Analysis on the heat transfer characteristics of a micro-channel type porous-sheets Stirling regenerator

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46 **Nomenclature**

47	a	thermal diffusivity, m^2/s
48	$A(r^*), B(r^*)$	functions defined in Eqs. (5a) and (5b)
49	A_o	dimensionless fluid oscillating amplitude, $A_o = x_{\max}/d_0$
50	A_R	relative fluid displacement ratio, $A_R = x_{\max}/L$
51	b	side length of a hexagonal cross section
52	Bi	Biot number, defined in Eq. (56)
53	c	specific thermal capacity, $\text{J} / (\text{kg K})$
54	c_p	specific thermal capacity at constant pressure, $\text{J} / (\text{kg K})$
55	C_1, C_2	constants defined in Eq. (6c)
56	C_f	Fanning flow friction coefficient, $C_f = \tau_w / \left(\frac{1}{2} \rho u_{\max}^2 \right)$
57	Cr	heat capacity ratio of solid matrix to fluid flow, defined in Eq. (64)
58	d_0, r_0	channel inner diameter and inner radius, m
59	D_1, D_2	constants defined in Eq. (31b)
60	d_h	hydraulic diameter, m
61	E_1, E_2	constants defined in Eqs. (21a) and (21b)
62	$f(r^*), g(r^*)$	functions defined in Eqs. (22a) and (22b)
63	Fo	Fourier number, defined in Eq. (60)
64	h_x	heat transfer coefficient, $\text{W} / (\text{m}^2 \text{K})$
65	$\text{Imag}[]$	imaginary component of a complex number
66	K	variation amplitude of pressure gradient multiplied by $-1/\rho$, m/s^2
67	k	thermal conductivity, $\text{W}/(\text{m K})$
68	L	regenerator length, m

69	L_f	hydrodynamic entrance length, m
70	L_t	thermal entrance length, m
71	$\dot{m}$	mass flow rate, kg/s
72	n	rotational velocity, rpm
73	N_s	number of porous sheets
74	N_{TU}	number of heat transfer units, defined in Eq. (62)
75	Nu	Nusselt number, $Nu = hd_h/k_f$
76	p	pressure, Pa
77	Pr	Prandtl number, $Pr = \nu/a$
78	q_w	wall heat flux, W/m ²
79	r^*	dimensionless radial coordinate, $r^* = 2r/d_0$, m
80	$\text{Real}[\]$	imaginary component of a complex number
81	Re_ω	kinetic Reynolds number, $Re_\omega = \omega d_0^2/\nu$
82	S	total heat transfer area between matrix and fluid, m ²
83	$\dot{S}_g$	total entropy generation rate, W/K
84	$\dot{S}_{g,f}$	entropy generation rate due to flow friction loss, W/K
85	$\dot{S}_{g,h}$	entropy generation rate due to irreversible heat transfer, W/K
86	t	time, s
87	T	temperature, K
88	T_b	bulk (mixed mean) temperature, $T_b = \int_0^{r_0} uTrdr / \int_0^{r_0} urdr$, K
89	t_{blow}	blow time for half cycle, s
90	T_m	cross-sectional average temperature, defined in Eq. (24)
91	u	axial velocity, m/s

92	u^*	dimensionless axial velocity, $u^* = u/u_{\max}$
93	$X(r^*)$	function defined in Eq. (18a)
94	$Y(r^*)$	function defined in Eq. (18b)
95	x^*	dimensionless axial position, $x^* = 2x/d_0$
96	$x_{\max}$	amplitude of fluid oscillation
97		
98	<i>Greek symbols</i>	
99	α	specific surface area, defined in Eq. (57) , 1/m
100	β	constants defined in Eq. (6c)
101	γ	modified Womersley number, $\gamma = \frac{1}{2}\sqrt{\text{Re}_\omega \text{Pr}}$
102	Δ	$\Delta = \delta/\delta_t = \sigma/\xi$
103	δ	thickness of hydrodynamic boundary layer, m
104	δ_s	width of solid rib between adjacent pores, m
105	δ_t	thickness of thermal boundary layer, m
106	ε	regenerator effectiveness
107	ζ	dimensionless y coordinate in flow boundary layer, $\zeta = y/\delta$
108	η	dimensionless y coordinate in thermal boundary layer, $\eta = y/\delta_t$
109	Θ	dimensionless fluid temperature, $\Theta = (T - T_c)/(T_h - T_c)$
110	θ	excess fluid temperature, $\theta = T - T_w$
111	λ	Womersley number, $\lambda = \frac{1}{2}\sqrt{\text{Re}_\omega}$
112	ν	kinetic viscosity, m ² /s
113	ξ	dimensionless thickness of thermal boundary layer, $\xi = \delta_t/r_0$

114	ρ	density, kg/m ³
115	σ	dimensionless thickness of flow boundary layer, $\sigma = \delta/r_0$
116	τ	shear stress, Pa
117	φ	crank angle, $\varphi = \omega t$
118	Φ	matrix outer diameter, m
119	ϕ	regenerator porosity
120	ψ	dimensionless temperature in solid rib, $\psi = (T - T_0)/(T_m - T_0)$
121	ω	rotational velocity, rad/s
122	μ	dynamic viscosity, Pa s
123	$\mathcal{A}_1$	phase shift between the mean velocity and pressure gradient
124	$\mathcal{A}_2$	phase shift between heat flux and temperature difference
125		
126	<i>Superscript and subscript</i>	
127	*	dimensionless parameter
128	—	cycle averaged parameter
129	→	complex parameter
130	Ac	cross section
131	c	cold end of regenerator
132	e	external core flow
133	f	fluid
134	h	hot end of regenerator
135	m	mean
136	max	maximum
137	s	solid

138	w	wall
139	w_i	channel inner wall
140	x	axially local parameters
141	Γ	the boundary curve of channel cross section
142		

Abstract

To avoid the high flow frictional loss associated with conventional wire mesh Stirling regenerators, a micro-channel type stacked porous-sheets Stirling regenerator is investigated. An analytical solution is derived for the transient heat transfer characteristics of the fully developed reciprocating laminar flow under prescribed wall temperature profiles. The complex Nusselt number (Nu) is expressed as a function of kinetic Reynolds number (Re_ω) and Prandtl number (Pr). At low Re_ω of less than 10, the real part of Nu has an almost constant value of 6.0, approximately equal to the known real-valued Nu for the fully developed unidirectional laminar flow under constant wall heat flux, while the imaginary part is negligible, thus “scaling effect” can be utilized to enhance heat transfer. At higher Re_ω , both the real and imaginary parts of Nu increase with the increase of Re_ω and Pr , and the phase shift between the temperature difference and the heat flux gradually increases and approaches 45° . Approximate analytical solutions are also deduced for the entrance region from the integral boundary layer equations in both cases of “Thermally developing flow” and “Simultaneously developing flow”. The heat transfer is enhanced in the entrance region and the local Nu in the flow direction approaches the corresponding values of fully developed flow. The analytical results are confirmed by dynamic mesh CFD results, and the obtained $Nu \sim Re_\omega$ data and patterns generally agree with available analytical and experimental data from published literatures. Application of the analytical results to the design and optimization of Stirling regenerator are also shown.

Key Words: Stirling engine; Porous-sheets regenerator; Micro-channel; Heat transfer characteristics; Analytical solution; Dynamic mesh Computational Fluid Dynamics.

164 **1 Introduction**

165 The application of Stirling engine is promising in the power generation field due to its many
166 advantages including adaptability to versatile heat sources, high thermal efficiency and environmental
167 friendliness [1]. As a central and crucial component of the Stirling engine, the regenerator has received
168 extensive research for improving the engine performance [2]. Up to date the wire mesh type regenerator
169 is most popularly adopted in Stirling engines in virtue of its large heat transfer area, high convective heat
170 transfer coefficient brought by numerous cross flow around cylindrical wires, and low axial thermal
171 conductance. However, the associated high flow friction due to flow separation, wakes, eddies and
172 stagnation zones might impair the power output and engine efficiency [3]. Theoretically a regenerator
173 with heat transfer surfaces parallel to the oscillating flow has better performance [4].

