Optimal Growth Through Product Innovation*

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Abstract

In Lentz and Mortensen (2008), we formulate and estimate a market equilibrium model of endogenous growth through product innovation. In this paper, we provide quantitative equilibrium solutions to the model based on our parameter estimates, and compare them with a social planner's solution. We find that the socially optimal growth rate is double that of the market equilibrium growth rate when firm differences in the ability to create productive products are highly persistent as a consequence of the "business stealing" externality present in the model. The welfare loss of the decentralized economy relative to that of the planner is equivalent to a 21% tax on the planner consumption path. The planner's solution differs from the decentralized economy in that it discourages innovation by low ability innovators as well as entry.

We introduce two mechanisms that temper the strength of the negative externality: Transitory firm types and the possibility of buyouts. We show that both sharply reduce the inefficiency due to innovation by low ability innovators. But with caveats: If firm types are completely transitory, then the notion of firm heterogeneity is for practical purposes lost. Buyouts improve efficiency, but the efficiency gain depends significantly on the strength of the innovator's ability to extract rents from incumbents through the buyout.

Keywords: Optimal growth, planner's problem, product innovation, innovation spill overs, creative-destruction externality.

JEL codes: E22, E24, J23, J24, L11, L25.

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1 Introduction

In Lentz and Mortensen (2008), we formulate and estimate a structural market equilibrium model of growth through product innovation. The model is an extended version of that proposed by Klette and Kortum (2004) originally designed to explain the link between innovation investment and the size distribution of firms. Their framework in turn is an elaboration of the Aghion and Howitt (1992) and Grossman and Helpman (1991) model of endogenous growth through creative-destruction combined with the firm dynamics insights in Hopenhayn (1992).

In our version of the model, firms differ with respect to the quality of the intermediate products they create. We find that heterogeneity in this sense is needed to explain the cross firm correlations between valued added, labor force size and labor productivity. One implication of this form of heterogeneity is that labor reallocation from slower growing firms to faster growing firms that create more profitable higher quality products plays an important role in determining the aggregate growth rate.

The purpose of this paper is to explore the quantitative welfare implications of our estimated model. Namely, we formulate and compute the socially optimal R&D strategy for the modeled environment and compare its implications for the performance with that implied by the market equilibrium. The analysis simplifies elements of the Lentz and Mortensen (2008) estimate that are of secondary importance for the welfare properties of the model. To this we add the possibility of transitory firm types as well as transactions between firms to buy out innovations in protection of existing markets. While the Danish data are supportive of firm types being persistent, it is plausible that they are not permanent. We demonstrate that the efficiency properties of the equilibrium are sensitive to the degree of type persistence. Likewise, the analysis also shows that protection of existing markets through buyouts can also greatly affect the performance of the market equilibrium, giving another dimension to the analysis of mergers and acquisitions.

The market equilibrium solution to the model need not be socially optimal for three different reasons spelled out in Grossman and Helpman (1991). First, the most recent product innovator has monopoly power and use it to set prices above the marginal cost of production. Second, every innovation replaces an older version of some product and by doing so truncates the stream of quasi rents accruing to its previous innovator. Finally, because each new improvement builds on those of previous innovators, innovation has a positive “spill-over” effect on future productivity which is not fully captured by the innovator in a market equilibrium. The net deviation of the equilibrium growth rate from that which is socially optimal is unclear. One of the contributions of a quantitative equilibrium model is its ability to reflect light on the relative magnitudes of these effects.

We begin by characterizing the market and planner’s solution. We then use the parameter estimates obtained by Lentz and Mortensen (2008) to compute the implications of both for aggregate growth and economic welfare. If

firm differences with respect to creative ability are highly persistent, then the planner's optimal innovation investment strategy yields a growth rate which is much larger than that obtained in market equilibrium. However, the difference between market equilibria and optimal growth rates are much smaller if firm heterogeneity is not very persistent.

Finally, the analysis considers the possible market response to inefficient innovation across firms where incumbents buy out low type innovators. We show that the buy out option can greatly improve welfare when types are persistent.

2 The Model

Firms come in an amazing range of shapes and sizes. This fact cannot be ignored in any analysis of the relationship between firm size and factor productivity. Furthermore, an adequate theory must account for entry, exit, and firm evolution in order to explain observed size distributions. Klette and Kortum (2004) construct a stochastic model of firm product innovation and growth that is consistent with stylized facts regarding the firm size evolution and distribution. In Lentz and Mortensen (2008), we find that an extension of their model that allows for cross firm heterogeneity in the quality of innovations is needed to explain our Danish data.

2.1 Preferences and Technology

The utility of the representative household at time t is given by

$$U_t = \int_t^\infty \ln C_s e^{-\rho(s-t)} ds \quad (1)$$

where $\ln C_t$ denotes the instantaneous utility of the single consumption good at date t and ρ represents the pure rate of time discount. Each household is free to borrow or lend at interest rate r_t . Nominal household expenditure at date t is $E_t = P_t C_t$. Optimal consumption expenditure must solve the differential equation $\dot{E}/E = r_t - \rho$. Following Grossman and Helpman (1991), we choose the numeraire so that $E_t = 1$ for all t without loss of generality, which implies $r_t = r = \rho$ for all t . Note that this choice of the numeraire also implies that price of the consumption good, P_t , falls over time at a rate equal to the rate of growth in consumption.

The quantity of the consumption produced is determined by the quantity and quality of the economy's intermediate inputs. Specifically, there is a unit continuum of intermediate good inputs and consumption is determined by the production function

$$\ln C_t = \int_0^1 \ln(A_t(j)x_t(j))dj = \ln A_t + \int_0^1 \ln x_t(j)dj \quad (2)$$

where $x_t(j)$ is the quantity of input $j \in [0, 1]$ at time t , $A_t(j)$ is the productivity of input j at time t , and A_t represents aggregate productivity. The level of

productivity of each input and aggregate productivity are determined by the number of technical improvements made in the past. Specifically,

$$A_t(j) = \prod_{i=1}^{J_t(j)} q_i(j) \text{ and } \ln A_t \equiv \int_0^1 \ln A_t(j) dj. \quad (3)$$

where $J_t(j)$ is the number of innovations made in intermediate input j up to date t and $q_i(j) > 1$ denotes the quantitative improvement (step size) in productivity attributable to the i^{th} innovation in product j . Innovations arrive at rate δ which is endogenous in market equilibrium and the same for all intermediate products.

2.2 The Behavior of a Firm

Because intermediate goods of the same type are perfect substitutes, the creator of the latest most productive version is the sole supplier at any point in time. The price charged is limited by the ability of suppliers of previous versions to supply one of equal value. In Nash-Bertrand equilibrium, any successful innovator takes over the market for its good type by setting the price just below that at which final good producers are indifferent between the new more productive product supplied by the innovator and the alternative supplied by the previous provider. The price charged is the product of the relative productivity of the innovation and the previous producer's marginal cost of production. Given the symmetry of demands for the different good types and the assumption that future innovation likelihoods are independent of the type of good, one can drop the good subscript without confusion. Given stationarity of quantities along the equilibrium growth path, the time subscript can be dropped as well.

Labor and capital, in fixed proportions, are used in the production of intermediate inputs to the final goods production process. Labor productivity is the same across all intermediate products and is set equal to unity. The required capital expressed in units of output, a constant κ , is also the same for all products. For the equilibrium wage w per unit of labor, the operating profit per unit obtained from supplying an intermediate product at price p is $p(1 - \kappa) - w$ which implies that the lowest price that the previous supplier is willing to charge, that which yields no profit, is $w/(1 - \kappa)$. Let the current quality leader's productive advantage relative to the previous supplier be $q > 1$, where firm type heterogeneity is reflected in q dispersion across firms. The quality leader will charge price $p = qw/(1 - \kappa)$ because consumers are exactly indifferent between buying from the quality leader at this price and the zero profit price of the previous supplier. As profit maximizing by the competitive suppliers of the consumption good requires

$$P_t \frac{\partial C_t}{\partial x_t(j)} = \frac{P_t C_t}{x_t(j)} = p_t(j)$$

and $P_t C_t = E_t = 1$ by choice of the numeraire, the demand function for any product and the labor requirement for its production is

$$x = \frac{1}{p} = \frac{1 - \kappa}{wq}. \quad (4)$$

Hence, the gross profit associated with supplying the good of quality q is

$$\pi(q) = p(1 - \kappa)x - wx = (1 - \kappa)(1 - q^{-1}). \quad (5)$$

The quality conditional profit level is the core determinant of the firm's incentives to innovate. A greater q type firm expects a greater ex post profit flow from an innovation and will invest more heavily in its realization.

