.032 m positions in the channel without VGs, where secondary flow is very weak. Figs. 6(e)-(h) show secondary flow on the cross section at the position $x=0.015$ m, $x=0.019$ m, $x=0.024$ m and 0.032 m in the channel with VGs, where the left vortices are rotated clockwise, and the right vortices are rotated counterclockwise. Comparing Figs. 6(e), 6(f), 6(g) and 6(h), it is found that the intensity of longitudinal vortices decreases along the main flow direction.

Figure 6 Velocity fields on the sections for the cases with VGs and without VGs: (a) and (e) $x=0.015$ m; (b) and (f) $x=0.019$ m; (c) and (g) $x=0.024$ m; (d) and (h) $x=0.032$ m; (a)-(d) the case without VGs; (e)-(h) the case with VGs.

The vorticity fields on different cross sections of the channel are presented in Fig.7, which shows the vorticity decay process along the main stream direction. These series of the vorticity fields are consistent with the distributions of Nu_s or $Nu_s/Nu_{s,plain}$ along the x direction shown in Fig. 5. It shows that the heat transfer enhancement is strong when the value of vorticity is large, and the heat transfer enhancement is weak when the value of vorticity is small.

Figure 7 Vorticity fields on the sections for the cases with VGs: (a) $x=0.015$ m; (b) $x=0.019$ m; (c) $x=0.024$ m; (d) $x=0.032$ m.

The three-dimensional stream traces are shown in Fig. 8. VGs produce rotation while the fluid is in forward motion, so that the flow has velocity components in the y -direction and z -direction. The fluid in cold and heat regions has more chance to mix, and then more heat can be transferred away from the bottom and the top surfaces. This process enhances the heat transfer.

Figure 8 The stream traces in 3-D

5.1.4 The difference of the main flow velocity distribution on the $y=B/3$ and $y=B/2$ planes

Figure 9 shows a comparison of the velocity fields on $y=B/3$ and $y=B/2$ plane of the channel with VGs and without VGs. When Figs. 9(a) and (c) are compared, the boundary layer characteristics are changed by longitudinal vortices on the plane $y=B/3$. As shown in Fig. 9(b), the velocity field is nearly the same as shown in Fig. 9(d), this is because longitudinal vortices do not greatly change the boundary layer characteristics on $y=B/2$ plane, which indicates that the change of the convective heat transfer characteristics enforced by the change of boundary layer characteristics is weak.

Figure 9 Comparison of the velocity fields in the channels with and without VGs: (a) and (c) $y=B/3$; (b) and (d) $y=B/2$; (c) and (d) the case with VGs.

5.2 The contributions of the velocity and the velocity gradient to the transport of $\mathbf{q}$

5.2.1 The local contributions of the velocity and the velocity gradient to the transport of $\mathbf{q}$

To specify clearly the mechanism of the heat transfer enhancement enforced by VGs, the contributions of the velocity and the velocity gradient to the transport of $\mathbf{q}$ should be addressed. In order to facilitate presentation, we take four cross sections at $x_1=0.0168$ m, $x_2=0.0214$ m, $x_3=0.0265$ m, $x_4=0.0321$ m. Here, the distributions of $W_{e-x}/\Delta T$, $W_{c-x}/\Delta T$, $(W_{e-x}+W_{c-x})/\Delta T$ and $q_x/\Delta T$, $W_{e-y}/\Delta T$, $W_{c-y}/\Delta T$, $(W_{e-y}+W_{c-y})/\Delta T$ and $q_y/\Delta T$, $W_{e-z}/\Delta T$, $W_{c-z}/\Delta T$, $(W_{e-z}+W_{c-z})/\Delta T$ and $q_z/\Delta T$ on the special lines that are formed by the intersections of the planes $x=x_1$, $x=x_2$, $x=x_3$, $x=x_4$ and the plane $y_1=B/2$ are inspected.

As shown in Fig. 10(a), on the inspected lines, $W_{e-x}/\Delta T$ value is nearly zero for the case without VGs. While for the case with VGs, at different x positions, W_{e-x} value is negative near the bottom

surface and positive near the top surface, whereas $W_{e-x}/\Delta T$ changes greatly along z . As x position is farther away from VGs, $W_{e-x}/\Delta T$ decreases. $W_{e-x}/\Delta T$ is very close to the value of the case without VGs at $x=x_4$ position. This is mainly because $W_{e-x}/\Delta T$ indicates the velocity gradient contribution to the transport of q_x , the velocity gradient contribution becomes very small downstream. It is found that near the vortex generator position, the velocity gradient contribution to the transport of q_x is large, and the largest velocity gradient contribution occurs at $z=0.001$ m and $z=0.003$ m. This figure shows that the great difference between the cases with and without VGs is that $W_{e-x}/\Delta T$ is large for the case with VGs.

The contribution of the velocity to the transport of q_x is shown in Fig. 10 (b). For the case without VGs, at different x position, $W_{c-x}/\Delta T$ changes from zero on the wall surfaces to a maximal negative value at $z=0.002$ m, and it is slightly dependent on x position. For the case with VGs, the value of W_{c-x} changes greatly compared to the case without VGs, and it is greatly dependent on x position. At $x=x_1$ position, starting from the top surface $W_{c-x}/\Delta T$ decreases greatly, and reaches its minimal value at $z=0.0025$ m, then increases to its maximal value at $z=0.0012$ m, and further decreases to zero value at the bottom surface. At $x=x_2$, $x=x_3$, and $x=x_4$ positions, starting from the top surface $W_{c-x}/\Delta T$ increases slightly, and reaches its maximal value at $z=0.0025$ m, then decreases to its maximal negative value at $z=0.001$ m, 0.00125 m and 0.0015 m for $x=x_2$, $x=x_3$, and $x=x_4$ positions, respectively. It further increases to zero value at the bottom surface. This figure shows that longitudinal vortices change the velocity contribution to the transport of q_x .

Figure 10(c) shows the combined contributions of the velocity gradient and the velocity to the transport of q_x . It is found that $(W_{e-x}+W_{c-x})/\Delta T$ for the case with VGs has great difference with $(W_{e-x}+W_{c-x})/\Delta T$ for the case without VGs. $(W_{e-x}+W_{c-x})/\Delta T$ has negative value in a large region, and

decreases as x increases. This figure indicates that longitudinal vortices strongly change the combined contributions of the velocity gradient and the velocity to the transport of q_x .