174 With the emerging micro-fabrication techniques, properly designed regular-shaped micro-
175 channel type regenerator can be fabricated to obtain extremely low flow friction while maintaining high
176 thermal effectiveness. The main features of the regular micro-channel type regenerator include: (1) the
177 heat transfer surface is smooth; (2) the flow acceleration rates are controlled; (3) the flow separation is
178 minimized; (4) the axial thermal conduction is reduced by interrupting the axial continuity of solid
179 structure, for example, using porous sheets with intermediate gaps or clearance. Other advantages
180 include improved structural durability, no gas leakage or short-circuit loss owing to tight tolerance, low
181 cost for mass production [5]. Ibrahim et al. [6] investigated a micro-channel type segmented-involute-
182 foil regenerator under NASA support, in which the oscillating-flow rig test showed the highest figures of
183 merit ever recorded, demonstrating a shift strongly in the direction of the theoretical performance of
184 ideal parallel-plate regenerators. Takizawa et al. [7] developed a 3-kW Stirling engine installed with a
185 porous-sheets regenerator with electrically etched holes, and the engine performance was improved by
186 about 5 to 10% compared to that with wire mesh regenerator. A series of engine tests were done by

187 Matsuguchi et al. [8] to optimize the geometrical parameters of the porous-sheets regenerator. The
188 recent work of the present authors, via dynamic mesh Computational Fluid Dynamics (CFD) method
189 and experimental validation, indicates that the regular-shaped micro-channel type porous-sheets
190 regenerator has extremely low flow friction while maintaining high thermal effectiveness, thus
191 achieving significantly lower total entropy generation rate and leading to higher comprehensive
192 performance [9].

193 Analytical method possess some recognized advantages both in theory and application, such as
194 clear physical concepts, logical reasoning, rigorous mathematical base, able to reveal various
195 influencing factors and convenient for use, so as to substantially improve the efficiency of design
196 optimization process. Uchida [10] derived an analytical solution for fully developed laminar pulsating
197 flow in a circular pipe. Later, Zhao [11] presented an analytical expression for reciprocating laminar
198 flow with zero mean velocity, then validated with pressure drop experiment.

199 In this work, analytical and approximate-analytical solutions for the heat transfer characteristics
200 of the reciprocating laminar flow in a micro-channel type porous-sheets regenerator including the
201 thermal entrance region are derived, and compared with the dynamic mesh CFD results. Then the effects
202 of various factors on the heat transfer characteristics are discussed in detail.

203

204 **2 Description of the micro-channel type porous-sheets Stirling regenerator**

205 The regenerator is stacked by hundreds of porous, stainless steel sheets fabricated through an
206 etching process, each having hundreds of through holes, as shown in Fig. 1, the holes of all sheets being
207 aligned with each other and inserted into a cylindrical container, so as to form hundreds of micro-
208 channel flow passages for the internal reciprocating working gas. One of the flow channels including the
209 entrance region is illustrated in Fig. 2. The axial continuity of the solid matrix is interrupted by the
210 intermediate clearance between adjacent sheets while maintaining the flow smoothness, which is

211 intended to effectively increase the longitudinal contact resistance, and thus to reduce the axial heat
 212 conduction loss. The cross section of the flow channel is assumed as circular for the ease of analysis.

213

214

215 **Fig. 1.** One porous sheet of the micro-channel type Stirling regenerator.

216

217

218 **Fig. 2.** Schematic of the channel entrance region. L_t - Thermal entrance length; L_f - Hydrodynamic
 219 entrance length; FD - Fully developed region.

220

221 **3 Analytical derivation of the heat transfer characteristics**

222 **3.1 Analytical solution for the fully developed reciprocating laminar flow**

223 The conservative equations of mass and momentum for hydrodynamically fully developed
 224 reciprocating laminar flow in a circular pipe is given as [10]

$$225 \quad \frac{\partial u}{\partial x} = 0 \quad (1a)$$

$$226 \quad \frac{\partial u}{\partial t} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u}{\partial r} \right) \quad (1b)$$

227 where $-\frac{1}{\rho} \frac{\partial p}{\partial x} = K \cos \omega t$, K is the variation amplitude of pressure gradient multiplied by $-\frac{1}{\rho}$, and ν is
 228 the kinetic viscosity. Constant thermophysical properties are assumed in this paper for the ease of
 229 reaching an analytical solution, and the applicability of this assumption will be evaluated later in the
 230 result discussion section. The axisymmetric and non-slip wall boundary conditions are expressed as

$$231 \quad r = 0: \quad \frac{\partial u}{\partial r} = 0 \quad (1c)$$

$$232 \quad r = d_0/2: \quad u = 0 \quad (1d)$$

233 Defining the dimensionless radial coordinate and the crank angle as $r^* = 2r/d_0$, and $\varphi = \omega t$, the
 234 velocity profile is assumed to take a “separation of variables” form as presented by [11]

$$235 \quad u(r^*, \varphi) = \frac{K}{\omega} \left\{ B(r^*) \cos \varphi + [1 - A(r^*)] \sin \varphi \right\} \quad (2)$$

236 Substituting the above velocity expression into Eq. (1b), gives

$$237 \quad -B(r^*) \sin \varphi + [1 - A(r^*)] \cos \varphi = \cos \varphi + \frac{4}{\text{Re}_\omega} \left\{ \frac{1}{r^*} [r^* B'(r^*)]' \cos \varphi - \frac{1}{r^*} [r^* A'(r^*)]' \sin \varphi \right\} \quad (3)$$

238 where Re_ω is the kinetic Reynolds number defined as $\text{Re}_\omega = \omega d_0^2 / \nu$. Comparing the two sides of Eq.
 239 (3), it can be followed that

$$240 \quad \frac{1}{r^*} [r^* A'(r^*)]' = \lambda^2 B(r^*) \quad (4a)$$

$$241 \quad \frac{1}{r^*} [r^* B'(r^*)]' = -\lambda^2 A(r^*) \quad (4b)$$

242 By utilizing Eq. (2), the boundary conditions 1(c) and 1(d) are reduced to

$$A'(0) = 0, B'(0) = 0 \quad (4c)$$

$$B(1) = 0, A(1) = 1 \quad (4d)$$

where λ is Womersley number defined as $\lambda = \frac{1}{2}\sqrt{\text{Re}_\omega}$. The solution of the differential equation set (4) can be constructed by a pair of linear combinations of Kelvin functions as follows according to the characteristics of Kelvin functions, considering the boundary conditions (4c) and (4d).

$$A(r^*) = \frac{\text{ber}\lambda\text{ber}(\lambda r^*) + \text{bei}\lambda\text{bei}(\lambda r^*)}{\text{ber}^2\lambda + \text{bei}^2\lambda} \quad (5a)$$

$$B(r^*) = \frac{\text{bei}\lambda\text{ber}(\lambda r^*) - \text{ber}\lambda\text{bei}(\lambda r^*)}{\text{ber}^2\lambda + \text{bei}^2\lambda} \quad (5b)$$

where $\text{ber}(\lambda r^*)$ and $\text{bei}(\lambda r^*)$ are Kelvin functions of the zeroth order that can be expanded to power series (see Appendix A). It can be proved that the above expression (5a) and (5b) satisfy the equation set (4). This solution can also be obtained from Uchida [10]'s analytical solution for fully developed laminar pulsating flow by letting the steady component of the velocity profile be zero.

As shown in Appendix A, the cross-sectional mean velocity, u_m , can be obtained by integrating Eq. (2) over the cross section as

$$u_m = u_{\max} \cos(\varphi - \Lambda_1) \quad (6a)$$

where $u_{\max}$ is the amplitude of cross-sectional mean velocity, and Λ_1 is the phase difference between the mean velocity and the pressure gradient,

$$u_{\max} = \frac{K\lambda^2\beta}{8\omega}, \Lambda_1 = \tan^{-1}\left(\frac{\lambda - 2C_1}{2C_2}\right) \quad (6b)$$

where β , C_1 and C_2 are constants determined by Re_ω ,

$$\beta = \frac{8}{\lambda^3} \sqrt{(\lambda - 2C_1)^2 + 4C_2^2}, C_1 = \frac{\text{ber}\lambda\text{bei}'\lambda - \text{bei}\lambda\text{ber}'\lambda}{\text{ber}^2\lambda + \text{bei}^2\lambda}, C_2 = \frac{\text{ber}\lambda\text{ber}'\lambda + \text{bei}\lambda\text{bei}'\lambda}{\text{ber}^2\lambda + \text{bei}^2\lambda} \quad (6c)$$

262 where $\text{ber}'\lambda = [d(\text{ber}\lambda)/d\lambda]$, $\text{bei}'\lambda = [d(\text{bei}\lambda)/d\lambda]$. $u_{\max}$ can also be calculated from the kinetics of
 263 crank and connecting rod with sinusoidal movement, $u_{\max} = x_{\max}\omega/2$, where $x_{\max}$ is the amplitude of
 264 fluid oscillation. By defining $u^* = u/u_{\max}$, the velocity profile can be normalized as

$$265 \quad u^*(r^*, \varphi) = \frac{32}{\beta \text{Re}_\omega} \left\{ B(r^*) \cos \varphi + [1 - A(r^*)] \sin \varphi \right\} \quad (7)$$

266 The equation of energy conservation for fully developed laminar flow is given as

$$267 \quad \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} = a \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u}{\partial r} \right) \quad (8a)$$

