Efficient height measurements in single images based on the detection of vanishing points

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Abstract

Surveillance cameras have become a customary security equipment in buildings and streets worldwide. It is up to the field of Computational Forensics to provide automated methods for extracting and analyzing relevant image data captured by such equipment. In this article, we describe an effective and semi-automated method for detecting vanishing points, with their subsequent application to the problem of computing heights in single images. With no necessary camera calibration, our method iteratively clusters segments in the bi-dimensional projective space, identifying all vanishing points – finite and infinite – in an image. We conduct experiments on images of man-made environments to evaluate the output of the proposed method and we also consider its application on a photogrammetry framework.

Keywords: Photogrammetry, Vanishing point, Height measurement, Projective space

1. Introduction

By analyzing certain image properties – known object size, occlusion, stereoscopic vision, focus, gradient, texture, vergence – we can infer three-dimensional

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(3D) information of the geometry portrayed in the images. This inferred 3D data
5 can be employed in various computer vision applications: image-based rendering [1, 2], automated driving [3], object detection [4], and forensic science [5].

The process of extracting, from images, geometric properties, such as heights, areas, and angles, is denominated *photogrammetry*. Photogrammetry methods are widely used in forensic investigations, where it can help corroborate pieces
10 of evidence [5]. Analysis of car accidents and of human height are the two main examples of forensic activities that involve photogrammetry [5].

Even in low quality images, like the ones captured by surveillance cameras, with the aid of photogrammetry we can estimate the height of objects and people, useful identification characteristic when the face of the suspects cannot
15 be identified or the details in their clothes are not relevant. Conversely, the suspects are often not on the crime scene at the time of the investigation, or the scene has changed. In these cases, the image itself is the only source of information.

When only one image depicting the scene is available, the essential problem
20 is the recovery of the third dimension, given that this information was not captured in the acquisition process, in which the 3D scene was projected onto the 2D image plane. In particular, perspective distortions also occur. For example, objects that are away from the camera appear smaller in the image than objects that are closer.

25 To solve this problem, the first photogrammetrists assumed some *a priori* information: internal camera parameters – focal length, optical center, scale, distortions, and skew factor –, and the camera position in relation to the scene. However, this is only valid when the accuracy of these values is high, since any deviation can generate large measurement errors [6].

30 Recent works on photogrammetry can be generally classified into two categories: the ones that exploit 3D information from multiple images and the ones that analyze geometric properties in a single image. The first category deals with 3D reconstruction of the scene, from multiple views, to estimate homography or to calibrate the camera [7, 8, 9]. Single-view based methods can only

35 rely on the analysis of geometric properties [10, 11, 12, 13] but, in this scenario,
they often use previously positioned markers for calibration purposes [12, 11].
Although these methods attain high accuracy, they cannot be used in forensic
applications, where often a previously taken image is the only source of infor-
mation.

40 Here we propose a single-view method that detects vanishing points – in-
variant geometric features that can aid photogrammetry. A *vanishing point* can
be defined as an image point where the projection of a set of real-world parallel
lines converges, assuming perspective projection. A *vanishing line* is a line that
contains two vanishing points.

45 The credit for discovering linear perspective is given to renaissance artist
Leon Battista Alberti and architect Filippo Brunelleschi [14]. Alberti’s trea-
tise, *De pictura* [15], published in 1435, encloses the first scientific study of
perspective. By the 1470s, several artists were able to produce their works of
art demonstrating a full understanding of the principles of linear perspective.
50 Leonardo da Vinci, beginning in 1481, also studied and employed perspective
in his earlier paintings (Figure 1).

Figure 1: Study for the painting *Adoration of Magi*, by Leonardo da Vinci showing the use of perspective.

In this article, we use a geometric approach to effectively estimate the lo-
cation of vanishing points – finite and infinite ones – in images of urban and

indoor spaces, with the Manhattan-world assumption. We are assuming that
55 the scene has a natural cartesian 3D coordinate system, which is plausible for
indoor, outdoor city, and even some country scenes [16].