The heat flux $q_x/\Delta T$ is shown in Fig. 10 (d). For the case without VGs $q_x/\Delta T$ is zero on the top and the bottom surfaces, it increases from zero to a maximal value when $z=0.002$ m, its maximal value is about $1.0 \text{ W/}^\circ\text{C}$, it is symmetrically distributed around $z=0.002$ m, and it is less dependent on x . For the case with VGs, $q_x/\Delta T$ is zero on the top and the bottom surfaces also, it increases from zero to a maximal value at certain z value that is dependent on x , its maximal value is about $1.35 \text{ W/}^\circ\text{C}$. It is found that at all positions inspected, when $z>0.00225$ m $q_x/\Delta T$ is less than the corresponding value of the case without VGs. These characteristics show clearly that longitudinal vortices enhance the process of the transport of q_x only in a certain region on the line inspected, and in the other region the process of the transport of q_x is deterred. This figure indicates that longitudinal vortices locally change the intensity of the transport of q_x . And at the same time, as indicated by Figs. 10(c) and 10(d), large negative values of $(W_{e-x}+W_{c-x})/\Delta T$ will promote the transport of q_x .

On the inspected lines, Fig. 10 shows that longitudinal vortices enforce the velocity gradient contribution on the transport of q_x in the region $z<0.0015$ m, and the velocity gradient has a negative effect on the transport of q_x in the other region; except at x_1 position, longitudinal vortices enhance the velocity contribution on the transport of q_x in the region $z<0.00175$ m, but deter the transport of q_x in the other region. The combined contribution of the velocity gradient and the velocity to the transport of q_x are positive only in the region $z<0.0022$ m on the inspected line.

Figure 10 The distributions of $W_{e-x}/\Delta T$, $W_{c-x}/\Delta T$, $(W_{e-x}+W_{c-x})/\Delta T$ and $q_x/\Delta T$ along the intersection lines of $y=y_1$ and $x=x_1, x_2, x_3, x_4$ in cases with VG and without VG: (a) $W_{e-x}/\Delta T$; (b) $W_{c-x}/\Delta T$; (c) $(W_{e-x}+W_{c-x})/\Delta T$; (d) $q_x/\Delta T$

As shown in Fig. 11(a), on the inspected line, $W_{e-y}/\Delta T$ is zero in the channel without VGs, it is independent of x , while in the channel with VGs, at different x position, $W_{e-y}/\Delta T$ has a wavy distribution along z , it is negative when $z < 0.0015$ m, $W_{e-y}/\Delta T$ is positive when $z > 0.0015$ m. And as x position is far away from VGs, $W_{e-y}/\Delta T$ decreases. Because $W_{e-y}/\Delta T$ indicates the velocity gradient contribution to the transport of q_y , it is found that for the case without VGs the velocity gradient has no contribution to the transport of q_y , longitudinal vortices enforce the velocity gradient contribution to the transport of q_y for the case with VGs.

The velocity contribution to the transport of $q_y/\Delta T$ is shown in Fig. 11 (b). For the case without VGs, at different x position, $W_{c-y}/\Delta T$ is zero. For the case with VGs, the value of $W_{c-y}/\Delta T$ changes greatly compared to the case without VGs, and it is strongly dependent on x position. At $x=x_1$ position, starting from the top surface $W_{c-y}/\Delta T$ increases, and reaches its maximal value at $z=0.0022$ m, then decreases to its minimal value at $z=0.0006$ m, and further increases to zero value at the bottom surface. At $x=x_2$, $x=x_3$, and $x=x_4$ positions, starting from the top surface, $W_{c-y}/\Delta T$ increases slightly, and reaches its maximal value at $z=0.0025$ m, 0.0023 m and 0.0022 m, then decreases to its maximal negative value at $z=0.0009$ m, 0.001 m and 0.0012 m for $x=x_2$, $x=x_3$, and $x=x_4$ positions, respectively. It further increases to zero value at the bottom surface. This figure shows that longitudinal vortices change the velocity contribution to the transport of q_y .

Figure 11 (c) shows the combined contributions of the velocity gradient and the velocity to the transport of q_y . It is found that $(W_{e-y}+W_{c-y})/\Delta T$ is zero for the case without VGs. For the case with VGs, $(W_{e-y}+W_{c-y})/\Delta T$ has a wavy distribution along z . On the inspected line in most regions $W_{e-y}/\Delta T$ and $W_{c-y}/\Delta T$ have the same sign, which means that the velocity gradient contribution and the velocity contribution are both positive for the transport of $q_y/\Delta T$. $(W_{e-y}+W_{c-y})/\Delta T$ decreases along the main stream direction. This figure indicates that longitudinal vortices strongly change the combined contribution of the velocity gradient and the velocity to the transport of q_y .

Figure 11 The distributions of $W_{e-y}/\Delta T$, $W_{c-y}/\Delta T$, $(W_{e-y}+W_{c-y})/\Delta T$ and $q_y/\Delta T$ along the intersection lines of $y=y_1$ and $x=x_1, x_2, x_3, x_4$ in the cases with VGs and without VGs: (a) $W_{e-y}/\Delta T$; (b) $W_{c-y}/\Delta T$; (c) $(W_{e-y}+W_{c-y})/\Delta T$; (d) $q_y/\Delta T$

On the inspected line, the heat flux q_y per unit temperature difference is shown in Fig. 11(d). For the case without VGs, $q_y/\Delta T$ is zero every where. For the case with VGs $q_y/\Delta T$ is zero on the top and the bottom surfaces, it increases from zero to a maximal value at $z=0.0006$ mm, then decreases to its minimal value at $z=0.00225$ m-0.00275 m that is dependent on x . These characteristics show clearly that longitudinal vortices enhance the process of the transport of q_y in all regions on the line inspected. This figure indicates that longitudinal vortices locally change the intensity of the transport of q_y .

On the inspected line, Fig.11 shows that VGs enforce the velocity gradient contribution and velocity contribution on the transport of q_y . The combined contribution of the velocity gradient and the velocity to the transport of q_y are positive on the inspected line.