268 where a is the thermal diffusivity. The thermal capacity of solid matrix is usually far larger than that of
 269 the internal working gas, the local matrix temperature is ideally invariable with time, and keeps a
 270 constant axial gradient in the cyclic-steady state, so both the matrix and gas temperature are generally
 271 assumed to vary linearly in the longitudinal direction [12], i.e.

$$272 \quad \partial T / \partial x = (T_h - T_c) / L \quad (8b)$$

273 where T_h and T_c are the hot and cold end temperature of the solid matrix, respectively. The
 274 axisymmetric and prescribed wall temperature boundary conditions are expressed as

$$275 \quad r = d_0/2: \quad T = T_w = T_c + (T_h - T_c)x/L \quad (8c)$$

$$276 \quad r = 0: \quad \partial T / \partial r = 0 \quad (8d)$$

277 Defining the dimensionless parameters, $\Theta = (T - T_c) / (T_h - T_c)$, $\text{Pr} = \nu/a$, $x^* = 2x/d_0$, $A_o = x_{\max}/d_0$,
 278 then the equation set (8) is reduced to

$$279 \quad \frac{\partial \Theta}{\partial \varphi} + A_o u^* \frac{\partial \Theta}{\partial x^*} = \frac{4}{\text{Re}_\omega \text{Pr}} \frac{1}{r^*} \frac{\partial}{\partial r^*} \left(r^* \frac{\partial \Theta}{\partial r^*} \right) \quad (9a)$$

$$280 \quad r^* = 0: \quad \partial \Theta / \partial r^* = 0 \quad (9b)$$

$$281 \quad r^* = 1: \quad \Theta = d_0 x^* / 2L \quad (9c)$$

282 Assuming the dimensionless temperature profile to take a “separation of variables” form as

$$283 \quad \Theta = \frac{d_0}{2L} x^* + f(r^*) \cos \varphi + g(r^*) \sin \varphi \quad (10)$$

284 Substituting the above expression into Eq. (7a), yields

$$285 \quad \begin{aligned} & -f(r^*) \sin \varphi + g(r^*) \cos \varphi + \frac{d_0}{2L} A_o \frac{32}{\beta \text{Re}_\omega} \left\{ B(r^*) \cos \varphi + [1 - A(r^*)] \sin \varphi \right\} \\ & = \frac{4}{\text{Re}_\omega \text{Pr}} \left\{ \frac{1}{r^*} [r^* f'(r^*)]' \cos \varphi + \frac{1}{r^*} [r^* g'(r^*)]' \sin \varphi \right\} \end{aligned} \quad (11)$$

286 Comparing the two sides of the above equation, we will obtain

$$287 \quad \frac{1}{r^*} [r^* f'(r^*)]' = \gamma^2 g(r^*) + 4 \frac{\text{Pr} A_R}{\beta} B(r^*) \quad (12a)$$

$$288 \quad \frac{1}{r^*} [r^* g'(r^*)]' = -\gamma^2 f(r^*) + 4 \frac{\text{Pr} A_R}{\beta} [1 - A(r^*)] \quad (12b)$$

289 By utilizing the expression (10), the boundary condition (9b) and (9c) are reduced to

$$290 \quad f'(0) = 0, \quad g'(0) = 0 \quad (12c)$$

$$291 \quad f(1) = 0, \quad g(1) = 0 \quad (12d)$$

292 where γ is modified Womersley number defined as $\gamma = \frac{1}{2} \sqrt{\text{Re}_\omega \text{Pr}}$, $A_R = x_{\max}/L$. It can be followed

293 from Eq. (12a) that

$$294 \quad g(r^*) = \frac{1}{\gamma^2} \frac{1}{r^*} [r^* f'(r^*)]' - \frac{16 A_R}{\beta \text{Re}_\omega} B(r^*) \quad (13)$$

295 Taking derivative of the above equation twice, yields

$$296 \quad \frac{1}{r^*} (r^* g')' = \frac{1}{\gamma^2} \frac{1}{r^*} \left\{ r^* \left[\frac{1}{r^*} (r^* f')' \right]' \right\}' + 4 \frac{A_R}{\beta} A(r^*) \quad (14)$$

297 Substituting the above equation into Eq. (12b), we obtain

$$\frac{1}{r^*} \left\{ r^* \left[\frac{1}{r^*} (r^* f')' \right] \right\}' + \gamma^4 f = -\frac{\text{Re}_\omega}{\beta} A_R \text{Pr} (1 + \text{Pr}) A(r^*) + \frac{\text{Re}_\omega}{\beta} A_R \text{Pr}^2 \quad (15)$$

The solution of the above inhomogeneous equation is assumed to take the form of

$$f(r^*) = E_1 X(r^*) + E_2 Y(r^*) + f^*(r^*) \quad (16)$$

where $E_1 X(r^*) + E_2 Y(r^*)$ is the general solution of the corresponding homogeneous equation,

$$\frac{1}{r^*} \left\{ r^* \left[\frac{1}{r^*} (r^* f')' \right] \right\}' + \gamma^4 f = 0 \quad (17)$$

and $f^*(r^*)$ is a particular solution of the inhomogeneous equation (15).

In a way similar to that of the equation set (4), the solutions of the homogeneous equation (17) can also be composed by the following couple of linear combinations of Kelvin functions of the zeroth order.

$$X(r^*) = \frac{\text{ber} \gamma \text{ber}(\gamma r^*) + \text{bei} \gamma \text{bei}(\gamma r^*)}{\text{ber}^2 \gamma + \text{bei}^2 \gamma} \quad (18a)$$

$$Y(r^*) = \frac{\text{bei} \gamma \text{ber}(\gamma r^*) - \text{ber} \gamma \text{bei}(\gamma r^*)}{\text{ber}^2 \gamma + \text{bei}^2 \gamma} \quad (18b)$$

A particular solution of Eq. (15) is assumed to take the form of $f^*(r^*) = MA(r^*) + N$ considering the characteristics of Kelvin functions, where M and N are unknown constants to be determined. Substituting it into Eq. (15), one of the particular solution can be found as

$$f^*(r^*) = \frac{16}{\beta \text{Re}_\omega} \frac{\text{Pr}}{1 - \text{Pr}} A_R A(r^*) + \frac{16}{\beta \text{Re}_\omega} A_R \quad (19)$$

Therefore, $f(r^*)$ can be written as

$$f(r^*) = E_1 X(r^*) + E_2 Y(r^*) + \frac{16}{\beta \text{Re}_\omega} \frac{\text{Pr}}{1 - \text{Pr}} A_R A(r^*) + \frac{16}{\beta \text{Re}_\omega} A_R \quad (20a)$$

315 Substituting the above expressions into Eq. (13), $g(r^*)$ is obtained as

$$316 \quad g(r^*) = E_1 Y(r^*) - E_2 X(r^*) + \frac{16}{\beta \text{Re}_\omega} \frac{\text{Pr}}{1 - \text{Pr}} A_R B(r^*) \quad (20b)$$

317 Substituting Eqs. (20a) and (20b) into the boundary conditions (12c) and (12d), the constants are
318 determined as

$$319 \quad E_1 = -\frac{16}{\beta \text{Re}_\omega} \frac{1}{1 - \text{Pr}} A_R \quad (21a)$$

$$320 \quad E_2 = 0 \quad (21b)$$

321 Therefore, $f(r^*)$ and $g(r^*)$ is derived as

$$322 \quad f(r^*) = \frac{16 A_R}{\beta \text{Re}_\omega} \left[1 + \frac{\text{Pr}}{1 - \text{Pr}} A(r^*) - \frac{1}{1 - \text{Pr}} X(r^*) \right] \quad (22a)$$

$$323 \quad g(r^*) = \frac{16 A_R}{\beta \text{Re}_\omega} \left[\frac{\text{Pr}}{1 - \text{Pr}} B(r^*) - \frac{1}{1 - \text{Pr}} Y(r^*) \right] \quad (22b)$$

324 Finally the temperature profile (10) is fully determined as

$$325 \quad \Theta = \frac{d_0}{2L} x^* + \frac{16 A_R}{\beta \text{Re}_\omega} \left\{ \left[1 + \frac{\text{Pr}}{1 - \text{Pr}} A(r^*) - \frac{1}{1 - \text{Pr}} X(r^*) \right] \cos \varphi + \left[\frac{\text{Pr}}{1 - \text{Pr}} B(r^*) - \frac{1}{1 - \text{Pr}} Y(r^*) \right] \sin \varphi \right\} \quad (23)$$

326 For steady channel flow, conventionally the bulk (mixed mean) temperature, $T_b = \frac{\int_0^{r_0} u T r dr}{\int_0^{r_0} u r dr}$, is