By representing image segments in the bi-dimensional projective space $\mathbb{RP}^2$,
our method clusters them into groups of segments that converge to a unique
vanishing point locality. The detected features are then used to identify the
60 ground plane and the vertical direction of the scene. This information is finally
inserted into a photogrammetry algorithm proposed by Criminisi [10], with the
ultimate goal of measuring the height of objects and people in single images.

This article starts by presenting, in Section 2, a background on the measure-
ment of heights in single images based on vanishing points. In Section 3, we
65 present our vanishing point detector. In Section 4, we show how to estimate the
scene vertical direction and also how to detect the ground plane vanishing line.
The experiments and their results are provided in Section 5. Finally, Section 6
states the conclusions of this work.

2. Background

The projection of a world point $X \in \mathbb{R}^3$ into an image point $x \in \mathbb{R}^2$, con-
sidering perspective projection, is described by projection matrix $\mathbf{P} \in \mathbb{R}^{3 \times 4}$ as

$$\tilde{x} = \mathbf{P}\tilde{X} = \mathbf{K}[\mathbf{R}|\mathbf{T}]\tilde{X} = [p_1 \ p_2 \ p_3 \ p_4]\tilde{X}, \quad (1)$$

70 where $\tilde{X}$ and $\tilde{x}$ are the points X and x in homogeneous coordinates, respectively;
 $\mathbf{K}$ is the matrix representing the intrinsic parameters of the camera; the extrinsic
parameters are $\mathbf{R}$ – rotation matrix – and $\mathbf{T}$ – translation vector from the world
to the camera system; p_1 , p_2 , p_3 and p_4 are the columns of $\mathbf{P}$; and the equality
is up to scale.

75 In [17], the authors prove that p_1 , p_2 and p_3 are the orthogonal vanishing
points corresponding to the world coordinate system, and that p_4 is the image
of the world origin. Here we denote them as v_x , v_y , v_z .

Considering that v_x and v_y are the two vanishing points on the vanishing line, we can say that p_4 must not be on the same line. If it does, then v_x , v_y , and p_4 are linearly dependent. Hence the fourth column can be set to $p_4 = m/\|m\| = \overline{m}$, where m is the vanishing line [6]. The final projection matrix is

$$\mathbf{P} = [v_x \ v_y \ \alpha v_z \ \overline{m}], \quad (2)$$

where α is an unknown scalar referred as *metric factor*. If v_z and m are available, then the metric factor α is the only unknown value.

To measure heights in images, we must compute the distance between points in two different planes. Let v_z be the vanishing point that indicates the scene vertical direction, and m the ground vanishing line. Considering the projection matrix $\mathbf{P}$ (Equation 2), Criminisi [18] proved that for an arbitrary object λ , with height Z_λ and delimited by image points t_λ and b_λ (top and bottom points), it holds that

$$\alpha Z_\lambda = -\frac{\|b_\lambda \times t_\lambda\|}{(\overline{m} \cdot b_\lambda) \|v_z \times t_\lambda\|}. \quad (3)$$

80 Figure 2 illustrates Equation 3. Point o_λ represents the intersection between vanishing line m and the line that contains t_λ and b_λ . This intersection point helps to define a ratio of distances between planes and, using this value, one can compute $\frac{Z_\lambda}{Z_c}$, where Z_c is the camera’s distance. However, it is simpler to compute Z_λ via a reference measurement in the image with a known length [6].

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Thus, if we want to measure height Z_{obj} of an object obj , and Z_{ref} is a known distance on the same image, i.e., it is a reference distance between points t_{ref} and b_{ref} , then Equation 3 allows the computation of the scale factor α and subsequently the distance Z_{obj} between t_{obj} and b_{obj} . The following steps can
90 be used to compute height Z_{obj} :

1. Detect vanishing points in I .
2. Identify vanishing point v_z associated with the scene vertical direction.
3. Estimate vanishing line m associated with the ground plane.