As shown in Fig. 12(a), $W_{e-z}/\Delta T$ has a wavy distribution for the case without VGs, the amplitude of waviness is about 200, on the bottom and on the top surfaces $W_{e-z}/\Delta T$ is zero, and at $z=0.002$ m while $W_{e-z}/\Delta T$ is also zero. As for the case with VGs, at different x positions, $W_{e-z}/\Delta T$ value is larger than the corresponding value of the case without VGs when $z<0.00125$ m. In the region $0.00125 \text{ m}<z<0.002$ m, $W_{e-z}/\Delta T$ is smaller than the corresponding value of the case without VGs. Near the top surface ($z=0.004$ m), $|W_{e-z}/\Delta T|$ value is larger than the corresponding value of the case without VGs. at $x=x_1$ in the region $0.002 \text{ m}<z<0.00325$ m and at $x=x_2$ in the region $0.002 \text{ m}<z<0.003$ m, $W_{e-z}/\Delta T$ value is larger than the corresponding value of the case without VGs. It is found that near the vortex generator position, the velocity gradient contribution to the transport of $q_z/\Delta T$ is large, and the largest velocity gradient contribution occurs at $z=0.00075$ m as $x=x_1$. This figure shows that longitudinal vortices change $W_{e-z}/\Delta T$ value.

On the inspected lines, the contribution of the velocity to the transport of q_z is shown in Fig. 12(b). For the case without VGs, at different x position, $W_{c-z}/\Delta T$ changes from zero on the bottom wall surface ($z=0$) to a maximal value at $z=0.001$ m, decreases to a minimal value at $z=0.003$ m, and then increases to zero value at the top surface. $W_{c-z}/\Delta T$ is slightly dependent on x position. For the case with VGs the value of $W_{c-z}/\Delta T$ changes greatly comparing to the case without VGs, and it is greatly dependent on x position. At $x=x_1$ position, starting from the top surface $W_{c-z}/\Delta T$ decreases slightly, as $z<0.0025$ m decreases and reaches its minimal value at $z=0.0015$ m, then increases to its maximal value at $z=0.0005$ m, and further decreases to zero value at the bottom surface. At $x=x_2$, $x=x_3$, and $x=x_4$ positions, starting from the top surface W_{c-z} decreases, and

reaches its minimal value at $z=0.00175$ m, 0.002 m, 0.00225 m for $x=x_2$, $x=x_3$, and $x=x_4$ positions, respectively, then increases to its maximal value at $z=0.001$ m, 0.0009 m, 0.00075 m for $x=x_2$, $x=x_3$, and $x=x_4$ positions, respectively, it further increases to zero value at the bottom surface. This figure shows that longitudinal vortices change the velocity contribution to the transport of q_z . Longitudinal vortices make the amplitude of the wavy $W_{c-z}/\Delta T$ change from 250 to 550.

Figure 12 The distributions of $W_{e-z}/\Delta T$, $W_{c-z}/\Delta T$, $(W_{e-z}+W_{c-z})/\Delta T$ and $q_z/\Delta T$ along the intersection lines of $y=y_1$ and $x=x_1, x_2, x_3, x_4$ in the cases with VG and without VG: (a) $W_{e-z}/\Delta T$; (b) $W_{c-z}/\Delta T$; (c) $(W_{e-z}+W_{c-z})/\Delta T$; (d) $q_z/\Delta T$.

Figure 12(c) shows the combined contributions of the velocity gradient and the velocity to the transport of q_z . It is found that $(W_{e-z}+W_{c-z})/\Delta T$ for the case with VGs is of great difference with

$(W_{e-z}+W_{c-z})/\Delta T$ for the case without VGs. $(W_{e-z}+W_{c-z})/\Delta T$ has negative value in a large region, and decreases as x increases. $(W_{e-z}+W_{c-z})/\Delta T$ is larger in the case with VGs than in the case without VGs in the region near the bottom surface. This means that in the region near the bottom surface, the velocity gradient and the velocity have the same positive contributions to the transport of q_z . Considering Figs.12(a) and 12(b), in the region near the top surface, the velocity gradient improves the transport of q_z , but the velocity deters the transport of q_z , the combination of both contributions counterbalance, and $(W_{e-z}+W_{c-z})/\Delta T$ is nearly the same as the corresponding value of the case without VGs. This figure indicates that longitudinal vortices strongly change the combined contributions of the velocity gradient and the velocity to the transport of q_z .

The value of $q_z/\Delta T$ is shown in Fig. 12(d). For the case without VGs, $q_z/\Delta T$ is in maximal value on the top surface and is in minimal value on the bottom surfaces. it increases from minimal value to the maximal value as $0 \text{ m} < z < 0.004 \text{ m}$. its minimal value is about $-22 \text{ W/}^\circ\text{C}$ and its maximal value is $22 \text{ W/}^\circ\text{C}$, and it is less dependent on x . For the case with VGs $q_z/\Delta T$ is in maximal value on the top surface and is in minimal value on the bottom surfaces, it increases from minimal value to the maximal value as $0 \text{ m} < z < 0.004 \text{ m}$, its minimal value is less than $-22 \text{ W/}^\circ\text{C}$ and its maximal value is about $22 \text{ W/}^\circ\text{C}$, and it is obviously dependent on x , near the position of VGs, its maximal value is smaller than $22 \text{ W/}^\circ\text{C}$, downstream, its maximal value is slightly larger than $22 \text{ W/}^\circ\text{C}$. These characteristics show clearly that depending on x position longitudinal vortices enhance the process of the transport of q_z only in a certain region on the line inspected, and in the other region the process of the transport of q_z is deterred. This figure indicates that longitudinal vortices locally change the intensity of the transport of q_z . And at the same time, indicated by Figs.12(c) and (d), large negative value of $(W_{e-z}+W_{c-z})/\Delta T$ will promote

the transport of q_z in the region near the bottom surface, and large positive value of $(W_{e-z}+W_{c-z})/\Delta T$ will promote the transport of q_z in the region near the top surface.

On the inspected lines Fig. 12 shows that longitudinal vortices enforce the velocity gradient contribution on the transport of q_z in the region $z<0.001$ m and $z>0.0025$ m that is dependent on x position, and the velocity gradient has negative effect on the transport of q_z in the other region. Longitudinal vortices enhance the velocity contribution on the transport of q_z in the region $z<0.00125$ m that is dependent on x position, but deter the transport of q_z in the other region. The combined contribution of the velocity gradient and the velocity to the transport of q_z are positive in the region $z<0.00125$ m and in the region $z>0.00275$ m as $x=x_4$.