327 used to define the Nusselt number (Nu). However, in the unsteady reciprocating flow, the mass flow rate
328 or the mean velocity becomes zero twice per cycle when the flow reversals occur, meanwhile T_b will be
329 infinitely large and lose its physical meaning, which causes anomalies of Nu values. So in this work the
330 following cross-sectional average temperature, T_m , is selected as the reference temperature for the
331 definition of Nusselt number to avoid that error, as recommended by Bouvier [13] and Gedeon [14].

$$332 \quad T_m = \frac{2}{r_0^2} \int_0^{r_0} T r dr \quad (24)$$

333 In addition, the traditional definitions of Nu for steady flow might be inappropriate, especially at high
 334 Re_ω , since phase shift exists between the heat flux and the fluid-wall temperature difference in the
 335 reciprocating unsteady flow. Therefore in this work complex parameters are adopted to define the
 336 Nusselt number, as recommended by Kornhauser [15] and Gedeon [16]. The temperature profile is
 337 rearranged with a complex form as

$$338 \quad T(r, x, t) = T_w(x) + \text{Real}[\bar{T}(r, \varphi)e^{i\varphi}] \quad (25)$$

339 Comparing it with Eqs. (8c) and (10), the complex temperature should be written as

$$340 \quad \bar{T}(r, \varphi) = (T_h - T_c) [f(r^*) - g(r^*)i] \quad (26)$$

341 Substituting Eq. (26) into Eq. (24), the complex mean temperature becomes

$$342 \quad \bar{T}_m = 2(T_h - T_c) \int_0^1 [f(r^*) - g(r^*)i] r^* dr^* \quad (27)$$

343 The complex wall temperature becomes

$$344 \quad \bar{T}_w = \bar{T}(r, \varphi) \Big|_{r^*=1} = f(1) - g(1)i = 0 \quad (28)$$

345 The complex heat flux at the solid-fluid surface is obtained by taking derivative of Eq. (26) as

$$346 \quad \vec{q}_w = -\frac{k_f}{r_0} \frac{\partial \bar{T}}{\partial r^*} \Big|_{r^*=1} = -\frac{2k_f(T_h - T_c)}{r_0} [f'(1) - g'(1)i] \quad (29)$$

347 The complex Nusselt number is defined as

$$348 \quad \overline{Nu} = \frac{d_0}{k_f} \frac{\vec{q}_w}{\bar{T}_m - \bar{T}_w} \quad (30)$$

349 Substituting Eqs. (22) and (27) ~ (29) into Eq. (30), yields

$$350 \quad \overline{Nu} = \frac{-\text{Pr} \lambda C_2 + \gamma D_2 - (\text{Pr} \lambda C_1 - \gamma D_1)i}{\frac{1}{2}(1 - \text{Pr}) + \frac{\text{Pr} C_1}{\lambda} - \frac{D_1}{\gamma} - \left(\frac{\text{Pr} C_2}{\lambda} - \frac{D_2}{\gamma} \right) i} \quad (31a)$$

351 where D_1 and D_2 are constants determined by Re_ω and Pr ,

$$D_1 = \frac{\text{ber}\gamma\text{bei}'\gamma - \text{bei}\gamma\text{ber}'\gamma}{\text{ber}^2\gamma + \text{bei}^2\gamma}, D_2 = \frac{\text{ber}\gamma\text{ber}'\gamma + \text{bei}\gamma\text{bei}'\gamma}{\text{ber}^2\gamma + \text{bei}^2\gamma} \quad (31b)$$

The major formulas used in deriving Eq. (31) are listed in Appendix B. When $\overline{Nu}$ is expressed in the form of $\text{Real}[\overline{Nu}] + \text{Imag}[\overline{Nu}]i$, the real and imaginary components of $\overline{Nu}$ can be drawn from Eq. (31a) as

$$\text{Real}[\overline{Nu}] = \frac{(-\text{Pr}\lambda C_2 + \gamma D_2) \left[\frac{1}{2}(1 - \text{Pr}) + \frac{\text{Pr}C_1}{\lambda} - \frac{D_1}{\gamma} \right] + (\text{Pr}\lambda C_1 - \gamma D_1) \left(\frac{\text{Pr}C_2}{\lambda} - \frac{D_2}{\gamma} \right)}{\left[\frac{1}{2}(1 - \text{Pr}) + \frac{\text{Pr}C_1}{\lambda} - \frac{D_1}{\gamma} \right]^2 + \left(\frac{\text{Pr}C_2}{\lambda} - \frac{D_2}{\gamma} \right)^2} \quad (31c)$$

$$\text{Imag}[\overline{Nu}] = \frac{(-\text{Pr}\lambda C_2 + \gamma D_2) \left(\frac{\text{Pr}C_2}{\lambda} - \frac{D_2}{\gamma} \right) - (\text{Pr}\lambda C_1 - \gamma D_1) \left[\frac{1}{2}(1 - \text{Pr}) + \frac{\text{Pr}C_1}{\lambda} - \frac{D_1}{\gamma} \right]}{\left[\frac{1}{2}(1 - \text{Pr}) + \frac{\text{Pr}C_1}{\lambda} - \frac{D_1}{\gamma} \right]^2 + \left(\frac{\text{Pr}C_2}{\lambda} - \frac{D_2}{\gamma} \right)^2} \quad (31d)$$

When $\overline{Nu}$ is expressed in the form of $|\overline{Nu}|e^{i\Lambda_2}$, the phase shift between the heat flux and the fluid-wall temperature difference, Λ_2 , is given as

$$\Lambda_2 = \tan^{-1} \frac{(-\text{Pr}\lambda C_2 + \gamma D_2) \left(\frac{\text{Pr}C_2}{\lambda} - \frac{D_2}{\gamma} \right) - (\text{Pr}\lambda C_1 - \gamma D_1) \left[\frac{1}{2}(1 - \text{Pr}) + \frac{\text{Pr}C_1}{\lambda} - \frac{D_1}{\gamma} \right]}{(-\text{Pr}\lambda C_2 + \gamma D_2) \left[\frac{1}{2}(1 - \text{Pr}) + \frac{\text{Pr}C_1}{\lambda} - \frac{D_1}{\gamma} \right] + (\text{Pr}\lambda C_1 - \gamma D_1) \left(\frac{\text{Pr}C_2}{\lambda} - \frac{D_2}{\gamma} \right)} \quad (31e)$$

It is worthy to note that at low Re_w (usually less than 1.0) due to the small hydraulic diameter of the flow channels (usually less than 0.2 mm), which is the case for most porous-sheets regenerators in the practical design of Stirling engines, the unsteady effect may have been overestimated, thus the conventional definition of instantaneous Nusselt number based on the quasi-steady assumption as shown below, which is useful to understand the variation of heat transfer characteristics within one cycle, might still be applicable for the most portion of a reciprocating cycle when the advection is strong (excluding the times of flow reversal) [17].

$$Nu(\varphi) = \frac{d_0}{k_f} \left| \frac{q_w}{T_m - T_w} \right| = \left| \frac{d_0}{T_w - T_m} \left(\frac{\partial T}{\partial r} \right)_{r=r_0} \right| \quad (32)$$

Substituting Eq. (23) into Eq. (32), $Nu(\varphi)$ with real values is obtained as

$$Nu(\varphi) = \frac{(\text{Pr} \lambda C_2 - \gamma D_2) \cos \varphi - (\text{Pr} \lambda C_1 - \gamma D_1) \sin \varphi}{\left[\frac{1}{2}(1 - \text{Pr}) + \frac{\text{Pr} C_1}{\lambda} - \frac{D_1}{\gamma} \right] \cos \varphi + \left(\frac{\text{Pr} C_2}{\lambda} - \frac{D_2}{\gamma} \right) \sin \varphi} \quad (33)$$

3.2 Approximate analytical solutions for the thermal entrance region

3.2.1 For hydrodynamically fully developed, but thermally developing flow

The quasi-steady assumption is adopted in the evaluation of thermal entrance effects in this paper for the reasons explained above. This is valid for low Re_w of less than 1.0, which is the generally the case in most porous-sheets Stirling regenerators. When the stacked porous-sheets form an integrated 3-stage Stirling heat exchanger, i.e. incorporating the heating, regenerating and cooling sections within one single unit, the flow field usually has already fully developed at the start of the regenerating section. The integral energy equation of thermal boundary layer in the entrance region of a circular channel can be written as

$$\frac{\partial}{\partial x} \int_0^{\delta_t} u(T - T_e)(r_0 - y) dy = \frac{q_w r_0}{\rho c_p} \quad (34)$$