Figure 2: Distance between the planes that contain points t_λ and b_λ . Point o_λ represents the intersection between vanishing line m and line that contains points t_λ and b_λ .

4. Compute scale factor α by applying Equation 3 with object *ref*, i.e.,

$$\lambda = ref.$$

5. Compute Z_{obj} by applying Equation 3 with object *obj*, i.e., $\lambda = obj$, and α .

The process of computing heights in images is illustrated in Figure 3. According to Criminisi [10], efficient measurements in images can be done by accurately detecting the vanishing points, and estimating vertical vanishing point v_z and vanishing line m that represents the ground plane (steps 1, 2 and 3). Therefore, we can use the output of our detector to estimate v_z and m .

In the subsequent sections, we describe our method to detect vanishing points and how to estimate, from them, vertical vanishing point v_z , and ground vanishing line, m .

3. Estimating vanishing points in single images

The problem of locating vanishing points in 2D perspective projection images has been studied since the 80s. This task is generally seen as the computation of line intersections, but due to quantization errors, lines that correspond to a single vanishing point intersect inside an area denominated *vanishing region*.

Figure 3: Measuring the height of a real object depicted in an image. Height Z_{obj} can be estimated with the aid of known height Z_{ref} of a reference object.

To confront the *vanishing region* problem, methods often divide the vanishing point detection in three main stages: detection of segments in the image, clustering of these segments in groups that converge to a vanishing point, and vanishing point estimation for each cluster.

115 The first stage can be accomplished using an edge detector subsequently grouping the edges to form segments, e.g., Canny operator [19] and Hough transform [20]. Methods can perform the other stages in two ways: using accumulator spaces [21, 22, 23, 24, 25] or using the image plane directly [26, 27].

120 Since Barnard’s seminal work on the detection of vanishing points [21], methods have been employing different Hough transform techniques in quantized Gaussian spheres [22]. The problem in such methods are the artifacts often present in digital images, producing erroneous maxima on the quantized space [23].

125 The methods that use the image plane directly do not need accumulator techniques [26, 27]. In this case, the accuracy of the vanishing point location is not limited by the space and the distances are preserved. However, it may be

necessary to incorporate additional criteria to work with the infinite vanishing points.

Against these works, our method uses the bi-dimensional projective space $\mathbb{RP}^2$, or projective plane, transformed directly from the image space, to cluster the segments and detect all vanishing points, including the infinite ones, without additional criteria. Moreover, the space is not bounded, meaning it does not limit the location accuracy.

The proposed detector also has three stages:

1. Extracting line segments from the input image.
2. Clustering of the segments that converge to the same vanishing point (repeated until convergence):
 - 2.1. Selection of seeds.
 - 2.2. Grouping of segments based on the seeds and the intersection points.
3. Detection of a vanishing point for each final cluster.

Figure 4 illustrates the three steps to estimate the vanishing points, numerated as shown above. Each one of the steps is described in a subsection.

3.1. Line segment detection

We start by extracting line segments from the input image using a method proposed by Desolneaux et al. [28, 29]. They deal with the extracting problem by exploring the Helmholtz Principle [29]. Besides line segments, their method also produce, for each segment, its number of false alarms, which will be used to compute the quality of this segment.

The Helmholtz principle declares that if, in an image, the expectation of a certain observed configuration is small, then this grouping is a Gestalt, i.e, it is meaningful [29].

Definition 1 (ϵ -meaningful configuration). *A configuration is ϵ -meaningful if it occurs in an image in a number less than ϵ .*

Figure 4: Illustration of the three steps to detect vanishing points in an image. Each color (red, green and blue) represents a different cluster. In step 3, the vertical vanishing point is not shown, because it lies at infinity.