Following Klette and Kortum (2004), the discrete number of products supplied by a firm, denoted as k , is defined on the integers and its value evolves over time as a birth-death process reflecting product creation and destruction. In their interpretation, k reflects the firm's past successes in the product innovation process as well as current firm size. New products are generated by R&D investment. The firm's R&D investment flow generates new product arrivals at frequency γk . The total R&D investment cost is $wc(\gamma)k$ where $c(\gamma)k$ represents the labor input required in the research and development process. The function $c(\gamma)$ is assumed to be strictly increasing and convex. According to the creators of the model, the assumption that the total cost of R&D investment is linear in the number of existing product, given the innovation rate per product line, "captures the idea that a firm's knowledge capital facilitates innovation." In other words, the current state of innovation capital for a continuing firm is equal to the number of its current products that are at the technological frontier. In any case, this cost structure is needed to obtain firm growth rates that are independent of size, as typically observed in the data.

The market for any version of an intermediate product currently supplied is destroyed, as well as a unit of its creation capital, by the creation of a new more productive alternative by some other firm. This event occurs with frequency δ independent of the product type. Below we refer to γ as the firm's product innovation rate and to δ as the common creative-destruction rate faced by all firms. The firm chooses γ to maximize the expected present value of its future net profit flow. Of course, the R&D strategies of all incumbent firms and potential entrants determine the equilibrium value of δ .

There are persistent but generally temporary differences with respect to the improvement in productivity (the quality of an innovation q) offered by the products that a firm creates. For simplicity, we assume that the a firm's identity changes from time to time. Specifically, identity is a Markov chain defined on two states, denoted as $j = 0$ and 1. Without loss of generality, the quality of a product created by a firm in state $j = 1$ is larger. Formally, a firm creates higher quality products in state 1 in the sense that $q_1 > q_0$. Finally, the given exogenous transition rate out of type state j is λ_j . Because a firm's creativity type is transitory, the firm may be supplying products of both qualities at any point in time.

2.3 The Value Function of the Firm

Given that the innovator always takes over the market for the new version of the product it has created, the value of a firm that currently supplies k_0 products

of low quality and k_1 high quality products is the solution to the asset pricing equation

$$\begin{aligned} rV_0(k_0, k_1) = & \max_{\gamma \geq 0} \left\{ k_0\pi_0 + k_1\pi_1 - wc(\gamma)k + \gamma k [V_0(k_0 + 1, k_1) - V_0(k_0, k_1)] \right. \\ & + \delta [k_0V_0(k_0 - 1, k_1) + k_1V_0(k_0, k_1 - 1) - kV_0(k_0, k_1)] \\ & \left. + \lambda_0 [V_1(k_0, k_1) - V_0(k_0, k_1)] \right\} \end{aligned} \quad (6)$$

$$\begin{aligned} rV_1(k_0, k_1) = & \max_{\gamma \geq 0} \left\{ k_0\pi_0 + k_1\pi_1 - wc(\gamma)k + \gamma k [V_1(k_0, k_1 + 1) - V_1(k_0, k_1)] \right. \\ & + \delta [k_0V_1(k_0 - 1, k_1) + k_1V_1(k_0, k_1 - 1) - kV_1(k_0, k_1)] \\ & \left. + \lambda_1 [V_0(k_0, k_1) - V_1(k_0, k_1)] \right\} \end{aligned} \quad (7)$$

where $k \equiv k_0 + k_1$ is the total number supplied and the subscript represent's the firm's current type and $\pi_i = \pi(q_i)$ for $i = 0, 1$ with $\pi(q)$ defined in equation (5). In each case, the first three terms on the right sides of (6) and (7) represent the cash flow obtained by supplying the portfolio of current products net of the current expenditure on R&D. The next three terms are respectively the expected charge in value attributable to the arrival of a new product line, the destruction of the existing product line, and a transition to the other creativity state.

The unique solution to (6) and (7) takes the form

$$V_i(k_0, k_1) = \frac{k_0\pi_0 + k_1\pi_1}{r + \delta} + k\Psi_i, \quad i = 0, 1 \text{ where} \quad (8)$$

$$\Psi_i = \max_{\gamma \geq 0} \left\{ \frac{\gamma \left(\frac{\pi_i}{r + \delta} + \Psi_i \right) - wc(\gamma) + \lambda_i \Psi_j}{r + \delta + \lambda_i} \right\}, \quad j = 1 - i, \quad (9)$$

as one can verify by substitution. Obviously, the first term on the right side of (8) represents the expected present value of the future profit streams generated by the firm's current products. The second term is the value of the firm R&D operation. One can think of there being k research operations, each associated with an existing product line, and regard Ψ_j as the asset price assigned to each operation when in creativity state j . That price is the expected present value of the net profit generated by the choice of the current innovation frequency γ .

The fact that a type 1 innovation yields a higher profit rate implies that the value of R&D is higher when the firm is high creativity state.

Proposition 1. $\pi_1 > \pi_0$ implies $\Psi_1 > \Psi_0$ if $c(\gamma)$ is strictly convex and $c(0) = c'(0) = 0$.

Proof. To the contrary, suppose that $\Psi_0 - \Psi_1 \geq 0$. Under the supposition,

equations (8) and (9), and $\pi_1 > \pi_0$ imply the following contradiction,

$$\begin{aligned}\Psi_1 &= \max_{\gamma \geq 0} \left\{ \frac{\gamma \frac{\pi_1}{r+\delta} - wc(\gamma) + \lambda_1 (\Psi_0 - \Psi_1)}{r + \delta - \gamma} \right\} \\ &\geq \max_{\gamma \geq 0} \left\{ \frac{\gamma \frac{\pi_1}{r+\delta} - wc(\gamma)}{r + \delta - \gamma} \right\} > \max_{\gamma \geq 0} \left\{ \frac{\gamma \frac{\pi_0}{r+\delta} - wc(\gamma)}{r + \delta - \gamma} \right\} \\ &\geq \max_{\gamma \geq 0} \left\{ \frac{\gamma \frac{\pi_0}{r+\delta} - wc(\gamma) + \lambda_0 (\Psi_1 - \Psi_0)}{r + \delta - \gamma} \right\} = \Psi_0.\end{aligned}$$

□

As the value of doing so is always positive, the choice of the state contingent innovation rate satisfies

$$wc'(\gamma_i) = v_i \equiv \frac{\pi_i}{r + \delta} + \Psi_i, \quad (10)$$

under the hypothesis. The sufficient second order condition ($c''(\gamma) > 0$) and Proposition 1 imply that firm's innovate more frequently while in the high creativity state. In other words, $\gamma_1 > \gamma_0$.

2.4 Entry and Labor Market Clearing

The entry of a new firm requires a successful innovation by a potential entrant. Suppose that there is a constant measure m of identical potential entrants. The rate at which any one of them generates a new product is γ_e and the total cost is $mc(\gamma_e)$ where the cost function is the same as that faced by an incumbent. The new firm's type is unknown ex ante but is realized immediately after the arrival of an innovation. Since the aggregate entry rate is $\eta = m\gamma_e$, the entry rate satisfies the following free entry condition

$$wc'(\gamma_e) = V_0(1, 0)\phi_0 + V_1(0, 1)\phi_1 = v_0\phi_0 + v_1\phi_1 \quad (11)$$

where ϕ_i is the probability that the entrant will turn out to be of creativity type i initially.

There is a fixed measure of available workers, denoted by ℓ , seeking employment at any positive wage. In equilibrium, these are allocated across production and R&D activities, those performed by both incumbent firms and potential entrants. The number of workers required per product of type $i = 0, 1$ is $x_i = 1/p_i = (1 - \kappa)/q_i w$ from equation (4). The number of R&D workers employed per product by incumbent firms of type $j = 0, 1$ is $\ell_R(j) = c(\gamma_j)$. Because each potential entrant innovates at frequency γ_e the aggregate number of workers engaged by all m in R&D is $\ell_E = mc(\gamma_e)$. Hence, the equilibrium wage satisfies the labor market clearing condition

$$\ell = \frac{1 - \kappa}{w} \left(\frac{K^0}{q_0} + \frac{K^1}{q_1} \right) + K_0 c(\gamma_0) + K_1 c(\gamma_1) + mc(\gamma_e) \quad (12)$$

where $K^i \equiv K_{i0} + K_{i1}$ represents the fraction of products of productivity q_i and $K_j \equiv K_{0j} + K_{1j}$ denotes the fraction supplied by firms in state j .