Defining $\theta = T - T_w$, $\eta = y/\delta_t$, $\xi = \delta_t/r_0$, then $\theta_e = T_e - T_w$, where T_e is the external core fluid temperature, δ_t is the thickness of thermal boundary layer. According to the thermal boundary conditions, the temperature profile in the boundary layer is assumed as [18]

$$\frac{\theta}{\theta_e} = \frac{3}{2}\eta - \frac{1}{2}\eta^3 \quad (35)$$

Then the right side of Eq. (34) becomes

$$\frac{q_w r_0}{\rho c_p} = - \frac{k r_0}{\rho c_p} \frac{\partial T}{\partial y} \bigg|_{y=0} = - \frac{3}{2} \frac{a \theta_e}{\xi} \quad (36)$$

Substitutes Eqs. (35) and (36) into Eq. (34), we have

$$\begin{aligned} \frac{d}{dx} \left\{ \xi \int_0^1 B(1-\xi\eta) \cdot \left(\frac{3}{2}\eta - \frac{1}{2}\eta^3 - 1 \right) (1-\xi\eta) d\eta \right\} \cos \varphi \\ + \frac{d}{dx} \left\{ \xi \int_0^1 [1-A(1-\xi\eta)] \left(\frac{3}{2}\eta - \frac{1}{2}\eta^3 - 1 \right) (1-\xi\eta) d\eta \right\} \sin \varphi = - \frac{3}{2} \frac{a}{r_0^2 u_{\max} \xi} \end{aligned} \quad (37)$$

Integrate the above equation, an implicit solution of $\xi(x)$ is obtained as

$$\frac{x}{x_{\max}} = \frac{\text{Re}_\omega \text{Pr}}{8\beta} \xi^3 \left[\left(\frac{8}{45} - \frac{1}{8}\xi + \frac{4}{175}\xi^2 \right) \cos \varphi + \frac{\text{Re}_\omega}{32} \left(\frac{8}{45} - \frac{1}{24}\xi - \frac{12}{175}\xi^2 + \frac{5}{144}\xi^3 \right) \sin \varphi \right] \quad (38)$$

Substituting Eq. (36) into Eq. (32), the local Nusselt number is expressed as

$$Nu_x = \frac{3}{\xi} \left| \frac{\theta_e}{T_m - T_w} \right| \quad (39)$$

where the cross-sectional mean temperature according to the definition (24) is given as

$$T_m = \frac{2}{r_0^2} \left[T_e \int_0^{r_0-\delta_i} r dr + \int_{r_0-\delta_i}^{r_0} T r dr \right] \quad (40)$$

Substituting Eq. (35) into Eq. (40), yields

$$T_m = T_w + \theta_e \left[(1-\xi)^2 + \xi \int_0^1 (3\eta - \eta^3) (1-\xi\eta) d\eta \right] \quad (41)$$

Substituting Eq. (41) into Eq. (39), the transient local Nusselt number is finally obtained as

$$Nu_x = \frac{3}{\xi \left[(1-\xi)^2 + \xi \int_0^1 (3\eta - \eta^3) (1-\xi\eta) d\eta \right]} \quad (42)$$

400

3.2.2 For simultaneously developing flow

402 When the porous sheets act only as a regenerator and the Stirling engine has separate heater and
 403 cooler, the hydrodynamic and thermal boundary layers will simultaneously develop in the entrance
 404 region. The integral continuity equation of the hydrodynamic boundary layer can be written as

$$405 \quad 2 \int_0^\delta u(r_0 - y) dy + U_e (r_0 - \delta)^2 = 2u_{\max} r_0^2 \int_0^1 u^* r^* dr^* \quad (43)$$

406 where δ is the thickness of hydrodynamic boundary layer. Defining $\zeta = y/\delta$, $\sigma = \delta/r_0$, according to
 407 the flow boundary conditions, the velocity profile in the hydrodynamic boundary layer can be assumed
 408 as [18]

$$409 \quad \frac{u}{U_e} = \frac{3}{2}\zeta - \frac{1}{2}\zeta^3 \quad (44)$$

410 Substitute it into Eq. (43), the velocity of core potential flow is related to the boundary layer thickness as

$$411 \quad \frac{U_e}{u_{\max}} = \frac{\frac{1}{2}\cos\varphi + \frac{1}{48}\text{Re}_\omega \sin\varphi}{\frac{1}{2} - \frac{3}{8}\sigma + \frac{1}{10}\sigma^2} \quad (45)$$

412 The integral momentum equation of the hydrodynamic boundary layer can be written as

$$413 \quad \frac{\partial}{\partial x} \left[\int_0^\delta u(U_e - u)(r_0 - y) dy \right] + \frac{\partial U_e}{\partial x} \left[\int_0^\delta (U_e - u)(r_0 - y) dy \right] = \frac{\tau_w r_0}{\rho} \quad (46)$$

414 where

$$415 \quad \tau_w = \mu \frac{\partial u}{\partial y} \Big|_{y=0} = \frac{3\mu U_e}{2\delta} \quad (47)$$

416 Substituting Eq. (44), (45) and (47) into Eq. (46), we have

$$417 \quad \frac{\partial}{\partial x} \left[\sigma U_e^2 \int_0^1 \left(\frac{3}{2}\zeta - \frac{1}{2}\zeta^3 \right) \left(1 - \frac{3}{2}\zeta + \frac{1}{2}\zeta^3 \right) (1 - \sigma\zeta) d\zeta \right] + \sigma U_e \frac{\partial U_e}{\partial x} \int_0^1 \left(1 - \frac{3}{2}\zeta + \frac{1}{2}\zeta^3 \right) (1 - \sigma\zeta) d\zeta = \frac{3\mu U_e}{2\rho\sigma r_0^2} \quad (48)$$

418 Integrating the above equation, the implicit form of $\sigma(x)$ is obtained as

$$419 \quad \frac{x}{x_{\max}} = \frac{1}{12} \text{Re}_\omega \left(\frac{1}{2} \cos\varphi + \frac{1}{48} \text{Re}_\omega \sin\varphi \right) \left(\frac{39}{280} \sigma^2 + \frac{9}{28} \sigma^3 + \frac{1257}{8960} \sigma^4 \right) \quad (49)$$

420 The integral energy equation of the boundary layer in the entrance region is given by

$$421 \quad \frac{\partial}{\partial x} \left[\int_0^\delta u(T - T_e)(r_0 - y) dy + \int_\delta^{\delta_t} u(T - T_e)(r_0 - y) dy \right] = \frac{q_w r_0}{\rho c_p} \quad (50)$$

422 Defining $\Delta = \delta/\delta_t$, then $\Delta = \sigma/\xi < 1$ since Pr is around 0.7 for most working gas. Substituting it into Eq.
423 (50), yields

$$424 \quad \frac{\partial}{\partial x} \left\{ U_e \theta_e \delta_t \left[\Delta \int_0^1 \left(\frac{3}{2}\zeta - \frac{1}{2}\zeta^3 \right) \left(\frac{3}{2}\Delta\zeta - \frac{1}{2}\Delta^3\zeta^3 - 1 \right) (1 - \sigma\zeta) d\zeta + \int_\Delta^1 \left(\frac{3}{2}\eta - \frac{1}{2}\eta^3 - 1 \right) (1 - \xi\eta) d\eta \right] \right\} = -\frac{3}{2} \frac{a\theta_e}{\delta_t} \quad (51)$$

425 Integrating the above equation, the implicit solution of $\xi(x)$ is obtained as

$$426 \quad \frac{x}{x_{\max}} = -\frac{1}{12} \text{Re}_\omega \text{Pr} \left(\frac{1}{2} \cos \varphi + \frac{1}{48} \text{Re}_\omega \sin \varphi \right) \int_0^\xi \xi \frac{d}{d\xi} \left[\frac{-\frac{3}{8}\xi + \frac{1}{10}\xi^2 + \frac{3}{8}\Delta\xi - \frac{3}{20}\Delta^2\xi - \frac{1}{10}\Delta^2\xi^2 + \frac{1}{16}\Delta^3\xi^2}{\frac{1}{2} - \frac{3}{8}\sigma + \frac{1}{10}\sigma^2} \right] d\xi \quad (52)$$

427 The cross-sectional mean temperature, T_m and the instantaneous local Nusselt number, Nu_x have
428 the same form as Eqs. (39) ~ (42).