Let f be a $N \times N$ image, and $A \in f$ a segment formed by a set of pixels $\{x_i\}, i = 1, \dots, l$. Consider a random variable X_i where $X_i = 1$ if the direction of pixel¹ x_i is aligned to the direction of A , and $X_i = 0$ otherwise. Therefore, X_i has the following distribution with precision level p :

$$P[X_i = 1] = p \quad \text{and} \quad P[X_i = 0] = 1 - p. \quad (4)$$

The direction of x_i is aligned to the direction of A with precision p when

$$D_{ang}(dir(x_i), dir(A)) \leq \pi p, \quad (5)$$

155 where function $dir(\alpha)$ outputs the direction of a pixel or segment α , and D_{ang} outputs the smallest angle between the two directions.

The number of aligned pixels in segment A is represented by random variable $S_l = X_1 + X_2 + \dots + X_l$. Because random variables X_i are independent, S_l has

¹The direction of a pixel is defined by Desolneux et al. [29] as the direction orthogonal to the direction of the gradient at the pixel.

the following binomial distribution:

$$P[S_l = k] = \binom{l}{k} p^k (1-p)^{l-k}. \quad (6)$$

To consider segment A as a primitive for our method, we need to know if A is ϵ -meaningful among all segments in f .

Definition 2 (ϵ -meaningful segment). *A l -length segment A is ϵ -meaningful if it contains a minimum of $k(l)$ aligned pixels, where*

$$k(l) = \min \left\{ k \in \mathbb{N}, P[S_l \geq k] \leq \frac{\epsilon}{N^4} \right\}. \quad (7)$$

The value N^4 is the number of oriented segments (defined by their initial
160 and end pixels) in a $N \times N$ image.

Consider the i -th segment, with length l_i , and the event e_i meaning “the i -th segment is ϵ -meaningful”. Let χ_{e_i} denote the characteristic function of this event, so that

$$P[\chi_{e_i} = 1] = P[S_{l_i} \geq k(l_i)] = \sum_{k=k(l_i)}^{l_i} \binom{l_i}{k} p^k (1-p)^{l_i-k}. \quad (8)$$

Then the total of ϵ -meaningful segments is represented by variable $R = \chi_{e_1} + \chi_{e_2} + \dots + \chi_{e_{N^4}}$, and expectation $E(R)$ relates to the number of false alarms.

Definition 3 (number of false alarms in a segment). *Considering a l_0 -length segment A with at least k_0 aligned pixels, the number of false alarms of A is*

$$F(k_0, l_0) = N^4 P[S_{l_0} \geq k_0] = N^4 \sum_{k=k_0}^{l_0} \binom{l_0}{k} p^k (1-p)^{l_0-k}. \quad (9)$$

If we take into account all ϵ -meaningful segments as primitives for our
165 method, we will have several low quality segments. To avoid these equivocal segments, we finally take into account only some of the ϵ -meaningful segments, the maximal ones.

Definition 4 (Maximal segment). *A segment A is maximal if:*

1. $\forall B, B \subset A \Rightarrow F(B) \geq F(A)$.
- 170 2. $\forall B, B \supset A \Rightarrow F(B) > F(A)$.

The two parameters of the method by Desolneux et al. [29], threshold ϵ and precision level p , are not critical. Values $\epsilon = 1$ and $p = \frac{1}{16}$ work well for all images [24].

We group the detected maximal segments in a set $\mathcal{S} = \{s_1, \dots, s_{|\mathcal{S}|}\}$, and
 175 each maximal segment $s_i \in \mathcal{S}$ has a number of false alarms denoted by F_i .

Consider the end points (x_1, y_1) and (x_2, y_2) , in the image plane $\mathbb{R}^2$, that specify an arbitrary maximal segment. If we represent these points in $\mathbb{RP}^2$, we get $(x_1, y_1, 1)$ and $(x_2, y_2, 1)$, respectively, using homogeneous coordinates. Here we are also interested in the lines that correspond to the maximal seg-
 180 ments, $(x_1, y_1, 1) \times (x_2, y_2, 1)$. So, for each maximal segment s_i , we compute the correspondent line as shown above, resulting in l_i . The computed lines form a set $\mathcal{L} = \{l_1, \dots, l_{|\mathcal{S}|}\}$.