5.2.2 The span average the velocity and the velocity gradient contributions to the transport of $\mathbf{q}$

Above mentioned contributions of the velocity and the velocity gradient are only on the inspected lines, it is too local to build up a whole picture for heat transfer enhancement enforced by longitudinal vortices. Thus, it is necessary to present the y -direction average contributions enforced by the velocity and the velocity gradient to the transport of $\mathbf{q}$.

As shown in Fig. 13(a) , for the case without VGs, $SW_{e-x}/\Delta T$ is quite the same as $W_{e-x}/\Delta T$ on the inspected line shown in Fig. 10(a). For the case without VGs $SW_{e-x}/\Delta T$ is smaller than the corresponding value for the case with VGs, and is slightly dependent on x position. For the case with VGs, at different x position, $SW_{e-x}/\Delta T$ value is negative near the bottom surface, $SW_{e-x}/\Delta T$ value is positive near the top surface as $x=x_1$ and x_2 , $SW_{e-x}/\Delta T$ value is negative near the top surface as $x=x_3$ and x_4 . $SW_{e-x}/\Delta T$ changes greatly along z . As x position is far away from VGs, $SW_{e-x}/\Delta T$ decreases. $SW_{e-x}/\Delta T$ is very close to the value of the case without VGs at $x=x_4$ position. Except at $x=x_1$, the wavy distribution of $SW_{e-x}/\Delta T$ has three peak values, the first peak value is located at $z=0.00075$ m, the second peak value is located at $z=0.002$ m, and the third peak value is

located at $z=0.0031$ m. This figure shows that longitudinal vortices produce a great different $SW_{e-x}/\Delta T$ distribution compared with the case without VGs.

For the case without VGs, as shown in Fig. 13 (b), the y-direction average $SW_{c-x}/\Delta T$ is quite the same as $SW_{c-x}/\Delta T$ on the inspected line shown in Fig. 10 (b). For the case with VGs, the value of W_{c-x} changes greatly compared to the case without VGs, and it is greatly dependent on x position. At $x=x_1$ position, $SW_{c-x}/\Delta T$ has two peak values, and in the region $0.0005 \text{ m} < z < 0.00175 \text{ m}$ $SW_{c-x}/\Delta T$ has positive value, but in this region the corresponding value of the case without VGs has negative value. At $x=x_2$, $x=x_3$, and $x=x_4$ positions, $SW_{c-x}/\Delta T$ has three peak values, depending on x position, in the region $0.00125 \text{ m} < z < 0.0025 \text{ m}$ $SW_{c-x}/\Delta T$ is larger than the corresponding value of the case without VGs, in the other region $|SW_{c-x}|/\Delta T$ is larger than the corresponding value of the case without VGs. This figure shows that longitudinal vortices change greatly the velocity contribution to the transport of q_x .

Figure 13(c) shows the combined contributions of the velocity gradient and the velocity to the transport of q_x . It is found that $S(W_{e-x}+W_{c-x})/\Delta T$ for the case without VGs is the same as the $(W_{e-x}+W_{c-x})/\Delta T$ shown in Fig. 10(c), but $S(W_{e-x}+W_{c-x})/\Delta T$ for the case with VGs is quite different from $(W_{e-x}+W_{c-x})/\Delta T$ shown in Fig. 10 (c). For the case with VGs, $S(W_{e-x}+W_{c-x})/\Delta T$ has three peak values. Depending on x position, the first peak value locates at $0.0005 \text{ m} < z < 0.00105 \text{ m}$, and the second peak value locates at $0.0015 \text{ m} < z < 0.002 \text{ m}$, and the third peak value locates at $z=0.003 \text{ m}$. In the regions near the bottom and the top surfaces $|S(W_{e-x}+W_{c-x})|/\Delta T$ is larger than the corresponding value of the case without VGs, in the region $0.001 \text{ m} < z < 0.0025 \text{ m}$ $S(W_{e-x}+W_{c-x})/\Delta T$ is larger than the corresponding value of the case without VGs. This figure indicates that longitudinal vortices strongly change the contribution combination of the velocity gradient and the velocity to the transport of q_x .

As shown in Fig. 13(d), for the case without VGs, $Sq_x/\Delta T$ is nearly the same as $q_x/\Delta T$ shown in Fig. 10 (d). For the case with VGs $Sq_x/\Delta T$ is quite different from $q_x/\Delta T$ as shown in Fig. 10 (d). It is found that $Sq_x/\Delta T$ is larger than the corresponding value of the case without VGs in the regions near the bottom and the top surfaces, but in the flow core, $Sq_x/\Delta T$ is smaller than the corresponding value of the case without VGs. These characteristics show clearly that longitudinal vortices enhance the process of the transport of q_x only in a certain region, and in the other region the process of the transport of q_x is deterred. At the same time, as indicated by Figs. 13(c) and (d), large negative values of $S(W_{e-x}+W_{c-x})/\Delta T$ will promote the transport of q_x .

Figure 13 shows that longitudinal vortices enforce the velocity gradient contribution on the transport of q_x in the region $z < 0.0015$ m and in the region $z > 0.00275$ m at the positions of x_2 , x_3 , x_4 , and the velocity gradient has a negative effect on the transport of q_x in the other region. Except at x_1 position, longitudinal vortices enhance the velocity contribution on the transport of q_x in the regions $z < 0.0015$ m and $z > 0.002$ m, but deter the transport of q_x in the other region. The combined contribution of the velocity gradient and the velocity to the transport of q_x are positive in the regions $z < 0.00125$ m and $z > 0.00235$ m that dependent on x position.

As shown in Fig. 14, on the inspected lines, $SW_{e-y}/\Delta T$, $SW_{c-y}/\Delta T$, $S(W_{e-y}+W_{c-y})/\Delta T$, and $q_y/\Delta T$ are all zero for both the cases with and without VGs. Considering the characteristics of $SW_{e-y}/\Delta T$, $SW_{c-y}/\Delta T$, $S(W_{e-y}+W_{c-y})/\Delta T$, and $q_y/\Delta T$ shown in Fig. 14, Fig.11 shows that longitudinal vortices only produce the velocity gradient and the velocity contributions to the transport of q_y locally, but do not do so in an average measure. This is a compromise with the physical problem studied presently. In the y -direction, there is no net heat transfer in an average measure. It should be inferred from Fig. 11 and Fig. 14 that longitudinal vortices bring the local transport of q_y , which

changes the transport characteristics of q_x and q_z , and then the enhancement of the transport q_z causes the convective heat transfer enhancement.