429

430 **4 Discussion of the analytical results**

431 **4.1 Heat transfer characteristics of the fully developed reciprocating laminar flow**

432 From the above derivation, it is found that for fully developed reciprocating laminar flow, the major
433 factors that influence the transient Nusselt number, $Nu(\varphi)$ are the kinetic Reynolds number, Re_ω and the
434 Prandtl number, Pr. The effect of Re_ω on $Nu(\varphi)$ is illustrated in Fig. 3, plotted according to Eq. (33). Two
435 discontinuous points exist in each cycle shortly after the crank angles of 90° and 270° corresponding to
436 the time of flow reversal, since the fluid-wall temperature difference, $t_w - t_m$ approaches zero, and
437 meanwhile $h(\varphi)$ and $Nu(\varphi)$ become infinite. In addition, it is shown that with the increase of Re_ω , $Nu(\varphi)$
438 varies more abruptly with φ within one cycle, and the phase shift between $Nu(\varphi)$ and $\partial p/\partial x$ also
439 increase. The patterns of $Nu(\varphi)$ variation with φ are highly similar to those obtained by Ibrahim [6] and
440 Ibrahim [17] through experiment and CFD simulation, although the Nusselt number of the “involute

441 foil” regenerator described in those literature is generally a little higher than that of the circular channel
 442 regenerator because the shapes of the “involute foil” channels are closer to the parallel plates. As can be
 443 seen from Eqs. (31) and (33), A_R has no effect on Nusselt number for the fully developed laminar
 444 reciprocating flow since the internal fluid never experiences the inflow conditions and no entrance effect
 445 exists.
 446

447
 448 **Fig. 3.** Variation of $Nu(\varphi)$ with φ at low Re_ω values ($Pr = 0.7$).
 449

450 When Re_ω is less than 2.0, the cycle-averaged Nusselt number, defined as
 451 $\overline{Nu} = \frac{1}{2\pi} \int_0^{2\pi} Nu(\varphi) d\varphi$, varies little with Re_ω and approaches a constant value of 6.0, which is exactly
 452 coincident with that of the fully developed unidirectional laminar flow, which has a theoretical Nu value
 453 of $48/11$ under the constant wall heat flux boundary condition [18], noting that T_b is used in the
 454 definition of Nu for the unidirectional steady flow, where $|(T_b - T_w)/(T_m - T_w)| = 11/8$ is valid. This

might be explained by the similarity of their thermal boundary conditions. At low Re_ω , quasi-steady state can be assumed, and the commonness is that both cases have constant axial temperature gradient, $\partial T/\partial x$, although $\partial T/\partial x$ in the case of reciprocating flow also varies periodically around a mean value of $T_w(x)$. Therefore, both cases should have similar temperature profile and corresponding Nusselt number under the low Re_ω condition.

In the design of Stirling engines, the appropriate range of Re_ω is 0.01~1.0 based on the micro-channel hydraulic diameter, d_h . According to the definition of Nusselt number ($Nu = hd_h/k_f$), when d_h is decreased, the heat transfer coefficient h will be inverse-proportionally increased, in addition to the increase of the ratio of internal area to volume, therefore the so called “scaling effect” can be utilized to enhance heat transfer.

465

466

Fig. 4. Effect of Re_ω on Complex Nusselt number ($Pr = 0.7$). A – Circular channel, $Real[\overline{Nu}]$; B – Circular channel, $Imag[\overline{Nu}]$; C – Parallel plates, $Real[\overline{Nu}] \approx 10.0$, $Imag[\overline{Nu}] \approx 0$ for $Re_\omega \ll 1$, Gedeon [14]; D – Parallel plates, $Real[\overline{Nu}] = Imag[\overline{Nu}] \approx \frac{\sqrt{2}}{2} \sqrt{Re_\omega} \frac{\sqrt{Pr} - Pr}{1 - Pr}$, for $Re_\omega \ll 1$, Gedeon [14].

470 The variation of $\overline{Nu}$ with Re_ω are plotted in Fig.4 for the typical Pr of 0.7. At low Re_ω ,
 471 especially when Re_ω is less than 10, the real part of Nusselt number is almost a constant, which is
 472 approximately equal to the real-valued Nusselt number of 6.0 for the unidirectional steady flow, while
 473 the imaginary part is negligible, so the quasi-steady assumption is valid and the conventional real-valued
 474 Nusselt number is applicable. With the increase of Re_ω , the imaginary part of Nu increases, indicating
 475 the increase of the phase shift between the fluid-wall temperature difference and the wall heat flux, and
 476 the conventional real-valued Nusselt number becomes inadequate. At high Re_ω , especially when Re_ω is
 477 larger than 10^3 , $\text{Imag}[\overline{Nu}]$ gradually approaches to $\text{Real}[\overline{Nu}]$ with the increase of Re_ω , therefore the
 478 phase shift between the fluid-wall temperature difference and the wall heat flux gradually approaches
 479 45° . The variation of $\overline{Nu}$ with Re_ω have similar patterns with those obtained by Gedeon [14]'s analysis
 480 for parallel-plates channels and Kornhauser [15]'s experimental work for an annular dead-end heat
 481 exchanger space connected to a piston-cylinder space. The correlation of Gedeon [14] for parallel-plates
 482 channels is also plotted in Fig.4 for reference. The Nusselt number for parallel-plates channel is
 483 generally higher than that for circular channels, which is similar to that for unidirectional steady flow.

484
 485 **Fig. 5.** Effect of Prandtl number on complex Nusselt number.

486

487 The variation of $\overline{Nu}$ with Re_ω under different Pr are plotted in Fig. 5. Since the commonly used
488 working gases for Stirling engine, such as helium, hydrogen, nitrogen, argon and air, all have Prandtl
489 numbers of around 0.7, only a small range of Pr (0.1~2.0) is plotted. $\overline{Nu}$ under different Pr have similar
490 patterns as shown above. At low Re_ω , especially when Re_ω is less than 10, $Real[\overline{Nu}]$ is almost
491 independent of Pr and Re_ω , and has an almost constant value of 6.0, which is similar to the case of
492 unidirectional steady flow. Both the real and imaginary components of $\overline{Nu}$ increase with the increase of
493 Pr. At very high Re_ω when the unsteady effects are dominant, $Real[\overline{Nu}]$ and $Imag[\overline{Nu}]$ approaches to
494 a common value, therefore the phase shift between the heat flux and the fluid-wall temperature
495 difference, Λ_2 , approaches to a constant value of 45° under different Pr.

496 Strictly speaking, the constant thermal property assumption is valid only when the channel is
497 sufficiently short that the effect of temperature variation on thermal properties is negligible. However,
498 the above analysis shows that when Re_ω is less than 2.0, as is generally the case in practical micro-
499 channel regenerators, the cycle-averaged Nu is almost a constant independent of Re_ω and Pr, so it is not
500 necessary to consider the effect of varying thermal properties on Nu. Of course, temperature-dependent
501 fluid conductivity should be considered when converting the Nusselt number into the convective heat
502 transfer coefficient in use. However, at high Re_ω , the Nusselt number obtained in this work can only be
503 applied on a local basis. In other words, the regenerator can be divided into several length-wise sections,
504 while the local Re_ω and Pr containing temperature-dependent thermal properties can be approximately
505 treated as a constant in each section.

506

507 4.2 The thermal entrance effects

508 In Fig. 6, the relative thickness variation of the hydrodynamic and thermal boundary layers in the
509 regenerator entrance region are plotted from Eq. (38), (49) and (52). The thermal boundary layer of the
510 “Simultaneously developing flow” is much thinner than that of the “Thermally developing flow” due to
511 the thinner hydrodynamic boundary layer. The relative thermal entrance length $L_t / x_{\max}$ in the case of
512 “Thermally developing flow” is approximately 0.6%; while in the case of “Simultaneously developing
513 flow”, $L_t / x_{\max}$ is approximately 1.0%. In Fig. 7, the axial variation of local Nusselt number, Nu_x is
514 plotted from Eq. (42) for both cases. Nu_x is higher in the entrance region and the convective heat transfer
515 is enhanced due to the thinner thermal boundary layer. Additionally, Nu_x is much higher in the case of
516 “Simultaneously developing flow” than in the case of “Thermally developing flow” due to the thinner
517 hydrodynamic boundary layer. Furthermore, for both cases, Nu_x approaches a constant value along the
518 flow direction corresponding to that of the fully developed reciprocating laminar flow.

519

520

521 **Fig. 6.** Axial development of hydrodynamic and thermal boundary layers ($Re_\omega = 1.0$, $Pr = 0.7$, $\varphi = 0^\circ$).
522 A – σ , Simultaneously developing flow; B – ξ , Simultaneously developing flow; C – ξ , Thermally
523 developing flow.

Fig. 7. Axial variation of Nu_x ($Re_\omega = 1.0$, $Pr = 0.7$, $\varphi = 0^\circ$).

4.3 Comparison with the dynamic mesh CFD results

Fig. 8. Illustration of the single-channel dynamic-mesh CFD model. A – Cold cylinder; B – Hot cylinder; C – Cooling section; L – Regenerating section; H – Heating section; 1 – Cold piston; 2 – Hot piston.