3.2. Segment clustering

The clustering process consists in assigning each segment $s_i \in \mathcal{S}$ to a cluster
 185 h . Each cluster h is a set formed by line segments assigned to it.

The clustering process has three main steps: determination of the first seeds, assignment of segments to clusters, and update of the seeds.

3.2.1. First seeds

The selection of the first seeds is done by considering a quality value for segments. The quality value q_i of segment s_i is defined as

$$q_i = 1 - \left(\frac{F_i - \min F}{\max F - \min F} \right), \quad (10)$$

where $\max F$ and $\min F$ are the maximum and minimum numbers of false alarms
 190 among segments in $\mathcal{S}$, respectively. Note that the quality value of a segment is inversely proportional to its number of false alarms, i.e., the lower the number of false alarms, higher its quality.

The number of clusters, H , is chosen by the user. The $2H$ higher quality segments are randomly selected, in pairs, to be the seeds for the H clusters.

195 The two segment seeds of cluster h are denoted by α_h and β_h .

The method converges in fewer steps when the first seeds are selected based on the quality of the segments. If random segments are chosen as the first seeds instead, the method yields the same results, except that it takes more steps.

3.2.2. Assignment step

200 The assignment step is responsible for selecting a cluster for each segment in $\mathcal{S}$. This is done by computing the distance between pseudo-centroids $c_h = l_{\alpha_h} \times l_{\beta_h}$, defined for each cluster $h = 1, \dots, H$, and lines in $\mathcal{L}$.

The distance between an arbitrary point c and an arbitrary line l in $\mathbb{RP}^2$ can be computed as

$$D_{proj}(c, l) = \frac{|c \cdot l|}{\|c\| \|l\|}. \quad (11)$$

The formula for distance D_{proj} is obtained by considering the angle between the line that corresponds to c and the plane that corresponds to l in $\mathbb{RP}^3$.

205 Distance D_{proj} only measures the symmetry between points and lines, because it is not a complete metric. Note that D_{proj} can be used arbitrarily also between two points in $\mathbb{RP}^2$.

The cluster assigned to line segment s_i is the one with the closest pseudo-centroid., i.e,

$$cluster(s_i) = \underset{j \in [1, H]}{\operatorname{argmin}} D_{proj}(c_j, l_i). \quad (12)$$

3.2.3. Update step

In this step, the method selects new seeds α_h and β_h for each cluster $h =$
210 $1, \dots, H$.

New seed α_h is chosen as the segment in cluster h with closest orientation to the weighted circular mean [30] orientation $\bar{\theta}_h$ of the cluster, computed as

$$\bar{\theta}_h = \arctan\left(\frac{S_h}{C_h}\right), \quad (13)$$

where S_h and C_h correspond to

$$S_h = \sum_{s_i \in \text{cluster } h} q_i \sin(2\theta_i), \quad (14)$$

$$C_h = \sum_{s_i \in \text{cluster } h} q_i \cos(2\theta_i). \quad (15)$$

In Equations 14 and 15, θ_i is the orientation of segment s_i .

Formally,

$$\alpha_h = \underset{s_i \in \text{cluster } h}{\operatorname{argmin}} D_{ang}(\overline{\theta_h}, \theta_i), \quad (16)$$

where D_{ang} gives the minimum angle between the orientations.

By selecting α_h , the possible new pseudo-centroids for cluster h are the points $l_{\alpha_h} \times l_i$, with $s_i \in \text{cluster } h$. New seed β_h is selected as the one that permits new pseudo-centroid c_h to be the closest point to all other possible pseudo-centroids of the cluster. Formally,

$$\beta_h = \underset{s_i \in \text{cluster } h}{\operatorname{argmin}} \sum_{\substack{s_j \in \text{cluster } h \\ i \neq j}} D_{proj}(l_{\alpha_h} \times l_j, l_{\alpha_h} \times l_i). \quad (17)$$

Figure 5 illustrates a cluster h with its new seeds α_h and β_h . The dashed segments are the candidates prior to the selection of β_h .