Figure 13 The distributions of $SW_{e-x}/\Delta T$, $SW_{c-x}/\Delta T$, $S(W_{e-x}+W_{c-x})/\Delta T$ and $Sq_x/\Delta T$ along the z -direction at the different x position in the cases with VG and without VG: (a) $SW_{e-x}/\Delta T$; (b) $SW_{c-x}/\Delta T$; (c) $S(W_{e-x}+W_{c-x})/\Delta T$; (d) $Sq_x/\Delta T$

As shown in Fig. 14(a), the distribution of $SW_{e-z}/\Delta T$ is similar to the distribution of $W_{e-z}/\Delta T$ shown in Fig. 11(a). While for the case with VGs, at different x position, the distribution of $SW_{e-z}/\Delta T$ is quite different from the distribution of $W_{e-z}/\Delta T$ on the inspected lines shown in Fig. 11(a). $SW_{e-z}/\Delta T$ has four peak values. One positive peak value appears near the bottom surface and one negative peak value appears on the top surface, in the core region of fluid flow, there are

two peak values, one is positive and one is negative. Except at the position of $z=0.002$ m, it is clear that longitudinal vortices enlarge $SW_{e-z}/\Delta T$ near the bottom and the top surfaces, and in the core region of flow the longitudinal vortices decrease the value of $SW_{e-z}/\Delta T$. This figure shows that longitudinal vortices change $SW_{e-z}/\Delta T$ value greatly, and longitudinal vortices contribute to the transport of q_z near the bottom and the top surfaces, but deter the transport of q_z in the core flow region.

Figure 14 The distributions of $SW_{e-y}/\Delta T$, $SW_{c-y}/\Delta T$, $S(W_{e-y} + W_{c-y})/\Delta T$ and $Sq_y/\Delta T$ along the z -direction at the different x position in the cases with VG and without VG: (a) $SW_{e-y}/\Delta T$; (b) $SW_{c-y}/\Delta T$; (c) $S(W_{e-y} + W_{c-y})/\Delta T$; (d) $Sq_y/\Delta T$

The y-direction average velocity contribution to the transport of q_z is shown in Fig. 14(b). For the case without VGs, at different x positions, the distribution of $SW_{c-z}/\Delta T$ is the same as the distribution of $W_{c-z}/\Delta T$. For the case with VGs, the value of $SW_{c-z}/\Delta T$ changes greatly compared to $W_{c-z}/\Delta T$ on the inspected lines. $SW_{c-z}/\Delta T$ has four peak values. One positive peak value appears near the bottom surface and one negative peak value appears on the top surface, in the core region of fluid flow, there are two peak values, one is positive and one is negative. Except at the position of $z=0.002$ m, longitudinal vortices enlarge $SW_{c-z}/\Delta T$ near the bottom and the top surfaces, and in the core region of flow longitudinal vortices decrease the value of $SW_{c-z}/\Delta T$. These characteristics are very similar to the distribution of $SW_{e-z}/\Delta T$. It is found that longitudinal vortices contribute the transport of q_z near the bottom and the top surfaces, but deter the transport of q_z in the core flow region. Comparing the values of $SW_{c-z}/\Delta T$ and $SW_{e-z}/\Delta T$, it is found that longitudinal vortices enforce larger increment of $SW_{e-z}/\Delta T$ than of $SW_{c-z}/\Delta T$, which shows that VGs enhance the transport of q_z through enlarging the values of $SW_{c-z}/\Delta T$ and $SW_{e-z}/\Delta T$, but mainly through the velocity gradient contribution. Although longitudinal vortices enlarge the value $SW_{c-z}/\Delta T$ near the bottom and the top surfaces, they greatly counterbalance the value of $SW_{c-z}/\Delta T$ in the flow core. This figure shows that longitudinal vortices change the velocity contribution to the transport of q_z .

Figure 15(c) shows the combined contributions of the velocity gradient and the velocity to the transport of q_z . It is found that $S(W_{e-z}+W_{c-z})/\Delta T$ for the case with VGs is of great difference with $S(W_{e-z}+W_{c-z})/\Delta T$ for the case without VGs. $S(W_{e-z}+W_{c-z})/\Delta T$ has four peak values. Longitudinal vortices enlarge $S(W_{e-z}+W_{c-z})/\Delta T$ in the regions near the top and the bottom surfaces, but it counterbalances $S(W_{e-z}+W_{c-z})/\Delta T$ in the flow core, except for the position near $z=0.00085$ m,

0.002 m and 0.003 m . This figure indicates that longitudinal vortices strongly change the combined contributions of the velocity gradient and the velocity to the transport of q_z .

Figure 15 The distributions of $SW_{e-z}/\Delta T$, $SW_{c-z}/\Delta T$, $S(W_{e-z}+W_{c-z})/\Delta T$ and $Sq_z/\Delta T$ along the z -direction at the different x position in the cases with VG and without VG: (a) $SW_{e-z}/\Delta T$; (b) $SW_{c-z}/\Delta T$; (c) $S(W_{e-z}+W_{c-z})/\Delta T$; (d) $Sq_z/\Delta T$

The value of $q_z/\Delta T$ is shown in Fig. 15(d). For the case without VGs, $Sq_z/\Delta T$ has the same distribution as $q_z/\Delta T$ on the inspected lines shown in Fig.12 (d). For the case with VGs, $Sq_z/\Delta T$ is in slightly wavy distribution along z . depending on x , the value of $Sq_z/\Delta T$ on the bottom surface is obviously larger than the corresponding value of the case without VGs, the value of $Sq_z/\Delta T$ on the

bottom surface is obviously larger than the corresponding value of the case without VGs. On the top surface, the value of $Sq_z/\Delta T$ is slightly larger than its corresponding value of the case without VGs. Such characteristics mean that longitudinal vortices enhance the transport of q_z on the bottom and the top surfaces. Because, on the surfaces, $Sq_z/\Delta T$ is related to the heat transfer coefficient, thus these characteristics also indicate that the convective heat transfer on the bottom and top surfaces are enhanced.

If Figs.15(c) and (d) are considered together, it is found that larger positive values of $S(W_{e-z}+W_{c-z})/\Delta T$ near the bottom surface will further promote the convection of q_z , larger negative value of $S(W_{e-z}+W_{c-z})/\Delta T$ near the top surface will further promote the convection of q_z , in the region $0.00075 \text{ m} < z < 0.002 \text{ m}$ and the region $0.0002 \text{ m} < z < 0.003 \text{ m}$ more counterbalance of $S(W_{e-z}+W_{c-z})/\Delta T$ produced by longitudinal vortices will deter the convection of q_z .