2.5 Aggregate Dynamics

The overall state of the aggregate economy can be represented by the joint distribution of products over product types and firm states. Recall that K_{ij} represent the number of products of type $i \in \{0, 1\}$ supplied by firms in creativity state $j \in \{0, 1\}$. Because every firm leaves state i at rate λ_i , the following laws of motion hold.

$$\dot{K}_{11} = \lambda_0 K_{10} + \eta \phi_1 + \gamma_1 (K_{11} + K_{01}) - (\delta + \lambda_1) K_{11} \quad (13)$$

$$\dot{K}_{01} = \lambda_0 K_{00} - (\delta + \lambda_1) K_{01} \quad (14)$$

$$\dot{K}_{10} = \lambda_1 K_{11} - (\delta + \lambda_0) K_{10} \quad (15)$$

$$\dot{K}_{00} = \lambda_1 K_{01} + \eta \phi_0 + \gamma_0 (K_{00} + K_{10}) - (\delta + \lambda_0) K_{00} \quad (16)$$

where ϕ_j is the fraction of firms that enter in creativity state $j = 0, 1$. The change in a stock equals inflow minus outflow. The inflow into the K_{11} stock in equation (13) has three sources: First is the type $i = 1$ products formerly produced by creativity state $j = 0$ firms that have switched creativity state. Second is the entry of high type entrants. Third is the creation of new product lines by creativity state $j = 1$ firms where innovation is proportional to their product line size. The outflow consists of product destruction and type $i = 1$ product lines formerly produced by creativity state $j = 1$ firms where the firm has changed creativity state. The other laws of motion follow similar arguments.

By implication,

$$\dot{K}_1 = \dot{K}_{01} + \dot{K}_{11} = \lambda_0 K_0 + \eta \phi_1 + \gamma_1 K_1 - (\delta + \lambda_1) K_1 \quad (17)$$

$$\dot{K}_0 = \dot{K}_{00} + \dot{K}_{10} = \lambda_1 K_1 + \eta \phi_0 + \gamma_0 K_0 - (\delta + \lambda_0) K_0. \quad (18)$$

Since every new product induces the destruction of an existing one, $\dot{K}_0 + \dot{K}_1 = 0$, the aggregate creative-destruction rate is

$$\delta = \eta + \gamma_1 K_1 + \gamma_0 K_0. \quad (19)$$

To find the steady state solution, use equation (19) to substitute out δ in equation of (18). Then use the result

$$\dot{K}_0 = \lambda_1 K_1 - K_0 [\gamma_1 K_1 + \eta \phi_1 + \lambda_0] = 0,$$

which provides the steady state level of $K_1 = 1 - K_0$. As this expression defines a strictly increasing relationship between K_0 and K_1 that passes through the origin, the fact that $K_0 + K_1 = 1$ implies that a unique positive steady state solution for the pair (K_0, K_1) exists for any set of innovation and transitions rates.

For equations (2) and (3), the contribution of any innovation to the aggregate rate of consumption growth is the natural log of its improvement in productivity over the previous version. Hence, the growth rate is,

$$g = \frac{\dot{C}}{C} = m \gamma_e (\phi_0 \ln q_0 + \phi_1 \ln q_1) + \gamma_0 K_0 \ln q_0 + \gamma_1 K_1 \ln q_1. \quad (20)$$

2.6 Market Equilibrium

A steady state *market equilibrium* is a triple composed of a labor market clearing wage w , entry rate η , and creative destruction rate δ together with an optimal creation rate for each firm type, γ_j , $j \in (0, 1)$ and a distribution of products across product and firm types, K_{ij} , $(i, j) \in \{0, 1\}^2$, that satisfy equations (8), (11), (12), (19), and the steady state conditions implied by equations (13)-(16).

3 The Social Planner's Problem

3.1 Formulation

The social planner chooses a non-negative time paths for the production rate of both product types, x_i , the rate of new product creation for each firm creativity state, γ_j , and the rate of product innovation by potential entrants, γ_e to maximize the present discounted utility of the representative household's consumption subject to the fact that there are a fixed number of intermediate products, a labor resource constraint, and laws of motion for the state variables. Under complete and symmetric information, both the firm and the planner observes the realized productivity of any innovation. In anticipation of the buyout extension of the model, the social planner can also be given the option of not adopting an innovation. Denote by $a_j = 1$ the decision to adopt the innovation of a type j firm. Otherwise, $a_j = 0$. It will always be optimal for the planner to adopt type 1 products, that is, $a_1 = 1$. Thus, the analysis will only be concerned with the choice of $a_0 = a$.

The planner's strategy, the choice of $(x_0(t), x_1(t), \gamma_0(t), \gamma_1(t), \gamma_e(t), a(t))$ that maximizes the expected present value of the representative consumer's utility stream subject to a set of constraints. Formally, the criterion is

$$\int_0^\infty \ln C_t e^{-rt} dt = \int_0^\infty \left[\ln A_t + \sum_{i=0}^1 \sum_{j=0}^1 \ln(x_i(t)) K_{ij}(t) \right] e^{-rt} dt,$$

where A represents aggregate productivity and K_{ij} is the fraction of products of type i currently supplied by incumbent firms in creativity state j . In addition, assume that the planner's discount rate $\rho = r$. The constraints follow: Employment cannot exceed the available labor supply,

$$\ell \geq \sum_{i=0}^1 \sum_{j=0}^1 [x_i + c(\gamma_j)] K_{ij} + mc(\gamma_e).$$

The entry rate is $\eta = m\gamma_e$. As a consequence, by equation (20), the law of motion for aggregate productivity can be written

$$\frac{d \ln A}{dt} \equiv g = m\gamma_e [a\phi_0 \ln q_0 + \phi_1 \ln q_1] + a\gamma_0 K_0 \ln q_0 + \gamma_1 K_1 \ln q_1,$$

where the adoption decision is now explicitly included. By equations (17)-(19), the distribution of products across firm state evolves according to

$$\begin{aligned}\dot{K}_0 &= \lambda_1 K_1 + am\gamma_e \phi_0 + a\gamma_0 K_0 - (am\gamma_e \phi_0 + a\gamma_0 K_0 + m\gamma_e \phi_1 + \gamma_1 K_1 + \lambda_0) K_0 \\ \dot{K}_1 &= \lambda_0 K_0 + m\gamma_e \phi_1 + \gamma_1 K_1 - (am\gamma_e \phi_0 + a\gamma_0 K_0 + m\gamma_e \phi_1 + \gamma_1 K_1 + \lambda_1) K_1.\end{aligned}$$

Although the complete planner's problem includes laws of motion for every K_{ij} , only K_0 and K_1 are decision relevant state variables for reasons that will become apparent.

The planner's problem is a relatively standard one in dynamic control. The constraint augmented present value Hamiltonian for the problem can be written as,

$$\begin{aligned}H &= \ln A + \sum_i \sum_j \ln(x_i) K_{ij} \\ &+ \omega \left(\ell - \sum_i \sum_j [x_i + c(\gamma_j)] K_{ij} - mc(\gamma_e) \right) \\ &+ \Lambda [m\gamma_e (a\phi_0 \ln q_0 + \phi_1 \ln q_1) + a\gamma_0 K_0 \ln q_0 + \gamma_1 K_1 \ln q_1] \\ &+ v_0 [\lambda_1 K_1 + m\gamma_e a\phi_0 + \gamma_0 aK_0 - [m\gamma_e (a\phi_0 + \phi_1) + \gamma_0 aK_0 + \gamma_1 K_1 + \lambda_0] K_0] \\ &+ v_1 [\lambda_0 K_0 + m\gamma_e \phi_1 + \gamma_1 K_1 - [m\gamma_e (a\phi_0 + \phi_1) + \gamma_0 aK_0 + \gamma_1 K_1 + \lambda_1] K_1],\end{aligned}$$

where ω is the labor supply constraint multiplier, Λ is the shadow price of the state variable $\ln A$ and v_j is the shadow price associated with K_j , $j \in \{0, 1\}$.