Figure 5: Illustration of a cluster h with seeds α_h and β_h , and pseudo-centroid $c_h = l_{\alpha_h} \times l_{\beta_h}$. The dots represent all possible pseudo-centroids prior to the selection of β_h . The dashed segments are the candidates for seed β_h prior to its selection. Seed β_h is chosen based on the distance between the possible pseudo-centroids.

215 The method converges when pseudo-centroids are the same in two consecutive steps.

3.3. Vanishing point detection

After the convergence, the possible vanishing points associated with cluster h are all intersection points in the cluster: $l_i \times l_j$, with $s_i, s_j \in \text{cluster } h$. The vanishing point v_h is the closest intersection to all segments in the cluster, i.e.,

$$v_h = \underset{\substack{l_i \times l_j \\ s_i, s_j \in \text{cluster } h}}{\operatorname{argmin}} \sum_{s_k \in \text{cluster } h} D_{proj}(l_k, l_i \times l_j). \quad (18)$$

4. Estimating vertical vanishing point and ground vanishing line

According to [31], vertical vanishing points have two characteristics:

- 220 1. They usually are observed near the image y-axis, considering typical camera positions;
2. They are well separated from non-vertical vanishing points.

By considering these two characteristics as our premise, we can determine the vertical vanishing v_z , and the ground vanishing line m .

225 Regarding the first characteristic studied by [31], v_z should be related to the cluster that has the closest mean orientation (Equation 13) to the y-axis of the image. But this condition alone does not suffice.

Figure 6 shows two possible clusters with their mean orientation represented by a dashed line. Note that the mean orientation in both clusters has no deviation from the y-axis. Nonetheless, a cluster associated with a vertical direction
230 should have all segments with similar orientation, eliminating the possibility of choosing the first cluster.

Consequently, another characteristic that has to be analyzed is the distribution of the segments orientation in a cluster. For example, considering the segments orientation in Figure 6, the first cluster has greater standard deviation than the second cluster. This observation leads us to the second necessary condition for associating a cluster with the vertical direction: the segments orientation in the cluster must have low circular standard deviation [30], defined as

$$\sigma_h = \frac{\sqrt{-2\ln(\overline{R_h})}}{2}, \quad (19)$$

Figure 6: Illustration of two possible clusters. The mean orientation of each cluster is represented by a dashed line.

where $\overline{R_h}$ corresponds to

$$\overline{R_h} = \frac{\sqrt{S_h^2 + C_h^2}}{\sum_{s_i \in \text{cluster } h} q_i}. \quad (20)$$

Combining these two conditions, we can select cluster z associated with the vertical direction. Formally,

$$z = \underset{h \in [1, H]}{\operatorname{argmin}} D_{ang} \left(\overline{\theta}_h, \frac{\pi}{2} \right) + \sigma_h, \quad (21)$$

where $\overline{\theta}_h$ is the circular mean orientation of cluster h (Equation 13) and σ_h is the circular standard deviation of cluster h (Equation 19).

235 Then vertical vanishing point v_z is the one associated with cluster z .

To estimate the ground vanishing line m , we recall the second characteristic studied by [31]: all vanishing points are well separated from v_z . So, there are three cases to be considered, depending on the number of clusters chosen by the user:

- 240
- Two: the ground vanishing line is the one that contains the non-vertical vanishing point, and with orientation given by the mean orientation of its cluster.
 - Three: the ground vanishing line is the one that connects the two non-vertical vanishing points.

- Four or more: in this case, it is not clear through which two vanishing points the ground vanishing line passes. By graphically showing the possible vanishing lines, the user can select the one that corresponds to the ground plane.

5. Experiments

5.1. Vanishing point detector

We tested the effectiveness of the vanishing point detector in the York Urban database [32], an image database with a hundred images of man-made environments. Together with the images, the authors of the database also provided the intrinsic camera parameters, and the vanishing points computed with hand-detected segments and a Gaussian sphere method.