In our recent work [9], via a dynamic mesh CFD method, it is shown that for a micro-channel flow passage with a hexagonal shaped cross section in a porous-sheets Stirling regenerator, the spatial-cycle averaged Nusselt number has an almost constant value. In order for comparison, a series of dynamic mesh CFD simulation is performed in this work for a micro-channel flow passage with circular cross section in a porous-sheets Stirling regenerator. As shown in Fig. 8, the calculation domain includes a gas channel zone, a solid wall zone and two cylinder gas zones. Both the flow passage and its solid wall are divided into 3 sections, i.e. cooling, regenerating and heating. **Constant thermophysical**

properties that are the same with those used in the approximate-analytical solution are adopted in order to compare on an equal basis. Constant temperature boundary conditions of 950K and 350 K are set at the outer walls of the cooling and heating sections, respectively. All the inner walls of the 3 channel sections are set as “Coupled walls”, i.e., hydrodynamically non-slip wall satisfying the temperature continuity and heat flux continuity conditions at both sides of the wall, which are described as

$$(T_s)_w = (T_f)_w \quad (53a)$$

$$(q_s)_w = (q_f)_w \quad (53b)$$

So the conjugate heat transfer problem both in the solid matrix and in the reciprocating gas flow are simultaneously solved. All the other boundaries are set as hydrodynamically no-slip and thermally adiabatic walls. A “rigid body” motion is specified to each of the two piston walls with sinusoidal velocity profile with zero phase shift by using a compiled User Defined Function (UDF).

The calculation is performed using a commercial CFD code FLUENT 14.0 [19]. A “layering” scheme is used for the dynamic mesh method, in which the cell quantity is varying and the cell shape is deforming with time in both cylinders. The PISO scheme is used for the pressure - velocity coupling because of its high efficiency in transient flow simulation. A constant time step corresponding to a crank angle of 1° is adopted during the simulation. A grid independence test shows that when the initial mesh is created with 1.7 million cells, the resulting pressure - time curve varies less than 0.1% if further increasing the initial cell quantity. After about 10 cycles of iteration, the pressure amplitude varies less than 0.1% relative to that of the previous cycle, so the cyclic steady state can be considered as having been reached. A residual of 1×10^{-4} for the continuity equation and the momentum equations in 3 directions, and a residual of 1×10^{-7} for the energy equation are set as the convergence criteria. The local convective heat transfer coefficients are obtained by data post-processing for a specific time step using

$$h_x = \left| \frac{q_{wi}(x)}{T_{wi}(x) - T_m(x)} \right| \quad (54a)$$

where $q_{wi}(x)$ is the local average heat flux through the inner channel wall calculated by

$$q_{wi}(x) = \frac{1}{\pi d_0} \int_{\Gamma} q_{wi}(x, y, z) dl \quad (54b)$$

where Γ is the circular boundary curve of the channel cross section; $T_{wi}(x)$ is the local average temperature of the inner channel wall calculated by

$$T_{wi}(x) = \frac{1}{\pi d_0} \int_{\Gamma} T_{wi}(x, y, z) dl \quad (54c)$$

and $T_m(x)$ is the cross-sectional mean temperature calculated by

$$T_m(x) = \frac{1}{\pi r_0^2} \int_{A_c} T(x, y, z) dA \quad (54d)$$

The local Nusselt number is calculated by

$$Nu_x = \frac{h_x d_0}{k_f(x)} \quad (55)$$

Fig. 9. Axial variation of local Nusselt number when the working gas enters the micro-channel at the cold end of Stirling regenerator taken from dynamic mesh CFD simulation. ($Pr = 0.7$, $\varphi = 0^\circ$)

577 The local Nusselt number, Nu_x in the channel entrance region at both ends of the porous-sheets
578 Stirling regenerator under different Re_ω values are plotted from the CFD simulation data in Figs. 9 and
579 10, respectively. The CFD simulation results generally agree with the analytical solution for “Thermally
580 developing” reciprocating laminar flow in that the relative thermal entrance length, $L_t / x_{\max}$ has a
581 maximum deviation of only 3.3% between the two methods, while the deviation of Nu_x values are within
582 8.9% between the two methods. The relatively large difference is mainly due to the simplification made
583 in the approximate-analytical solution, in which the complicated real distribution of dimensionless
584 temperature is replaced by the polynomial profile, and also due to the assumption of constant
585 temperature (T_e) in the core flow at the thermal entrance region. All in all, the approximate-analytical
586 solution is only a convenient method to estimate the thermal entrance effect, and a full dynamic-mesh
587 CFD method is recommended when conditions are allowed. In addition, both the relative thermal
588 entrance length, $L_t/x_{\max}$ and the local Nusselt number, Nu_x increase with the increase of Re_ω , but the
589 Nusselt number has almost the same value when the fully developed state is reached for the
590 reciprocating laminar flow.
591

592

Fig. 10. Axial variation of local Nusselt number when the working gas re-enters the micro-channel at the hot end of Stirling regenerator taken from dynamic mesh CFD simulation. ($Pr = 0.7$, $\varphi = 180^\circ$)

5. Application to regenerator design and optimization

An optimization procedure adapted for the micro-channel type porous-sheets regenerator is summarized as follows, which is modified from the procedure originally presented by Hamaguchi etc. [20] that had been validated with real engine tests by Miyabe et al. [21]. The $\varepsilon \sim N_{TU}$ relation is a bridge that connects the Nusselt number obtained in this work with the regenerator optimization process, where ε is the regenerator effectiveness, and N_{TU} is the number of heat transfer unit. Taking a micro-channel type regenerator with hexagonal porous-sheets illustrated in Fig. 11 as an example, the formulas and equations used in the optimization are listed in Table 1. Constant Nusselt number is adopted since the suitable hydraulic diameter is small enough so that Re_w is usually less than 1.0, and the thermal entrance length is usually less than 1% in practical regenerator design.

Fig. 11. Geometric parameters of the hexagonal cross-sections in a porous-sheets regenerator.

(1) For any given operating conditions, including the mean pressure p_m , the hot and cold end temperature T_h and T_c , the rotational velocity n and the matrix diameter Φ , calculate the thermal properties ρ_s , ρ_f , $c_{p,f}$, c_s , k_s , k_f , μ , the mean mass flow rate $\dot{m}$, the mean fluid temperature T_m and

614 the blow time t_{blow} for half cycle.

615 (2) Assume a rib width, δ_s , then calculate Nusselt number, Nu , Biot number, Bi and Fourier number, Fo .

616 (3) Select an appropriate δ_s , to ensure that the dimensionless temperature in the solid rib, ψ satisfies

617 $0.95 \leq \psi(Bi, Fo) \Big|_{x=0, \text{ or } r=0} \leq 1$, so the thermal penetration depth is equivalent to the half rib width.

618 (4) Select the number of stacked porous sheets, N_s to satisfy $\varepsilon(N_{\text{TU}}, Cr) \geq 0.95$.

619 (5) Readjust N_s to fulfil the “Minimum total entropy generation rate” principle, or the “Maximum power

620 output” or “Maximum engine thermal efficiency” principle. The total entropy generation rate,

621 $\dot{S}_g = \dot{S}_{g,h} + \dot{S}_{g,f}$, is a measure of the available work loss within the regenerator due to irreversible heat

622 transfer and flow friction, which could be minimized for optimal engine performance. The latter

623 principle is widely described in literatures concerning the “Second order analysis” of Stirling engine

624 performance [22-25], in which the convective heat transfer between the solid matrix and the fluid flow

625 can be easily calculated via the Nusselt number obtained in this work.