As the optimal controls maximize the Hamiltonian given state and co-state variables, the first order necessary conditions for all the continuous choice variables are

$$\frac{\partial H}{\partial x_i} = \left(\frac{1}{x_i} - \omega \right) K^i = 0, \text{ for all } i \in \{0, 1\}. \quad (21)$$

$$\frac{\partial H}{\partial \gamma_0} = [a\Lambda \ln q_0 + av_0 - av_0 K_0 - av_1 K_1 - \omega c'(\gamma_0)] K_0 \leq 0 \text{ (with } = \text{ if } \gamma_0 > 0) \quad (22)$$

$$\frac{\partial H}{\partial \gamma_1} = [\Lambda \ln q_1 + v_1 - v_1 K_1 - v_0 K_0 - \omega c'(\gamma_1)] K_1 \leq 0 \text{ (with } = \text{ if } \gamma_1 > 0) \quad (23)$$

$$\begin{aligned}\frac{\partial H}{\partial \gamma_e} &= \left[\Lambda [a\phi_0 \ln q_0 + \phi_1 \ln q_1] + v_0 [(1 - K_0) a\phi_0 - K_0 \phi_1] \right. \\ &\quad \left. + v_1 [(1 - K_1) \phi_1 - K_1 a\phi_0] - \omega c'(\gamma_e) \right] m \leq 0 \text{ (with } = \text{ if } \gamma_e > 0). \quad (24)\end{aligned}$$

The assumption that the cost of R&D is convex in the innovation rate is sufficient to guarantee that the second order necessary conditions are satisfied.

Because they enter symmetrically in the production of the consumption good and they cost the same at the margin, the first equation of (21) implies that all intermediate products should be supplied at the same rate. Formally,

$$x_i = x = \frac{1}{\omega}, \quad i \in (0, 1). \quad (25)$$

Given this result, the co-state (Euler) equations are

$$\frac{\partial H}{\partial \ln A} = 1 = r\Lambda - \dot{\Lambda} \quad (26)$$

$$\begin{aligned} \frac{\partial H}{\partial K_0} = & \ln x - \omega[x + c(\gamma_0)] + (\Lambda \ln q_0 + v_0(1 - K_0) - v_1 K_1) a \gamma_0 \\ & + (v_1 - v_0) \lambda_0 - \delta v_0 = r v_0 - \dot{v}_0 \end{aligned} \quad (27)$$

$$\begin{aligned} \frac{\partial H}{\partial K_1} = & \ln x - \omega[x + c(\gamma_1)] + (\Lambda \ln q_1 + v_1(1 - K_1) - v_0 K_0) \gamma_1 \\ & + (v_0 - v_1) \lambda_1 - \delta v_1 = r v_1 - \dot{v}_1 \end{aligned} \quad (28)$$

Of course, v_i represents the social value of creating a quality i product. The labor resource constraint is represented by,

$$\omega \left[\ell - \sum_j [x + c(\gamma_j)] K_j - m c(\gamma_e) \right] = 0.$$

Finally, the necessary transversality conditions require that Λe^{-rt} and $v_j e^{-rt}$ converge to zero as $t \rightarrow \infty$.

Equations (22)-(24) can be rewritten as

$$\omega c'(\gamma_0) = a [\Lambda \ln q_0 + (v_0 - v_0 K_0 - v_1 K_1)] \text{ if } \gamma_0 > 0 \quad (29)$$

$$\omega c'(\gamma_1) = \Lambda \ln q_1 + (v_1 - v_0 K_0 - v_1 K_1) \text{ if } \gamma_1 > 0 \quad (30)$$

$$\omega c'(\gamma_e) = a \phi_0 \Lambda \ln q_0 + \phi_1 \Lambda \ln q_1 + (a \phi_0 v_0 + \phi_1 v_1) - (v_0 K_0 + v_1 K_1) \text{ if } \gamma_e > 0 \quad (31)$$

In all cases, the marginal cost of innovation is equal to the expected social return. We discuss the planner's innovation incentives below.

The planner's steady state is obtained by setting $(\dot{\Lambda}, \dot{v}_0, \dot{v}_1) = (0, 0, 0)$ in the above equations, which gives the following steady state expressions for v_j ,

$$v_j = \frac{\ln x - 1}{r + \delta} + \Psi_j^p, \quad j = 0, 1,$$

where

$$\Psi_j^p = \max_{\gamma} \frac{\left[\frac{\ln q_j}{r} + \Psi_j^p - \Psi_0^p K_0 - \Psi_1^p K_1 \right] \gamma - \frac{c(\gamma)}{x} + \lambda_j \Psi_{1-j}^p}{r + \delta + \lambda_j}, \quad j = 0, 1.$$

Ψ_j^p is the social planner's valuation of the innovation capital embodied in a type j firm's product line. This formulation can be directly compared to the innovation incentives in the market equilibrium. The planner's optimal incumbent innovation choice satisfies,

$$\omega c'(\gamma_j) = \frac{\ln q_j}{r} + [\Psi_j^p - \Psi_0^p K_0 - \Psi_1^p K_1], \quad j = 0, 1, \quad (32)$$

whereas the market equilibrium innovation choice satisfies,

$$wc'(\gamma_j) = \frac{\pi_j}{r + \delta} + \Psi_j, \quad j = 0, 1,$$

which is equation (10), repeated here for exposition.

For a firm in the more creative state $j = 1$, the social return to innovation is the sum of the value of the spill-over plus the difference between the innovation ability value of a high quality product and the expected value of the innovation ability value in the product it replaces. The deviation between the socially optimal and private incentive to invest in R&D is apparent by comparing these with the market equilibrium first order conditions represented in equation (10). The first term on the right hand sides of equation (32) is the present value (expressed in utility terms) of an innovation's contribution to future productivity. The second term is the difference between the expected present value of the innovation and the product that it will replace. The subtraction of the value of the innovation ability in the replaced product represents the planner's internalization of the "business stealing" externality.

The private gains to innovation are increasing in q . However, they do not reflect the innovation's contribution to the productivity of all future versions of the product, as is captured by the "spill over" term in the planner expression. Furthermore, private incentives are distorted by monopoly pricing effects. The private innovator values the innovation capital embodied in a product but does not internalize the destruction of the innovation capital embodied in the product that is replaced, the "business stealing" externality.

3.2 Steady State Comparison Planner

An alternative social planner problem commonly applied is the one where the planner maximizes steady state welfare. We provide solutions for this problem as well. It is stated as follows: In steady state, $d \ln A_t / dt = g$ for all t . Consequently, $\ln A_t = gt + \ln A_0$, where $\ln A_0$ is the initial quality level. Furthermore, because the optimal production choice is the same for both product types; $x_i(t) = x$. Thus, the steady state social planner criterion is,

$$\begin{aligned} \max \int_0^\infty \ln C_t e^{-rt} dt &= \max \int_0^\infty \left[\ln A_t + \sum_i \sum_j \ln(x_{ij}(t)) K_{ij}(t) \right] e^{-rt} dt \\ &= \max \int_0^\infty [gt + \ln A_0 + \ln x] e^{-rt} dt \\ &= \max \left[g \int_0^\infty t e^{-rt} dt + \frac{\ln x + \ln A_0}{r} \right] \\ &= \max \left[\frac{g}{r^2} + \frac{\ln x + \ln A_0}{r} \right] \\ &= \frac{\ln A_0}{r} + \max [r \ln x + g] / r^2. \end{aligned}$$

The full steady state planner problem reduces to a choice of the real vector $(\gamma_0, \gamma_1, \gamma_e)$ that solves

$$\begin{aligned}
\max \quad & \{r \ln x + g\} \\
st \quad & : \quad x = \ell - c(\gamma_0)(1 - K_1) - c(\gamma_1)K_1 - mc(\gamma_e) \\
& \quad g = m\gamma_e(a\phi_0 \ln q_0 + \phi_1 \ln q_1) + a\gamma_0(1 - K_1) \ln q_0 + \gamma_1 K_1 \ln q_1 \\
\dot{K}_0 = \dot{K}_1 = 0 \quad & = \lambda_1 K_1 + m\gamma_e a\phi_0 - (m\gamma_e(a\phi_0 + \phi_1) + \lambda_0 + (\gamma_1 - a\gamma_0)K_1)(1 - K_1),
\end{aligned} \tag{33}$$

where the constraints are simple restatements of the constraints on the planner problem above given the choice of controls in the current formulation.

4 Numerical Solutions

In this section, we use model parameter estimates reported by Lentz and Mortensen (2008) to compare the quantitative properties of the market and the planner's solutions. We conclude the section by computing the welfare gain that could be achieved by a switch to the optimal solution when the market steady state solution characterizes the initial conditions.