In Figure 7, we provide some visual examples of the obtained results. The first row presents the input images together with the line segments in black, detected as shown in Subsection 3.1. The second row presents the results of the segment clustering algorithm associated with each input image of the first row, computed as shown in Subsection 3.2. Each cluster is represented in a different color (red, green, and blue) and the colored squares represent the finite vanishing points, detected as shown in 3.3.

Note that we are considering the Manhattan-world assumption, i.e., the images must depict scenes with a cartesian 3D coordinate system. Scenes with few parallel lines or lines not aligned with the coordinate system axis cannot be considered here.

Besides the visual inspection of the results, we conducted two experiments. First, we computed the *orthogonality error* to quantify the deviation of the most orthogonal vanishing points² from the actual orthogonality.

The second experiment measured the *focal length error* using the camera

²The *most orthogonal vanishing points* are the ones with the lowest orthogonality error (Equation 25).

Figure 7: The first row presents three input images together with the detected segments in black. The second row shows the correspondent final segment clusters and colored squares representing the detected finite vanishing points. Red parallel segments represent a cluster associated with a infinite vanishing point.

intrinsic parameters, provided by the database. We compared the focal length calculated using the most orthogonal vanishing points to the real expected one.

5.1.1. Orthogonality error

The authors of the database provided the intrinsic parameters of the camera: focal length f , pixel dimension (m_x, m_y) , principal point (p_x, p_y) , and skew factor ς . With this information, we can construct the camera intrinsic matrix:

$$\mathbf{K} = \begin{bmatrix} f/m_x & \varsigma & p_x \\ 0 & f/m_y & p_y \\ 0 & 0 & 1 \end{bmatrix}, \quad (22)$$

Given that we have matrix $\mathbf{K}$, it is trivial to obtain the Image of the Absolute

Conic ω :

$$\omega = \mathbf{K}^{-T} \mathbf{K}^{-1}. \quad (23)$$

Considering two arbitrary orthogonal vanishing points v_p and v_q , it is true that

$$v_p \omega v_q = 0. \quad (24)$$

Therefore, to find the most orthogonal vanishing points among all detected vanishing points $v_i, i = 1, \dots, M$, we select the triplet (v_p, v_q, v_r) that minimizes the orthogonality error

$$e_{p,q,r} = (v_p \omega v_q)^2 + (v_q \omega v_r)^2 + (v_r \omega v_p)^2. \quad (25)$$

For each image on the York Urban database, we selected the most orthogonal
 275 vanishing point triplet. We then constructed a histogram of the cumulative
 orthogonality error for these triplets, shown in Figure 8. We compared our
 method with the one provided in the database [32], named here as *York Urban*
 method, where the segments are hand detected and the computation is done by
 considering a cumulative space represented in a Gaussian Sphere.

Figure 8: Orthogonality error cumulative histogram computed in the York Urban Database, with the most orthogonal vanishing point triplet in each image. Point (x, y) represents $y\%$ of images with orthogonality error lower than x .

280 5.1.2. Focal length error

For this experiment, consider that the focal length f is unknown, but instead we have the most orthogonal vanishing point triplet for each image. With this information, we can estimate the unknown f and compare it with the real value. In order to do this, it is necessary to retrieve matrix K by decomposing matrix

285 ω .

For each image of the database, we selected the most orthogonal vanishing point triplet, and estimated a focal length from them. We then constructed a histogram of the cumulative focal length, shown in Figure 9. We compared our obtained focal length with the one obtained by *York Urban* method and two
290 other detectors:

- Almansa et al. [24] considers the Helmholtz principle to detect segments and estimate vanishing regions. We selected the center of these regions to locate the vanishing points. This extension is named here as *Almansa et al. + vpe*.
- 295 • Tardif [27] considers a Canny edge detector and a flood fill algorithm to extract segments. Vanishing points are detected by a J-Linkage algorithm.