5.3 The parametrical effect on the velocity and the velocity gradient contributions to the transport of $\mathbf{q}$

Figures 16(a) and (b) show the distributions of $S(W_{e-z}+W_{c-z})/\Delta T$ and $Sq_z/\Delta T$ at different attack angles of VGs. The results show that $S(W_{e-z}+W_{c-z})/\Delta T$ and $Sq_z/\Delta T$ have nearly the same distribution shapes. As β increases, the amplitude of $S(W_{e-z}+W_{c-z})/\Delta T$ distribution wave increases, as shown in Fig. 16 (a), which results in an increase of the absolute value of $Sq_z/\Delta T$ in the region near the top and the bottom surfaces. This indicates that the heat transfer enhancement ability of VGs increases with increasing β . Such results have been observed by available literature. It is found also that a large attack angle of VGs contributes to larger increase of $Sq_z/\Delta T$ in the region near the bottom surface than in the region near the top surface.

Figure 16 The distributions of $S(W_{e-z}+W_{c-z})/\Delta T$ and $Sq_z/\Delta T$ along the z -direction at the different attack angle β : (a) $S(W_{e-z}+W_{c-z})/\Delta T$; (b) $Sq_z/\Delta T$.

The distributions of $S(W_{e-z}+W_{c-z})/\Delta T$ and $Sq_z/\Delta T$ along the z -direction at different height of VGs are shown in Figs. 17(a) and (b). It is found that when $h=H/4$, VGs change slightly the distribution shape of $S(W_{e-z}+W_{c-z})/\Delta T$ and $Sq_z/\Delta T$ s, and the value of $Sq_z/\Delta T$ is not changed greatly compared to the case without VGs. This implies that exceedingly weak vortices are produced by VGs. VGs do not bring a significant change of the contribution to heat transfer enhancement. As h increases to $H/2$ and $3H/4$, the distributions of $S(W_{e-z}+W_{c-z})/\Delta T$ and $Sq_z/\Delta T$ along the z -direction change greatly compared to the case without VGs, the amplitude of $S(W_{e-z}+W_{c-z})/\Delta T$ distribution wave increases, the absolute value of $Sq_z/\Delta T$ in the region near the top and the bottom surfaces increases. This indicates that the heat transfer enhancement ability of VGs increases with increasing their height. Large height of VGs contributes to larger increase of $Sq_z/\Delta T$ in the region near the bottom surface than in the region near the top surface.

Figure 17 The distributions of $S(W_{e-z}+W_{c-z})/\Delta T$ and $Sq_z/\Delta T$ along the z -direction at the different the height of VGs: (a) $S(W_{e-z}+W_{c-z})/\Delta T$; (b) $Sq_z/\Delta T$.

In Figs. 18(a) and (b), the distributions of $S(W_{e-z}+W_{c-z})/\Delta T$ and $Sq_z/\Delta T$ at different base lengths of VGs are described. It is found that the base length of VGs has less effect on the distributions of $S(W_{e-z}+W_{c-z})/\Delta T$ and $Sq_z/\Delta T$. This implies that $l=2.5H$ is enough to produce nearly the same distributions of $S(W_{e-z}+W_{c-z})/\Delta T$ and $Sq_z/\Delta T$, and to obtain nearly the same heat transfer enhancement on the bottom and the top surfaces.

Figure 18 The distributions of $S(W_{e-z}+W_{c-z})/\Delta T$ and $Sq_z/\Delta T$ along the z -direction at different base lengths of VGs: (a) $S(W_{e-z}+W_{c-z})/\Delta T$; (b) $Sq_z/\Delta T$; (c) f/f_{plain} ; (d) Nu/Nu_{plain}

6. The mechanism of the heat transfer enhancement caused by longitudinal vortices

In the present study, the convective heat transfer enhancement using VGs means that the transport ability of the heat flux q_z is increased by longitudinal vortices compared to the case without VGs. Considering that the heat transfer is related to the temperature difference between the wall and fluid, where q_z is put into per unit temperature difference form $q_z/\Delta T$.

Comparing the above results of the cases with and without VGs, the most significant changes occur for the local transport characteristics of q_y . The existence of longitudinal vortices makes the local terms that can promote the transport of q_y appear, but the existence of longitudinal vortices does not make the corresponding average terms (averaged along the y -direction) appear. As stated above, this result is a compromise with the physical problem studied presently, in the y direction there is no heat transfer in an average measure. When longitudinal vortices exist, the local terms that promote the local transport of q_y intensify the local transport of q_y , and due to the enhancement of the transport of q_y the transports of q_x and q_z are promoted through very complex couples of the velocity contribution terms and velocity gradient contribution terms that are indicated by enlarging the values of $(W_{e-x}+W_{c-x})/\Delta T$ and $(W_{e-z}+W_{c-z})/\Delta T$, respectively. When longitudinal vortices exist, the values of $(W_{e-x}+W_{c-x})/\Delta T$ and $(W_{e-z}+W_{c-z})/\Delta T$, or the values of $S(W_{e-x}+W_{c-x})/\Delta T$ and $S(W_{e-z}+W_{c-z})/\Delta T$ are enlarged by longitudinal vortices compared to the corresponding values of the case without VGs. The enlargements of the values of these terms result in the increase of $q_z/\Delta T$ on the top and the bottom surfaces, and hence the heat transfer enhancement is enforced by longitudinal vortices.

7. Conclusions

The acknowledged mechanisms of heat transfer enhancement produced by longitudinal vortices are too vague to be discovered in detail, they have not been strictly proven by some

suitable terms in an available equation describing convective heat transfer. To discover the mechanisms in detail, the present paper tries from another view point to address the mechanisms. Based on the convective transport equation of heat flux, a description of convective heat transfer process in the process parameter of $\mathbf{q}$, the heat transfer mechanism is explained by the contributions of the combination of flux with velocity gradient and velocity with flux gradient to the heat transfer enhancement. To demonstrate the differences of these contributions for the cases with and without VGs, the heat transfer process in the channel formed by two planes is studied using a numerical method. Two cases are considered, one case is that on the channel bottom wall VGs are mounted and the other case is that there is no VG on the channel bottom wall. After comparing the velocity gradient and the velocity contributions to convective heat transfer of the heat fluxes for the cases with and without VGs, the following conclusions can be made.