4.1 Model parameters

The parameter estimates obtained by Lentz and Mortensen (2008) derived from Danish firm data using a simulated methods of moments method are reported in Table 1. In the empirical version of the model estimated, the R&D cost function is assumed to take the power form $c(\gamma) = c_0 \gamma^{1+c_1}$. The parameter Z is the average real value added per product line and L is the total labor force. Since real value added per product is normalized at unity and the total measure of products is set to unity as well in the model, the total labor supply per product line is $\ell = L/Z$. The estimated model is based on 3 types of firms where the type conditional quality realization is based on Weibull distributions. The model in this paper is simplified to two types and the type conditional quality realization distribution is degenerate. The 3 types in Lentz and Mortensen (2008) can be grouped into one high type and two low types. The parameter values in Table 1 are based on such a grouping. Specifically, the type conditional quality improvement is chosen so as to match $E \ln(q_i)$ across the two models. Finally, the model in Lentz and Mortensen (2008) is estimated under the assumption that a firm's initial type realization is permanent ($\lambda_0 = \lambda_1 = 0$).

The extreme right skew in the distribution of firm types at entry reflects the fact that very few innovations by potential entrants are of high quality. Indeed, around 85% of the entrants create innovation of only marginal value. These might be interpreted as simply "imitators."

4.2 Comparison of planner and market outcomes

This section will present a comparison of the steady state equilibrium outcomes of economies that differ in the degree of type permanence. The market out-

Table 1: Model Parameter Values

Cost scale parameter	c_0	175.800
Cost curvature parameter	c_1	3.728
Capital cost per product	κ	0.410
Interest rate	r	0.050
Labor supply	L	45.734
Value added	Z	16,859.000
Entrant mass	m	1.341
Low type quality improvement	q_0	1.001
High type quality improvement	q_1	1.600
Low type entrant probability	ϕ_0	0.850
High type entrant probability	ϕ_1	0.150

comes are compared to the steady state planner solution. Welfare in the market solution takes a similar form to the social planner except here products are not supplied at the same rates. Normalizing the initial quality level at unity, welfare in the steady state market equilibrium is given by

$$\hat{U} = r \sum_i \ln \left(\frac{1 - \kappa}{q_i w} \right) K^i + g$$

where K^i is the aggregate supply of type i products in steady state. Here, the manufacturing labor resources are allocated evenly across all products. Otherwise, the market equilibrium resource allocations are kept the same.

In Table 2, L_X and L_R are respectively the amount of labor engaged in product manufacturing and innovation. Welfare, $\hat{U}$, in the market equilibria outcomes are compared to the planner problem by means of determining a tax, τ , on planner consumption such that agents are indifferent between the market outcome and the planner's solution,

$$r \ln (x^p (1 - \tau)) + g^p = \hat{U},$$

where x^p and g^p are the planner consumption and growth levels, respectively.

In order to quantify the relative magnitude of the efficiency loss due to monopoly pricing and that of inefficient innovation resource allocation, define the hypothetical market welfare where consumption goods are efficiently provided subject to aggregate manufacturing resources,

$$\hat{U}_X = r \ln L_X + g.$$

Define the tax τ_X as the tax on planner consumption such that agents would be indifferent between the planner solution and the market outcome where consumption goods are provided efficiently,

$$r \ln (x^p (1 - \tau_X)) + g^p = \hat{U}_X.$$

Table 2: Social planner and market outcomes for permanent types ($\lambda_0 = \lambda_1 = 0.0$).

	SS Planner	SS Planner $L_R = 1.583$	Market	Planner SS	Planner SS (monopoly markup)
w			189.551		
δ	0.060	0.047	0.069	0.060	0.060
η	0.000	0.000	0.043	0.000	0.000
γ_0	0.000	0.000	0.005	0.000	0.000
γ_1	0.060	0.047	0.054	0.060	0.060
γ_e	0.000	0.000	0.032	0.000	0.000
K_0	0.000	0.000	0.577	0.000	0.000
K_1	1.000	1.000	0.423	1.000	1.000
K^0	0.000	0.000	0.577	0.000	0.000
K^1	1.000	1.000	0.423	1.000	1.000
v_0			0.001	24.699	23.515
v_1			2.886	28.734	28.734
Ψ_0			0.000	0.000	0.000
Ψ_1			1.026	4.035	4.035
L_X	40.879	44.151	44.151	40.879	40.879
L_R	4.856	1.583	1.583	4.856	4.856
g	0.028	0.022	0.014	0.028	0.028
U	0.214	0.212	0.202	0.214	0.214
τ		0.041	0.211	0.000	0.000
τ_X			0.190		

Table 2 considers the case of permanent types. The steady state maximizing planner solution is referred to as “SS planner” whereas the steady state of the planner solution is referred to as “Planner SS”. The outcomes of the two planner solution concepts turn out identical. That is of course not generally the case; for greater interest rates, the particular far sightedness of the steady state welfare maximizing planner results in a maintained emphasis on the selection effect, whereas the planner steady state equilibrium will begin to allow some entry of new firms. We provide an example in the appendix.

A comparison of the planner’s solution to the market equilibrium reveals that the optimal growth rate is twice that of the decentralized equilibrium. Although selection takes place in the sense that more creative firms innovate more frequently in equilibrium as well as in the planner’s solution, more creative firms account for only 42% of the products supplied in steady state equilibrium as compared with 100% in the planner’s solution. The planner obtains this outcome through a gradual lowering of the innovation rates of the less creative firm type and of the potential entrants toward zero. In the high persistence case an increase in steady state K_1 is sufficiently valuable that it outweighs any gains from innovations through γ_0 and γ_e . In other words, the expected contribution

to future aggregate productivity of an innovation by a low creativity firm or a potential entrant does not compensate for transfer of R&D capacity from the more to the less creative firms. Consequently, innovation is done only by firms in the more creative state and selection into the more creativity firms is almost perfect. The large difference between private and social values of innovation by less creative firms and entrants translates into a welfare difference between the planner and market solutions equivalent to a tax of 21% of planner consumption. Only 2 percentage points of this tax is attributable to the monopoly pricing distortion in the market equilibrium. If consumption goods are provided efficiently in the market but all else is held constant, the market inefficiency reduces only to the equivalent of a 19% tax on planner consumption.

Table 2 also contains two constrained planner solutions. Column 2 illustrates the welfare gains from redistribution of aggregate innovation resources across firms by requiring the SS planner to use the same level of innovation resources as the market equilibrium. The innovation resource constrained planner is able to achieve an outcome that is equivalent to a 4% tax on the unconstrained planner consumption. This, despite the fact that the unconstrained planner dictates a threefold increase in aggregate innovation resources relative to the market outcome. In light of the arguments in Atkeson and Burstein (2014), this result should not be all that surprising given the concavity of the relationship between productivity growth and aggregate innovation resources in the model. Unlike Atkeson and Burstein (2014), the results in this paper highlights the significant welfare gains from redistribution of aggregate innovation resources across heterogeneous firms.

Finally, the fifth column in Table 2 shows another constrained planner outcome. In this case, the planner is forced to maintain the monopoly markups present in the market solution. This is a relevant exercise as one considers implementation of the planner solution through innovation policies that are not able to address monopoly pricing. We show the monopoly markup constrained planner problem in the appendix. As can be seen, the steady state of the monopoly constrained planner is identical to the unconstrained planner solutions, reflecting that there is only one product type being provided in equilibrium and consequently, there is no distortion from constrained markups. It is an interesting feature that the monopoly markup constrained planner comes to behave more like the steady state maximizing planner for higher interest rates because co-existence of low and high type products is associated with an immediate distortion in consumption good provision. In the appendix, we show that it is possible that the unconstrained planner obtains a lower steady state welfare level than the monopoly markup constrained planner for higher interest rates.

In the low type persistence case reported in Table 3, the less creative type's negative innovation externality on the R&D capacity of the creative type firms is less dramatic. The type transition rates of $\lambda_0 = \lambda_1 = 0.2$ imply that a firm switches type at a rate more than triple that by which it develops new products. In this case, the type switching by itself remedies a substantial part of the skew toward imitator firms at birth. The fraction of products held by low and

Table 3: Social planner and market outcomes for transitory types ($\lambda_0 = \lambda_1 = 0.2$).