Note that *York Urban* method [32] provides in general a better estimation of the focal length. This is justified by the fact that, in this method, the segments are hand detected, eliminating a major source of errors.

300 5.2. Measuring heights

With the knowledge of the vanishing points locality, we can select which one is vanishing direction v_z and also determine the ground line m , as detailed in Section 4. This information can be inputed into Criminisi’s photogrammetry framework [6], as described in 2, to make height measurements in single images.

305 Because we are dealing only with a single view of the scenes, it is necessary to obtain the height of reference objects in order to compute absolute values. This cannot be done with images of scenes we don’t have access to make reference measurements. Therefore we constructed a database of twenty images (750×563

Figure 9: Focal length error cumulative histogram computed in the York Urban Database, with the most orthogonal vanishing point triplet in each image. Point (x, y) represents $y\%$ of images with focal length error lower than x .

pixels) with reference objects in the real world. The vanishing points detected
 310 on these images would generate focal length errors of less than 25 pixels.

In order to measure the real height of an object obj depicted in the image, the user must inform its top and bottom points, t_{obj} and b_{obj} , and the top and bottom points of a reference ref , t_{ref} and b_{ref} , with known height Z_{ref} .

In an error-free case, the points t_α and b_α , $\alpha = obj, ref$, have to be aligned
 315 with v_z [6]. Therefore, to compute height Z_{obj} , we replace the location of the selected points b_{obj} and b_{ref} with their perpendicular projection onto lines $\overleftrightarrow{v_z t_{obj}}$ and $\overleftrightarrow{v_z t_{ref}}$, respectively, generating the scenario depicted in Figure 3.

The errors associated with the choice of points t_{obj} , b_{obj} , t_{ref} and b_{ref} were considered by computing the height using all possible combinations of points
 320 inside a range of $1\% \cdot |t_\alpha b_\alpha|$ around each point, for $\alpha = obj, ref$. For example, Figure 10 shows the considered range around t_{obj} (top of object obj). Each point inside the depicted range was considered once as t_{obj} . The final height is the average between every obtained height.

The height of a person was measured comparatively to a known reference

Figure 10: Range of $1\% \cdot |t_{obj} b_{obj}|$ around t_{obj} .

height in twenty images taken in urban environments. The ground truth height
 is $171.5cm$ and the mean observed error across all images was $\pm 0.58cm$. Some of
 the results are shown in Figures 11, 12, and 13. In each one of these figures, the
 first column contains the input image with the segment clustering result, where
 each color represents a cluster associated with a vanishing point. The second
 column shows a zoomed crop with the vanishing line in green, the reference
 object height, and the computed height.

6. Conclusion

In this article, we described a method to detect vanishing points in an image
 and showed how to apply it to make efficient height measurements. The pro-
 posed method works with uncalibrated cameras, and can detect all vanishing
 points. Since it is performed in a bi-dimensional projective space, the detected
 points have accurate locations with no loss of information in transformations
 between spaces.

However, the method is only effective with images of man-made environ-
 ments, in which it is possible to extract straight segments corresponding to
 different 3D orientations.

Several applications can benefit from the proposed method. In forensic inves-
 tigations, for example, images from CCTV cameras – Closed-circuit television
 – can be used to estimate the height of suspects. CCTV cameras are widely
 employed in streets of major cities like New York and London.

Figure 11: Measuring the height of a person. The first column contains the input image with the segment clustering result. The second column presents a crop with the computed height.

The results show visually and experimentally the effectiveness of the method and its application to a photogrammetry framework, allowing the estimation of heights in images.

The presented method is useful in computing the height of rigid objects or people standing in a straight position. However, the height of animated objects
 350 tend to change while they move. This study will be included in a future work.

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Figure 12: Measuring the height of a person. The first column contains the input image with the segment clustering result. The second column presents a crop the computed height.

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Figure 13: Measuring the height of a person. The first column contains the input image with the segment clustering result. The second column presents a crop the computed height.

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