- (1) Longitudinal vortices greatly enhance the local terms that promote the local transport of q_y , and these local terms intensify the local transport of q_y .
- (2) Due to the enhancement of the transport of q_y the transports of q_x and q_z are promoted through very complex couples of the velocity contribution terms and velocity gradient contribution terms, which are indicated by enlargements of the values of the convection contribution terms.
- (3) The enlargements of the convection contribution terms result in the increase of $q_z/\Delta T$ on the top and the bottom surfaces, and hence the heat transfer enhancement is enforced by longitudinal vortices.
- (4) Longitudinal vortices do not enhance the transport of q_y in an average measure, that is, the values of corresponding average convection contribution terms (averaged

along the y -direction) are zero, which compromises with the physical problem studied: along the y direction no heat is transferred in an average measure.

- (5) The mechanism of heat transfer enhancement produced by longitudinal vortices can be stated as longitudinal vortices promote the local transport of q_y efficiently, and with the help of the transport of q_x the strong intensified local transport of q_y enforces the heat transfer enhancement of q_z .
- (6) The height of VGs and the attack angle of VGs are the main parameters that affect the velocity contribution terms and velocity gradient contribution terms to the local transport of q_y , and then affect the heat transfer enhancement of q_z .

It should be pointed out that the above conclusions are strongly dependent on the coordinate used. Even for the same problem, when the coordinate is changed, the coordinate direction mentioned in the above conclusions will be changed accordingly.

Appendix

A general form of the three-dimensional steady state equation in computational domain is:

$$\begin{aligned} & \frac{\partial}{\partial \xi} (\rho U \phi) + \frac{\partial}{\partial \eta} (\rho V \phi) + \frac{\partial}{\partial \zeta} (\rho W \phi) \\ &= \frac{\partial}{\partial \xi} \left[\frac{\Gamma_{\phi\alpha}}{J} \frac{\partial \phi}{\partial \xi} \right] + \frac{\partial}{\partial \eta} \left[\frac{\Gamma_{\phi\beta}}{J} \frac{\partial \phi}{\partial \eta} \right] + \frac{\partial}{\partial \zeta} \left[\frac{\Gamma_{\phi\gamma}}{J} \frac{\partial \phi}{\partial \zeta} \right] + JS^\phi + JS^\psi \end{aligned} \quad (A1)$$

Parameter ϕ can be u , v , w , T , respectively. S^ψ is the additional source term introduced by the non-orthogonal grid.

$$\begin{aligned} JS^\phi &= \frac{\partial}{\partial \xi} \left[\frac{\Gamma_{\phi\alpha_1}}{J} \frac{\partial \phi}{\partial \eta} \right] + \frac{\partial}{\partial \xi} \left[\frac{\Gamma_{\phi\alpha_2}}{J} \frac{\partial \phi}{\partial \zeta} \right] + \frac{\partial}{\partial \eta} \left[\frac{\Gamma_{\phi\beta_1}}{J} \frac{\partial \phi}{\partial \xi} \right] \\ &+ \frac{\partial}{\partial \eta} \left[\frac{\Gamma_{\phi\beta_2}}{J} \frac{\partial \phi}{\partial \zeta} \right] + \frac{\partial}{\partial \zeta} \left[\frac{\Gamma_{\phi\gamma_1}}{J} \frac{\partial \phi}{\partial \xi} \right] + \frac{\partial}{\partial \zeta} \left[\frac{\Gamma_{\phi\gamma_2}}{J} \frac{\partial \phi}{\partial \eta} \right] \end{aligned} \quad (A2)$$

JS^Ψ is the source item in the computational domain, for energy equation, $JS^\Psi = 0$, for u_i equation

this term is:

$$JS^\Psi = -\left(\frac{\partial p}{\partial \xi} \alpha_i + \frac{\partial p}{\partial \eta} \beta_i + \frac{\partial p}{\partial \zeta} \gamma_i\right), \quad i = 1, 2, 3 \quad (A3)$$

Where

$$\begin{cases} U = \alpha_1 u + \alpha_2 v + \alpha_3 w \\ V = \beta_1 u + \beta_2 v + \beta_3 w \\ W = \gamma_1 u + \gamma_2 v + \gamma_3 w \end{cases} \quad (A4)$$

$$\begin{cases} \alpha_1 = (y_\eta z_\zeta - y_\zeta z_\eta) \\ \alpha_2 = (x_\zeta z_\eta - x_\eta z_\zeta) \\ \alpha_3 = (x_\eta y_\zeta - x_\zeta y_\eta) \end{cases}, \begin{cases} \beta_1 = (y_\zeta z_\xi - y_\xi z_\zeta) \\ \beta_2 = (x_\xi z_\zeta - x_\zeta z_\xi) \\ \beta_3 = (x_\zeta y_\xi - x_\xi y_\zeta) \end{cases}, \begin{cases} \gamma_1 = (x_\xi z_\eta - x_\eta z_\xi) \\ \gamma_2 = (x_\eta z_\xi - x_\xi z_\eta) \\ \gamma_3 = (x_\xi y_\eta - x_\eta y_\xi) \end{cases} \quad (A5)$$

$$\begin{cases} \alpha_1^2 + \alpha_2^2 + \alpha_3^2 = \alpha \\ \beta_1^2 + \beta_2^2 + \beta_3^2 = \beta \\ \gamma_1^2 + \gamma_2^2 + \gamma_3^2 = \gamma \end{cases}, \begin{cases} \alpha_1 \beta_1 + \alpha_2 \beta_2 + \alpha_3 \beta_3 = \varphi_1 \\ \beta_1 \gamma_1 + \beta_2 \gamma_2 + \beta_3 \gamma_3 = \varphi_2 \\ \alpha_1 \gamma_1 + \alpha_2 \gamma_2 + \alpha_3 \gamma_3 = \varphi_3 \end{cases}$$

$$J = x_\xi y_\eta z_\zeta + x_\eta z_\xi y_\zeta + y_\xi z_\eta x_\zeta - x_\zeta y_\eta z_\xi - x_\xi y_\zeta z_\eta - x_\eta y_\xi z_\zeta \quad (A6)$$

ξ 、 η 、 ζ are the coordinates axes in computational space, x_ξ , x_η , x_ζ , y_ξ , y_η , y_ζ , z_ξ , z_η , z_ζ

are the geometric factors in the coordinate transformation from the physical space to the computational space. The subscript means the derivation respecting coordinate. J represents Jacobi factor.