	SS Planner	Market	Planner SS
w		184.555	
δ	0.076	0.084	0.076
η	0.045	0.046	0.046
γ_0	0.000	0.028	0.000
γ_1	0.061	0.049	0.061
γ_e	0.034	0.034	0.034
K_0	0.501	0.524	0.501
K_1	0.499	0.476	0.499
K^0	0.507	0.547	0.507
K^1	0.493	0.439	0.493
v_0		0.248	23.269
v_1		2.040	24.173
Ψ_0		0.247	1.434
Ψ_1		0.385	2.338
L_X	42.578	44.285	42.579
L_R	3.156	1.449	3.156
g	0.018	0.014	0.018
U	0.205	0.203	0.205
τ		0.036	0.000
τ_X		0.027	

high type firms, respectively, is not all that different from the high persistence solution. However, this is now more a result of the firm type shifting rather than pure selection. In the planner solution, the selection effect is stronger and the fraction of high type products increases. Firm types exist in roughly equal proportion with slightly more selection in the planner solution. But even in the planner solution, the more creative firms only supply 50% of the products. Finally, it is seen that the product pricing distortion in the market is again fairly small, equivalent to 0.9% of planner consumption.

The imitator's negative innovation externality is less important when firm type is less persistent simply because firm type is now a less meaningful distinction and the betting on known winners is a less effective strategy. The value of moving R&D capacity to firms in the more creative state is significantly lower in the low persistence case, because a firm's creativity is subject to rapid change. The planner solution increases productivity growth from 1.4% annually to 1.8%. Product pricing is also more efficient. In all, the welfare loss in the decentralized equilibrium corresponds to a 3.6% tax on planner consumption, a substantially more moderate efficiency loss relative to the high type persistence case.

Finally, the last column in Table 3 shows the planner's steady state outcome which again is identical to the steady state welfare maximizing planner.

5 Buyout

Innovation in the Klette and Kortum (2004) and Lentz and Mortensen (2008) model is associated with a negative impact on the innovation technology of existing product leaders: Because innovation ability is embodied in product leadership, once a firm loses its leadership for a given product, it also loses the innovation ability embodied in it. Thus, once another firm moves the knowledge frontier forward, the previous leader's innovation production technology associated with the market in question is rendered obsolete. The possible inefficiency associated with this effect can be very strong. When a low ability innovator takes over product leadership from a high ability innovator, innovation capacity is shifted away from a more efficient producer of innovations toward a less efficient innovator. As shown in this paper, the planner can increase the growth rate in a Lentz and Mortensen (2008) calibrated model from 1.4% to almost 3% annually with a welfare difference corresponding to a 20% tax on planner consumption. The planner achieves this by almost shutting down entry and in particular innovation by low innovation ability types.

The market can by itself attempt to address the inefficiency. One way is for the high innovation ability firms to defensively buy out an innovator's innovation into its market so as to protect its leadership and thereby its innovation production scale. A defensive buyout is modeled as the purchase of the innovator's technology which is then scrapped. This reflects the assumption that knowledge cannot be transferred to other firms. If it could, the most efficient innovators would simply purchase all product lines and there would be no firm type dispersion in equilibrium.¹

In the model above with two transitory types, denote by v_{ij} the value of quality i product line owned by a type j firm, $i = 0, 1$ and $j = 0, 1$. The value of a new innovation to a low type firm is v_{00} . To keep the market, an incumbent of type j with a quality i product line is willing to pay v_{ij} . It is straightforward to show that $v_{00} \leq v_{ij}$ and $v_{11} \geq v_{ij}$ for $i, j = 0, 1$. Hence, assuming efficient bargaining between innovators and incumbents, a high ability innovator is never bought out, and a low ability innovator is always bought out except when innovation is into a low quality product owned by a low ability incumbent. The surplus of the buyout is split by a lottery: With probability α the incumbent makes the innovator a take-it-or-leave-it offer, and with probability $1 - \alpha$ the innovator makes the take-it-or-leave-it offer. The value functions for the two types of firms for given product portfolios (k_0, k_1) can be written as,

$$\begin{aligned} V_0(k_0, k_1) &= \frac{k_0(\pi_0 - \delta\alpha\beta v_0) + k_1(\pi_1 - \delta\alpha\beta v_0)}{r + \delta(1 - \alpha\beta)} + k\Psi_0 \\ V_1(k_0, k_1) &= \frac{k_0(\pi_0 - \delta\alpha\beta v_0) + k_1(\pi_1 - \delta\alpha\beta v_0)}{r + \delta(1 - \alpha\beta)} + k\Psi_1, \end{aligned}$$

¹Anyway, the efficiency issue associated with scrapping the innovation is irrelevant given the model estimate where the productivity increase associated with imitators' innovations is small.

where the firm type conditional innovation ability value embodied in a product, Ψ_i is given by,

$$\begin{aligned}\Psi_0 &= \max_{\gamma_0} \frac{\gamma_0 v^* - wc(\gamma_0) + \lambda_0 \Psi_1}{r + \delta(1 - \alpha\beta) + \lambda_0} \\ \Psi_1 &= \max_{\gamma_1} \frac{\gamma_1 v_1 - wc(\gamma_1) + \lambda_1 \Psi_0}{r + \delta(1 - \alpha\beta) + \lambda_1},\end{aligned}$$

where β is the fraction of overall product creation by type $j = 0$ firms,

$$\beta = \frac{\eta\phi_0 + K_0\gamma_0}{\eta + K_0\gamma_0 + K_1\gamma_1},$$

and $v^* = \alpha v_{00} + (1 - \alpha) \sum_i \sum_j K_{ij} v_{ij}$ is the expected value of an innovation to a low ability firm. Appendix A describes the asset equations in detail. They are long and tedious and the algebra involved in reaching the simplified expressions above is not particularly interesting.

The product dynamics that follow the same logic as that of equations (13)-(16) are as follows,

$$\begin{aligned}\dot{K}_{11} &= \lambda_0 K_{10} + \eta\phi_1 + \gamma_1 (K_{11} + K_{01}) - [\gamma_1 (K_{11} + K_{01}) + \eta\phi_1 + \lambda_1] K_{11} \\ \dot{K}_{01} &= \lambda_0 K_{00} - [\gamma_1 (K_{11} + K_{01}) + \eta\phi_1 + \lambda_1] K_{01} \\ \dot{K}_{10} &= \lambda_1 K_{11} - (\gamma_1 (K_{11} + K_{01}) + \eta\phi_1 + \lambda_0) K_{10} \\ \dot{K}_{00} &= \lambda_1 K_{01} + (\eta\phi_0 + \gamma_0 K_{00}) K_{00} - (\delta + \lambda_0) K_{00}.\end{aligned}$$

They reflect that $i = 0$ innovations are not adopted, except if they replace another $i = 0$ owned by a creativity state $j = 0$ firm. Thus, instead of representing product destruction by δ , the product destruction outflow stems only from product innovation that is not bought out. For example the destruction of K_{11} products is the combination of innovation by type $j = 1$ entrants and creativity state $j = 1$ incumbents. Innovation by low type entrants or incumbents will not result in product destruction.

In the permanent types case, $\lambda_0 = \lambda_1 = 0$ the system becomes,

$$\begin{aligned}\dot{K}_{11} &= \eta\phi_1 + \gamma_1 K_{11} - \delta(1 - \beta) K_{11} \\ \dot{K}_{00} &= -\delta(1 - \beta) K_{00}.\end{aligned}$$

The stable solution to this system has $K_{11} = 1 - K_{00} = 1$. That is, with permanent types, the buyout regime eventually eliminates all low types. Given that the estimation in Lentz and Mortensen (2008) calls for a mix of types in steady state, this suggests that as an empirical model, this case is of limited interest. It is however notable how the decentralized steady state product distribution across types coincides with the desired planner solution in this case, although depending on α , the low ability firm may be doing too much innovation in the decentralized solution.

The transitory types case does deliver a steady state with coexistence of low and high type innovators. Suppose to the contrary that $K_{00} + K_{10} =$

Table 4: Buyout outcomes.