626

Table 1. Equations adopted in the optimization

Parameter	Porous-sheets regenerator
ϕ (Porosity)	$\phi = \frac{b^2}{\left(b + \frac{2}{\sqrt{3}}\delta_s\right)^2} \quad (56)$
α (Specific surface area)	$\alpha = \frac{4b}{\sqrt{3}\left(b + \frac{2}{\sqrt{3}}\delta_s\right)^2} \quad (57)$
d_h (Hydraulic diameter)	$d_h = \sqrt{3}b \quad (58)$
$\overline{Nu} = hd_h / k_f$ (Nusselt number)	$Nu \approx 6.0$
Bi (Biot number)	$Bi = h\delta_s / k_s \quad (59)$
Fo (Fourier number)	$Fo = a_s t_{\text{blow}} / \delta_s^2 \quad (60)$
$\psi = (T - T_0) / (T_m - T_0)$ (Dimensionless temperature in the solid rib)	$\psi _{x=0, \text{ or } r=0} = 1 - \sum_{n=1}^{\infty} C_n \exp(-\mu_n^2 Fo) \quad (61)$ where $C_n = \frac{2 \sin \mu_n}{\mu_n + \cos \mu_n \sin \mu_n}$, $\mu_n \tan \mu_n = Bi, \quad n = 1, 2, \dots$
N_{TU} (Number of heat transfer units)	$N_{TU} = (hS) / (\dot{m}c_p) \quad (62)$
ε (Regenerator effectiveness)	$\varepsilon = \frac{1}{N_{TU}} \left\{ N_{TU} - \frac{N_{TU}}{Cr} - 1 + e^{-\left(N_{TU} + \frac{N_{TU}}{Cr}\right)} I_0\left(\frac{2N_{TU}}{\sqrt{Cr}}\right) + \int_0^{\frac{N_{TU}}{Cr}} \left(2 + \frac{N_{TU}}{Cr} - z\right) e^{-(N_{TU}+z)} I_0\left(2\sqrt{N_{TU}z}\right) dz \right\} \quad [20] \quad (63)$
Cr (Heat capacity ratio of the solid matrix to the fluid flow)	$Cr = m_s c_s / (\dot{m}c_p t_{\text{blow}}) \quad (64)$
$\Delta T = T_m - T_w $ (Fluid-wall temperature difference)	$\Delta T = \frac{Q}{S} \frac{d_h}{Nuk_f} \quad (65)$
$\dot{S}_{g,h}$ (Entropy generation rate due to irreversible heat transfer)	$\dot{S}_{g,h} = Q \left \frac{1}{T_w} - \frac{1}{T_m} \right \approx Q \frac{\Delta T}{T_m^2} \quad (66)$
$C_f = \frac{\tau_w}{\frac{1}{2}\rho u_{\text{max}}^2}$ (Fanning flow friction coefficient)	$C_{f,\infty,\text{ave}} = \frac{64F_\omega}{\pi A_o}, \quad F_\omega = \frac{\sqrt{C_1^2 + C_2^2}}{16\sqrt{(\alpha - 2C_1)^2 + 4C_2^2}} \quad [11] \quad (67)$
$(\Delta p)_f$ (Flow friction loss)	$(\Delta p)_f = \frac{4L}{\sqrt{3}b} C_{f,\infty,\text{ave}} \frac{1}{2} \rho u_{\text{max}}^2 \quad (68)$
$\dot{S}_{g,f}$ (Entropy generation rate due to flow friction loss)	$\dot{S}_{g,f} = \frac{1}{2} \rho u_{\text{max}}^2 C_{f,\infty,\text{ave}} S u_m \frac{1}{T_m} \quad (69)$
$\dot{S}_g$ (Total entropy generation rate)	$\dot{S}_g = \dot{S}_{g,h} + \dot{S}_{g,f} \quad (70)$

630 Taking an arbitrary example, given the operating condition of $T_h = 900$ K, $T_c = 400$ K, $p_m = 1.5$
 631 MPa, $n = 3000$ rpm, $\Phi = 25$ mm. The optimized geometry and performance parameters for a hexagonal
 632 porous-sheets regenerator are listed in [Table 2](#).
 633

634 **Table 2** The geometric parameters of optimized porous sheets. Sheet thickness $\delta_t = 0.1$ mm.

$2\delta_w$ [mm]	b [mm]	ϕ	α [mm ² /mm ³]	N_s	ε	$(\Delta p)_f$ (kPa)	$\dot{S}_{g,f}$ (W/K)	$\dot{S}_{g,h}$ (W/K)	$\dot{S}_g$ (W/K)
0.022	0.060	0.681	26.22	147	0.985	7.875	0.0215	0.0214	0.0429

635

636 6. Conclusion

637 In this work, the heat transfer characteristics of the reciprocating laminar flow in a micro-channel
 638 type porous-sheets Stirling regenerator including the entrance effects are analytically or quasi-
 639 analytically investigated to facilitate the efficient design and optimization of Stirling engines, from
 640 which the following major conclusions can be drawn.

641 (1) For fully developed reciprocating laminar flow, $\overline{Nu}$ is expressed as a function of Re_ω and Pr .
 642 At low Re_ω of less than 10, $Real[\overline{Nu}]$ and has an almost constant value of 6.0, approximately equal to
 643 the known real-valued Nu for the fully developed unidirectional laminar flow under constant wall heat
 644 flux, while $Imag[\overline{Nu}]$ is negligible, thus “scaling effect” can be utilized to enhance heat transfer. At
 645 higher Re_ω , both $Real[\overline{Nu}]$ and $Imag[\overline{Nu}]$ increase with the increase of Re_ω and Pr , and the phase
 646 shift between the fluid-wall temperature difference and the wall heat flux gradually increases and
 647 approaches 45°.

648 (2) Heat transfer is enhanced in the entrance region, and Nu_x axially approaches the constant
 649 value of the fully developed flow. Compared to the “Thermally developing flow”, the “Simultaneously

developing flow” has much thinner thermal boundary layer, higher Nu_x and longer thermal entrance length.

(3) The analytical solutions **generally** agree with the dynamic mesh CFD results, and the obtained $Nu \sim Re_\omega$ data and patterns generally agree with available analytical and experimental data from published literatures.

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Appendix A

The 0-order Kelvin functions can be expanded into the following power series.

$$\text{ber}(\lambda r^*) = \sum_{n=0}^{\infty} \frac{(-1)^n \left(\frac{\lambda r^*}{2}\right)^{4n}}{[(2n)!]^2}, \quad \text{bei}(\lambda r^*) = \sum_{n=0}^{\infty} \frac{(-1)^n \left(\frac{\lambda r^*}{2}\right)^{4n+2}}{[(2n+1)!]^2} \quad (\text{A.1})$$

The following characteristics can be proved by using the above formulas.

$$\lambda \int_0^1 r^* \text{ber}(\lambda r^*) dr^* = \text{bei}'\lambda = \lambda \sum_{n=0}^{\infty} \frac{(-1)^n \left(\frac{\lambda}{2}\right)^{4n}}{[(2n)!]^2 (4n+2)} \quad (\text{A.2})$$

$$-\lambda \int_0^1 r^* \text{bei}(\lambda r^*) dr^* = \text{ber}'\lambda = -\lambda \sum_{n=0}^{\infty} \frac{(-1)^n \left(\frac{\lambda}{2}\right)^{4n+2}}{[(2n+1)!]^2 (4n+4)} \quad (\text{A.3})$$

Then the following integrations can be obtained.

$$\begin{aligned}
\int_0^1 B(r^*) r^* dr^* &= \frac{\text{bei}\lambda \int_0^1 r^* \text{ber}(\lambda r^*) dr^* - \text{ber}\lambda \int_0^1 r^* \text{bei}(\lambda r^*) dr^*}{\text{ber}^2\lambda + \text{bei}^2\lambda} \\
&= \frac{1}{\lambda} \frac{\text{ber}\lambda \text{ber}'\lambda + \text{bei}\lambda \text{bei}'\lambda}{\text{ber}^2\lambda + \text{bei}^2\lambda} \\
&= \frac{1}{\lambda} C_2
\end{aligned} \tag{A.4}$$

$$\begin{aligned}
\int_0^1 A(r^*) r^* dr^* &= \frac{\text{ber}\lambda \int_0^1 r^* \text{ber}(\lambda r^*) dr^* + \text{bei}\lambda \int_0^1 r^* \text{bei}(\lambda r^*) dr^*}{\text{ber}^2\lambda + \text{bei}^2\lambda} \\
&= \frac{1}{\lambda} \frac{\text{ber}\lambda \text{bei}'\lambda - \text{bei}\lambda \text{ber}'\lambda}{\text{ber}^2\lambda + \text{bei}^2\lambda} \\
&= \frac{1}{\lambda} C_1
\end{aligned} \tag{A.5}$$

Then [Eq. \(6\)](#) can be derived as follows.

$$\begin{aligned}
u_m &= \frac{1}{\pi r_0^2} \int_0^{r_0} u \cdot 2\pi r dr \\
&= \frac{2K}{\omega} \left\{ \cos\varphi \int_0^1 B(r^*) r^* dr^* + \sin\varphi \left[\frac{1}{2} - \int_0^1 A(r^*) r^* dr^* \right] \right\} \\
&= \frac{K}{\omega\lambda} [2C_2 \cos\varphi + (\lambda - 2C_1) \sin\varphi] \\
&= \frac{K}{\omega\lambda} \sqrt{(\lambda - 2C_1)^2 + 4C_2^2} \cos(\varphi - \Lambda_1) \\
&= u_{\max} \cos(\varphi - \Lambda_1)
\end{aligned} \tag{A.6}$$

Appendix B

The major formulas used in deriving [Eq. \(31\)](#) in addition to [Eqs. \(A.4\)](#) and [\(A.5\)](#) are as follows.

$$\int_0^1 X(r^*) r^* dr^* = \frac{1}{\lambda} D_1, \quad \int_0^1 Y(r^*) r^* dr^* = \frac{1}{\lambda} D_2 \tag{A.7}$$

$$A'(1) = \lambda C_2, \quad B'(1) = -\lambda C_1 \tag{A.8}$$

$$X'(1) = \gamma D_2, \quad Y'(1) = -\gamma D_1 \tag{A.9}$$

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