In the Cartesian coordinate system, the convective transfer equation of the heat flux can be written as follows.

$$\frac{\partial q_x}{\partial t} + \left(u \frac{\partial q_x}{\partial x} + v \frac{\partial q_x}{\partial y} + w \frac{\partial q_x}{\partial z} \right) + \left(q_x \frac{\partial u}{\partial x} + q_y \frac{\partial v}{\partial x} + q_z \frac{\partial w}{\partial x} \right) = \alpha \nabla^2 q_x \quad (A7)$$

$$\frac{\partial q_y}{\partial t} + \left(u \frac{\partial q_y}{\partial x} + v \frac{\partial q_y}{\partial y} + w \frac{\partial q_y}{\partial z} \right) + \left(q_x \frac{\partial u}{\partial y} + q_y \frac{\partial v}{\partial y} + q_z \frac{\partial w}{\partial y} \right) = \alpha \nabla^2 q_y \quad (A8)$$

$$\frac{\partial q_z}{\partial t} + \left(u \frac{\partial q_z}{\partial x} + v \frac{\partial q_z}{\partial y} + w \frac{\partial q_z}{\partial z} \right) + \left(q_x \frac{\partial u}{\partial z} + q_y \frac{\partial v}{\partial z} + q_z \frac{\partial w}{\partial z} \right) = \alpha \nabla^2 q_z \quad (\text{A9})$$

Eqs. (A7) to (A9) can be written further as

$$\frac{\partial q_x}{\partial t} + W_{c-x} + W_{e-x} = \alpha \nabla^2 q_x \quad (\text{A10})$$

$$\frac{\partial q_y}{\partial t} + W_{c-y} + W_{e-y} = \alpha \nabla^2 q_y \quad (\text{A11})$$

$$\frac{\partial q_z}{\partial t} + W_{c-z} + W_{e-z} = \alpha \nabla^2 q_z \quad (\text{A12})$$

W_{c-x} , W_{c-y} , W_{c-z} , W_{e-x} , W_{e-y} and W_{e-z} are defined by

$$W_{c-x} = \left(u \frac{\partial q_x}{\partial x} + v \frac{\partial q_x}{\partial y} + w \frac{\partial q_x}{\partial z} \right) \quad (\text{A13a})$$

$$W_{c-y} = \left(u \frac{\partial q_y}{\partial x} + v \frac{\partial q_y}{\partial y} + w \frac{\partial q_y}{\partial z} \right) \quad (\text{A13b})$$

$$W_{c-z} = \left(u \frac{\partial q_z}{\partial x} + v \frac{\partial q_z}{\partial y} + w \frac{\partial q_z}{\partial z} \right) \quad (\text{A13c})$$

$$W_{e-x} = \left(q_x \frac{\partial u}{\partial x} + q_y \frac{\partial v}{\partial x} + q_z \frac{\partial w}{\partial x} \right) \quad (\text{A14a})$$

$$W_{e-y} = \left(q_x \frac{\partial u}{\partial y} + q_y \frac{\partial v}{\partial y} + q_z \frac{\partial w}{\partial y} \right) \quad (\text{A14b})$$

$$W_{e-z} = \left(q_x \frac{\partial u}{\partial z} + q_y \frac{\partial v}{\partial z} + q_z \frac{\partial w}{\partial z} \right) \quad (\text{A14c})$$

Except for the local characteristics of $W_{c-x}/\Delta T$, $W_{c-y}/\Delta T$, $W_{c-z}/\Delta T$, $W_{e-x}/\Delta T$, $W_{e-y}/\Delta T$, $W_{e-z}/\Delta T$, $q_x/\Delta T$, $q_y/\Delta T$ and $q_z/\Delta T$, their y-direction average values are defined as followings.

$$Sq_x / \Delta T = \int_0^B \frac{q_x(x, y, z)}{B \Delta T} dy \quad (\text{A15a})$$

$$Sq_y / \Delta T = \int_0^B \frac{q_y(x, y, z)}{B \Delta T} dy \quad (A15b)$$

$$Sq_z / \Delta T = \int_0^B \frac{q_z(x, y, z)}{B \Delta T} dy \quad (A15c)$$

$$SW_{e-x} = \int_0^B \frac{1}{B \Delta T} (q_x \frac{\partial u}{\partial x} + q_y \frac{\partial v}{\partial x} + q_z \frac{\partial w}{\partial x}) dy \quad (A16a)$$

$$SW_{e-y} = \int_0^B \frac{1}{B \Delta T} (q_x \frac{\partial u}{\partial y} + q_y \frac{\partial v}{\partial y} + q_z \frac{\partial w}{\partial y}) dy \quad (A16b)$$

$$SW_{e-z} = \int_0^B \frac{1}{B \Delta T} (q_x \frac{\partial u}{\partial z} + q_y \frac{\partial v}{\partial z} + q_z \frac{\partial w}{\partial z}) dy \quad (A16c)$$

$$SW_{c-x} / \Delta T = \int_0^B \frac{1}{B \Delta T} (u \frac{\partial q_x}{\partial x} + v \frac{\partial q_x}{\partial y} + w \frac{\partial q_x}{\partial z}) dy \quad (A17a)$$

$$SW_{c-y} / \Delta T = \int_0^B \frac{1}{B \Delta T} (u \frac{\partial q_y}{\partial x} + v \frac{\partial q_y}{\partial y} + w \frac{\partial q_y}{\partial z}) dy \quad (A17b)$$

$$SW_{c-z} / \Delta T = \int_0^B \frac{1}{B \Delta T} (u \frac{\partial q_z}{\partial x} + v \frac{\partial q_z}{\partial y} + w \frac{\partial q_z}{\partial z}) dy \quad (A17c)$$

$$S(W_{c-x} + W_{e-x}) / \Delta T = (SW_{c-x} + SW_{e-x}) / \Delta T \quad (A18a)$$

$$S(W_{c-y} + W_{e-y}) / \Delta T = (SW_{c-y} + SW_{e-y}) / \Delta T \quad (A18b)$$

$$S(W_{c-z} + W_{e-z}) / \Delta T = (SW_{c-z} + SW_{e-z}) / \Delta T \quad (A18c)$$

Acknowledgement

This research was supported by the National Science Foundation of China (Grant No. 51306086).

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