	$\lambda_0 = \lambda_1 = 0.0$			$\lambda_0 = \lambda_1 = 0.2$		
	$\alpha = 0$	$\alpha = 1$	Planner SS	$\alpha = 0$	$\alpha = 1$	Planner SS
w	151.715	152.201		151.715	147.995	
δ	0.057	0.066	0.067	0.034	0.039	0.041
η	0.067	0.047	0.048	0.067	0.055	0.049
γ_0	0.050	0.005	0.000	0.050	0.035	0.000
γ_1	0.050	0.058	0.060	0.050	0.059	0.061
γ_e	0.036	0.036	0.036	0.037	0.037	0.037
K_0	0.000	0.000	0.000	0.458	0.454	0.454
K_1	1.000	1.000	1.000	0.542	0.546	0.546
K^0	0.000	0.000	0.000	0.000	0.000	0.000
K^1	1.000	1.000	1.000	1.000	1.000	1.000
v_{00}	0.410	0.000	23.112	0.410	0.436	32.360
v_{10}	1.740	1.916	23.112	1.740	2.885	32.360
v_{01}	0.410	1.272	26.892	0.410	0.710	33.324
v_{11}	1.740	3.187	26.892	1.740	3.159	33.324
Ψ_0	0.410	0.000	0.000	0.410	0.738	2.126
Ψ_1	0.410	1.271	3.781	0.410	1.012	3.090
L_X	40.977	40.846	40.376	40.977	42.007	42.165
L_R	4.758	4.889	5.358	4.758	3.728	3.569
g	0.028	0.031	0.031	0.017	0.019	0.019
U	0.214	0.216	0.216	0.203	0.206	0.206
τ	0.050	0.001		0.062	0.007	
τ_X	0.050	0.001		0.062	0.007	

$1 - K_{11} - K_{01} = 0$. Steady state, $\dot{K}_{10} + \dot{K}_{00} = 0$ then implies, $\lambda_1 K_{11} + \lambda_1 K_{01} = 0 \Leftrightarrow \lambda_1 = 0$. But this is in violation of the assumption of transitory types. Hence, it must be that the steady state has $K_{00} + K_{10} > 0$.

In Table 4 we show the counterfactual outcome of the estimated Lentz and Mortensen (2008) economy if buyouts are allowed. In addition we show buyouts in the transitory types economy. We show the two bargaining extremes: $\alpha = 0$ where the innovator makes the incumbent a take-it-or-leave it offer, and $\alpha = 1$ where the bargaining power is fully with the incumbent who makes the innovator a take-it-or-leave-it offer. In addition we also include the planner's steady state solution which again coincides with the steady state welfare maximizing social planner solution (not shown). In this we give the planner the option as to whether an innovation should be implemented or discarded. The optimal planner behavior is the same as buyout behavior, that is, a low firm type's innovation is only implemented if it is into the market of a low quality product held by a low type firm.

As can be seen, the two buyout cases differ primarily in the incentives for a low type to innovate. In the case where the innovator has the full bargaining power ($\alpha = 0$), innovation incentives are identical across firm types. In steady

state, there are no low quality products. A low quality product can only enter a market of another low quality product. Once all low quality products have been eliminated, a low quality product is always bought out. Hence, a low type innovator will always be bought out, and the market equilibrium with permanent types is fully populated by high type firms, only. In this respect the planner and the market solutions come to look very similar.

It is a natural yet somewhat striking result that in the $\alpha = 0$ case, the innovation ability value embodied in a product is identical across the type of the owner. In steady state, the product mass is composed of type $i = 1$ products, only. The value of an innovation depends on the incumbent's valuation of the product, and in the steady state, all existing products are valued the same by incumbents. Consequently, the low type innovator does much more innovation in this equilibrium than in the no buyout case. Relative to the social planner solution, the $\alpha = 0$ case is equivalent in welfare terms to a 5% tax on planner consumption in the case of permanent types and a 6% tax in the case with transitory firm types.

The other extreme where the bargaining power rests fully with the incumbent ($\alpha = 1$), is considerably more attractive in terms of efficiency. In this case, the low type innovator's incentives to innovate are kept at a minimum. Furthermore, low type innovations never take away innovation ability from the high type incumbents. As a result, this equilibrium is very close to the social planner. In the permanent firm types case, the tax on planner consumption that makes the two equivalent is 0.1% and in the transitory case it is 0.7%.

The buyout equilibria also have the feature that they eliminate the product pricing distortion from the market equilibrium since there are now no low type products in production.

The planner's option to not adopt low type innovations allows the planner to exploit the entrants innovation technology and select on high quality innovations. Because of the convexity of the innovation cost function, it is in this case optimal to have entrants do a strictly positive amount of innovation.

Thus, a market that facilitates buyouts can go a long way to address the inefficiency due to innovation by low ability innovators, but with the caveat that a substantial fraction of the efficiency gains can be lost as a function of the innovator's ability to extract the surplus of the buyout.

6 Concluding remarks

Optimal innovation policy in the Lentz and Mortensen (2008) model with permanent firm types is one that discourages innovation by low type innovators. To the extent that the policy can sufficiently discourage creative destruction by low types the policy will also want to discourage entry, because the incumbents will not need replacement. This is the design of the optimal planner solution. The paper thus provides the core insights into what a optimal policy design must try to obtain. An example of such a policy is a fixed patent fee combined with R&D subsidies. The fee must sit in between the low ability innovator's

valuation of an innovation and that of the high ability innovator. This will perfectly discourage low ability innovators from engaging in innovation since they will not find it optimal to implement innovations upon discovery. The R&D subsidy is then used to offset the diminished innovation incentives of the high ability innovators due to the patent fee. Monopolistic competition remains an inefficiency problem as incumbents will choose a different output level relative to that of the planner, but the results in the paper document that it is a minor efficiency loss relative to the issue of allocating innovation resources optimally across heterogeneous innovators.

The buyout analysis highlights the value of selecting which innovations to implement; in particular the low value innovations should not be adopted. There exists a simple policy instrument for this purpose: Patent fees. Simply, set the patent fee in excess of the value of a low type innovation and it can no longer be used to extract rents from incumbents and the innovator will choose to scrap it. The patent fee of course diminishes the ex post returns to innovation for high type innovators, but the addition of R&D subsidies to the policy mix can then be used to fine tune the innovation intensity of the high type innovators to the socially optimal level.²

The paper documents the substantial inefficiency in the decentralized economy due to innovation ability heterogeneity across firms. A similar mechanism arises in Acemoglu et al. (2013) where substantial inefficiencies related to heterogeneity are also documented. Thus, in both cases, the analyses emphasize the importance of designing optimal policy with a view of targeting incentives conditional on firm heterogeneity.

Galaasen and Irarrazabal (2014) estimate the Lentz and Mortensen (2008) model on Norwegian data and come to much the same conclusions regarding the strength of the selection. If anything, once R&D expenditure at the firm level is included in the estimation, they find a stronger selection effect and therefore contribution to growth. They find that innovation policy design such as R&D subsidies can have very large impacts on growth rates depending on how they are targeted to the innovation ability heterogeneity across firms.

Atkeson and Burstein (2014) discuss optimal innovation policy in models related to Klette and Kortum (2004), leaving out the issue of heterogeneity in innovation ability across firms. In contrast to the results in this paper, they find that innovation policy changes must have very moderate impacts on productivity growth. In their paper, the impact is naturally bounded by the growth rate of the economy and the social depreciation of innovation expenditures. Thus, in the absence of firm heterogeneity, the design of optimal innovation policy in their analysis comes to depend crucially on the existence of social depreciation of innovation expenditures. Our paper highlights that firm heterogeneity may well be of first order importance for the design of optimal innovation policy.

²This particular policy implementation scheme assumes the knowledge outcome of innovation can be legally protected by for example patents, copyright, trade marks, or the like. Undoubtedly, a substantial part of innovation outcomes fit into this category, but it does represent a narrowing of our general view of innovation activity in the paper.

A Buyout asset equations

With probability α the incumbent makes a take it or leave it offer and with probability $1 - \alpha$ the innovator makes the take it or leave it offer. Hence, the value functions are,

$$\begin{aligned}
 rV_0(k_0, k_1) &= \max_{\gamma \geq 0} \left\{ k_0\pi_0 + k_1\pi_1 - wc(\gamma)k + \lambda_0[V_1(k_0, k_1) - V_0(k_0, k_1)] \right. \\
 &\quad + \gamma k \alpha [V_0(k_0 + 1, k_1) - V_0(k_0, k_1)] \\
 &\quad + \gamma k (1 - \alpha) [K_{00}[V_0(k_0 + 1, k_1) - V_0(k_0, k_1)] + K_{01}v_{01} + K_{11}v_{11} + K_{10}v_{10}] \\
 &\quad + \delta(1 - \beta) [k_0V_0(k_0 - 1, k_1) + k_1V_0(k_0, k_1 - 1) - kV_0(k_0, k_1)] \\
 &\quad + \delta\beta k_0 [V_0(k_0 - 1, k_1) - V_0(k_0, k_1)] \\
 &\quad \left. + \delta\beta k_1 [\alpha(-v_0) + (1 - \alpha)[V_0(k_0, k_1 - 1) - V_0(k_0, k_1)]] \right\} \\
 rV_1(k_0, k_1) &= \max_{\gamma \geq 0} \left\{ k_0\pi_0 + k_1\pi_1 - wc(\gamma)k + \gamma k [V_1(k_0, k_1 + 1) - V_1(k_0, k_1)] \right. \\
 &\quad + \lambda_1[V_0(k_0, k_1) - V_1(k_0, k_1)] \\
 &\quad + \delta(1 - \beta) [k_0V_1(k_0 - 1, k_1) + k_1V_1(k_0, k_1 - 1) - kV_1(k_0, k_1)] \\
 &\quad \left. + \delta\beta [\alpha k [-v_0] + (1 - \alpha) [k_0V_1(k_0 - 1, k_1) + k_1V_1(k_0, k_1 - 1) - kV_1(k_0, k_1)]] \right\}.
 \end{aligned}$$

The flow value equation for the type 0 firm type is as follows: The first line is the usual profit flow from the firm's k_0 low type products and k_1 high type products minus innovation labor expenses and the net value of the type switching option. The second line represents the value of innovation in case the incumbent will make a take it or leave it offer. Whether or not the firm is bought out, it receives a value equal to an additional low type product innovation. The third line represents the value of innovation in the case where the innovating firm gets to make the take-it-or-leave it offer. Conditional on making an innovation, with probability K_{ij} the firm innovates into a type i product owned by a type j firm. In the case where the firm innovates into a $(i, j) = (0, 0)$ market, the innovation is not bought out and the firm takes the asset value of the innovation. In any other case, the innovation is bought out at take-it-or-leave it offer v_{ij} . The remaining 3 lines represent the asset loss from product destruction. β is the fraction of overall product creation by type 0 firms. Thus, at rate $\delta(1 - \beta)$ one of the firm's product lines is innovated into by a high type firm. In this case, there is no buyout and the firm loses the product line in question. At rate $\delta\beta k_0$, one of the firm's low type product lines is innovated upon by a low type firm. Again, there is no buyout, and the firm loses the product line. Finally, at rate $\delta\beta k_1$ one of the firm's high type product lines is threatened by a low type innovator. In this case, the firm buys out the innovator. With probability α , the buy out price is v_0 and with probability $1 - \alpha$ the firm pays its own valuation of the product line.

The flow value equation for the type 1 firm follows the same logic. The value functions maintain proportionality in number and type of product lines, which results in the simplification of the expressions shown in the main text.

B The monopoly distortion constrained planner

Consider a planner that is forced to supply products subject to the decentralized market's monopoly pricing distortions. The supply of product is in the decentralized solution given by, $x_i = (MC \cdot q_i)^{-1}$, where MC is the marginal production cost and the markup is q_i . The planner problem where the decentralized monopoly supply distortion across product is imposed can be stated as,

$$\begin{aligned}
H = & \ln A + \sum_i \sum_j [\ln x - \ln q_i] K_{ij}(t) \\
& + \omega \left(\ell - \sum_i \sum_j \left[\frac{x}{q_i} + c(\gamma_j) \right] K_{ij} - mc(\gamma_e) \right) \\
& + \Lambda [m\gamma_e (a\phi_0 \ln q_0 + \phi_1 \ln q_1) + a\gamma_0 K_0 \ln q_0 + \gamma_1 K_1 \ln q_1] \\
& + v_0 \left[\begin{array}{c} \lambda_1 K_1 + am\gamma_e \phi_0 + a\gamma_0 K_0 \\ - (am\gamma_e \phi_0 + a\gamma_0 K_0 + m\gamma_e \phi_1 + \gamma_1 K_1 + \lambda_0) K_0 \end{array} \right] \\
& + v_1 \left[\begin{array}{c} \lambda_0 K_0 + m\gamma_e \phi_1 + \gamma_1 K_1 \\ - (am\gamma_e \phi_0 + a\gamma_0 K_0 + m\gamma_e \phi_1 + \gamma_1 K_1 + \lambda_1) K_1 \end{array} \right].
\end{aligned}$$

The optimal product supply choice, x , must now satisfy,

$$\frac{1}{x} = \omega \frac{K^0 q_1 + K^1 q_0}{q_0 q_1}$$

and the steady state value of a type i product line is,

$$(r + \delta + \lambda_i) v_i = \ln x - \ln q_i - \frac{q_{1-i}}{K_0 q_1 + K_1 q_0} + \varphi_i + \lambda_i v_{1-i},$$

where φ_i is as before.

C Planner solutions for higher interest rate.

The distinction between the steady state welfare maximizing planner and the planner's steady state becomes apparent as the interest rate increases. In this example, the interest rate is set at the rather extreme annual rate of $r = 0.5$. Table 5 shows solutions for the permanent types case. The first four columns do not allow non-adoption of low type innovations. The last two columns allow non-adoption of innovations. It is seen that for this higher interest rate, the steady state welfare maximizing planner solution differs from the planner steady state. The core difference is due to positive entry in the planner steady state. For the given product type mix, the planner value some entry because of the chance that the entrant happens on a high type innovation. The steady state welfare maximizing planner sets entry to zero because of its negative impact on the steady state selection. Notice that the monopoly markup constrained

Table 5: Social planner and market outcomes for permanent types ($\lambda_0 = \lambda_1 = 0.0$).

	SS Planner	Market	Planner SS	Planner SS (monopoly markup)	SS Planner (no low type adoption)	Planner SS (no low type adoption)
w		183.876				
δ	0.033	0.042	0.042	0.033	0.037	0.037
η	0.000	0.026	0.026	0.000	0.027	0.027
γ_0	0.000	0.004	0.000	0.000	0.000	0.000
γ_1	0.033	0.032	0.033	0.033	0.033	0.033
γ_e	0.000	0.019	0.019	0.000	0.020	0.020
K_0	0.000	0.581	0.529	0.000	0.000	0.000
K_1	1.000	0.419	0.471	1.000	1.000	1.000
K^0	0.000	0.581	0.000	0.000	0.000	0.000
K^1	1.000	0.419	1.000	1.000	1.000	1.000
v_0		0.000	5.204	5.039		5.242
v_1		0.429	5.251	5.328		5.287
Ψ_0		0.000	0.000	0.000		0.000
Ψ_1		0.020	0.047	0.046		0.046
L_X	45.435	45.589	45.557	45.435	45.399	45.399
L_R	0.299	0.146	0.178	0.299	0.335	0.335
g	0.016	0.008	0.009	0.016	0.017	0.017
U	1.924	1.905	1.919	1.924	1.925	1.925
τ		0.037	0.010	0.000		

planner steady state coincides with the SS planner solution. The monopoly markup constrained planner understands that coexistence of different product types implies a distortion in consumption good production. It is a strong enough distortion that the planner ends up in the steady state welfare maximizing solution.

The last two columns show that as soon as the planner is allowed to not adopt low type innovations, then the planner steady state coincides with the steady state welfare maximizing solution.

References

- Acemoglu, D., Akcigit, U., Bloom, N., Kerr, W. R., 2013. Innovation, reallocation and growth. NBER Working Paper (18993).
- Aghion, P., Howitt, P., March 1992. A model of growth through creative destruction. *Econometrica* 60 (2), 323–51.
- Atkeson, A., Burstein, A., September 2014. Aggregate implications of innovation policy. Working Paper.
- Galaasen, S. M., Irarrazabal, A., 2014. R&d heterogeneity and implications for growth. Ph.D. thesis, University of Oslo.
- Grossman, G. M., Helpman, E., 1991. *Innovation and Growth in the Global Economy*. MIT Press, Cambridge, Massachusetts and London, England.
- Hopenhayn, H. A., September 1992. Entry, exit, and firm dynamics in long run equilibrium. *Econometrica* 60 (5), 1127–50.
- Klette, T. J., Kortum, S., October 2004. Innovating firms and aggregate innovation. *Journal of Political Economy* 112 (5), 986–1018.
- Lentz, R., Mortensen, D. T., November 2008. An empirical model of growth through product innovation. *Econometrica* 76 (6), 1317–73.